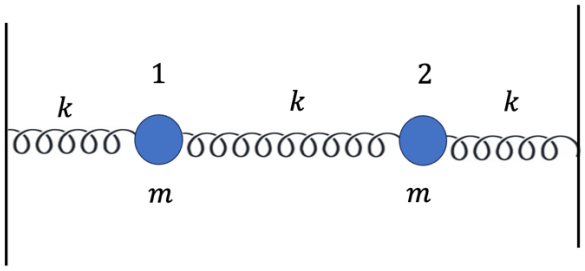
Assessment 1 – Coupled oscillators.

In this first assessment, you will write a program able to find the different oscillation frequencies of a coupled harmonic oscillator like the one in the figure below:



The equations of motion of such a pair of coupled oscillators are:

$$m\ddot{x}_1 = -2kx_1 + kx_2,$$

$$m\ddot{x}_2 = +kx_1 - 2kx_2.$$

To find the frequencies, one can assume solutions of the form $x_1 = A_1 e^{-i\omega t}, x_2 = A_2 e^{-i\omega t}$ and rewrite the previous differential equations as a linear algebraic system of equations:

$$-m\omega^2 A_1 = -2kA_1 + kA_2,$$

$$-m\omega^2 A_2 = +kA_1 - 2kA_2,$$

which can be transformed into an eigenvalue problem:

$$M\boldsymbol{A} = -\omega^2 \boldsymbol{A},$$

where:

$$M = \begin{pmatrix} -2k/m & +k/m \\ +k/m & -2k/m \end{pmatrix}, \text{ and } \boldsymbol{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}.$$

By using the QU decomposition method explained in class, you will write an algorithm able to find the eigenvalues of *M*. The different tasks that you will do in this assessment will be:

a) Write a code able to find eigenvalues of square matrices using the QU method **[40 marks]**

b) Validate it by solving by hand the characteristic equation, and comparing the analytical result with your numerical ones. **[30 marks]**

c) Then, you will use your program to investigate how the oscillation frequencies change as a function of the mass of the particles. You will present this in the form of a frequency vs mass plot. **[30 marks]**

You will present your appropriately commented code and a report showing the results of your validation and investigation (questions b and c). A more detailed set of expectations for the report will be updated on ELE for this assessment.