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CS 324
Homework 4
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Problem 1

Prove that the subset relation “⊆” on all subsets of Z is a partial order but not a total order.

Proof: The subset relation is reflexive

Let A be a subset of Z. Every element in A also exists in A because they are the same. Therefore, A must be a subset of itself. There will be no value in A that is not also in A.

Proof: The subset relation is antisymmetric

Let A and B be subsets of Z such that A a subset of B and B is a subset of A.
So, A ⊆ B and B ⊆ A.
Let a be some element in A.
Because A ⊆ B, a exists in A and B for all values in A.
Because B ⊆ A, b exists in B and A for all values in B.
Every value in A is in B, and every value in B is in A.
Therefore, for every value in a, there is a value b such that a = b and vice-versa.
Therefore A = B.

Proof: The subset relation is transitive

Let A, B, and C be subsets of Z.
Let A ⊆ B.
Let B ⊆ C.
Let a be some value that exists in A.
Because A ⊆ B, a exists in B.
Because B ⊆ C, a exists in C.
Therefore, because every value of A is in C, A ⊆ C.

Proof: The subset relation is not a total relation

Let A = {0, 1, 2} and B = {3, 4, 5}.
Both A and B are subsets of Z, but no values of A are in B, so A is not a subset of B.
And no values of B are in A, so B is not a subset of A.
Therefore for any two subsets of Z, it is not the case that either A ⊆ B or B ⊆ A.

<u>Problem 2</u>
Give examples of relations that are:

a. reflexive and symmetric, but not transitive

R = {(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)} on the set {1,2,3}

b. reflexive and transitive, but not symmetric

The ≤ relation is reflexive and transitive, but not symmetric.

c. symmetric and transitive but not reflexive

R = {(1,2),(2,1),(1,1),(2,3),(3,2),(2,2),(3,3),(1,3),(3,1)} on the set {1,2,3,4}. Note that the pair {4,4} never appears, and thus, R is not reflexive.

<u>Problem 3</u>
In how many ways can we choose three distinct numbers from the set {1,2,…,99} so that their sum is even?

Two possibilities:
- two odds and an even
- three evens

Number of ways to get three evens = 50C3
>> 19600
Number of ways to get two odds and an even = 50C2 * 50C1
>> 1225 * 50 = 61250

19600 + 61250 = 80850

Problem 4

Suppose we shuffle a deck of 10 cards, each bearing a distinct number from 1 to 10, to mix the cards thoroughly. We then remove three cards, one at a time, from the deck. What is the probability that we select the three cards in sorted (increasing) order?

Number of permutations of 3 cards = 3! = 6
Only one permutation of drawing 3 cards will be in increasing order, therefore, probability = 1/6

Problem 5

Suppose we roll two ordinary, 6-sided dice. What is the expectation of the sum of the two values showing?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2*(1/36) + 3*(2/36) + 4*(3/36) + 5*(4/36) + 6*(5/36) + 7*(6/36)
+ 8*(5/36) + 9*(4/36) + 10*(3/36) + 11*(2/36) + 12*(1/36)
= (2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12)/36
= 252/36
= 7

Whats is the expectation of the maximum of the two values showing?

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

(1*1 + 2*3 + 3*5 + 4*7 + 5*9 + 6*11)/36
= (1 + 6 + 15 + 28 + 45 + 66)/36
= 161/36 ≈ 4.472

Problem 6

Verify axiom 2 of the probability axioms for the geometric distribution.

Need to prove that the probability distribution of the sample space is 1.

Proof:
sum of the probabilities from 1 to infinity is equal to:

$$q^{(0)}*p + q^{(1)}*p + q^{(2)}*p + \dots$$

$$= p * [q^{(0)} + q^{(1)} + \dots]$$

$$[q^{(0)} + q^{(1)} + \dots] = 1/(1-q)$$

$$1-q = p$$

$$[q^{(0)} + q^{(1)} + \dots] = 1/p$$

$$p * [q^{(0)} + q^{(1)} + \dots] = p * (1/p) = 1$$

Problem 7
How many times on average must we flip 6 fair coins before we obtain 3 heads and 3 tails?

Probability of 3 heads on 6 flips
= $(6C3)*(0.5)^3(0.5)^3$
= 20 * (0.125) * (0.125)
= 0.3125

E(x) = 1/p
p = 0.3125
E(x) = 3.2 times

Problem 8

Using Figure 6.4 as a model, illustrate the operation of Heapsort on the array A=[5,13,2,25,7,17,20,8,4]. Sort from lowest to highest—so use a max-heap. Also, show the heap being built (as in Figure 6.3), as well as being taken apart.

A=[5,13,2,25,7,17,20,8,4]

```
step 1:
         5
      /     \
     i13      2
    /  \    /  \
  25    7  17  20
 /  \
8    4

step 2:
         5
      /     \
     25      i2
    /  \    /  \
  13    7  17  20
 /  \
8    4

step 3:
         5
      /     \
     25      20
    /  \    /  \
  13    7  17   2
 /  \
8    4

step 4:
         i5
      /     \
     25      20
    /  \    /  \
  13    7  17   2
 /  \
8    4
```

```
step 5:
         25
      /     \
     i5      20
    /  \    /  \
  13    7  17   2
 /  \
8    4

step 6:
         25
      /     \
     13      20
    /  \    /  \
  i5    7  17   2
 /  \
8    4

step 7:
         25
      /     \
     13      20
    /  \    /  \
   8    7  17   2
 /  \
5    4

step 8:
         20
      /     \
     13      17
    /  \    /  \
   8    7  4    2
 /
5   i25

step 9:
         17
      /     \
     13      5
    /  \    /  \
   8    7  4    2

i20   25
```

```
step 10:
         13
      /     \
     8       5
    /  \    /
   2    7  4    i17

20   25

step 11:
         8
      /     \
     7       5
    /  \
   2    4  i13   17

20   25

step 12:
         7
      /     \
     4       5
    /
   2   i8  13   17

20   25

step 13:
         5
      /     \
     4       2

   i7    8  13   17

20   25

step 14:
         4
      /
     2       i5

   7    8  13   17

20   25
```

```
step 15:
         2

     i4       5

   7    8  13   17

20   25
```

step 16:
A = [2,4,5,7,8,13,17,20,25]

<u>Problem 9</u>
Illustrate the operation of Heap-Extract-Max on the heap A = [15,13,9,5,12,8,7,4,0,6,2,1]. Only illustrate a single call (i.e. removal of one element), but for that call, show each step (i.e. at each level), not just the beginning and ending configurations of the heap.

```
step 1:
            15
       /          \
     13            9
   /    \         /  \
  5     12       8    7
 / \    / \     /
4   0  6   2   1

step 2:
            i1
       /          \
     13            9
   /    \         /  \
  5     12       8    7
 / \    / \
4   0  6   2

step 3:
            13
       /          \
      i1           9
   /    \         /  \
  5     12       8    7
 / \    / \
4   0  6   2

step 4:
            13
       /          \
     12            9
   /    \         /  \
  5     i1       8    7
 / \    / \
4   0  6   2
```

```
step 5:
            13
       /          \
     12            9
   /    \        /   \
  5      6      8     7
 / \    / \
4   0  1   2
```