# Detection of Intima-Media Layer of Common Carotid Artery with Dynamic Programming Based Active Contour Model

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**Abstract:** Ultrasound measurements of the carotid artery wall in image are usually obtained by manually tracing. In this paper, we present an automatic segmentation method to detect the intima-media layer in far wall of the common carotid artery. The energy definition of active contour model is used. Different from the traditional approach applied in snake techniques, we treat the optimization problem as finding the shortest cost path in a directed graph. Dynamic programming is selected to search the shortest path. The external force and internal force in snake model are modified to be suitable for our approach. To reduce the effect of speckle noise, a new method in speckle reduction by anisotropic diffusion is adopted. At last, we compare the result of our method with other two methods. Results show that our method can detect the intimal and adventitia layers as well as other methods, the two layers will not cross with each other as traditional dynamic programming does. Moreover, our method needs less manual input than others.

**Key Words:** Intima-media layer; IMT; artery wall; anisotropic diffusion; active contour model; dynamic programming.

## 1 INTRODUCTION

The intima-media thickness (IMT) of the common carotid artery (CCA) can usually be an indicator of cardiovascular disease [1][2]. Therefore the measurement of the IMT is important to diagnosis. As the quality of ultrasound imaging is improved, B-mode images provide a non-invasive approach to obtaining the IMT, see figure 1. In the past, physicians usually trace the IMT manually, but it is time consuming and unreliable. A tracing result may be subjective for the operator.

To overcome these problems, many automatic methods have been developed for measuring IMT. Several methods are based on active contour model. In [3], a modified Cohen's snake algorithm [4] is used. They describe the intimal and adventitial wall by energy functions, and solve the function using the procedure proposed by Kass [5]. In Ceccarelli's paper [6], another snake approach is proposed; the minimizing procedure is a greedy minimization algorithm [7]. In these methods, an initial layer must be given and if it is not good enough, it will take more time to converge to the final contour. To set a high quality initial contour, however, is quite time consuming, e.g., Loizou [8] introduced a complex IMT initialization approach.

In our paper, according to the features of IMT, we proposed a new detection approach to get the optimized contour. Active contour model is used to define the cost function. Dynamic programming is adopted to search the final contour in a selected region. We improved the minimizing procedure appeared in Luo's paper [9], and make it suitable to detect intimal-media layers.

Because speckle noises always affect the boundary detection on ultrasound image, we reduce noise by using anisotropic diffusion. Based on a new approach introduced by [10], the noise can be reduced and, at the same time, preserve the boundary information. Compared with other anisotropic diffusion methods, this approach is faster.

This paper is organized as follows: Section 2 introduces our noise reduction method as image pre-processing; in Section 3 we present our approach to detect IMT, experiment and results will be shown in Section 4, followed by conclusions in Section 5.

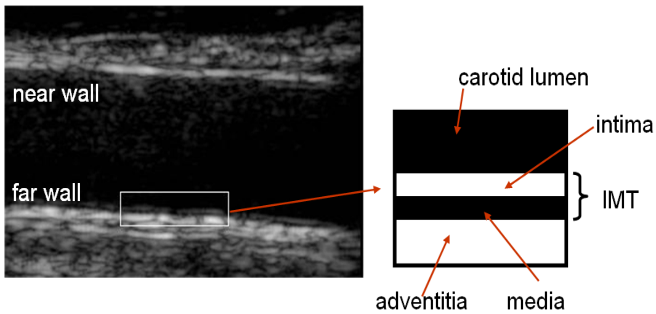



Figure 1，B-mode common carotid artery ultrasound image from a 10MHz linear probe for IMT measurement, acquired by Saset iMago color ultrasound system which is designed and made in our lab.

## 2 PRE-PROCESSING

The presence of speckle introduces noise to edge detection, and traditional methods such as Gaussian may be used to reduce noise. In recent years, anisotropic diffusion methods were developed which could reduce speckle and preserve edges at the same time. Since 2000, some researchers have introduced anisotropic diffusion to speckle reduction in ultrasound images. We adopt Local Coherence based Fast Speckle Reducing Anisotropic Diffusion (LCFSRAD) [10] which can improve the previous real-time speckle reduction anisotropic diffusion method (RTAD) [11].

In the paper of RTAD, the authors adopt tensor based anisotropic diffusion to do speckle reduction, and they use both explicit and implicit schemes to discretize the diffusion equation. RTAD [11] is then

$$\frac{\partial I}{\partial t} = div[D\nabla I] = div[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} I_x \\ I_y \end{pmatrix}] \tag{7}$$

where $D$ is the diffusion tenor based on the structure tensor.

In [10], Wang adopts a scalar diffusivity in its anisotropic diffusion. Wang uses semi-implicit AOS scheme as Weickert did in [12] to discretize the diffusion equation. Because it employs local coherence to detect speckle noise which is the same way as RTAD and reduces the computation amount, this method is not only as good as RTAD in speckle reduction, but also faster than RTAD in speed. The diffusion equation of LCFSRAD [10] is

$$\frac{\partial I}{\partial t} = div[l \cdot \nabla I] \tag{8}$$

$$l(|\mu_1 - \mu_2|) = 1/1 + (|\mu_1 - \mu_2|/K)^2 \tag{9}$$

where $\mu_1$ and $\mu_2$ are the eigenvalues of structure matrix at each point, structure matrix is as $J(\nabla I_\sigma) = (\nabla I_\sigma \cdot \nabla I_\sigma^T)$. Here, $\nabla I_\sigma$ is the gradient of a smoothed version of $I$ which is obtained by convolving I with a Gaussian of standard deviation $\sigma$. Converting (8) to a matrix-vector notation and adopting AOS scheme, Eqn (8) comes down to the following iteration scheme

$$I^{t+1} = \frac{1}{2}\sum_{l=1}^{2}[U + 2\tau T_l(I^t)]^{-1} I^t \tag{10}$$

where $U$ is a $KL$ by $KL$ identity matrix, $\tau$ is the time step size (TSS), $T_l(I^t)$ is a tridiagonal matrix, with $T_l(I^t) = [t_{ij}(I^t)]$ and

$$t_{ij}(I^t) = \begin{cases} l_j^t / 2h^2 & [j \in \mathrm{N}(i)], \\ -\sum_{n \subset \mathrm{N}(i)} l_n^t / 2h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{11}$$

where $l$ is the diffusion coefficient. $\mathrm{N}(i)$ is the set of the two neighbors of pixel $i$. In [10], Wang demonstrates that LCFSRAD satisfies the criteria for discrete nonlinear diffusion scale-spaces [12]. So we could use larger TSS to do anisotropic diffusion. In contrast with explicit scheme based anisotropic diffusion method, in order to get the same smoothing result, we can to do less iteration.

There are three parameters in LCFSRAD. The parameter $K$ in (9) affects the monotonous decreasing bell-shape function, $\tau$ in (10) is the TSS, and user controllable iteration number of (10). In general, we could designate $K$ as 110, this is appropriate for our application. Using larger TSS, we could use less iteration number to get the desirable smoothing result. However, too large TSS will bring some artifacts because we use AOS scheme to do discretization. From our experiments, in order to get good anisotropic smoothing results for later segmentation processing, users could designate TSS about 1, and do 3 to 7 iterations.

## 3 PROPOSED SNAKE MODEL

### 3.1 Snake model

Active contour model, also named snake technique, is proposed by Kass et al. at [5], and they define an energy function for the contour as:

$$\begin{aligned} E_{snake} &= \int_0^1 E_{snake}(v(s))ds \\ &= \int_0^1 E_{\text{int}}(v(s)) + E_{image}(v(s)) + E_{con}(v(s))ds \end{aligned} \tag{12}$$

where $v(s)$ represents the contour, $s \in (0,1)$.

$$E_{\text{int}} = (\alpha(s)|v_s(s)|^2 + \beta(s)|v_{ss}(s)|^2)/2 \tag{13}$$

is the internal energy of the spline due to bending, $E_{con}$ is the external constraint and $\alpha(s), \beta(s)$ are constants to present the weight of $v_s(s)$ and $v_{ss}(s)$. $E_{image}$ gives rise to image forces which can be defined by different ways with the edge having a lower energy, such as $-(G_\sigma * \nabla^2 I)^2$ where $G_\sigma$ is the Gaussian filter.

To solve the energy function and get the minimum energy contour, Kass represented the techniques by the method of variational calculus. But some problems in this method are pointed out by Amini[13], such as problems of the stability and a tendency for points to bunch up on a strong portion of an edge. Amini proposed an algorithm for the active contour model using dynamic programming. By the traditional method of variational calculus or dynamic programming, the processing can be summarized as follow:

Step 1 Set an initial contour.

Step 2 Move the initial contour towards the low energy contour step by step.

Step 3 When match the condition of convergence, stop, else return to step 2

Here, an initial contour is a must, and a good initial edge is very important. Good quality means initial contour needs to be as close to the true one as possible. If the initial contour is far from the true one, it may not reach convergence. In this paper, we present a new algorithm to find the minimum energy of active contour models directly. The initial edge is only used to locate a searching area. A new energy definition is also given to be suitable for the intima-media detection.

### 3.2 Proposed snake model for IMT detection

For detection of IMT, as (12), the $E_{\text{snake}}$ of the contour can be divided into two parts $E_{\text{int}}$ and $E_{image}$, $E_{con}$ in (12) is ignored. $E_{\text{int}}$ is decided by the contour. $E_{image}$ is decided by the tendency of the gradient.

In a $M \times N$ region, a polyline contains $N$ nodes on the contour, Figure 2. In each column, one point will be selected on the contour. The points on the contour are considered as 8-adjacencies.

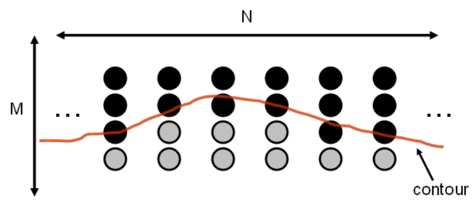



Figure 2, for every column only one point is needed on the contour

The discretization of the energy function for contour is:

$$E_{total} = \sum_{j=1}^{N} E_{image}(I_j) + \sum_{j=1}^{N} E_{\text{int}}(I_j) \tag{14}$$

Since the points on the contour are 8-adjacent, Figure 3 shows all the candidate contours in the region. Each candidate contour is associated with one $E_{total}$, and the minimizing procedure can be modeled as to find the shortest path in a directed graph.

Every pixel in the region is a node of the graph, start nodes are on the left side of the region, end nodes are on the right side of the region. The path will pass through $N$ nodes and ($N$-$1$) edges among the nodes. Weighting on every node is $E_{image}$. Weighting for every edge between two nodes is $E_{int}$.

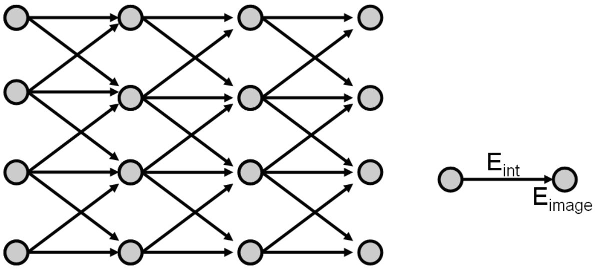



Figure 3, the possible paths in the selected area.

### 3.3 Procedures to get the searching area

For our proposed procedures of searching a path to get the minimum energy contour, the searching area is very important. Basic area is the ROI (region of interest) given by the user. But in this area, there are two edges, intimal edge and adventitial edge. Sometimes, intimal edge is stronger than adventitial edge, but sometimes it is not. Then we need to divide the area into two parts: one contains intimal edge, and the other contains adventitial edge only.

If we know the position of an initial edge, we can make the searching area smaller. To get an initial intimal edge, there are many methods proposed by researchers, such as methods by Cheng [3], Loizou [8], and Ceccarelli [6]. Here we propose a new method to get a rough initial intimal edge which is simple with less computation.

Divide the points in the ROI into two clusters according to the gray level of every point by c-means method. $C_{dark}$ is the center of the points with dark gray and $C_{light}$ is the center of the points with light gray.

Make $T_g=(C_{dark}+C_{light})/2$ as a threshold. Let P(i,j) be selected point on the edge.

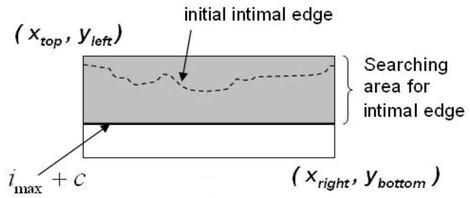
Step 1 Let j = 0, increase i from 0 to M until P(i,0) > $T_g$, P(i,0) is the starting point on the edge;

Step 2 If the last selected point to the edge is P($i_1$,$j_1$), let $j_2 = j_1+1$, increase $i_2$ from 1 to M-1, find the first point with P($i_2$,$j_2$)> $T_g$, and then record the value $i_2$;

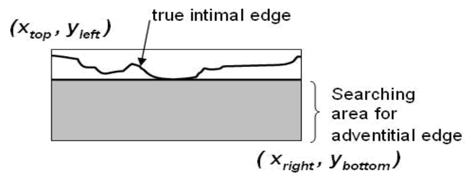
Step 3 If $i_2>i_1$ select the point P($i_1$+1, $j_2$) as new point on the edge. If $i_2<i_1$, select the point P($i_1$-1,$j_2$), if $i_2= i_1$ select P($i_1$, $j_2$) as the new point;

Step 4 If $j_2+1 = N$ output the initial intimal edge; else go to Step 2.

After getting the initial contour, we can find the maximum $i_{max}$ value from all the points of P(i, j) on the initial contour. Set ($x_{top}$, $y_{left}$) as the coordinates of the top-left point in the ROI, ($x_{bottom}$, $y_{right}$) as the coordinates of the bottom-right point in the ROI. The searching area, see Figure 4.a, for intimal edge will be from ($x_{top}$, $y_{left}$) to ($I_{max}$+$c$, $y_{right}$) where $c$ is a constant. Let $s$ be the true size of every pix in the image, the constant $c$ needs to meet the condition 0.05cm > c*s > (0.05/2) cm, based on the knowledge that the intima-media thickness is usually thicker than 0.05cm. After the area is determined, we can get the final intimal edge by procedures in Section 3.2. The searching area for adventitial edge is below the final intimal edge and above the bottom line of ROI as shown in Figure 4.b. By looking at a series of images frame by frame, the initial intimal edge will be defined by the last neighboring frame. The searching region can be more accurate which is helpful for detection.




(a)




(b)

Figure 4, illustrate the two areas for intimal and adventitia edges

### 3.4 The definition of $E_{int}$

We modify the definition of $E_{int}$ and $E_{image}$ for better representing the energy in intimal-media detection. To reduce the computational cost, we can do approximation of the derivative in (13) where $E_{\text{int}}$ can be defined as:

$$E_{\text{int}} = (\alpha(s)\,|\,v_s(s)\,| \;+\beta(s)\,|\,v_{ss}(s)\,|\,)/2 \qquad (15)$$

The first term is associated with the elasticity of contour and defined as:

$$\begin{aligned} &|\,v_s(t,t-1)\,| \approx |\,v_t - v_{t-1}\,| = |\,i_t - i_{t-1}\,| + |\,j_t - j_{t-1}\,| \\ &t = 0,1,2,3,...N \end{aligned} \qquad (16)$$

where i ,j represent the position. The second term in (15) is associated with the stiffness of the contour and defined as

$$v_{ss}(s) \approx |\,v_{t-1} - 2v_t + v_{t+1}\,| \qquad (17)$$

where the three points $v_{t-1}, v_t, v_{t+1}$ on the contour are too close. In order to represent the stiffness of contour better, we consider the three points $v_{t-2}, v_t, v_{t+2}$ for

$$v_{ss}(s) \approx |\,v_{t-2} - 2v_t + v_{t+2}\,| \qquad (18)$$

In the algorithm, when we have point $v_t$, the two points $v_{t+1}$, $v_{t+2}$ are not decided yet. The $v_{ss}(s)$ can then be modified as:

$$\begin{aligned} v_{ss}(s) &= v_{ss}(t,t-1) \approx |\,v_{t-4} - 2v_{t-2} + v_t\,| \\ &= |\,i_{t-4} - 2i_{t-2} + i_t\,| + |\,j_{t-4} - 2j_{t-2} + j_t\,|; \;\; t = 4,5,...N \end{aligned} \qquad (19)$$

A priori knowledge has been assumed that the direction of the artery wall on the image is in horizontal. To avoid corners from occuring, we add an external constraint: for points on the contour, $|j_t - j_{t-2}|$ should be less than 2. Since we select point column by column, $|j_{t-4} - 2j_{t-2} + j_t|$ always equals to 0. When $t$<4, we can't get $v_{ss}(s)$, we then set $v_{ss}(s)$ to be 0. So the formula becomes:

$$v_{ss}(t,t-1)=\begin{cases} |i_{t-4}-2i_{t-2}+i_t|, & if\ |j_t-j_{t-2}|<2\ and\ t\geq 4 \\ \infty \quad , & else \\ 0, \quad , & t<4 \end{cases} \tag{20}$$

### 3.5 The definition of $E_{image}$

$E_{int}$ is mainly associated with the spline of the contour and $E_{image}$ is decided by forces such as gradient, and other forces given by user according to the application. In traditional way, $E_{int}$ is the force to keep the shape of the curve; different $E_{image}$ gives rise to the force pushing the curve to the significant line. $E_{image}$ is usually defined as:

$$E_{image} = -(G_\sigma * \nabla^2 I)^2 \tag{21}$$

Here we introduce one dimensional edge operator [14]:

$$\nabla(I_k) = h_k + h_{k+1} + h_{k+2} - h_{k-1} - h_{k-2} - h_{k-3}, \tag{22}$$

where $I_k$ is the $k$-$th$ pixel along the direction and $h_k$ is its amplitude, this operator is not very sensitive to noise and can get good gradient to the actual edge. Since the direction of the layer is in horizontal, the gradient can be computed as:

$$\nabla(I_{i,j}) = I_{i,j} + I_{i+1,j} + I_{i+2,j} - I_{i-1,j} - I_{i-2,j} - I_{i-3,j} \tag{23}$$

where $I_{i,j}$ is the intensity at point *(i, j)*. We need to detect two edges from the image; intimal edge and adventitial edge, see Figure 1, and the region of interest (ROI) can be divided into four parts: carotid lumen, intimal layer, media layer, and adventitial layer (from up to down). Distribution of the gray for the four parts is: dark, bright, dark, bright. There are two edges we need to detect: the edge between lumen of carotid and intimal layer, the edge between media and adventitial layers. Common feature for the edges is that from one side to the other; gray turns from dark to bright.

Consider $-\nabla(I_{i,j})$ as one part of $E_{image}$: when the gray turns from dark to light on the edge, $-\nabla(I_{i,j})$ is negative. When the gray turns from light to dark on the edge, $-\nabla(I_{i,j})$ is positive. In areas with flat gray, the value is around zero. Therefore, $-\nabla(I_{i,j})$ can be used to represent the edge well.

Even we get the gradient by one dimensional operator (23), the effect of noise still exists, we add the variance of intensity to $E_{image}$ as *S(i,j),* which is introduced by [9], the formula is as follow:

$$S(i,j) = \underset{1\leq q\leq i-1}{mean}(|I(q,j) - \underset{1\leq p\leq i-1}{mean}(I(p,j))|) + \underset{i+1\leq q\leq m-1}{mean}(|I(q,j) - \underset{i+1\leq q\leq m-1}{mean}(I(p,j))|) \tag{24}$$
$$i = 0,1,2,....M;\ j = 0,1,2......N$$

*S(i,j)* is not sensitive to noise and can make dark pixels on one side and light pixels on the other side. The definition of $E_{image}$ is:

$$E_{image}(i,j) = \gamma\cdot(-\nabla'(i,j) + S'(i,j) + \eta\cdot I(i,j)) \tag{25}$$

where $\nabla'(i,j)$, $S'(i,j)$ are the normalized results of $\nabla(i,j)$, $S(i,j)$, respectively, and $I(i,j)$ is the intensity of every point. During the detection of the intimal edge, the adventitial edge could be stronger than intimal edge which may result in the crossing of each other. Since the intimal edge is close to lumen area and points around the edge are darker than the points on adventitial edge, *I(i,j)* is added to the energy to avoid the influence of adventitial layer. For detecting intimal and adventitial edges, the value of $\eta$ could be different and $\gamma$ in (25) is a weighting of $E_{image}(i,j)$.

### 3.6 Minimization procedure

To solve this problem, we adopt the algorithm of dynamic programming. For every node in the region, we can represent the energy by formula:

$$E_{total}^*(i,j) = \min[E_{int}(i,j;i+a,j-1) + E_{image}(i,j) + E_{total}^*(i+a,j-1)] \tag{26}$$
$$i = 1,2,...M-1;\quad j = 1,2,...N;\quad a = -1,0,1$$

where *(i, j)* represents the position of the node. Different values for $a$ mean different choices and $E_{int}$ is associated with two nodes *(i, j)* and *(i+a, j-1)*.

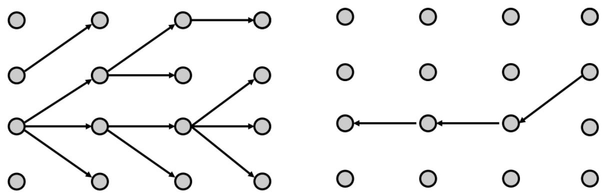
For every node $(i,j)$ in column $j$, the minimum energy and the position of pre-node in column $j-1$ should be recorded. Here we use a matrix $pre-node(i,j)$ to record the position:

$$pre-node(i,j) = (i^*, j-1) \tag{27}$$

The paths will be built among the nodes. Every node in column j should find a node in column *(j-1)* directing to it, see Figure 6.a. At the last column, the minimum $E_{total}$ and the end point can be selected as follows:

$$E_{total}(i,j) = \min[E_{image}(i,j) + E_{total}^*(i,j)] \tag{28}$$

When the end point is selected, the shortest cost path can be backtracked by the matrix $pre-node(i,j)$, see Figure 6.b for the tracking path.



(a) (b)

Figure 6.a shows an example for the path matrix $pre-node$, 6.b is the last path by backtracking.

## 4 RESULTS

The original ultrasound image is shown in Figure 7a. Firstly, a region of interest (ROI) should be selected by the user. Apply LCFSRAD to reduce noise in ROI. In experiments with 4 iterations and set TSS =1, we can get a good result. Then an initial intimal edge will be set by the selection algorithm in Section 3.3. The initial intimal edge is used to decide the searching region for true intimal edge. Afterwards, the minimized energy of the intimal edge can be obtained by dynamic programming with parameters $\alpha(s) = 1$, $\beta(s) = 1$, $\gamma=5$, η =0.05. Finally, set the region for adventitial edge and get the adventitial edge with parameters $\alpha(s) = 1$, $\beta(s)=1$, $\gamma=5$, $\eta=0.00$. The result is shown in Figure 7.b. Figure 7.c is the result using the method appeared in [15][3]. From Figure 7.b and Figure 7.c, we can find out that our proposed method works well as [15][3]. But for methods in [15][3], besides ROI, two points on the intimal edge should be defined manually by the user from beginning. For our proposed method, however, only ROI is needed.

In [16], Cheng has pointed out that, using the traditional dynamic programming to detect the intimal and adventitial edges, the two curves may cross with each other. In our approach, this will not happen, see Figure 8. The energy definition and policy to divide different areas in our method, can overcome this problem.

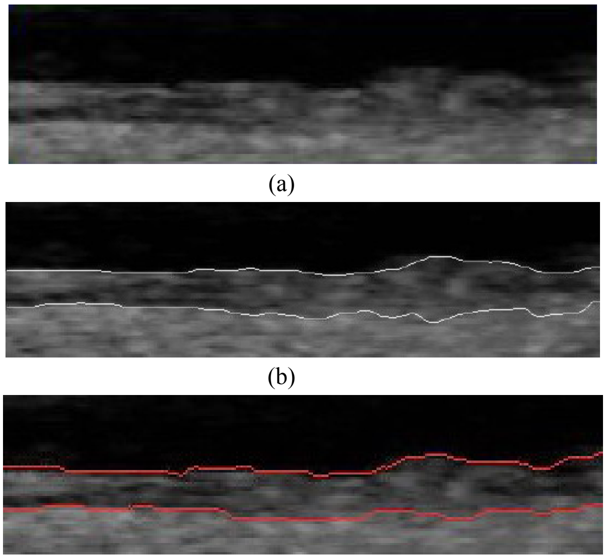



Figure 7.a is initial image appears in [15], 7.b is the result given by [15], 7.c is the result by our method

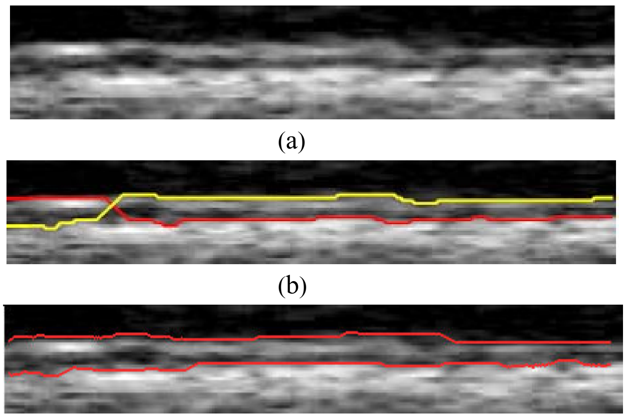



Figure 8.a is an example given by [16], 8.b is the result using traditional dynamic programming where the yellow line is intimal edge and the red line is adventitial edge; they cross with each other. 8.c is the result by our proposed method

Some other cases for different images are shown in Figure 9.

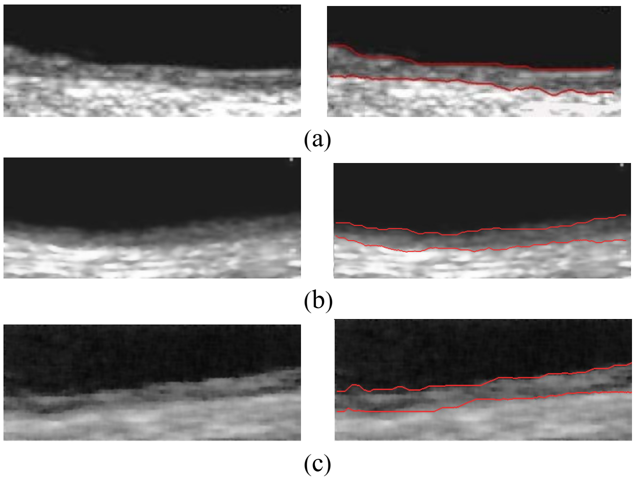



Figure 9 Pictures on the left are the initial images; on the right are results

## 5. CONCLUSIONS

In this paper, we propose a new method to detect the intimal and adventitial edges based on active contour model and solve the model by dynamic programming. Definition of energy and the searching region for the edge will affect the result significantly and our improved way in dynamic programming can avoid the edges-crossing from the direct implementation of the dynamic programming. We compare our method with other methods published in the literature. Our method is automatic and works well.

Usually, the snake model is practiced by moving the initial contour to the destination contour. For enough priori knowledge, such as the local area of the contour resides, we can apply this model using our proposed method where the formulated energy can be global minimum. The time complexity of our method is *MN* where N is the number of points on the contour and *M* is the possible position of every point in the selected area.

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