Using LaTeX in Schools: Simplifying Inclusive STEM Education

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1 Introduction

We report on experience of the use of LaTeX in Schools to teach mathematics to blind and visually impaired children. For the last 20 years the German education has phased out the use of sophisticated mathematical Braille notations and replace it by teaching children from year one LaTeX (or LaTeX-like) notation in mathematics. This notation made tactile by translating it into 8-dot Euro Braille thus removing the use of indicators and the need to express single characters with multiple Braille cells. The aim was to reduce the learning curve for students as well as to improve their abilities to author and communicate mathematics with peers and teachers that are not trained in Braille, which is paramount for inclusive education.

There has been previous work on using LaTeX as the basis for assistive technologies, such as text-to-speech (TTS) translation [11] or for transcriptions of Braille textbooks [10]. These works would transform LaTeX into dedicated math speech systems or specialist math Braille dialects like Marburg or Nemeth, rather then using the power of LaTeX directly and making it available as a communication medium for blind and visually impaired (BVI) learners. We present how our approach can technically be supported via web technology that can not only render LaTeX, but also provide 8-dot Braille output for expressions as well as compute meaningful LaTeX commands for sub-expression and support for direct copying from web sites.

Finally, we will discuss our initial ideas to transfer this model to other education systems, namely the Dutch system, where the current approach is ambivalent, in that there exists an AsciiMath-like linearized math notation for teaching to BVI students, while tactile math text-books use an outdated Braille notation, that is effectively no longer used in teaching. As a result, students are actively discouraged from studying mathematics.

2 LaTeX in Education for the Blind

The choice of mathematical notation in education for blind and visually impaired (BVI) students vary widely throughout the world. While there exist many national and regional variations in regular mathematical notation, these differences are particularly strong for primary and secondary education and become less pronounced in advanced mathematics and higher education. For BVI learners however this is generally not the case.

Traditionally, mathematics is presented in tactile format on the basis of highly specialized Braille notations, which increases in complexity for more advanced mathematical notation and more unconventional symbols. In particular, in traditional 6-dot Braille systems symbols have to be embellished with a plethora of indicators (e.g., for numbers, capital letter, fonts) and modifiers (for accents, positioning etc.). For a comparison of notational systems see [13].

This approach has a number of drawbacks: (1) The learning curve for mathematics, already steep for sighted students, becomes even steeper for BVI students. (2) Students can not easily communicate their Braille math with their sighted peers or teachers not trained in the formalism, which is an obstacle for inclusive education. (3) Math Braille notation is usually easier to read but less suitable for writing mathematics. This is particularly compounded by the use of computers even with Braille input devices. (4) Many traditional Math Braille notations have a 2D variant for complex elements like nested fractions or matrices, which are difficult to display on a computer Braille display. (5) Different Math Braille notations are generally not compatible and often even mutually intelligible. Not only have different countries different notations, but even the same language has different Math Braille dialects, e.g., in English there exist Nemath and UEB as well as formerly British Math Braille etc.

As a consequence more than 20 years ago Germany has chosen a different path for specialist education for BVI students [6, 7, 9], replacing the teaching of the local mathematical Braille notation (Marburg System) based on 6-dot Braille by adopting a uniform LaTeX notation for writing and transcribing that notation into 8-dot Euro Braille.

Since the early 1990s, the number of blind students in mainstream schools in Germany rose significantly. This increase was mainly due to the technical developments, with students working consistently with a PC and a connected Braille display, using 8-dot Braille, as opposed to the traditional six dots. Employing 8-dot font did not only prove less difficult for use with a computer, but also advantageous for inclusive education, as it turned out that 8-dot Braille was more compatible with the world of the sighted classmates.

For every character, letter, and number that had to be learned by the sighted student, there was exactly one equivalent in 8-dot Braille. There was no need for duplicate meanings of characters or indicators and modifiers as every single ASCII character can be translated into its equivalent Euro Braille cell.

Consequently, in the 2000s, a debate broke out in Germany between the competing writing systems of 6-dot and 8-dot Braille. While 6-dot script with and without contraction were mainly used in schools for the blind, the 8-dot script was the Braille system for integrated education and inclusion [4]. This long-standing practice was scientifically underpinned by the Zubra study conducted by Lang et al [8, 14], which demonstrated that sufficient reading speed is also possible with 8-dot Braille. Moreover, in combination with screen reader output, reading speed is much higher than reading contracted 6-dot Braille on paper.

In mathematical writing the script that goes hand in hand with 8-dot Braille is the LaTeX typesetting system as every single character of the LaTeX code corresponds to exactly one Braille cell. In elementary school mathematics, LaTeX code is only necessary in a few places, as the digits and arithmetic symbols are already available in the 8-dot system. However, some characters, such as the element symbol, are introduced in LaTeX notation. With the transition to secondary school, the LaTeX commands for fractions, root signs and exponent notation are added. Thus, on the way to the secondary school examination (Abitur), the students are gradually taught all the necessary LaTeX.

For teachers in mainstream schools, LaTeX is not a major hurdle, as many mathematics teachers are already familiar with this system from University. Those who are new to it can quickly learn it due to its intuitive and semantic notation. In contrast, both 6-dot contracted Braille and the 6-dot mathematical notation in Germany — the Marburg system — are much more complex to learn. As a result, learning these notations is hardly feasible for teachers in mainstream schools. The 8-dot Braille system, together with the linear notation of LaTeX on the one hand and the PC in conjunction with a screen reader and a Braille display on the other, represent an ideal bridging technology between the sighted and blind worlds. LaTeX not only helps students to communicate mathematics with their teachers but also with their sighted peers. Simple expressions are easy to understand even for those not yet exposed to LaTeX syntax. And if that fails it can be visually rendered for easier reading.

With the increasing digitalization in schools for blind students and the introduction of computers in every classroom, the 8-dot system and LaTeX have become the widely accepted standard. Many students thus achieve the University entrance qualification and can continue to work seamlessly at Universities with LaTeX [1]. As LaTeX is the lingua franca among mathematicians and as such internationally understood notation, students going on to higher education are already equipped with the most important tool to communicate mathematics with their professors.

3 LaTeX to Braille

As already mentioned, in early primary education very little LaTeX is actually needed. And even then the main goal is to use as much as possible symbols that are available on the keyboard. For example, asterisk `*` is used instead of `\cdot` for multiplication and simple fractions are written in beveled notation e.g., $\frac{1}{2}$ would be written as `1/2` instead of `\frac{1}{2}`. Nevertheless, some symbols like $\in$ are already introduced in their LaTeX notation `\in`.

Other conventions are adopted for easier reading, such as avoiding curly braces as much as possible, and inserting spaces before operators and relation symbols. Consider the example $n! = n * (n-1)!$ that is written by this convention as `n! =n *(n -1)!` In addition, certain command abbreviations are introduced, primarily with the goal of reducing spatial requirements on the Braille display. E.g., writing `\f` instead of `\frac`, or `\ol` instead of `\overline` . Also commands can be replaced by meaningful symbol combinations, such as `\le` being replaced by `<=` (see also `https://augenbit.de/`).

Initial online support for working with LaTeX and Euro Braille was developed at the SBBZ Ilvesheim as an extension of the MathJax v2.7 [2] library. The extension allowed to expose the LaTeX sources in the web page from which the MathJax would compute the visual rendering as textual underlay for the expression. This allowed easy selection and copying and pasting into a text editor or word processor from which the expression could be automatically translated into 8-dot Braille by a correctly configured screen reader. While this technique was sufficient for students to work with, in particular during the Covid pandemic, it had the drawback that, while some cleanup on the LaTeX source could be done, the expression would usually not follow all the rules described above.

In MathJax version 4 [3] we will support these features natively, in particular, copying and Euro Braille translation and rewriting LaTeX sources into the required format. Figure 1 shows the Braille translation for the commands describing the quadratic equation:

```
x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}
```

As an additional feature MathJax v4 exposes correct LaTeX sub expressions for parts of formulas that can be interactively explored. While this sounds straightforward, these sub expressions are non-trivial to compute. Firstly, LaTeX is a touring complete language, which requires a recursive stack automaton for parsing and thus partial expressions are not as readily available as in a simple LR-parser. Secondly, the exploration is based on a semantic model that is computed using Speech Rule Engine [12], which produces a canonical purely semantic representation of the math expression regardless of the incoming syntax, i.e., LaTeX, MathML, or AsciiMath..

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Figure 1: Translation of the quadratic equation into 8-dot Euro Braille

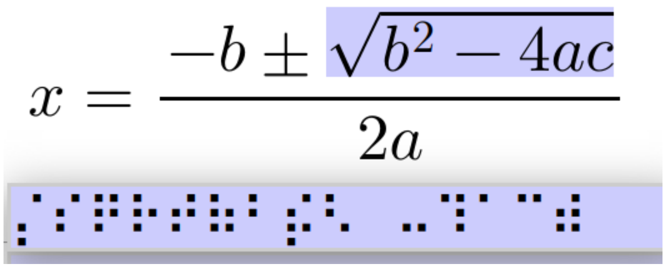

Figure 2: Translation of the quadratic equation into 8-dot Euro Braille

Figure 2 shows the Braille for the square root sub expression in the quadratic formula.[1] Note that there are nevertheless limits to the LaTeX sub expression MathJax can produce. For example, AMS matrix environments like `pmatrix`, `bmatrix`, or `vmatrix` implicitly generate fences, that are rendered. However, at the moment the parsing algorithm cannot produce corresponding LaTeX output.

4 Math Notation in the Netherlands

While similar approaches to Math Braille notation have been taken in other European countries using established mathematical syntax notations (e.g., LaTeX in Slovenia and AsciiMath in Sweden), other countries have pursued a different approach.

We will examine the situation in The Netherlands. Over twenty years ago in The Netherlands math had become practically inaccessible, the primary problem being the lack of a Math Braille code. Consequently, a linear notation was developed and introduced by Dedicon[2], which eases math communication in particular in inclusive education, but did not follow any pre-existing standards. In fact, the Dedicon notation is a mix of several linear notations, borrowed from symbolic calculation languages, like Maple, Mathematica, Excel. Pragmatic choices were made to make the notation as intuitive as possible; e.g. using SQR, SQRT or the dutch word WORTEL for abbreviations of square root.

In [5], Dorine in't Veld and Davy Kager presented the status of this notation, which in practice works quite well for students, since it meets all the needs summed up above for LaTeX:

1. It allows direct communication with sighted peers and teachers.

[1] Additional examples of the technique can be found here: https://mathjax.github.io/MathJax-demos-web/euro-Braille/

[2] http://Braille.dedicon.nl/wiskunde

2. It remains consistent notwithstanding in what application it is used. It can be converted from Microsoft Word or HTML to plain text or Markdown without any problems, because, like LaTeX, the notation only uses ASCII characters that can be typed with a qwerty keyboard.

3. It requires no additional assistive or conversion software. The screenreader translates the notation to 8-dot Braille and TTS.

The weaknesses are that the notation initially only covered primary and secondary education, not higher education. And while it was good for writing, it was too ambiguous for more complex expressions, mainly due to its use of spaces for delineation. For example, there is a rule that spaces terminate superscript and subscript text, which is usually very convenient but can go wrong when combining the two: Consider `x_2 ^2` versus `x_2^2`, where the former is x_2^2, while the latter represents x_{2^2}. Note that the space between the 2 and the caret symbol makes the difference, which can lead to easy mistakes. The main issue is the lack of clear notation that defines argument boundaries, which is particularly concerning with more complex expressions containing fractions, roots, vectors etc. As a consequence, automatic conversion is difficult and there currently exists no implementation of a renderer that can visualize expressions.

As already mentioned, the Dedicon notation is quite popular with students and teachers in The Netherlands and it works sufficiently well in practice. And since Dedicon has the responsibility to make textbooks accessible, they must use formats that are actually taught to students, which currently means that books are transcribed into Word documents with math in Dedicon notation.

As a consequence the necessary improvements indicated in 2016 have not been made yet. Likewise there is no pressure to change to AsciiMath or LaTeX for schools or to extend the current Dedicon notation to become suitable for more advanced mathematics. However, this leads to a disconnect on the other stages of the education ladder:

1. Students in higher education have no support of specialized itinerant teachers who are experts in a discipline and in the use of screenreaders. This means for example that those students who need to learn LaTeX at their University, have to find a practical accessible tutorial on their own. Additionally, if they do not have the luck to find someone in their IT-department who can help out, they have to find out what software to use and how it can be installed in the network of their University, etc.

2. Students in secondary pre academic education already have no support of specialized itinerant teachers. In fact they too are pretty much on their own with their laptop and screenreader. The last couple of years there is much more attention for science education; there is a working group that develops (mainly tactile) materials and there is a helpdesk. But there was no attention for standard mathematics notation like LaTeX so far; there were many other priorities and – again – the Dedicon notation works well enough.

This matter is compounded by another peculiar problem in primary schools: children often want printed Braille, also for math. In these tactile books an old 6-dot Braille math code is used, that is not taught to the students. When they start from scratch they will easily learn and accept that spacing rules are different in Braille and if there might be a notation they do not understand, they can ask their teacher or peers what the original book says or they can look up things in a symbols list. For secondary and higher education, we no longer print Braille. The educational institutes for the BVI students do not want it since the code for more advanced math is too complex and nobody learns it. For tactile images this means, that it is often not possible to transcribe formulas into Braille, if they cannot be expressed in 6-dot Braille.

Moreover math is mostly done electronically on the laptop in secondary school. Until fairly recently students would choose their own preferred 8-dot Braille table on their Braille display, mostly the 'German' or 'American' table. While students are now encouraged to mostly use Euro Braille, those who are used to other Braille tables, are often reluctant to change to Euro Braille.

Due to the fact that students are very much on their own finding their way and there is no uniform notation or code, still only very few blind students choose science studies where they need more advanced mathematical notation and specialized software applications. Because of the same reasons, many students experience big obstacles in disciplines that require statistics. Here R becomes more prevallent with strong community support and improving accessibility support based on LaTeX. This, together with the advantages to be learned from the German system, in combination with the latest MathJax developments, should help to improve the situation in The Netherlands and bring about a feeling of urgency to have books from which you can simply copy (human readable) LaTeX that one can communicate, render or paste into accessible software thereby increasing the number of BVI students taking up mathematical and scientific subjects in secondary and higher education. Hopefully this article will raise awareness and involve experts from the above mentioned working group to push this agenda forward.

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