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Local Coherence based Fast Speckle Reducing Anisotropic Diffusion

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Abstract. In ultrasound image, because of the presence of speckle, the image contrast resolution is reduced. And the image speckle noise also limits the effective application of image processing and analysis methods such as edge detection and segmentation methods. In recent years, a class of new methods called anisotropic diffusion has been developed in the field of ultrasound image speckle reduction. From the aspect of the quality of ultrasound speckle reduction, this class of method is desirable, however, most of this kind of methods are slow in speed. In this paper, we propose a new anisotropic diffusion based speckle reduction method which uses local coherence to control diffusivity in each pixel. We use additive operator splitting method to discretize our method to reduce the time consumption, Comparing with traditional methods, our method is faster in speed and, at the same time, its smoothing result is acceptable in terms of image quality.

Keywords: Anisotropic diffusion, speckle reduction, ultrasound imaging, local coherence.

1. Introduction

Ultrasound images are widely used now by doctors to diagnose diseases of patients. Due to the nature of ultrasound imaging, speckle as a dominant noise decrease the image contrast resolution of ultrasound image. Finding appropriate ways to reduce speckle noise to help the doctor's diagnosis is a hot area for researchers in medical image processing. Since 2000, after Yu and Acton, Krissian and some other researchers introduced anisotropic diffusion to the field of speckle reduction, many literatures discuss and develop this class of methods such as speckle reducing anisotropic diffusion (SRAD) [1], detail preserved anisotropic diffusion (DPAD) [2], speckle constrained anisotropic diffusion (SCAD) [3], and oriented speckle reducing anisotropic diffusion (OSRAD) [4] and real-time speckle reduction (RTAD) [10].

Most of the above methods except RTAD are very slow in speed because they use an explicit scheme to discretize the anisotropic diffusion. In SRAD and DPAD, the authors use the statistics of speckle to control the diffusivity at each point according to the similarity of that area with the fully developed speckle area. Although the methods are slow, they are very good in reducing speckle. RTAD is a fast anisotropic diffusion method based on semi-implicit and explicit discretizations whose time consumption we can further reduce.

In this paper, we propose a new fast local coherence based anisotropic diffusion method for speckle reduction. At each pixel, we use the local coherence which can be obtained by eigenvalue decomposition of structure tensor to estimate whether this pixel is in speckle region, according to the value of local coherence, the diffusivity function of our method will smooth the pixel. Moreover, we use semi-implicit Additive Operator Splitting (AOS) scheme [6] to do discretization. Because of this method, our discrete diffusion method is unconditionally stable. So the time step size (TSS) which controls the extent of blurring can be assigned values much larger than in the explicit Scheme. As a result, our method can be much faster than traditional methods.

This paper is organized as follows. In Section 2, we review the traditional slow anisotropic diffusion based speckle reduction methods. In Section 3, we present our anisotropic diffusion model and the fast discretization method. Experiments and Testing results of our method from *in vivo* images are shown in Section 4. Conclusions and future directions are given in Section 5.

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2. Slow Anisotropic Diffusion based Speckle Reduction Methods

2.1. Anisotropic Diffusion of Perona and Malik

In the image processing field, Perona and Malik [8] firstly proposed nonlinear partial differential equation (PDE) for smoothing image at the same time preserving edges. On continuous domain, the anisotropic diffusion method proposed by Perona and Malik is as follows,

$$\begin{cases} \dfrac{\partial I}{\partial t} = div[c(|\nabla I|)\cdot \nabla I] \\ I(t=0) = I_0 \end{cases} \tag{1}$$

where ∇ is the gradient operator, div is the divergence operator, || denotes the magnitude, $c(\cdot)$ is the diffusivity function, and I_0 is the initial image. They said that the diffusivity functions should be monotonous decrease bell-shape function. The dicretization formation used in Perona and Malik is an explicit scheme, it is as follows

$$I_s^{t+1} = I_s^t + \lambda \sum_{p\in\overline{\eta}_s} c(\nabla I_{s,p}^t)\nabla I_{s,p}^t \tag{4}$$

where I_s^t denotes the gray value of a pixel which position is s in a 2-D image. λ is the TSS which value is from 0 to 0.25, due to (4) is a explicit discretization scheme, in order to keep the stability of above equation, the value of TSS must be very small. In fact, this requirement is too restrictive, and the speed of above method is very slow. We need to iterate hundreds times to get desirable smoothing results.

2.2. Speckle Reducing Anisotropic Diffusion

Yu and Acton derived SRAD from Lee filter [9], they took Lee filter as an isotropic diffusion filter, then they modified the equation to an anisotropic diffusion formulation. The Lee filter is as

$$\hat{I} = \overline{I}_s + \frac{C_s^2 - C_n^2}{C_s^2}\cdot(I_s - \overline{I}_s) = \overline{I}_s + k_s\cdot(I_s - \overline{I}_s) \tag{5}$$

with $k = (C_s^2 - C_n^2)/C_s^2$, and C_n^2 is the spatial variance of a noise area, C_s^2 is the local variance of a window which center is s. Transforming (5) to a PDE formation, it is

$$\begin{cases} \partial I(t)/\partial t = k\cdot div(\nabla I) \\ I(0) = I_0 \end{cases} \tag{6}$$

where I_0 is the initial image. It is obvious to see that above equation is an isotropic diffusion. Yu and Acton put the diffusivity coefficient into the divergence operator and modified the value of k, so, SRAD can be written as

$$\begin{cases} \partial I(t)/\partial t = div(c(q)\nabla I) \\ I(0) = I_0 \end{cases} \tag{7}$$

where $c(q)$ is the diffusivity function which is a function of local statistics in the image, where

$$c(q) = \frac{1}{1+[q^2 - q_0^2]/[q_0^2(1+q_0^2)]} \tag{8}$$

or where $q(\,)$ and $q_0(\,)$ are all the ratio of gray intensity variance and mean in windows. The ratio of $q_0(\,)$ is calculated in the fully developed speckle area, and the window of $q(\,)$ is current moving window. We can analyze the behavior of SRAD in different cases. In the fully developed speckle area, $q \to q_0$ and $c(q) \to 1$, so SRAD in this kind of area process like a isotropic diffusion, it will smooth the speckle noise. On the other hand, if current window is at edges or borders, $q >> q_0$ and $c(q) \to 0$, then the filter can have enhancing effect close to the contours. However, because SRAD uses explicit discretization scheme, so, as Perona and Malik's method, the speed of SRAD is very slow.

3. Fast Speckle Reducing Anisotropic Diffusion Model

3.1. Semi-implicit AOS Scheme for Discretizing Anisotropic Diffusion Equation

Weickert proposed the discretization criteria [5], [6] for discretizing anisotropic diffusion equation. The general discrete diffusion equation is

$$\begin{cases} I^0 = I_0 \\ I^{k+1} = Q(I^k)I^k \quad \forall k \in [0, +\infty] \end{cases} \tag{9}$$

where I^k is the image vector which size is the number of pixels of the image, denoted n, and $Q(I^k)$ is a $n \times n$ matrix, and $Q(I^k) = [q_{ij}(I^k)]$. Weickert has proved that the discretization schemes satisfied the discretization criteria create smoothing scale-spaces, e.g., they obey a maximum-minimum principle, have a large class of smoothing Lyapunov functions, and converge to a constant steady-state. The details of the discretization criteria is as

- D1 continuity in its arguments ($Q \in C(\mathrm{R}^n, \mathrm{R}^n \times \mathrm{R}^n)$);
- D2 symmetry ($q_{ij}=q_{ji}$ $\forall\, i, j \in J$);
- D3 unit row sum ($\sum_{i \subset J} q_{ij}=1$ $\forall\, i \in J$);
- D4 non-negativity ($q_{ij} \geqslant 0$ $\forall\, i \in J$);
- D5 positive diagonal($q_{ii} > 0$ $\forall\, i \in J$);
- D6 irreducibility (any two pixels can be connected by a path with nonvanishing diffusivities)

He also proposed AOS scheme [6] to do fast anisotropic diffusion, and proved that this dicretization scheme satisfies his criteria. The AOS scheme based anisotropic diffusion is based on CLMC filter [7], the 1-D CLMC filter is as

$$\frac{I_i^{t+1} - I_i^t}{\tau} = \sum_{j \subset \mathrm{N}(i)} \frac{g_j^k + g_i^k}{2h^2}(I_j^t - I_i^t) \tag{10}$$

where I_i^{t+1} and I_i^t are the image pixel values of current and next steps respectively, τ the TSS. The diffusivity function $g(\cdot)$ is as

$$g_i^k := g[\frac{1}{2} \sum_{p,q \subset \mathrm{N}(i)} (\frac{I_p^k - I_q^k}{2h})^2] \tag{11}$$

$$g(s) = \begin{cases} 1, (s \le 0) \\ 1 - \exp[\frac{-3.315}{s/\lambda}], (s > 0) \end{cases} \tag{12}$$

where $\mathrm{N}(i)$ is the set of the two neighbours of pixel i (boundary pixels have only one neighbor). Weickert integrated (10) to the following matrix-vector notation as

$$\frac{I^{t+1} - I^t}{\tau} = A(I^t)I^t \tag{13}$$

where I^{t+1} and I^t are the image gray value vectors of the current and next time steps respectively, obtained by stacking each column vector of the K by L image on top of another into one long column vector of length KL. $A(I^t)$ is a tridiagonal matix, with $A(I^t) = [a_{ij}(I^k)]$ and

$$a_{ij}(I^t) = \begin{cases} (g_i^k + g_j^k)\big/2h^2 & [j \in \mathrm{N}(i)], \\ -\sum_{n\subset \mathrm{N}(i)} (g_j^k + g_n^k)\big/2h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{14}$$

So (13) could be written as the iteration explicit scheme, the 2-D explicit scheme is

$$I^{t+1} = [U + \tau \sum_{l=1}^{2} A_l(I^t)] I^t \tag{15}$$

where U is a KL by KL identity matrix. The above equation is as the Perona and Malik's discretization method is an explicit scheme. So the requirement of the range of TSS is very restricted. In order to reduce the time consumption, Weickert proposed the semi-implicit AOS scheme, the 2-D formulation is as

$$I^{t+1} = \frac{1}{2}\sum_{l=1}^{2} [U + 2\tau A_l(I^t)]^{-1} I^t \tag{16}$$

It is unconditionally stable. Inspired by this idea, we develop our method.

3.2. Local Coherence based Fast Speckle Reducing Anisotropic Diffusion

In the paper of RTAD [10], the author used tensor based anisotropic diffusion to do speckle reduction, and used AOS scheme to discretize their diffusion method. In order to reduce the time consumption, we use scalar diffusivity in our anisotropic diffusion. We also use AOS scheme to discretize our method. Because we use local coherence to evaluate speckle noise which is the same way as RTAD and reduces the computation amount, our method is not only faster than RTAD but also just as good in speckle reduction. In order to reduce the influence of speckle noise, the structure tensor used in RTAD and our method is multiscale structure tensor, it is

$$\begin{pmatrix} K_\rho * I_x^2 & K_\rho * (I_x I_y) \\ K_\rho * (I_x I_y) & K_\rho * I_y^2 \end{pmatrix} \tag{17}$$

where I_x and I_y are partial derivatives at certain point in the image, K_ρ is the Gaussian kernel and $*$ the convolution. Using eigenvalue decomposition, (17) could be written as

$$J(I) = (\omega_1 \quad \omega_2)\begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix}\begin{pmatrix} \omega_1^T \\ \omega_x^T \end{pmatrix} \tag{18}$$

The diffusivity function used in RTAD is as

$$D(I) = (\omega_1 \quad \omega_2)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} \omega_1^T \\ \omega_x^T \end{pmatrix} \tag{19}$$

$$\begin{aligned} \lambda_1 &= \begin{cases} a \cdot (1 - \dfrac{(\mu_1 - \mu_2)^2}{s^2}), & if\,((\mu_1 - \mu_2)^2 \le s^2) \\ 0, & else \end{cases} \\ \lambda_2 &= a. \end{aligned} \tag{20}$$

where a is a coefficient controlled by users, and μ_1 and μ_2 are eigenvalues in (18). The discretization equation of RTAD is

$$\frac{\partial I}{\partial t} = div[D\nabla I] = div[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} I_x \\ I_y \end{pmatrix}]; \tag{21}$$

In our method, we only use local coherence to control the diffusion, and we use a scalar value instead of above $2*2$ matrix to control diffusion. Our diffusion equation is as

$$\frac{\partial I}{\partial t} = div[l \cdot \nabla I] \tag{22}$$

Where $l(\cdot)$ is the local coherence function which is the squared difference of eigenvalues of multiscale structure tensor, it is as

$$l(|\mu_1 - \mu_2|) = 1/1 + (|\mu_1 - \mu_2|/K)^2 \tag{23}$$

where μ_1 and μ_2 are the eigenvalues of structure matrix at each point, structure matrix is as $J(\nabla I_\sigma) = (\nabla I_\sigma \cdot \nabla I_\sigma^T)$. Here, ∇I_σ is the gradient of a smoothed version of I which is obtained by convolving I with a Gaussian of standard deviation σ. Converting (22) to a matrix-vector notation and adopting AOS scheme, (24) comes down to the following iteration scheme

$$I^{t+1} = \frac{1}{2}\sum_{l=1}^{2}[U + 2\tau T_l(I^t)]^{-1} I^t \tag{25}$$

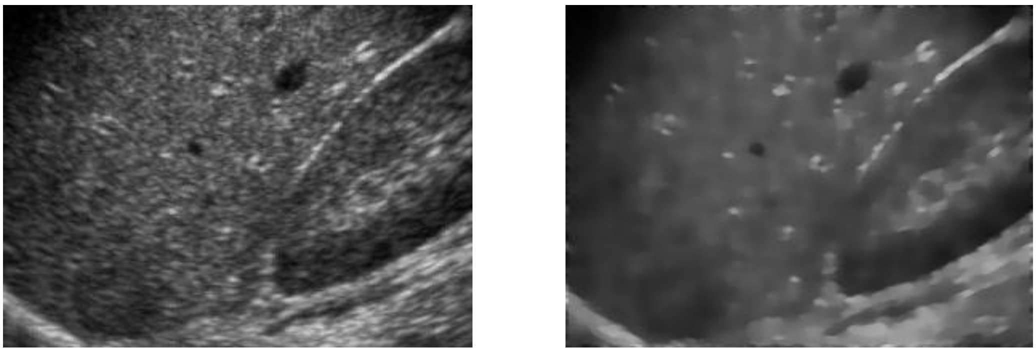
where U is a *KL* by *KL* identity matrix, τ is the time step size (TSS), $T_l(I^t)$ is a tridiagonal matrix, with $T_l(I^t) = [t_{ij}(I^t)]$ and

$$t_{ij}(I^t) = \begin{cases} \bar{l}_j^k / 2h^2 & [j \in \mathrm{N}(i)], \\ -\sum_{n \subset \mathrm{N}(i)} \bar{l}_n^k / 2h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{26}$$

where $\bar{l}$ is the average local coherence values between the current and next pixels. Because it has the same formation and idea as CLMC filter, it satisfies the discretization criteria D1-D6 unconditionally. So we could use very large TSS to do anisotropic diffusion. It means that in order to get the same smoothing result, using our method we need to do much less steps than explicit scheme ones.

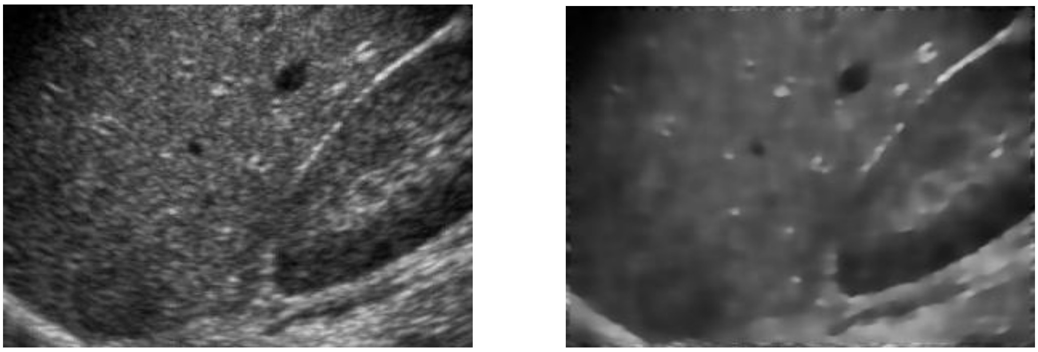
4. Experiment and Result

To verify our proposed algorithms, we have tested lots of ultrasound phantom and *in vivo* images which contains diverse shapes and sizes of structural details. We used Saset iMago color ultrasound scanner for data acquisition which is the system we designed in our lab.



(a) (b)

Fig. 1. (a) Original liver and kidney ultrasound image. (b) Explicit SRAD method, 100 iterations, TSS = 0.25



(a) (b)

Fig. 2. (a) Original liver and kidney ultrasound image. (b) our method, 4 iterations, TSS = 1.5

We can see that in order to get the same result of speckle reduction, the number of iteration of our method is much less than explicit SRAD method. Compare with RTAD, the number of iteration of our method is the same with the one of RTAD. However, in each iteration, the computation amount of our method is less than RTAD.

5. Conclusion

Anisotropic diffusion is an edge-preserving smoothing algorithm that was successfully adapted to ultrasonic speckle reduction by using local statistics to determine the extent of blurring. However, numerical solution of the underlying PDE was very slow because of explicit discretization. The semi-implicit RTAD scheme was introduced to substantially increase the calculation speed, but it used a matrix to control the diffusion. We propose a scalar based diffusivity using the eigenvalues of the structure matrix that not only matches RTAD in quality but further improves speed.

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