Van der Waals Parameters for Cations in AMBER Format

The AMBER force field contains parameters for a variety of ions; however, if we are interested in studying a system containing a species not included in the standard parameter set we must append the force field with suitable quantities. The "state of the art" for monovalent and divalent cations is attributable to Aqvist [J. Aqvist, *J. Phys. Chem.* **94**, 8021 (1990)]. We are charged with determining the parameters r_{ij}^* (the internuclear separation of the *ij* pair at the potential minimum) and **e**, the potential well depth for the ion at this minimum value.

Given is the Lennard-Jones potential:

$$[1]\ U(r) = \mathbf{e}\,(r^*/r)^{12} - 2\mathbf{e}\,(r^*/r)^6$$

which is commonly rewritten for the *ij* pair of ions as:

$$[2]\ U(r_{ij}) = (A_iA_j / r_{ij}^{12}) - (B_iB_j / r_{ij}^6)$$

where the A and B parameters are given in Aqvist. To find r_{ij}^*, we take the derivative of [2] with respect to r_{ij}:

$$[3]\ dU(r_{ij}) / dr_{ij} = -12A_iA_jr_{ij}^{-13} + 6B_iB_jr_{ij}^{-7}$$

and set the righthand side of [3] to zero:

$$[4]\ -12A_iA_jr_{ij}^{-13} + 6B_iB_jr_{ij}^{-7} = 0$$

then solve for r_{ij} at this minimum which is what we call r_{ij}^*:

$$\mathbf{[5]\ r_{ij}^* = (2A_iA_j / B_iB_j)^{1/6}}$$

Using [5] in concert with Aqvist's paper, we can proceed to determine r_{ij}^* for Mg^{2+}, Ca^{2+}, Sr^{2+}, and Ba^{2+} (the monovalent species are already in the AMBER parameter set) where species *i* is one of the cations and species *j* is the oxygen in water. As an example, we will calculate r_{ij}^* for Ca^{2+}.

From Aqvist, we have $A_{Ca^{2+}} = 264.1$ and $B_{Ca^{2+}} = 18.82$. The water model used in AMBER is TIP3P [W. L. Jorgensen, J. Chandrasekhar, J. D. Madura, R. W. Impey, and M. L. Klein, *J. Chem. Phys.* **79**, 926 (1983)]; the cation interacts with the oxygen in water, thus we need the A and B parameters for the oxygen in TIP3P water ($A_O = 762.89$ and $B_O = 24.39$).

$$[6]\ r_{O\text{-}Ca^{2+}}^* = (2 \times 762.89 \times 264.1 / 24.39 \times 18.82)^{1/6} = 3.09437\ \text{Å}$$

We are almost finished. In [6], $r_{O\text{-}Ca^{2+}}^*$ is the sum of r_O^* and $r_{Ca^{2+}}^*$:

$$[7]\ r_{O\text{-}Ca^{2+}}^* = r_O^* + r_{Ca^{2+}}^*$$

Rearranging [7], and using $r_O^* = 1.768$ Å from Aqvist (note that this number includes the hydrogens implicitly!):

$$[8]\ r_{Ca^{2+}}^* = r_{O\text{-}Ca^{2+}}^* - r_O^* = 3.09437 - 1.768 = 1.3264\ \text{Å}$$

For **e**, we just compare [1] to [2]: It is clear that

$$[9]\ A_iA_j = \mathbf{e}_{ij}\,(r_{ij}^*)^{12}$$

and

$$[10]\ B_iB_j = 2\,\mathbf{e}_{ij}\,(r_{ij}^*)^6$$

Since *i* = *j* for each species, we will drop the subscripts in the following discussion. If we square [10]:

$$[11]\ B^4 = 4\,\mathbf{e}^2\,(r^*)^{12}$$

we can solve [11] for $(r^*)^{12}$:

$$[12]\ (r^*)^{12} = B^4 / 4\,\mathbf{e}^2$$

Rearrange [9] to

$$[13]\ (r^*)^{12} = A^2 / \mathbf{e}$$

We can now equate [12] to [13] and solve for **e**:

$$\mathbf{[14]\ e = B^4 / 4\,A^2}$$

Returning to the Ca^{2+} example, recall from Aqvist we have $A_{Ca^{2+}} = 264.1$ and $B_{Ca^{2+}} = 18.82$. Plugging these quantities into [14]:

$$[15]\ \mathbf{e}\,(Ca^{2+}) = 18.82^4 / 4 \times 264.1^2 = 0.44966\ \text{kcal/mol}$$

Finally, we are ready to make our file that includes these data. All we need to do is name the file - let's choose frcmod_Ca.in - and set it up as follows (there are a total of 6 lines in the file, including the blank line):

```
\# First line; parameters for Ca++.
MASS
CA 40.08

NONB
CA 1.3264 0.44966 (adjusted, from Aqvist)
```

That's all. In tLEaP, we would do the following:

```
> loadamberparams frcmod_Ca.in
Loading parameters: ./frcmod_Ca.in
Reading force field mod type file (frcmod)
>
```

and so on. Questions and comments can be directed to Todd J. Minehardt (tjm@princeton.edu).