

$$\textbf{Customer C Spend on Order X} = c * r * t + \epsilon$$

c = Customer's mean order amount in dollars where $c \sim \mathcal{N}(\$100, \$25)$.

r = Scalar corresponding to mean retailer order amount where $r \sim \mathcal{N}(1.0, 0.05)$.

t = Whether or not the customer recieved the experimental treatment $t \in [1, 1.01]$.

ϵ = Random noise where $\epsilon \sim \mathcal{N}(\$10, \$1)$.

The number of orders placed by Customer C $\sim \text{exp}(\lambda = 1)$.

	Method I	Method II	Method III	Method IV	Method V	Method VI	Method VII	Method VIII	Method IX
Points Used	3	5	7	3	5	7	3	5	7
Simulations Per Point	100	100	100	200	200	200	300	300	300
Verification Simulations	500	500	500	500	500	500	500	500	500
Total Simulations Required	800	1,000	1,200	1,100	1,500	1,900	1,400	2,000	2,600
Mean Run Time (Min.)	10.3	10.2	14.9	14.5	17.8	25.0	18.7	23.1	35.0
Mean Error: Est. Power - Desired Power	0.012	0.048	0.046	0.043	0.033	0.041	0.025	0.026	0.042
Std. of Error Est. Power - Desired Power	0.094	0.057	0.043	0.07	0.052	0.037	0.084	0.034	0.028

Simulations were conducted using a single thread (Intel Xeon CPU E5-2670).

If we were implementing a difference of means t-test, then we could simply apply the formula below to get the appropriate sample size.

$$n = 2 \left(\frac{Z_{1-(\alpha/2)} + Z_{1-\beta}}{ES} \right)^2 \text{ where } ES = \frac{|\mu_1 - \mu_2|}{\sigma}$$

Here, α is the probability of a Type I error (false rejection of the NULL hypothesis, i.e. a false positive) and β is the probability of a Type II error (i.e. a false negative). Observe how if we want to make our test more rigorous and reduce the false positive rate, the the left side of the numerator increase and the resulting recommended sample size - n , also increases. Similarly, if we want to reduce the probability for failing to detect the difference across groups, then we will need to increase the right-hand side of the numerator - also increasing the recommended sample size.

$$y = \beta_0 + \epsilon$$

$$y = \beta_0 + X_1\beta_1 + \epsilon$$

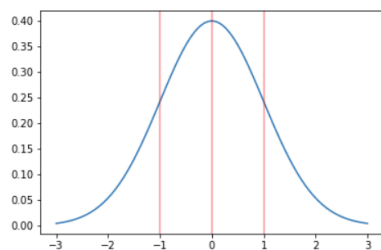
$$y = \beta_0 + X_1\beta_1 + X_2\beta_2 + \epsilon$$

$$y = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + \epsilon$$

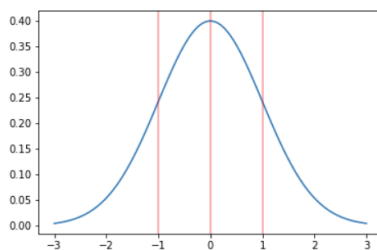
$$y = \beta_0 + \dots + \beta_n X_n + \epsilon \quad \text{where} \quad \hat{\epsilon} = y - \hat{y}$$

$$Y = \text{Signal} + \text{Noise}_{\text{clustered}} + \text{Noise}_{\text{unclustered}}$$

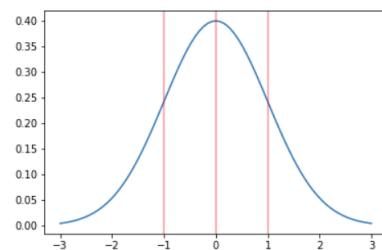
$$Y = \text{Signal} + \text{Explained Noise}_{\text{clustered}} + \text{Unexplained Noise}_{\text{clustered}} \tag{1}$$



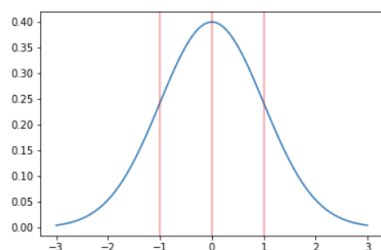
(a) $\mu \pm [1]\sigma$
with 500 iterations.



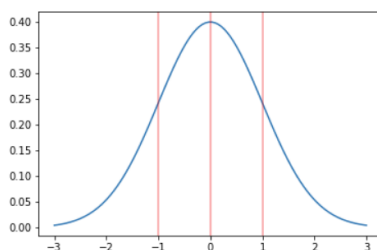
(d) $\mu \pm [1]\sigma$
with 1,000 iterations.



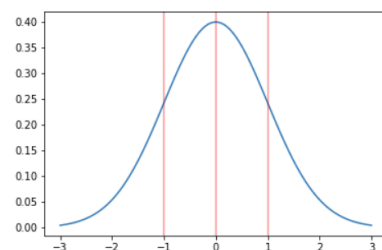
(g) $\mu \pm [1]\sigma$
with 2,000 iterations.



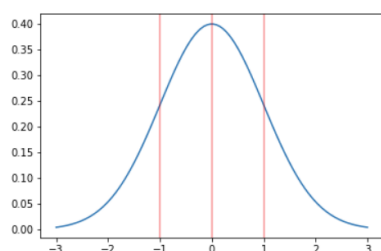
(b) $\mu \pm [1,2]\sigma$
with 500 iterations.



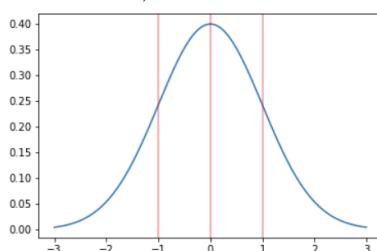
(e) $\mu \pm [1,2]\sigma$
with 1,000 iterations.



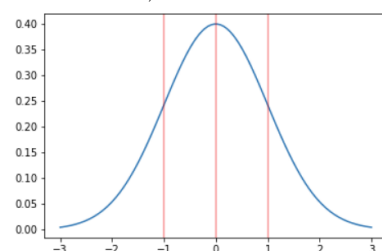
(h) $\mu \pm [1,2]\sigma$
with 2,000 iterations.



(c) $\mu \pm [1,2,3]\sigma$
with 500 iterations.



(f) $\mu \pm [1,2,3]\sigma$
with 1,000 iterations.



(i) $\mu \pm [1,2,3]\sigma$
with 2,000 iterations.