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Semi-Implicit Scheme based Nonlinear Diffusion Method in Ultrasound Speckle Reduction

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***Abstract*—Because of the presence of speckle noise in ultrasound image, the image resolution is not so desirable. Recently, a class of anisotropic diffusion based methods has been developed, which can reduce the speckle noise, and at the same time, preserve and enhance the edge/borders. Speckle Reducing Anisotropic Diffusion (SRAD) is an important representative of these methods. It is an explicit scheme based method with very slow processing speed due to the severe restriction in time step size (TSS), and it is sensitive to noise due to its small window size to calculate ratio of mean and variance. Using semi-implicit scheme (e.g., additive operator splitting AOS) method to discretize SRAD (called ASRAD), we could use large TSS for speed-up. However, artifacts brought by ASRAD degrade the image. In this paper, we aim to solve above problems. Our method could be generalized to solve other explicit scheme based nonlinear diffusion speckle reduction methods.**

Keywords-nonlinear diffusion; speckle reduction; ultrasound imaging; image restoration;

I. INTRODUCTION

Ultrasound images are widely used now by doctors to diagnose diseases of patients because of its noninvasive and low cost nature. Due to the limitation of ultrasound imaging, speckle as a dominant noise decreases the image contrast resolution of ultrasound image. Reducing speckle noise to help doctors' diagnosis is a hot area for researchers in medical image processing. Since 2000, many researchers studied anisotropic diffusion (AD) based speckle reduction methods, such as speckle reducing anisotropic diffusion (SRAD) [1], and oriented speckle reducing anisotropic diffusion (OSRAD) [2], nonlinear diffusion in Laplacian pyramid domain for ultrasonic speckle reduction (LPND) [3] and local coherence based fast speckle reducing anisotropic diffusion (LCFSRAD) [4].

Most of the above methods except [4] are very slow in speed because they use an explicit scheme to discretize the anisotropic diffusion equation. In SRAD, the author employed the statistics characteristics of speckle to control the diffusion coefficients at each point in their AD methods. It is very good in reducing speckle; however, its speed is very slow. LCFSRAD is a fast AD method based on semi-implicit discretization method, but it utilized local coherence to control the diffusivity which is not as exact as SRAD.

In this paper, we use a new semi-implicit scheme to discretize the AD equation, and analyze the relationship between time step size (TSS) and the image quality. We also propose a modified AD method to increase the image quality. At each pixel, we still use the ratio of standard deviation over mean as SRAD to estimate whether this pixel is in speckle region; the diffusivity function of our method will smooth the pixel according to the ratio. Using semi-implicit Additive Operator Splitting (AOS) scheme [5] to do discretization (called ASRAD), the equation is unconditionally stable. So the time step size (TSS) which controls the extent of blurring at each iteration can be assigned values much larger than in the explicit scheme. However, from our experiments we can see that ASRAD brings many artifacts in filtered image. In order to solve this problem we modify the equation of ASRAD to reduce the artifacts.

This paper is organized as follows. In Section II, we introduce SRAD and ASRAD. After that, we present our anisotropic diffusion model. Experiments and Testing results of our method from *in vivo* images are shown in Section III. Conclusions and future directions are given in Section IV.

II. METHOD

A. Explicit scheme based SRAD

Yu and Acton derived SRAD from Lee filter [6], they took Lee filter as an isotropic diffusion filter and modified the equation to an anisotropic diffusion formulation. The Lee filter is as

$$\hat{I} = \overline{I}_s + \frac{C_s^2 - C_n^2}{C_s^2} \cdot (I_s - \overline{I}_s) = \overline{I}_s + k_s \cdot (I_s - \overline{I}_s) \qquad (1)$$

with $k = (C_s^2 - C_n^2)/C_s^2$, and C_n^2 is the spatial variance of a noise area, C_s^2 is the local variance of a window which center is *S*. Transforming (3) to a PDE formation, it is.

$$\begin{cases} \partial I(t)/\partial t = k \cdot div(\nabla I) \\ I(0) = I_0 \end{cases} \qquad (2)$$

where I_0 is the initial image. It is obvious to see that above equation is an isotropic diffusion. Yu and Acton put the diffusivity coefficient into the divergence operator and modified the value of *k*, so, SRAD can be written as

$$\begin{cases} \partial I(t)/\partial t = div(c(q)\nabla I) \\ I(0) = I_0 \end{cases} \tag{3}$$

where $c(q)$ is the diffusivity function which is a function of local statistics in the image, where

$$c(q) = \frac{1}{1+[q^2-q_0^2]/[q_0^2(1+q_0^2)]} \tag{4}$$

or where $q(\cdot)$ and $q_0(\cdot)$ are all the ratio of gray intensity variance and mean in windows. The ratio of $q_0(\cdot)$ is calculated in the fully developed speckle area, and the window of $q(\cdot)$ is current moving window. We can analyze the behavior of SRAD in different cases. In the fully developed speckle area, $q \to q_0$ and $c(q) \to 1$, so SRAD in this kind of area process like a isotropic diffusion; it will smooth the speckle noise. On the other hand, if current window is at edges or borders, $q >> q_0$ and $c(q) \to 0$, then the filter can have enhancing effects close to the contours. However, because SRAD uses explicit discretization scheme, so, as Perona and Malik's method [7], the speed of SRAD is very slow. The discrete equation is,

$$\begin{aligned} I_{i,j}^{t+1} = I_{i,j}^t + \frac{\tau}{4h^2}[c_{i+1,j}^t(I_{i+1,j}^t - I_{i,j}^t) + c_{i,j}^t(I_{i-1,j}^t - I_{i,j}^t) \\ + c_{i,j+1}^t(I_{i,j+1}^t - I_{i,j}^t) + c_{i,j}^t(I_{i,j-1}^t - I_{i,j}^t)] \end{aligned} \tag{5}$$

In order to compare the explicit scheme and the semi-implicit scheme, we can write the 2-D explicit scheme in matrix-vector notation as

$$I^{t+1} = I^t + \tau[\sum_{l=1}^{2} A_l(I^t)]I^t \tag{6}$$

The $A_l = (\alpha_{ijl})_{ij}$ corresponds to derivatives along the lth coordinate axis. For example,

$$a_{ij}(I^t) = \begin{cases} c_j^t/h^2 & [j \in N(i)], \\ -\sum_{n \subset N(i)} c_n^t/h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{7}$$

where $N(i)$ is the set of the two neighbors of pixel i (boundary pixels have only one neighbor).

B. Semi-implicit AOS Scheme SRAD

The continuous formulation of ASRAD is the same as (3). The difference is in the discretization method. The AOS scheme is as,

$$I^{t+1} = \frac{1}{2}\sum_{l=1}^{2}[U + 2\tau A_l(I^t)]^{-1} I^t \tag{8}$$

where U is a KL by KL identity matrix, τ is the time step size (TSS), $A_l = (\alpha_{ijl})_{ij}$ is the same as (7). Above equation can be solved by AOS scheme which is much faster than explicit based scheme because (6) satisfies the criteria for discrete nonlinear diffusion scale-spaces [6] so that we can use large TSS.

C. Modified ASRAD

In order to increase the robustness, we utilize the idea of CLMC filter [8] to modify the ASRAD. We adopt the following formulation of diffusion coefficient to increase the robustness, at the same time, let it satisfy the criteria for discrete nonlinear diffusion scale-spaces.

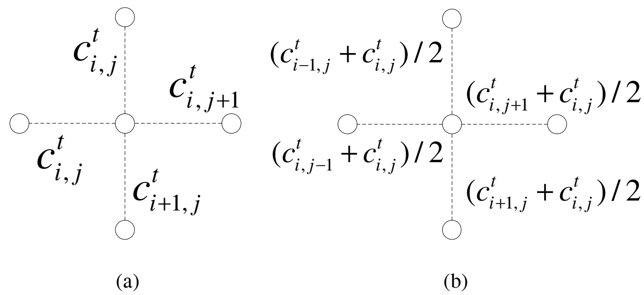


Figure 1. Formulation of diffusion coefficient. (a) Location of diffusion coefficients of ASRAD. (b) Formulation of diffusion coefficient of our method.

The formulation of diffusion coefficients in [2] is similar to the one in Fig. 1(b). However, the author didn't use semi-implicit AOS scheme to do discretization. We aim to utilize semi-implicit AOS scheme to discretize this formulation, our semi-implicit scheme matrix-vector notation is the same as (6), $A_l = (\alpha_{ijl})_{ij}$ is defined as

$$a_{ij}(I^t) = \begin{cases} (c_i^k + c_j^k)/2h^2 & [j \in N(i)], \\ -\sum_{n \subset N(i)} (c_j^k + c_n^k)/2h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{9}$$

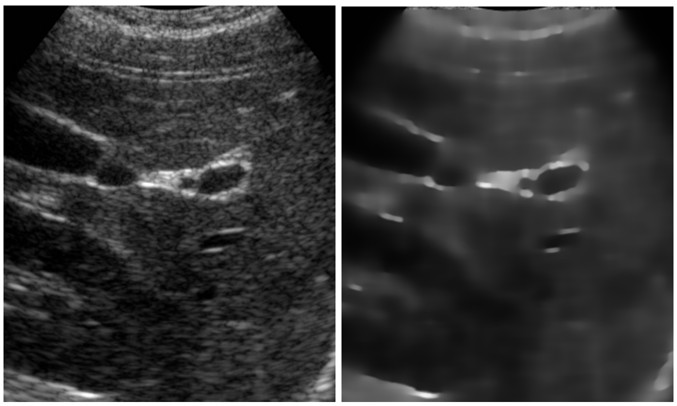
The notations in above equation are the same as (7). The above diffusion equation satisfies the criteria for discrete nonlinear diffusion scale-spaces (e.g., the criteria D1-D6) [6]. So we can use very large TSS to do anisotropic diffusion. It means that in order to get the same smoothing result, using our method we need to do much less steps than explicit scheme ones. Therefore, as ASRAD and LCFSRAD, our diffusion equation is unconditionally stable and can be solved by AOS scheme. Moreover, in ASRAD, the discrete formulation of divergence operator is not accordingly dependent on four directions, this imbalance of divergence will bring artifacts in the nearby areas of edge/borders, however, our diffusion equation doesn't have this problem.

III. Experiments

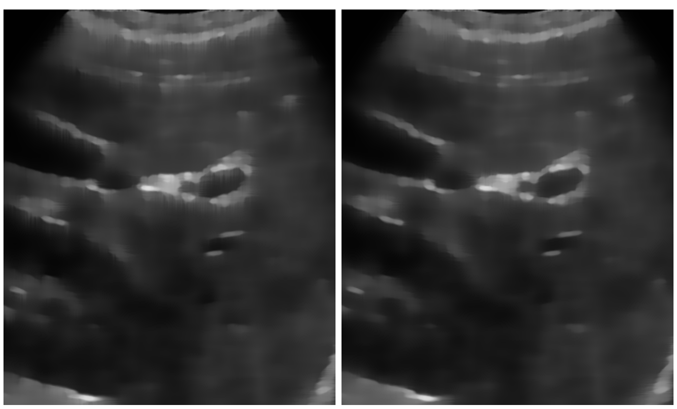
Studies were conducted on simulated and in-vivo ultrasound images to compare different methods. Some papers had compared and analyzed the performance of different explicit scheme based AD methods on simulated *B*-mode images [2], [3]. Therefore, in our experiments, we pay more attention to the advantages of our method compared with some other semi-implicit scheme based AD methods in in-vivo images. We used Saset iMago color ultrasound scanner for data acquisition which is the system we designed in our lab.

A. Analysis of Speckle Reduction From In-Vivo Image

In this experiment, our proposed method (MASRAD) was compared with other semi-implicit scheme based AD methods such as ASRAD, LCFSRAD.



(a) Original liver image (b) LCFSRAD



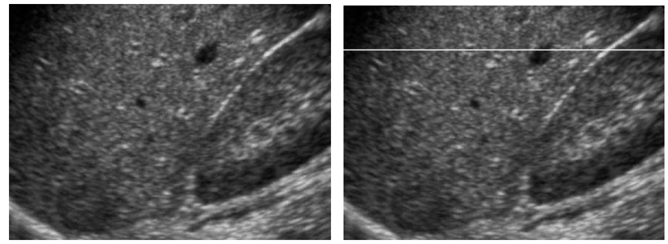
(c) ASRAD (d) Proposed method

Figure 2. Original liver image and its filtered result. The parameters used in above three experiments are the same, TSS = 5.0, Number of iteration = 6.

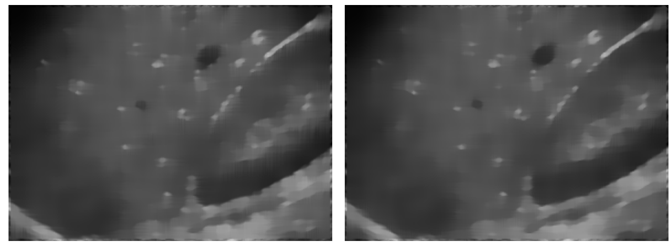
From above results we can see the shortcomings of ASRAD and LCFSRAD compared with our proposed method clearly. In the same condition (e.g., the same TSS and number of iteration), ASRAD produces some artifacts in the nearby areas of strong edges, and the noise brought by the large TSS is visible. In the result of LCFSRAD, the noise brought by the large TSS is not visible. However, the result of LCFSRAD preserves less details compared with our proposed method. Thus, our method is the best among these semi-implicit scheme based AD methods.

B. Analysis of Profiles From In-Vivo Image

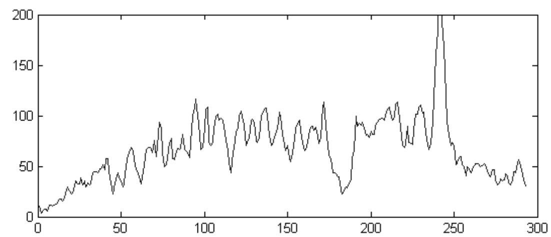
To know how well our proposed method can do in avoiding artifact and noise of large TSS, we conduct this experiment to analyze profiles of results. In the following, we show our method have better performance in avoid the artifacts brought by ASRAD.



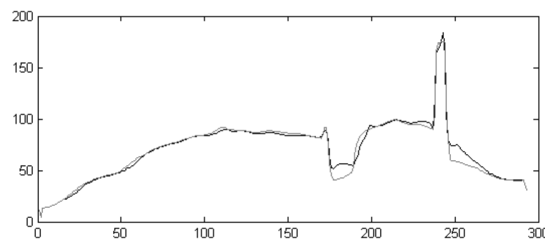
(a) Original noisy kidney image (b) Image indicating the highlight line



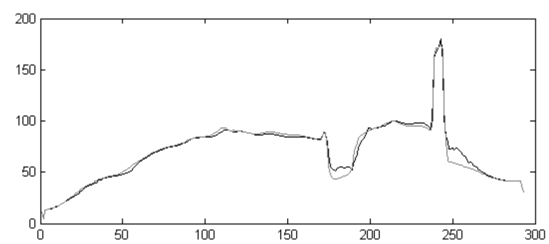
(c) ASRAD (d) Proposed method



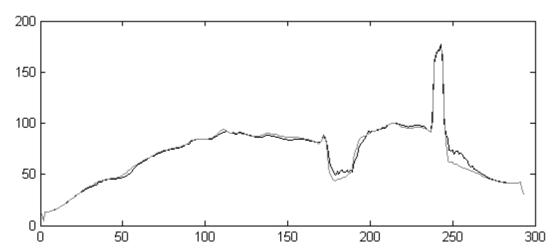
(e)



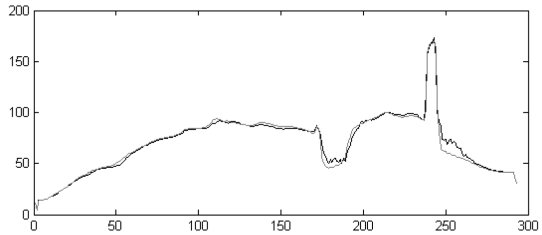
(f)



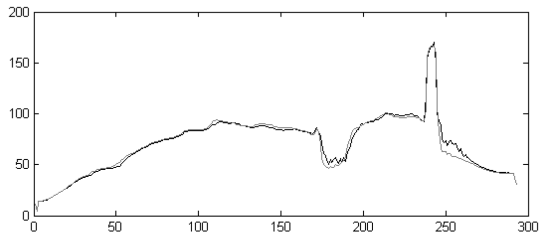
(g)



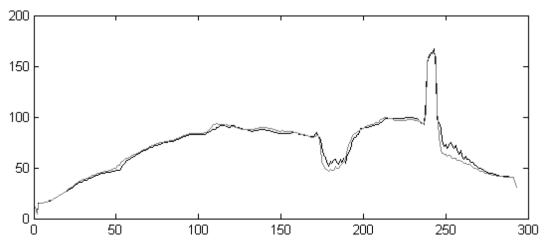
(h)



(i)



(j)



(k)

Figure 3. Profiles along the highlight line in the kidney and liver image. (a) Original noisy kidney image. (b) Image showing the highlight line. (c)-(d) Filtered results of ASRAD and the proposed method with the same parameters (TSS = 6.0, Number of iteration = 6). (e) Profile along the highlight line in the original noisy image. (f)-(k) Profiles along the highlight line in the images filtered by ASRAD and the proposed method, in each figure, there are two curves, the black one is the result of ASRAD, the brown one is the result of the proposed method. Parameters of ASRAD and the proposed method in above experiments are shown in the following table.

TABLE I. PARAMETERS USED IN THE EXPERIMENT OF FITURE 3

	The Number of Figure 3					
	(f)	***(g)***	***(h)***	***(i)***	***(j)***	***(k)***
TSS	1.0	2.0	3.0	4.0	5.0	6.0
Iteration Times	25	13	8	7	6	6

In order to show the differences between ASRAD and our proposed method, a line presenting the pixel gray values on row 41 is highlighted, and the pixel gray values on this line from the original and filtered images are shown in Fig. 3. From these Figures we can see the two weak points of ASRAD. The first problem of ASRAD is that the discrete formulation of divergence operator of ASRAD is not accordingly dependent on four directions, this imbalance of divergence brings artifacts near the edge/borders. This phenomenon can be noticed from Fig. 3 (c) to (h), in the right part of strong edge, the ASRAD filtered profiles are all higher than the ones of our proposed method, and the actual gray value is not that bright. Secondly, as we increase the TSS and reduce the iteration time, we can see that the curves of ASRAD vibrates more and more acutely in certain areas (i.e., in the vicinity of strong edge or lesion). In contrast, our proposed method produces neither the artifact in the nearby area of strong edge in all different cases, nor the vibration of gray value in the vicinity of strong edge or lesion when the TSS gets larger.

IV. CONCLUSION

Anisotropic diffusion is an edge-preserving smoothing algorithm that was successfully adapted to ultrasonic speckle reduction by using local statistics to determine the extent of blurring. However, numerical solution of the underlying PDE was very slow because of explicit discretization. Directly modifying SRAD to ASRAD will bring some new problems. In this paper, we study the possibility of solving these problems. From our experiments, we can see that our method performs well in the cases of using large TSS than previous methods. In the future, we would like to explore to use the GPU to do speedup of our method as [9]. Using the GPU parallel processing, we could further reduce the time consumption without bring any noise and artifacts.

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