## Abstract Algebra

| Subject | Objects | Subojects | Homomorphisms | Numerics |
|---|---|---|---|---|
| Group theory | [Group G] | (H : Subgroup G) | (f : G →* H) | Nat.card H<br>Subgroup.index H<br>Subgroup.relindex H K |
| Ring theory | [CommRing R] | (I : Ideal R) | (f : R →+* S) | Ideal.absNorm I |
| Field theory | [Field K] | (F : IntermediateField K L)<br>(F : Subfield K) | [Algebra K L]<br>[IsScalarTower K L M] | Module.finrank K L<br>IntermediateField.relfinkrank F E<br>Subfield.relfinrank F E |
| Linear algebra | [Module R M] | (S : Submodule R M) | (f : M $\to_l$[R] N) | Module.finrank R M |

## Galois Theory

- Gal(L/K) is defined as L $\simeq_a$[K] L for any field extension [Field K] [Field L] [Algebra K L]
- Given [FiniteDimensional K L], the theorem IsGalois.tfae states that the following are equivalent:
  - IsGalois K L
  - IntermediateField.fixedField (⊤ : Subgroup Gal(L/K)) = (⊥ : IntermediateField K L)
  - Nat.card Gal(L/K) = finrank K L
  - ∃ p : K[X], p.Separable ∧ p.IsSplittingField K L
- The Galois correspondence consists of the following inverse pair of inclusion-reversing functions:
  - IntermediateField.fixingSubgroup : IntermediateField K L → Subgroup Gal(L/K)
  - IntermediateField.fixedField : Subgroup Gal(L/K) → IntermediateField K L
  - IntermediateField.fixingSubgroup_fixedField : fixingSubgroup (fixedField H) = H
  - IsGalois.fixedField_fixingSubgroup : fixingSubgroup (fixedField K) = K
  - IntermediateField.fixingSubgroup_antitone : Antitone fixingSubgroup
  - IntermediateField.fixedField_antitone : Antitone fixedField
- IsGaloisGroup G K L allows groups other than Gal(L/K) to be Galois groups of L/K.