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A latent capture history model for digital aerial surveys

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SUMMARY: We anticipate that unmanned aerial vehicles will become popular wildlife survey platforms. Because detection of animals from the air is imperfect, we develop a mark-recapture line-transect method based on footage from two digital cameras, possibly mounted on a single aircraft, which cover the same area with a short time delay between them. Animal movement between the passage of the two cameras introduces uncertainty in individual identity, so individual capture histories are unobservable and are treated as latent variables. Our likelihood is obtained by enumerating all possibilities within segments of the transect containing ambiguous identities, without any idntity matching between the two cameras. We call this 'Latent Capture-history Enumeration', or LCE. We include an availability model for species that are periodically unavailable for detection, such as cetaceans which are undetectable while diving. External data are needed to estimate the availability cycle length but not mean the availability rate if the full availability model is employed.

We compare the LCE method with the recently-developed cluster capture-recapture method (CCR), which uses a Palm likelihood approximation, so providing the first comparison of CCR with maximum likelihood. Both methods are approximately unbiased with nominal confidence interval coverage and similar precision. The LCE method variance reduces below that of the CCR method as sample size increases. We illustrate the methods with semi-synthetic data from a harbour porpoise survey.

KEY WORDS: Availability bias; Double-observer survey; Line transect; Mark-recapture; Movement model; Poisson process.

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1. Introduction

Aerial surveys of wildlife populations allow large areas of land or sea to be surveyed at relatively low expense. We anticipate that aerial surveys with human observers will increasingly be replaced by unmanned aerial vehicle (UAV) surveys using digital video or still cameras. This presents some new statistical challenges. In this paper we address these challenges and develop a method of estimating animal density from cameras deployed from UAVs.

Traditional aerial surveys using human observers involve a reasonably wide field of view, perhaps as much as 1000m either side of the aircraft. Detections of animals decrease with distance from the aircraft, an effect that is modeled using a detection function. By contrast, aerial footage from UAV-mounted cameras has a much narrower field of view — perhaps 100m to either side — and detectability can often be assumed to be constant within this zone, in which case a distance-dependent detection function is not needed.

Conventional line transect analyses assume that animals are detected with certainty if they are at distance zero from the transect line, which corresponds to the aircraft's path in our case. If this assumption cannot be met, extensions based on mark-recapture methods are employed: see Burt et al. (2014) for an overview. The basis of mark-recapture extensions to line transect analyses is to have two observers who search the area independently of each other. The two observers serve as two "capture occasions", and animals detected by both observers are described as recaptures or duplicates. The mark-recapture design enables us to estimate the detection probability of each observer, conditional on detection by the other observer, and therefore to adjust for imperfect detection at distance zero. In the case of narrow-strip aerial surveys from UAVs, imperfect detection can result from animals being indistinct or obscured in digital images, so a mark-recapture design may be necessary even if detectability is constant with respect to distance.

Animals of some species may spend a proportion of their time entirely unavailable for detection. For example, whales are unavailable while diving; seals are unavailable at haul-out sites while they are at sea; burrowing animals are unavailable while underground, and birds or frogs may be available only when vocalising. If some animals are systematically unavailable to both observers, then the unavailable portion of the population is unsampled, so there is no information from which to estimate how large this portion is. An ideal sampling design ensures that all animals are subject to the same detection model, so that the sample is representative of the entire population. In the case of animals that are periodically unavailable, for example due to diving, this can be accomplished by incorporating an availability model into the analysis and sampling at more than one time.

There are various ways of sampling at two times. One option is to have two aircraft follow the same transect at a fixed time delay. Ensuring that the narrow search strips of two UAVs overlap adequately can be difficult in some environments and a cheaper alternative is to mount two cameras on a single aircraft: one forward-pointing, the other rear-pointing. These can be engineered so that the rear-pointing camera records the same area as the forward-pointing camera after a time delay of several seconds. This separation generates data with which we can model the availability cycle, as long as there is a chance that the availability status of an animal changes between the passage of the two cameras. For example, a whale might dive or surface during this time interval. In practice, the time delay will need to be sufficiently long relative to the duration of the diving cycle to ensure that the data are adequate to fit the availability model.

Mounting both cameras on the same UAV also has the advantage of creating a different viewing aspect for the two cameras: an animal that is obscured from one camera by a bush or shadow might be detectable from the other camera. Likewise, the longer time separation generated by running two UAVs in succession creates the opportunity for either camera to detect an animal that was undetected by the other, due to changes in the animal's position, sunlight, or wind. The two-camera design has general potential for doing mark-recapture line transect surveys from the air, regardless of whether or not an availability cycle is involved.

There are however two complications. Firstly, because animals will typically move between the passage of the two cameras, there is uncertainty in whether animals detected in similar locations by the two cameras correspond to the same animal or two different animals. We describe this as *uncertainty in capture history*. Each detected animal has a true capture history specifying which of the two cameras detected it, with capture histories (1, 0), (0, 1), and (1, 1) corresponding respectively to detection by only the first camera, only the second camera, or both cameras. When animals are detected from the air, there are usually inadequate visible features for distinguishing between individuals, so recaptures are determined purely on the basis of spatial location and detection time. The longer the time elapsed between the passage of the two cameras, the more difficult it is to distinguish between recaptures of a single individual, and captures of two different individuals. Rather than the capture histories being observed data, as they are in conventional capture-recapture studies, they are now latent variables.

Secondly, although separation in time allows us to deal with availability processes such as diving, there is likely to be dependence between the animal's availability state at the passage of the two cameras, so we are forced to adopt a model that accommodates this dependence. The dependence is reduced as the time delay between the passage of the cameras increases. However, we demonstrate below that the dependence never reduces to zero if animals are mobile, because animal movement in and out of the field of view of the cameras is itself an availability process. Moreover, while longer delays may reduce the dependence between cameras, they exacerbate the problem of capture-history uncertainty.

We develop an analysis framework suitable for two-camera aerial surveys. We explicitly model animal movement into and out of the detection strip between the passage of the two cameras, corresponding to an “in/out” availability process that induces dependence between the two cameras. For diving animals, we further consider an “up/down” availability process by modeling the diving cycle. As noted by Stevenson et al. (2018), two-observer survey data do not contain sufficient information to identify all parameters of the diving model, if the time delay between cameras is less than the mean dive-cycle duration. In that case, one parameter must be estimated from external data: we take this to be the mean dive-cycle duration itself. We derive our methods in generality including both in/out and up/down availability processes, but the methodology is equally applicable when only the in/out process is required, and in that case there is no need for external data.

To fit the two-camera model we use a full maximum-likelihood approach. We assemble the likelihood by identifying segments of the transect line that have ambiguous animal identities, and enumerating all possible matchings within each segment. As long as animal density is reasonably low, the enumeration is manageable within each segment, and our approach is computationally feasible using a constraint programming algorithm. Conditional on a particular set of matchings, we use a hidden Markov model formulation of the likelihood. This creates a general and extendable modeling framework for two-camera scenarios. We call our new approach the latent capture-history enumeration (LCE) method.

Previous literature has devoted substantial attention to each of the problems of availability and uncertain capture histories, but rarely together. Availability models for double-observer line transect surveys were developed by Borchers et al. (2013), Langrock et al. (2013), and Borchers and Langrock (2015). Most previous work on uncertain capture histories has focused on methods for resolving uncertainties before fitting conventional models. Pike and Doniol-Valcroze (2015) used a logistic regression technique to decide on an optimal dissimilarity score between pairs of detections, then established a threshold score within which pairs would be resolved as duplicates. Hamilton et al. (2018) devised estimates of the duplicate probability for each pair of detections, and repeatedly resampled from these probabilities within a bootstrap scheme to create a new resolved dataset at each iteration. Other work on latent capture histories has treated the case where observed histories are predictable, but non-invertible, transformations of the latent histories (e.g. Zhang et al. (2019); Bonner and Holmberg (2013); Link et al. (2010)). This contrasts with our case where no capture histories are observed: only a stream of detections from each camera in continuous time and space.

Our approach is most similar to that of Hiby and Lovell (1998) and Stevenson et al. (2018). Hiby and Lovell (1998) included both an availability model and uncertain capture histories in their analysis, and they maximized a log-likelihood obtained by summing over possible pairs of detections. They did not allow animal movement in the direction of aicraft travel and some aspects of their implementaion were not explict. Stevenson et al. (2018) derived an alternative approach using the new technique of cluster capture-recapture (CCR; see Fewster et al., 2016). In CCR, the locations of detections are treated as a clustered point-process and the model is fitted using a Palm likelihood approximation, which is an objective function fitted to the distribution of pairwise distances between all pairs of detections. The CCR method is asymptotically consistent and remains computationally efficient at high animal densities, unlike the LCE method we present here. However, it is not based on a true likelihood, and its performance has never been compared against that of maximum likelihood due to the difficulty of computing likelihoods in the scenarios for which CCR is intended. A key output of the present work is the first comparison between CCR and maximum likelihood.

2. Models for movement and availability

Two observers move along a transect line, one behind the other, searching a strip of half-width w. They move at constant speed v and are separated by a time-lag l and distance vl. We use the general term 'observers' and develop the model for lags l of any size, but we anticipate that the two observers are most likely to be two cameras on the same UAV.

We use two coordinates for location: the forward coordinate along the transect line, and the transverse coordinate perpendicular to the line. We say that an observer 'passes over' an animal at the instant that their forward coordinates coincide, regardless of the animal's transverse coordinate at that instant. We assume that observer speed exceeds animal speed, so the time at which each observer passes over each animal is well-defined.

Animals may move between the passage of the first and second observers. We model animal movement as a Brownian motion, such that the animal's displacement over time t follows a bivariate normal distribution with mean $(0, 0)$ and variance $\boldsymbol{\Sigma}(t) = \sigma^2 t \boldsymbol{I}$, where $\boldsymbol{I}$ is the 2×2 identity matrix.

2.1 *Forward movement*

For a given animal, the time elapsed between the passage of the first and second observers overhead is a random variable T. We show in Appendix A that T has probability density function (PDF)

$$f_T(t) = \frac{vl \exp\left\{-\frac{v^2(l-t)^2}{2\sigma^2 t}\right\}}{\sqrt{2\pi\sigma^2 t^3}}. \tag{1}$$

2.2 *Transverse movement and in/out availability*

The survey design involves two observers detecting animals within a strip of width w either side of the line. As animals may move into or out of the detection zone between the passage of the two observers, we consider the survey area to constitute a wider strip of width $b > w$ either side of the line, where the buffer b is chosen such that there is negligibly small probability that animals beyond b at the passage of the first observer will be within the searched strip w at the passage of the second. The buffered strip of width $2b$ therefore covers all animals that may be exposed to detection.

Let Y_0 be the signed distance of an animal to the right of the transect line when the first observer passes overhead, and Y_t be this distance time t later. We assume that $Y_0 \sim U(-b, b)$, independently for all animals. Our movement model implies that $Y_t - Y_0 \sim N(0, \sigma^2 t)$.

For each animal within the buffered strip b at the passage of the first observer, let the binary random variable Z_t be 1 if the animal is within the detection zone of width w at time t, and 0 otherwise. Thus Z_t describes the animal's in/out availability for detection at time t. The probability $\mathbb{P}(Z_t = 0 \mid Z_0 = 1)$ that an animal moves from inside to outside the detection zone during time t, and the probability $\mathbb{P}(Z_t = 1 \mid Z_0 = 0)$ that it moves from outside to inside the detection zone, are

$$p_{IO}(t) = \mathbb{P}(Z_t = 0 \mid Z_0 = 1) = \frac{1}{w}\int_0^w \left\{\Phi(y - w; \sigma^2 t) + \Phi(-y - w; \sigma^2 t)\right\} dy \qquad (2)$$

$$p_{OI}(t) = \mathbb{P}(Z_t = 1 \mid Z_0 = 0) = \frac{1}{b - w}\int_w^b \left\{\Phi(y + w; \sigma^2 t) - \Phi(y - w; \sigma^2 t)\right\} dy \qquad (3)$$

where $\Phi(\cdot; \sigma^2 t)$ is the cumulative distribution function of a normal random variable with mean zero and variance $\sigma^2 t$.

We model in/out availability as a two-state Markov process with transition probabilities over an interval of time t given by Eqns (2) and (3):

$$\boldsymbol{M}(t) = \begin{pmatrix} 1 - p_{IO}(t) & p_{IO}(t) \\ p_{OI}(t) & 1 - p_{OI}(t) \end{pmatrix}. \qquad (4)$$

The stationary distribution of the in/out Markov chain, which gives the long-term proportion of time spent in each state, is $(w/b,\ 1 - w/b)$.

2.3 *Diving behavior and up/down availability*

We model animal diving behavior using a two-state continuous-time Markov chain such that the time spent in state 1 (the near-surface state) is an exponential random variable with expected value κ, and the time spent in state 2 (the diving state) is an exponential random variable with expected value $\tau - \kappa$, where τ is the expected dive cycle duration. The Markov transition rate matrix $\boldsymbol{Q}$ is

$$\boldsymbol{Q} = \begin{pmatrix} -\frac{1}{\kappa} & \frac{1}{\kappa} \\ \frac{1}{\tau-\kappa} & -\frac{1}{\tau-\kappa} \end{pmatrix}. \tag{5}$$

The up/down state transition probability matrix at time separation t is $\boldsymbol{U}(t) = \exp(\boldsymbol{Q}t)$. The stationary distribution of the Markov chain is $(\gamma,\, 1-\gamma)$, where $\gamma = \kappa/\tau$.

2.4 *Combined availability model*

The possibilities of being in or out of the detection zone, and up or down with respect to diving, generate four states that animals can occupy: (up and in), (up and out), (down and in), and (down and out). We number the states 1 to 4 in that order. Assuming that the up/down state is independent of the in/out state, the matrix of transition probabilities between these states at time separation t is the Kronecker product $\boldsymbol{\Gamma}(t) = \boldsymbol{U}(t) \otimes \boldsymbol{M}(t)$. Using a matrix formulation for $\boldsymbol{\Gamma}(t)$ provides an extendable and computationally efficient way of dealing with the hidden states 2, 3, and 4. The stationary distribution for the four-state Markov process is

$$\boldsymbol{\delta} = \left(\gamma\frac{w}{b},\, \gamma\left(1-\frac{w}{b}\right),\, (1-\gamma)\,\frac{w}{b},\, (1-\gamma)\left(1-\frac{w}{b}\right)\right). \tag{6}$$

3. Detection model

We assume that the probability that an animal is in each state at the time the first observer passes over it is given by the stationary distribution $\boldsymbol{\delta}$, and hence that its state distribution after a waiting time t, when the second observer passes it, is $\boldsymbol{\delta\Gamma}(t)$.

Define the binary variable X_{ij} to be 1 if animal i is detected by observer j and zero otherwise. We model X_{ij} as a state-dependent Bernoulli random variable with parameter $p_j(c) = \Pr(X_{ij} = 1 \mid C_{ij} = c)$ where C_{ij} is the state of animal i when observer j passes over it, and $c \in \{1, 2, 3, 4\}$. It follows that X_{ij} $(j = 1, 2)$ are observations from a Markov modulated Bernoulli process.

It is convenient to arrange the state-dependent probability mass functions of X_{ij} in a diagonal matrix (see Zucchini et al., 2016, Eqn 2.13). For observer j, this matrix is

$$
\begin{array}{cc}
 & \begin{array}{cccc} \text{up,in} & \text{up,out} & \text{down,in} & \text{down,out} \end{array} \\
\boldsymbol{P}(x_{ij}) = & \begin{pmatrix}
\text{Bern}(x_{ij}; p_j(1)) & 0 & 0 & 0 \\
0 & 1 - x_{ij} & 0 & 0 \\
0 & 0 & \text{Bern}(x_{ij}; p_j(3)) & 0 \\
0 & 0 & 0 & 1 - x_{ij}
\end{pmatrix}
\end{array}
\tag{7}
$$

where $\text{Bern}\big(x_{ij}; p_j(c)\big) \equiv p_j(c)^{x_{ij}}\{1 - p_j(c)\}^{1-x_{ij}}$. The above matrix allows for animals to be detected in the 'down' state, but not in the 'out' state. We now assume that $p_j(3) = 0$, so that only animals in the 'up' state can be detected, but in general this need not be the case.

Let t_i be the time elapsed between the passage of the first and second observers over animal i. Conditional on t_i, the probability of observing capture history $\omega_i = (x_{i1}, x_{i2})$ for animal i can be expressed as the following matrix product, which efficiently sums over hidden states:

$$
\mathbb{P}(x_{i1}, x_{i2} \mid t_i) = \boldsymbol{\delta} \boldsymbol{P}(x_{i1}) \boldsymbol{\Gamma}(t_i) \boldsymbol{P}(x_{i2}) \mathbf{1}\,, \tag{8}
$$

where $\mathbf{1}$ is a column vector of ones. We label the three observable capture histories as $\omega_1 = (0, 1)$, $\omega_2 = (1, 0)$, and $\omega_3 = (1, 1)$, and define $p_k(t) = \mathbb{P}(\omega_k \mid t)$ as given in (8). The overall probability of capture history ω_k is then

$$
\tilde{p}_k = E_t\{p_k(t)\} = \int p_k(t) f_T(t)\, dt\,. \tag{9}
$$

4. Survey model

We assume that the number and locations of animals in the forward direction, within distance b of the transect line at the time that the first observer passes overhead, are governed by a Poisson process with intensity $D(s)$ at along-transect location s. We derive the likelihood supposing that the capture history ω of each animal is known. We will later revoke this requirement by marginalizing over all possible assignments of detections to capture histories.

Let $\boldsymbol{s} = (s_1, \ldots, s_n)$ be the observed forward locations of the n detected animals at the time of first detection. We can write $\boldsymbol{s} = \left(\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \boldsymbol{s}^{(3)}\right)$, where $\boldsymbol{s}^{(k)}$ corresponds to locations of animals with capture history k for $k = 1, 2, 3$. Each set of locations $\boldsymbol{s}^{(k)}$ arises from a thinned Poisson process with thinning probability $\tilde{p}_k$. Because multinomial splitting of a Poisson process produces independent Poisson subprocesses, the likelihood of $\boldsymbol{s}$ is the product of the three likelihoods from the thinned subprocesses. For animals with capture history $\omega_3 = (1, 1)$, there are additional observations on the time delay t between detection by the first and second observers, providing information about the movement parameter σ. The PDF of waiting time T, conditional on the capture history being ω_3, is

$$f_{T\,|\,\omega}(t \mid \omega_3) = \frac{f_T(t)\,\mathbb{P}(\omega_3 \mid t)}{\mathbb{P}(\omega_3)} = \frac{f_T(t)\,p_3(t)}{\tilde{p}_3}\,, \tag{10}$$

where the right-hand side of (10) is obtained from Equations (1), (8), and (9). This PDF is included as an auxiliary component to the Poisson process likelihood for $\boldsymbol{s}^{(3)}$.

Let L be the total transect length of the survey, and let n_k be the number of observations of capture history $k = 1, 2, 3$, with $n_1 + n_2 + n_3 = n$. Let $\tilde{p}. = \tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3$ be the overall probability of detection. We write $\boldsymbol{s}$, $\boldsymbol{\omega}$, and $\boldsymbol{t}$ for the locations, capture histories, and (where available) time delays for animals $i = 1, \ldots, n$. The parameter vector is $\boldsymbol{\theta}$. The likelihood is:

$$\mathcal{L}(\boldsymbol{\theta}\,;\,\boldsymbol{s}, \boldsymbol{\omega}, \boldsymbol{t}) = \frac{\exp\left\{-\int_0^L D(u)\tilde{p}.\,du\right\}}{n_1!\,n_2!\,n_3!} \left\{\prod_{i=1}^{n} D(s_i)\right\} \tilde{p}_1^{n_1}\,\tilde{p}_2^{n_2} \prod_{i\,:\,\omega=\omega_3} \left\{f_T(t_i)\,p_3(t_i)\right\}. \tag{11}$$

4.1 *Homogeneous density*

In the homogeneous case, where density is constant throughout the survey, we have $D(s) = D$. The likelihood is:

$$\mathcal{L}(\boldsymbol{\theta}\,;\,\boldsymbol{s}, \boldsymbol{\omega}, \boldsymbol{t}) = \frac{\exp\left(-LD\,\tilde{p}.\right)}{n_1!\,n_2!\,n_3!}\,D^n\,\tilde{p}_1^{n_1}\,\tilde{p}_2^{n_2} \prod_{i\,:\,\omega=\omega_3} \left\{f_T(t_i)\,p_3(t_i)\right\}. \tag{12}$$

4.2 *Model parameters*

The model has four kinds of parameters:

Density parameters: In the case of the homogenous Poisson process there is one parameter, θ, such that $D = e^{\theta}$. When density varies with covariates, θ is replaced by a linear predictor involving a parameter vector.

Dive cycle parameters: The two-state dive cycle model described above is parametrized in terms of the mean dive cycle length, τ, and the mean proportion of time in the near-surface state, γ, which are linked to parameters α_τ and α_γ via log and logit links: $\tau = e^{\alpha_\tau}$ and $\gamma = e^{\alpha_\gamma}/(1 + e^{\alpha_\gamma})$.

Movement parameters: The animal movement model has one parameter, σ, which we model using a log link: $\sigma = e^{\phi}$.

Detection parameters: Assuming that animals are only detectable when in state $c = 1$ (up,in), we have two Bernoulli parameters to model: $p_1(1)$ and $p_2(1)$. These can be modeled using logit link functions. If the observers are identical digital detectors, it may be reasonable to assume these two probabilities are identical, i.e. $p_1(1) = p_2(1) = p = e^{\beta}/(1 + e^{\beta})$.

As is the case for density, covariates can be incorporated into the other three models by replacing the corresponding scalar parameter on the link scale with a suitable linear predictor involving the covariates.

For the rest of this paper, we focus on the constant density model with identical detectors and no covariates, which hasfive parameters: $(\theta, \alpha_\gamma, \alpha_\tau, \phi, \beta)$. Stevenson et al. (2018) showed that they are not all identifiable from the two-observer survey design. For the detection model, they assume that $p = e^{\beta}/(1 + e^{\beta}) = 1$. This is reasonable for digital aerial surveys conducted in calm sea states, if we define the near-surface state to be "at or breaking the surface": a state that is easily observed. The field of view of a digital camera is such that objects towards the periphery of the image are as easily detected as objects in the centre of the image, so a detection function that drops off with distance from the line is not needed.

Stevenson et al. (2018) also show that even when p is known, only two of $(\theta, \alpha_\gamma, \alpha_\tau)$ are identifiable, so one of these parameters must be estimated using external data. We follow Stevenson et al. (2018) and Hiby and Lovell (1998) and assume that the mean dive cycle duration $\tau = e^{\alpha_\tau}$ is estimated separately, so we treat it as known in the present survey. In what follows, we therefore assume that detection of animals in the up/in state is certain ($p = 1$); we use external estimates to set τ; and we estimate the remaining three parameters. These constitute the density, D; the mean proportion of time in the near-surface state, γ; and the movement parameter, σ. The parameter vector is therefore $\boldsymbol{\theta} = (\theta, \alpha_\gamma, \phi)$.

5. Marginalising over the latent capture histories

The likelihoods (11) and (12) are formulated under the supposition that the capture histories ω_i are known for animals $i = 1, \dots, n$. However, the core problem when observers are separated in time is that the capture histories cannot be known with certainty: they are latent variables. Here we address this problem by enumerating all plausible combinations of latent capture histories. We marginalize the likelihood by summing over the individual likelihoods for every plausible capture history combination. We refer to each combination of capture histories as a "pairing", since once the pairs of detections with capture history $\omega_3 = (1,1)$ have been decided, the capture histories $(0,1)$ or $(1,0)$ of all other detections are determined, because we know which of the two observers made each detection.

Calling the mth set of pairings $\boldsymbol{\omega}^{(m)}$, and the associated vectors of first-detection locations and time delays $\boldsymbol{s}^{(m)}$ and $\boldsymbol{t}^{(m)}$ respectively, we obtain the likelihood for the parameters $\boldsymbol{\theta}$ as

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{m=1}^{M} \mathcal{L}\left(\boldsymbol{\theta}; \boldsymbol{s}^{(m)}, \boldsymbol{\omega}^{(m)}, \boldsymbol{t}^{(m)}\right) , \tag{13}$$

where M is the number of plausible pairings.

While this likelihood is easy to write down, it is challenging to evaluate because we need to enumerate all M plausible combinations $\boldsymbol{\omega}^{(m)}$. For any but very small sample sizes, the number M of possible pairings is very large. We tackle this problem by first partitioning the location vector $\boldsymbol{s}$ into subsets between which paired detections are impossible, to reduce the number of plausible pairings, and then using a constraint programming technique for efficient enumeration of all possible pairings within subsets.

5.1 *Subdivision of* $\boldsymbol{s}$

We partition $\boldsymbol{s}$ by "cutting" the transect line immediately after detections by observer j for which the distance to the next detection by the other observer is greater than a maximum possible distance that an animal could have moved between the two observers passing over it (d_{max}). This distance d_{max} must be decided using knowledge of the movement speed and behavior of the target species. A suitable value for d_{max} can be chosen by doing inference at a range of plausible values to find where estimates become insensitive to d_{max}. The cost of setting d_{max} too large is in computational speed; the cost of setting d_{max} too small is positive bias in estimation of D, since setting d_{max} too small will result in some animals with true capture history $(1, 1)$ being assigned capture history $(0, 1)$ or $(1, 0)$.

Having divided the transect line into R segments, we enumerate the possible pairings $\boldsymbol{\omega}^{(m_r)}$ for segments $r = 1, \ldots, R$. Let M_r be the number of possible pairings in segment r. We calculate the likelihood as

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{r=1}^{R} \sum_{m_r=1}^{M_r} \mathcal{L}\left(\boldsymbol{\theta}; \boldsymbol{s}^{(m_r)}, \boldsymbol{\omega}^{(m_r)}, \boldsymbol{t}^{(m_r)}\right) . \tag{14}$$

When d_{max} is substantially smaller than most of the distances between detections by different observers, segmentation can lead to a massive reduction in computation time, making it quite feasible to compute what would otherwise be an intractable likelihood.

5.2 *Constraint programming for enumerating all* $\boldsymbol{\omega}^{(m)}$

For efficient enumeration of the possible pairings within one segment, we define a simple *constraint satisfaction problem* (CSP) (Russell and Norvig, 2010, Chapter 6). A CSP is a triple $\mathcal{P} = \langle \mathcal{X}, \mathcal{D}, \mathcal{C} \rangle$. The CSP $\mathcal{P}$ has a set of decision variables $\mathcal{X}$, each of which has a set of possible values that it may take, called its *domain*, where $\mathcal{D}(x)$ is the domain of $x \in \mathcal{X}$. In addition there is a set of constraints $\mathcal{C}$ that restrict the combinations of values that may be taken by the variables. A constraint $c \in \mathcal{C}$ is a relation defined on a set of variables scope$(c) \subseteq \mathcal{X}$. A *solution* is an assignment of values to variables such that each variable is assigned a value from its domain, and all constraints are satisfied.

We define a CSP for a segment as follows. Two detections by different observers may be paired if and only if the distance between them is less than or equal to d_{max}. For each set $\{i, j\}$ of two observations that may be paired, we define one decision variable $x_{i,j}$ with domain $\{0, 1\}$. Variable $x_{i,j}$ is equal to 1 in a solution if and only if the two observations are paired.

Suppose we have two distinct sets, $s_1 = \{i, j\}$ and $s_2 = \{k, l\}$, where i may be paired with j, and k may be paired with l, but the two sets are not disjoint: in other words $s_1 \cap s_2 \neq \emptyset$. The two sets cannot both be paired simultaneously because they share an observation. In all such cases we add the constraint $(x_{i,j} = 0 \vee x_{k,l} = 0)$ to prevent such pairing.

We use a backtracking search procedure with forward checking (Russell and Norvig, 2010, Chapter 6) to enumerate all solutions to the CSP. The set of solutions to the CSP corresponds one-to-one to the set of valid pairings within the segment. When a solution is found, the part of the likelihood pertaining to that pairing is calculated, avoiding the need to store the set of pairings and allowing efficient calculation of $\sum_{m_r=1}^{M_r} \mathcal{L}\left(\boldsymbol{\theta}; \boldsymbol{s}^{(m_r)}, \boldsymbol{\omega}^{(m_r)}, \boldsymbol{t}^{(m_r)}\right)$.

5.3 *Interval estimation*

We estimate the variances of parameters using the inverse of the Hessian obtained in the fitting process. Confidence intervals for the parameters D, σ and γ are gained from the inverse log transformation of confidence intervals for θ and ϕ, and the inverse logit transformation of α_γ, assuming normality of the maximum likelihood estimators of these parameters.

6. Application

We use the term 'Latent Capture-history Enumeration' method, or LCE, to describe our framework. We developed this method in anticipation of digital aerial survey data becoming widely used, but, pending the availability of analysis methods such as the LCE method developed here, such data are not yet available. We therefore estimate density from the semi-synthetic data used by Stevenson et al. (2018). These data are taken from an aerial survey of harbor porpoise (*Phocoena phocoena*) in the North Sea using human observers, compiled from periods when the aircraft circled back over its transect after a lag of $l = 248$ seconds. The two observers correspond to the two passes of the aircraft. Only data in a narrow strip of half-width $w = 0.125$ km are included, to mimic the narrow field of view and perfect near-surface detection characteristic of digital observers. As noted by Stevenson et al. (2018), a lag of $l = 248$ seconds is longer than any plausible value for τ and as a consequence the surfacing states of an animal at the times the two observers pass are independent and the estimator is robust to unknown τ. For shorter lags τ needs to be speficied (see Section 7).

Following Stevenson et al. (2018), we use a buffer of $b = 2$ km, beyond which we assume no animal could enter the detection zone between the passage of the two observers. Using a Palm likelihood approximation termed cluster capture-recapture (CCR), Stevenson et al. (2018) obtained the following estimates, with 95% confidence intervals in brackets: $\hat{D} = 1.05$ (0.84, 1.60) pods per km^2; $\hat{\sigma}_{palm} = 0.15$ $(0.11, 0.19)$ km; and the expected proportion of time in the surface state, $\hat{\gamma} = 0.86$ (0.56, 1.00). Using the enumerated likelihood via our LCE formulation, we obtain $\hat{D} = 1.24$ (0.97,1.6) pods per km^2, $\hat{\sigma}_{palm} = 0.09$ (0.07,0.11) km, and $\hat{\gamma} = 0.73$ (0.55,0.91). The estimate $\hat{\sigma}_{palm} = 0.09$ corresponds to a mean rate of movement over $l = 248$ seconds of 0.58 m/s, with 95% confidence interval (0.47,0.71) m/s.

Stevenson et al. (2018) estimated the coefficients of variation (CV) of $\hat{D}$, $\hat{\sigma}_{palm}$ and $\hat{\gamma}$ to be

19%, 16%, and 13%, respectively. The corresponding estimated CVs from the LCE method are 13%, 10%, and 13%, respectively.

The estimates from the two methods are broadly consistent; the LCE method estimates there to be substantially less animal movement, slightly less time at the surface, and a higher animal density. As we cannot evaluate the relative merits of the methods on the basis of a single survey with unknown density, we investigate their performance by simulation.

7. Simulation study

Recall that X_{i1} and X_{i2} are detections of animal i by two observers separated by a time lag. With lags close to zero, X_{i1} and X_{i2} are highly correlated because animals available to one observer are almost certain to be available to the other. As lag increases, we expect this correlation to decrease. A pertinent question is whether dependence can be removed by choosing a suitably long lag. To investigate this we look at the correlation between X_{i1} and X_{i2} as a function of lag, with γ values from 0.1 to 0.9, and lags from 0 to 500 seconds.

In our model, X_{i1} and X_{i2} are Bernoulli random variables with expectation $\gamma w/b$. The correlation between these variables when there is a separation of t seconds between the two observers passing over an animal, is

$$\rho(t) \quad = \quad \frac{\sum_{x_{i1}=0}^{1} \sum_{x_{i2}=0}^{1} \left(x_{i1} - \gamma \frac{w}{b}\right) \left(x_{i2} - \gamma \frac{w}{b}\right) \mathbb{P}(x_{i1}, x_{i2} \mid t)}{\gamma \frac{w}{b} \left(1 - \gamma \frac{w}{b}\right)}, \tag{15}$$

where $\mathbb{P}(x_{i1}, x_{i2} \mid t)$ is given in (8).

The dark line in Figure 1 shows the correlation as a function of the lag (l) for $\tau = 110$ seconds and γ and σ equal to the estimates obtained in the previous section. It also shows the correlation for $\gamma \in \{0.1, 0.2, \ldots, 0.9\}$ and the correlation under the assumption that animals do not move but do become unavailable by diving.

[Figure 1 about here.]

It is clear that increasing the lag to τ or more reduces correlation to approximately zero if animals do not move (grey lines), although in the presence of animal movement there is still correlation between observers due to in/out availability. This corroborates the observation of Stevenson et al. (2018) that correlation between observers due to up/down availability can be removed by setting a lag greater than τ. With such long lags, the up/down availability model requires only the single parameter γ, corresponding to the proportion of time spent at the surface. This has the considerable advantage that there are no unidentifiable parameters, so no external data are needed to estimate density.

In practice, we are primarily interested in methods for surveys with two cameras on one aircraft, and with this configuration and fast-moving aircraft, lags of more than some tens of seconds are unlikely to be achievable. In light of this, and the results of Figure 1, we present simulations for (a) a scenario designed to imitate the porpoise survey above, and (b) scenarios with lag l of 10, 20, 50 and 80 seconds, and γ equal to 10%, 20%, 50% and 80%. We do this for σ equal to 1.5 m/s (the speed estimated by Hiby and Lovell, 1998), 0.95 m/s (the speed estimated by Westgate et al., 1995), and 0.5 m/s (a speed lower than that estimated above or by Stevenson et al., 2018). In all cases, we use the same τ in estimation as was used in simulation. For the short-lag scenarios in (b), we perform simulations with true density $D = 1.24$, as estimated in the previous section, and with an observer speed of 100 knots, which is around the typical speed of marine aerial surveys. We performed 1,000 simulations for each scenario.

7.1 *Simulation based on harbor porpoise data: lag of 248 seconds*

For this scenario we use the estimates of Stevenson et al. (2018) as the generating values, corresponding to $D = 1.05$, $\gamma = 0.86$ and $\sigma_{palm} = 0.15$. We investigate by simulation the bias and precision of the LCE estimator. In the light of our results in Section 6, where we obtained an LCE estimate that was 18% greater than the CCR estimate of Stevenson et al.

(2018), we also investigate whether this discrepancy is within the bounds expected due to estimator variability.

The empirical bias of the LCE and CCR density estimators from the simulations are 9.9% (CV=29.8%) and 12.7% (CV=37.9%), respectively. The biases reduce to 4.3% (CV=18.0%) and 5.1% (CV=21.4%), respectively when sample size is doubled while holding density constant, and to 2.9% (CV=14.6%) and 3.3% (CV=16.7%) when sample size is trebled.

The correlation between LCE and CCR density estimates from the simulations is 0.75, while the probability of getting a relative difference as large as, or larger than, that observed is approximately 20%, from which we conclude that the observed difference is not large enough to raise concerns about the validity of either estimator with the porpoise data.

The LCE estimator formulates the delay in encounter times between the two observers as a random variable, due to animal movement towards or away from the second observer, while the CCR method does not, and instead assumes these times to be equal to the lag time between the observers. We anticipate that this will cause the expected values of the two estimators to diverge for long lags, or for the case where animal speeds are non-negligible relative to observer speeds, and this may cause the CCR estimator of density to become biased. Here, however, with the observers moving some 50 times faster than the animals, and the standard deviation of the difference of encounter times from lag time between observers being only 2.4% of the lag time, the effect on the CCR estimator is very small.

7.2 *Short lag scenarios*

Here we investigate the bias and confidence interval coverage of the LCE density estimator under short lag scenarios, and compare these with those obtained from the CCR estimator of Stevenson et al. (2018). There are 36 simulation scenarios corresponding to all combinations of lag $l \in \{10, 20, 50, 80\}$ seconds, $\gamma \in \{0.1, 0.2, 0.5, 0.8\}$, and $\sigma \in \{1.5, 0.95, 0.5\}$ m/s.

Simulation results are summarized in Table 1. Boxplots of the density estimates for each of the 36 scenarios are shown in Figure 2.

[Figure 2 about here.]

[Table 1 about here.]

The LCE density estimator is unbiased or very nearly unbiased in all 36 scenarios. Figure 3 shows the empirical bias as a function of the mean number of detections by each observer, together with the empirical bias of the CCR estimator fitted to the same simulated data. The bias of the two estimators is very similar. The correlation between the two estimators varies from 0.888 to 0.995 across the 36 scenarios, and the mean difference of the estimator means from the true density, as a percentage of the true density, is 1.08% in the case of the LCE estimator, and -0.1% in the case of the CCR estimator.

[Figure 3 about here.]

The coefficients of variation of the LCE and CCR density estimators for all 36 scenarios are shown in Figure 4. The CVs decrease with sample size, as expected. The CV of the LCE estimator is greater than that of the CCR estimator for the smallest sample sizes but smaller for larger sample sizes, the more so the larger the sample size. We interpret this to be a consequnce of the fact that the CCR estimator is not a maximum likelihood estimator, being based instead on an approximation to the Palm likelihood of pairwise comparisons between detections, so it does not have the asymptotic efficiency of a maximum likelihood estimator. Nevertheless, the difference in precision of the two estimators is very small.

[Figure 4 about here.]

In all cases coverage probability is close to 95% (Table 1). We conclude that for these shorter-lag scenarios, the LCE estimator is approximately unbiased, with close to nominal confidence interval coverage. The performance of the LCE and CCR estimators is very similar, with the LCE estimator making a slight gain in precision as sample size increases.

8. Discussion

Burt et al. (2014) note that most MRDS models do not allow for animals to be at different distances from different observers, and say "A more satisfactory approach would be to develop models that incorporate movement, but this is not straightforward and remains to be done.". We have done that here, although our detection function is customized for digital aerial surveys, with certain detection within distance w of the transect. If capture histories *are* known, as is assumed with MRDS models, the LCE model provides a framework for extending MRDS aerial survey methods to incorporate animal movement and allow for the fact that the observers commonly observe the same animals at different perpendicular distances.

Looking at Figure 1, one might be tempted to think of the LCE model as a kind of M_b area search spatial capture-recapture model, in which capture probability is elevated by first capture and then decays slowly over time. The LCE model is not quite that though, because capture probability for the second observer changes with time since an animal was available for detection by the first observer, whether or not the first observer detected it.

The LCE method has some advantages over the CCR method, but it does not scale well as density increases. While we were able to deal with moderately large sample sizes above, this is because density is low enough that the transect line can be divided into many segments with relatively few possible combinations of capture histories within each segment. The number of possible capture histories increases very rapidly when each observer detects more than a few animals within a segment, and the LCE method computation will be infeasible in this case. The number of possible capture histories is

$$N_{CH} = \sum_{m=0}^{n_2^*} \binom{n_2^*}{m} {}^{n_1^*}P_m, \qquad (16)$$

where n_1^* is the larger of n_1 and n_2, and n_2^* is the smaller of them. For $n_1 = n_2 = 2, \ldots 10$, N_{CH} is 7; 34; 209; 1,546; 13,327; 130,922; 1,441,729; 17,572,114; 234,662,231. As n_1^* increases, the LCE estimation method will become too slow to be practically useful on typical desktop computers. The CCR method, by contrast, scales well and is able to deal with much larger numbers of detections within segments.

Being a maximum likelihood method, the LCE method has the advantage of being able to use the extensive inference results and machinery associated with maximum likelihood estimators, including likelihood-based model selection criteria such as AIC, asymptotic efficiency, and associated interval estimation methods. The CCR estimator requires interval estimation by bootstrap, is slightly less efficient than the LCE estimator for larger sample sizes and cannot take advantage of likelihood-based model selection methods. It is also not able to accommodate varying times between encounters of animals due to animal movement, although in the scenarios we considered this has negligible effect. Finally, the LCE method provides an inference framework that allows inclusion of covariates in all parameters mentioned in Section 4.2. (While covariates were not available for our application, we anticipate that they can be collected on future surveys.) It is not clear how easy it will be to include covariates that change continuously along the transect line in the CCR estimation framework, although it should easily accommodate covariates that are constant within, but different along, sections of transect.

As was shown by Stevenson et al. (2018), it is not possible to estimate all parameters of interest from two-observer data with unobserved capure histories. However, varying the lag, or having more than two cameras operating may make one more parameter identifiable. It is possible that incorporating covariates into some of the parameters will also make more parameters identifiable. This is an area worth exploring in future.

We anticipate that the framework provided by the LCE and CCR methods may facilitate substantial reduction in the cost of processing double-observer data by allowing the estimation process to be automated. To do this, we need only an adequate automatic identifier of the species in question in the video stream from each "observer" separately – we do not need recaptures to be identified. If false negatives affect only the availability process (e.g. remove animals underwater and partially visible), the only cost of using a strict identification criterion in order to avoid false positives, is reduced sample size. If, however, false negatives affect the detection process (e.g. remove some individuals because although they were as available as possible, a wave broke over them as the observer passed) then the assumption of $p = 1$ may be violated and bias may ensue if lag between observers is short. Surveying at different lags may allow estimation of p (in additon to D, σ and κ) so that it will likely be possible to automate inference from digital surveys by using automated object identification criteria that are sufficiently strict so as to reduce the probability of false positives to virtually zero, providing that the survey involves effort and detections at more than one lag.

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Appendix A. Derivation of $f_T(t)$

Define the time and forward coordinate at which observer 1 passes over an animal to be 0. The animal's forward coordinate at time t is σW_t, where W_t is a one-dimensional Brownian motion. The forward coordinate of observer 2 at time t is $-vl + vt$. The time at which observer 2 passes over the animal is therefore the minimum t such that

$$\begin{aligned} -vl + vt &= \sigma W_t \\ \Rightarrow \quad \frac{vt}{\sigma} - W_t &= \frac{vl}{\sigma}. \end{aligned} \tag{A.1}$$

The passage time for observer 2 is therefore $T = \inf\{t : vt/\sigma + B_t = vl/\sigma\}$, where $B_t = -W_t$ is also a Brownian motion. We use the following standard result for the first passage time of a Brownian motion with drift. Suppose a particle follows Brownian motion with drift parameter

c, such that its location at time t is $X_t = ct + B_t$. The random variable $T = \inf\{t : X_t = a\}$ is the first passage time to location a, which has probability density function

$$f_T(t) = \frac{a \exp\left\{\frac{-(a-ct)^2}{2t}\right\}}{\sqrt{2\pi t^3}}. \tag{A.2}$$

Substituting $c = v/\sigma$ and $a = vl/\sigma$, we obtain the probability density of the time T at which observer 2 passes over the animal:

$$f_T(t) = \frac{vl \exp\left\{\frac{-v^2(l-t)^2}{2\sigma^2 t}\right\}}{\sqrt{2\pi\sigma^2 t^3}}. \tag{A.3}$$

Appendix B. The relationship between σ_{palm}, σ and mean animal speed

The σ of Stevenson et al. (2018), which we call σ_{palm} here, is based on the displacement of animals from the midpoint of their two locations after time l has elapsed, being normally distributed with mean zero variance equal to σ^2_{palm}. If we let the signed distance between the first and second location be Y, then $Y/2 \sim N(0, \sigma^2_{palm})$ and hence $\sqrt{\{Y/(2\sigma_{palm})\}^2} = |Y|/(2\sigma_{palm}) \sim \chi(1)$. Using the fact that the expected value of a $\chi(1)$ random variable is $\sqrt{2}/\Gamma(0.5)$, we have that $E\{|Y|/(2\sigma_{palm})\} = \sqrt{2}/\Gamma(0.5)$, and hence $2\sigma_{palm} = E(|Y|)\Gamma(0.5)/\sqrt{2}$.

The distance Y between the initial location and the location after l seconds, of an animal following Brownian motion with rate parameter σ, has distribution $Y \sim N(0, \sigma^2 l)$, so that $E\left\{|Y|/(\sigma\sqrt{l})\right\} = \sqrt{2}/\Gamma(0.5)$ and $\sigma\sqrt{l} = E(|Y|)\Gamma(0.5)/\sqrt{2}$, and hence $\sigma = 2\sigma_{palm}/\sqrt{l}$.

As the average speed of an animal over a period of l seconds is $E(|Y|)/l$, the average speed over l seconds of an animal following Brownian motion with rate parameter σ can be written as $\sigma\sqrt{2}/\{\Gamma(0.5)\sqrt{l}\}$.

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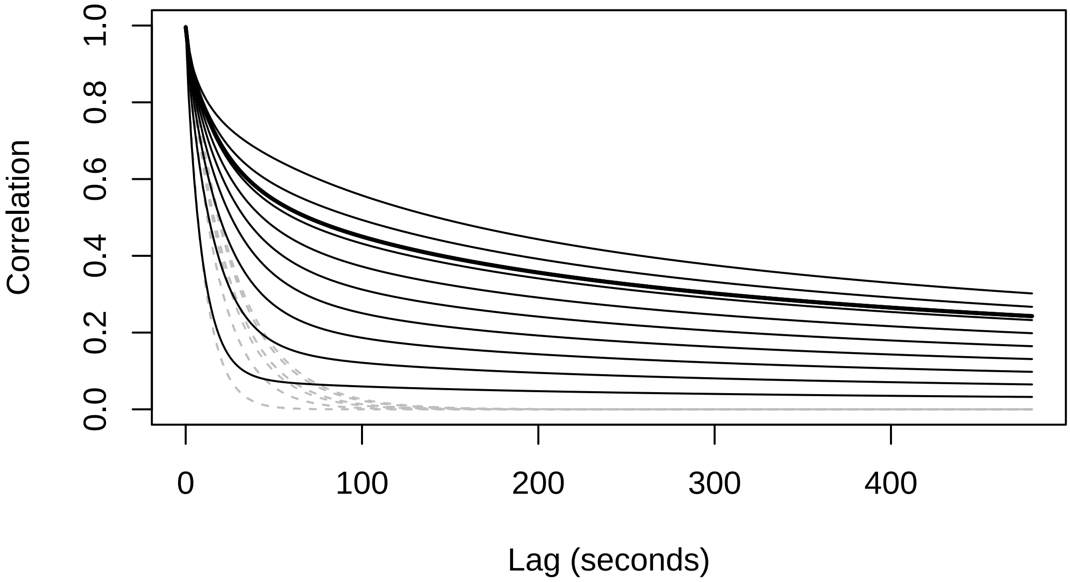

Figure 1. Correlation between detections by the two observers as a function of lag for mean proportions of time available $\gamma = 10\%$ (bottom black line), $20\%, 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, 90\%$ (top black line). The thick black line is for $\gamma = 73\%$. The grey dashed lines show the correlation under the assumption of no animal movement.

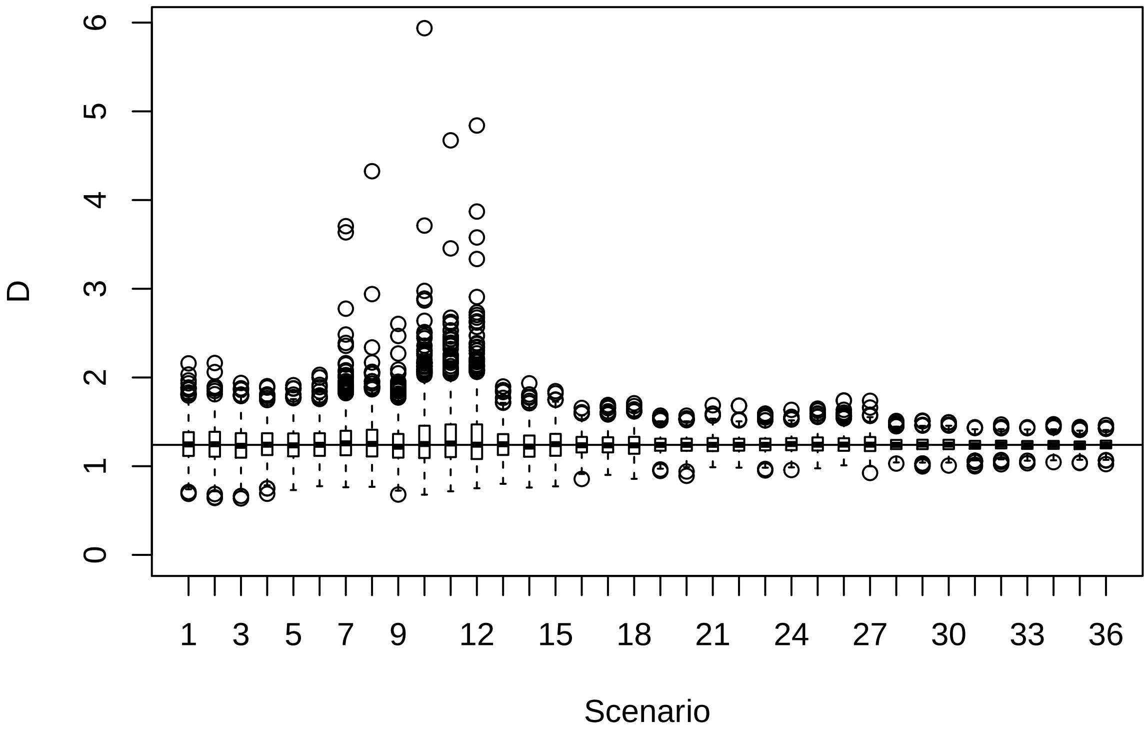

Figure 2. Box plots of estimated density for each of the 36 scenarios. The horizontal line is at true density $D = 1.24$.

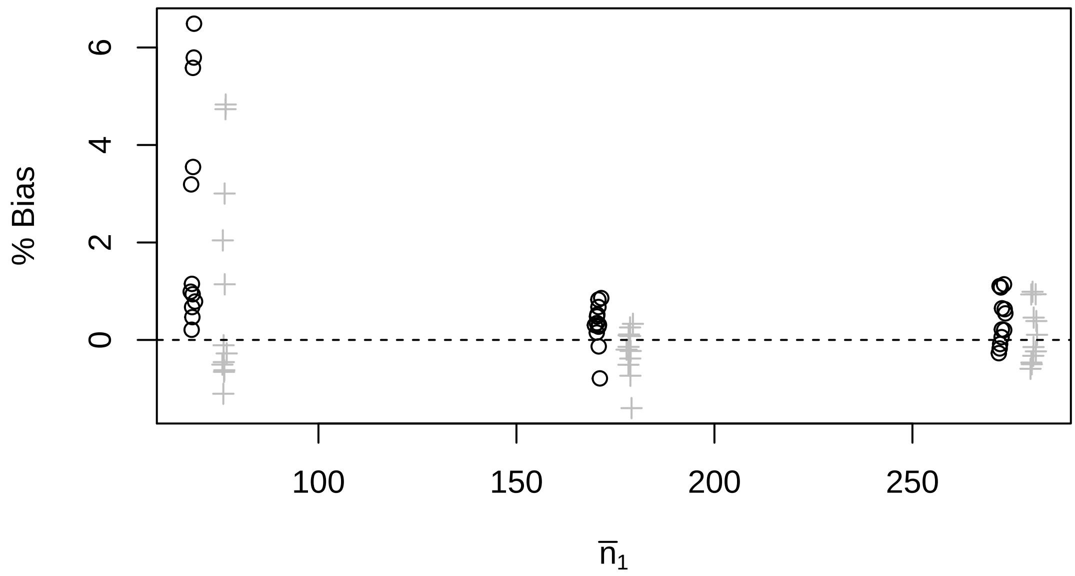

Figure 3. Percentage difference of estimated density from true density, as a function of mean number of detections by a single observer.The LCE estimator is represented by circles, the CCR estimator by crosses. Crosses are offset 8 points to the right, to avoid overlap with circles.

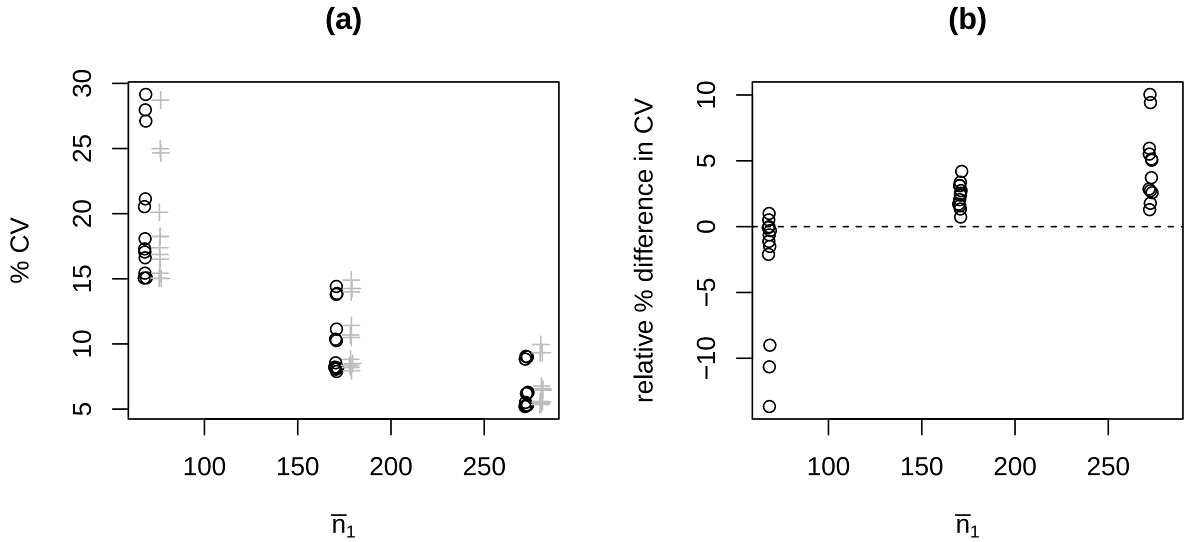

Figure 4. Percentage coefficient of variation (%CV), as a function of mean number of detections by a single observer. The LCE estimator is represented by circles, the CCR estimator by crosses. Crosses are offset 8 units to the right, to avoid overlap with circles. Panel (a) shows the %CV. Panel (b) shows the amount by which the CV from the CCR method exceeds that from the LCE method, expressed as a percentage of the LCE CV.

Table 1

Simulation results for LCE and Palm estimators from 1,000 simulations. Here gamma is the proportion of time animals are availabld, lag is the time between obsrvers, sigma is the animal diffusion rate parameter, mean(n) and mean(m) are the mean numbers of detections by one observer and the mean number of recaptures, across simulations.

	gamma	lag	sigma	%BiasLCE	%cvLCE	%CoverLCE	%BiasPalm	%cvPalm	mean(n)	mean(m)
1	0.20	10.00	0.65	0.94	16.61	0.95	-0.62	16.51	68.23	44.39
2	0.20	10.00	0.95	1.15	17.06	0.96	-0.11	16.87	68.05	43.88
3	0.20	10.00	1.50	0.21	17.30	0.95	-1.10	17.39	67.98	43.85
4	0.20	20.00	0.65	0.99	15.06	0.96	-0.51	15.05	67.74	30.67
5	0.20	20.00	0.95	0.79	15.07	0.96	-0.28	15.02	68.84	31.00
6	0.20	20.00	1.50	0.67	15.43	0.96	-0.45	15.43	68.10	30.56
7	0.20	50.00	0.65	3.55	21.14	0.96	1.14	18.25	68.33	16.68
8	0.20	50.00	0.95	3.19	20.54	0.96	2.04	20.11	67.85	16.38
9	0.20	50.00	1.50	0.47	18.07	0.95	-0.65	18.25	68.17	16.66
10	0.20	80.00	0.65	5.58	27.97	0.96	3.00	24.99	68.31	14.14
11	0.20	80.00	0.95	6.49	27.11	0.96	4.83	24.67	68.58	13.76
12	0.20	80.00	1.50	5.79	29.15	0.95	4.73	28.71	68.53	13.70
13	0.50	10.00	0.65	0.83	13.81	0.95	0.26	13.99	170.70	143.78
14	0.50	10.00	0.95	-0.79	13.88	0.95	-1.40	14.26	171.07	143.97
15	0.50	10.00	1.50	0.68	14.41	0.94	0.08	14.90	170.71	142.57
16	0.50	20.00	0.65	0.27	10.25	0.96	-0.38	10.50	170.74	125.56
17	0.50	20.00	0.95	0.15	10.36	0.96	-0.51	10.69	170.29	124.59
18	0.50	20.00	1.50	-0.13	11.14	0.94	-0.73	11.42	170.78	124.12
19	0.50	50.00	0.65	0.35	8.17	0.96	-0.14	8.34	170.33	97.85
20	0.50	50.00	0.95	0.30	8.22	0.96	-0.20	8.36	169.81	96.64
21	0.50	50.00	1.50	0.48	8.55	0.96	0.09	8.82	170.34	95.50
22	0.50	80.00	0.65	0.31	7.89	0.96	-0.23	7.94	170.84	88.42
23	0.50	80.00	0.95	0.52	8.06	0.95	0.11	8.18	170.43	87.16
24	0.50	80.00	1.50	0.86	8.15	0.95	0.33	8.49	171.42	86.45
25	0.80	10.00	0.65	1.14	9.00	0.95	0.94	9.33	273.13	247.68
26	0.80	10.00	0.95	1.10	8.83	0.96	0.93	9.32	272.00	246.19
27	0.80	10.00	1.50	1.08	9.05	0.96	0.99	9.96	272.34	244.71
28	0.80	20.00	0.65	0.55	6.29	0.96	0.11	6.45	273.47	234.23
29	0.80	20.00	0.95	0.63	6.26	0.97	0.39	6.57	273.30	232.95
30	0.80	20.00	1.50	0.65	6.19	0.98	0.46	6.77	272.62	230.38
31	0.80	50.00	0.65	-0.08	5.51	0.96	-0.50	5.59	272.14	217.32
32	0.80	50.00	0.95	0.21	5.30	0.96	-0.15	5.45	272.58	216.46
33	0.80	50.00	1.50	-0.17	5.21	0.97	-0.46	5.51	272.04	213.68
34	0.80	80.00	0.65	0.06	5.28	0.97	-0.33	5.37	272.52	214.10
35	0.80	80.00	0.95	-0.27	5.20	0.97	-0.59	5.35	271.82	211.78
36	0.80	80.00	1.50	0.20	5.29	0.96	-0.23	5.56	273.18	209.54