Ultrasound Speckle Reduction via Super Resolution and Nonlinear Diffusion

Bo Wang[1], Tian Cao[1], Yuguo Dai[1], and Dong C. Liu[1],

[1] Computer Science College, Sichuan University, China, 610064
{bowanghawk, ct.radiate, daiyuguo, dongcliu}@gmail.com

Abstract. Recently, some diffusion-based filtering methods have been developed such as anisotropic diffusion (AD) or nonlinear diffusion (ND), which can reduce the speckle noise, at the same time, preserve and enhance the edge/borders in ultrasound image. However, because of the granular pattern of speckle, it is quite difficult to reduce speckle exactly through diffusion-based methods only. In this paper, we propose a super resolution (SR) based ND method. We firstly reduce and compound speckle noise in a sequence of ultrasound images by using a fast SR method for ultrasound image. After this process, ultrasound speckle is much smaller, and the edge and structure are much clearer as complementary information of different images was used. To reduce the noise of the SR improved image, we use a local coherence based ND method. In the end, experimental results of the proposed method are compared with some other AD methods to demonstrate its effectiveness.

1 Introduction

Ultrasound imaging systems are widely used because of its real-time image formation, portability, low cost and noninvasive nature. However, due to the nature of ultrasound imaging, speckle as a dominant noise decreases the resolution of ultrasound image. Moreover, because of the presence of speckle, it is quite difficult to directly use common image processing methods in ultrasound image (such as feature detection, image segmentation and image registration). Therefore, finding appropriate method to reduce speckle noise in ultrasound image is a hot area for researchers in medical image processing. After Perona and Malik's seminal work [1], since 2000, many researchers studied anisotropic diffusion (AD) based ultrasound speckle reduction methods, such as speckle reducing anisotropic diffusion (SRAD) [2], and oriented speckle reducing anisotropic diffusion (OSRAD) [3] and semi-implicit scheme based nonlinear diffusion method in ultrasound speckle reduction (SIND) [4]. These methods have similar results in reducing speckle noise and preserving edge in ultrasound image.

However, all of above AD methods process ultrasound B mode image directly without utilizing an advanced image restoration method. In this way, affecting by the granular speckle, they all cannot reduce ultrasound speckle noise exactly. In order to get better denoising result, we firstly use a new ultrasound image fast super resolution (SR) method [5] to restore ultrasound image. This method can reconstruct an enhanced ultrasound image from a sequence of ultrasound images by using a maximum a-posteriori framework. Moreover, this method can automatically escape the errors generated by outliers in a sequence of input images. After the image restoration, we will get an improved ultrasound image in which the speckle noise is much smaller and structures are much clearer. To reduce the noise in the restored image, we propose to employ local coherence to control diffusion coefficients in our ND method. In addition, our ND method can be discretized by additive operator splitting (AOS) scheme [6] so that we can use large time step size (TSS) in the process of iteration to do speedup. To our knowledge, this is the first paper to address the speckle reduction of SR restored ultrasound images.

This paper is organized as follows. Section 2 shortly introduces the ultrasound image fast SR method [5]. Section 3 presents our AD method in which we use local coherence to control diffusion coefficients at each point of image. Section 4 demonstrates the experimental results. Section 5 presents the conclusion.

2 Fast Super Resolution for Ultrasound Image Reconstruction

The SR problem is an ill-posed inverse problem. In [5], the authors used a maximum a-posteriori (MAP) approach with transformation information, and they utilized AD [1] for regularization. During this process, a frequency domain approach to registration [7] is used for get better registration result, because incorrect registration may severely affect final result. In addition, the authors proposed a robust and efficient implementation.

2.1 Super-Resolution Model

The goal of SR is to improve the spatial resolution of an image. This type of problem is an inverse problem, wherein the source of information, or high-resolution (HR) image, is estimated from the observed data, or low-resolution (LR) images. Each of the LR images $\{Y_k, k=1,2,\ldots,N\}$ $[M\times M]$ can be modeled by a sequence of geometric warping, blurring, and downsampling operations on the high resolution $L\times L$ image X, followed by additive noise. We can represent Y_k and X as column vectors with length M^2 and L^2 respectively. The model can be formulated as [8]:

$$Y_k = D_k H_k F_k X + V_k \quad k = 1,2,\ldots,N \tag{1}$$

where D_k is the downsampling matrix of size $[M^2\times L^2]$, H_k is the blurring matrix of size $[L^2\times L^2]$ representing the ultrasound system's point spread function (PSF), F_k is the geometric warp matrix of size $[L^2\times L^2]$, V_k is the additive noise, and N is the number of available input LR images.

In [5], the author introduced that using a maximum a-posteriori (MAP) estimator of X to maximize the probability density function (PDF) $P(X|Y_k)$, taking the log function and Bayes rule to the conditional probability, and assuming the V_{k} is additive Gaussian noise with mean value of zero and variance of σ_k^2 , the conditional probability in (1) can be written as:

$$P(Y_1, Y_2, \ldots, Y_N \mid X) == \frac{1}{\left(2\pi\sigma^2\right)^{\frac{M}{2}}} \exp\left\{-\sum_{k=1}^{N} \frac{1}{2\sigma^2} \left\| Y_k - D_k H_k F_k X \right\|^2 \right\} \quad \mathbf{(2)}$$

The noise is assumed to be identically distributed with variance of σ^2, which is absorbed by the parameter $\lambda=1/2\sigma^2$. Letting $dE/dX=0$, the steepest descent (SD) algorithm is an efficient method to reach the solution X by the following iterative process until an iteration convergence criterion is met:

$$X_{i+1} = X_i - \mu\left[\lambda \sum_{k=1}^{N} F_k^T H_k^T D_k^T \left(D_k H_k F_k X_i - Y_k\right) - \nabla \bullet \left[C \nabla X\right]\right] \quad \mathbf{(3)}$$

where μ is the step size, which should be small enough. C is the diffusion coefficient of the anisotropic diffusion method [1]. The second term is the detail recovery term that combines information from different frames to update X, and the last term is a smoothing term, or regularization factor, that suppresses instability. The second term can be implemented by convolution with some appropriate kernels. In this paper, a faster and robust implementation is applied [9].

2.2 The Fast and Robust Implement

Before introducing the implementation, there are several assumptions:

- All the input images are the same size, and all the decimation operations are also the same, i.e., $\forall k,\ D_k=D$.
- The sequence input images are acquired by the same ultrasound system at the same depth so all the blur operations are assumed equal, i.e., $\forall k,\ H_k=H$. Moreover, the region-of-interest (ROI) is sufficiently small so that H is assumed to be linear space invariant (LSI), and the matrix H is block circulant.
- The ROI needed to reconstruct the result is small enough so that translational displacement at all points inside may be considered equal. A rigid registration will be applied for motion estimation [7]. Therefore, the matrices F_k are all block circulant and linear space invariant.

With these assumptions, H and F_k are block circulant matrices which commute ($F_kH=HF_k$ and $F_k^T H_k^T = H_k^T F_k^T$). So, the second term in (3) can be written as [10]:

$$\sum_{k=1}^{N} F_k^T H_k^T D_k^T \left(D_k H_k F_k X_i - Y_k\right) = H^T R_0 H X_i - H^T P_0 \quad \mathbf{(4)}$$

where

$$R_0 = \sum_{k=1}^{N} F_k^T D^T D F_k^T \ \text{ and } \ P_0 = \left(\sum_{k=1}^{N} F_k^T D^T Y_k\right) \quad \mathbf{(5)}$$

To compensate for potential outlier images in the sequence, we propose replacing the summation in (5) with a scaled pixel-wise median to increase robustness [9].

$$R_0 = N \cdot median\{F_k^T D^T D F_k^T\}_{k=1}^N \text{ and } P_0 = N \cdot median\{F_k^T D^T Y_k\}_{k=1}^N \quad \textbf{(6)}$$

Using (6), manually pre-selecting proper frames from a cine loop is no longer necessary. The improvement in (6) gives a fast and robust method to implement the updates of X corresponding to the second term in (3), but the results will be degraded by speckle noise. So the author adopted a Perona and Malik's AD [1] in their method to achieve an edge-enhancing regularization during the SR process [5].

3 Nonlinear Diffusion Method

After using above SR method to do image restoration, we get an enhanced and compounded ultrasound image from a sequence of ultrasound images. In the restored image, speckle has been compounded so that it is no longer a granular pattern. It turns to be much smaller. To reduce the image noise in the restored ultrasound image, we propose to use a local coherence based ND method. Local coherence is defined from structure matrix, it is as,

$$\begin{pmatrix} I_x^2 & (I_x I_y) \\ (I_x I_y) & I_y^2 \end{pmatrix} \quad \textbf{(7)}$$

where I_x and I_y are partial derivatives at certain point in the image, before compute structure matrix at each point, we convolute the image I with a Gaussian mask. Using eigenvalue decomposition, (7) could be written as

$$J(I) = (w_1 \quad w_2)\begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix}\begin{pmatrix} w_1^T \\ w_x^T \end{pmatrix} \quad \textbf{(8)}$$

where u_1 and u_2 are eigenvalues, w_1 and w_2 are eigenvectors.

Our diffusion equation is as

$$\frac{\partial I}{\partial t} = div[l \cdot \nabla I] \quad \textbf{(9)}$$

Where l is the local coherence function, the independent variable of l is the absolute value of the difference of two eigenvalues, the definition of l is,

$$l(|\mu_1 - \mu_2|) = 1/1 + (|\mu_1 - \mu_2|/K)^2 \quad \textbf{(10)}$$

where μ_1 and μ_2 are the eigenvalues of structure matrix at each point, structure matrix is as $J(\nabla I_\sigma) = (\nabla I_\sigma \cdot \nabla I_\sigma^T)$. Here, ∇I_σ is the gradient of a smoothed version of I which is obtained by convolving I with a Gaussian of standard deviation σ. Converting

(9) to a matrix-vector notation and adopting AOS scheme, (9) comes down to the following iteration scheme

$$I^{t+1} = \frac{1}{2}\sum_{l=1}^{2}[U + 2\tau T_l(I^t)]^{-1} I^t \tag{11}$$

where U is a *KL* by *KL* identity matrix (the size of image is $K \times L$), τ is the time step size (TSS), $T_l(I^t)$ is a tridiagonal matrix, with $T_l(I^t) = [t_{ij}(I^t)]$ and

$$t_{ij}(I^t) = \begin{cases} l_j^t / h^2 & [j \in \mathrm{N}(i)], \\ -\sum_{n \subset \mathrm{N}(i)} l_n^t / h^2 & (j = i), \\ 0 & (\text{else}). \end{cases} \tag{12}$$

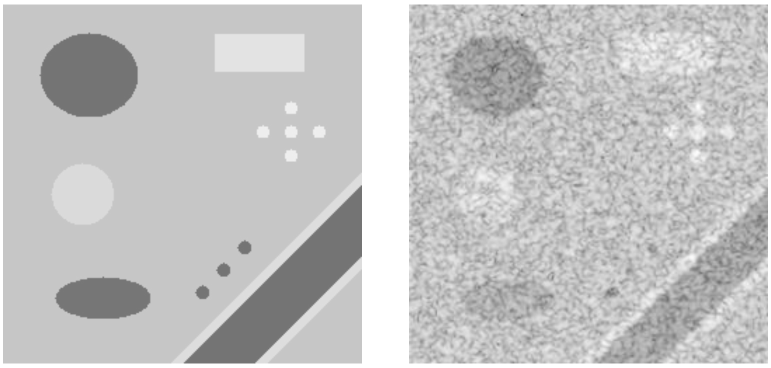
where l is the diffusion coefficient. $\mathrm{N}(i)$ is the set of the two neighbors of pixel i. From the criteria for discrete nonlinear diffusion scale-spaces proposed by Weickert [6], we find that (9) can by discretized by AOS scheme to do speedup.

4 Experiment and Results

To evaluate the effectiveness of the proposed method, we conducted simulation and *in vivo* experiments. In the experiment of simulated ultrasound image speckle reduction, we analyzed different kinds of AD methods quantitatively. In each experiment, we compared the proposed method with Parona and Malik's method (P&M AD) [1], SRAD [2] and semi-implicit scheme based nonlinear diffusion (SIND) [4].

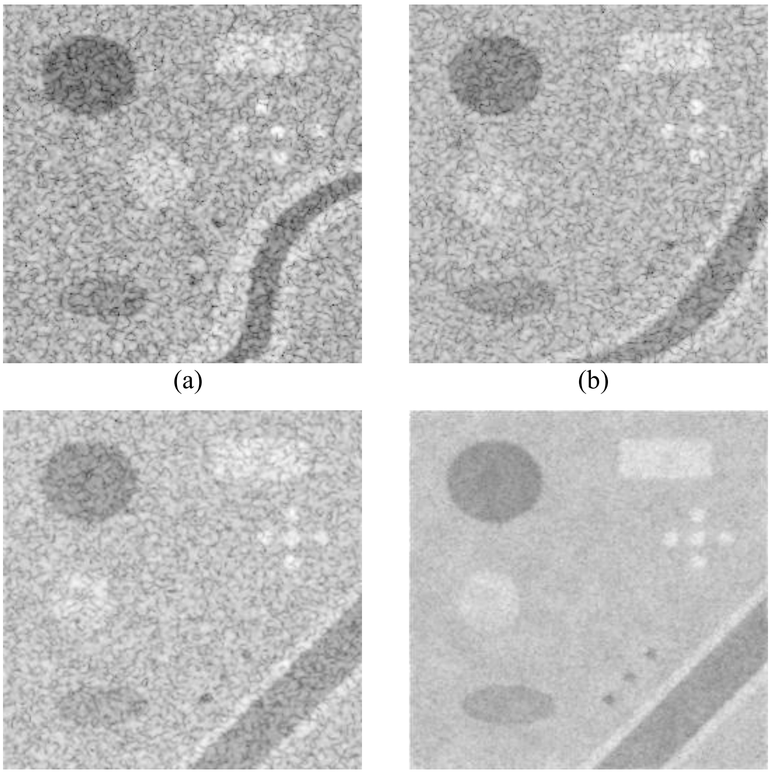
4.1 Results of Simulation Experiment

In this experiment, we firstly simulated ultrasound B mode image as [2, 11]. In this method, ultrasound radio-frequency (RF) image was generated by convoluting a 2-D point spread function with a 2-D echogeneity image. The parameters used in our simulation are: the pulse width was 1.2, the lateral beam width was 1.5, the center frequency was 5 MHz. The size of echogeneity image was 256×256. The gray values of different objects in the echogeneity image are: the dark circular at top left corner was 2, the rectangular target at top right corner was 25, the bright small circular at middle left area was 18, the dark circular at bottom left corner was 4, the simulated artery interior was 3, the simulated vascular wall was 20, the three small cysts were 2, the five bright point targets were 40, the background was 10. The following are the echogeneity image and simulated ultrasound image (both images are log-compressed and normalized with the same way for better displaying).



(a) (b)

Fig. 1. Ultrasound B mode image simulation result. (a) The echogeneity image. (b) The generated ultrasound simulated image



(a) (b)

(c) (d)

Fig. 2. The result of SR method on a sequence of 16 ultrasound B mode simulated images. (a)-(b) two outliers generated by different echogeneity images. (c) One simulated image generated by the echogeneity image as figure 1.a. (d) The result of the SR method

Because in the SR method we needed to use a sequence of ultrasound images to generate one restored image, so we used the simulated ultrasound image in figure 1.b as one in the set of simulated ultrasound images needed for restoration. Moreover, we generated another 15 simulated ultrasound images. Nine of them are generated by using the same echogeneity image as figure 1.a. To generate the other six simulated outliers, we used six different echogeneity images as outliers. Two of the simulated outliers are showed as figure 2.a - 2.b. The fast SR method we used was about six times than traditional SR methods. In the SR method, the size of final restored image was as, $M_{new} \times N_{new} = k \cdot (M_{old} \times N_{old})$, where k is the square root of the number of images in the data set. In our experiment, we had 16 simulated images in the data set, so the size of the restored image is 4 times of the size of original ultrasound simulated images. In addition, the authors used a Gaussian kernel instead of PSF in the blurring processing [5]. The result of SR method is as figure 2.d.

The following are the filtering results of the proposed method and some other AD methods.

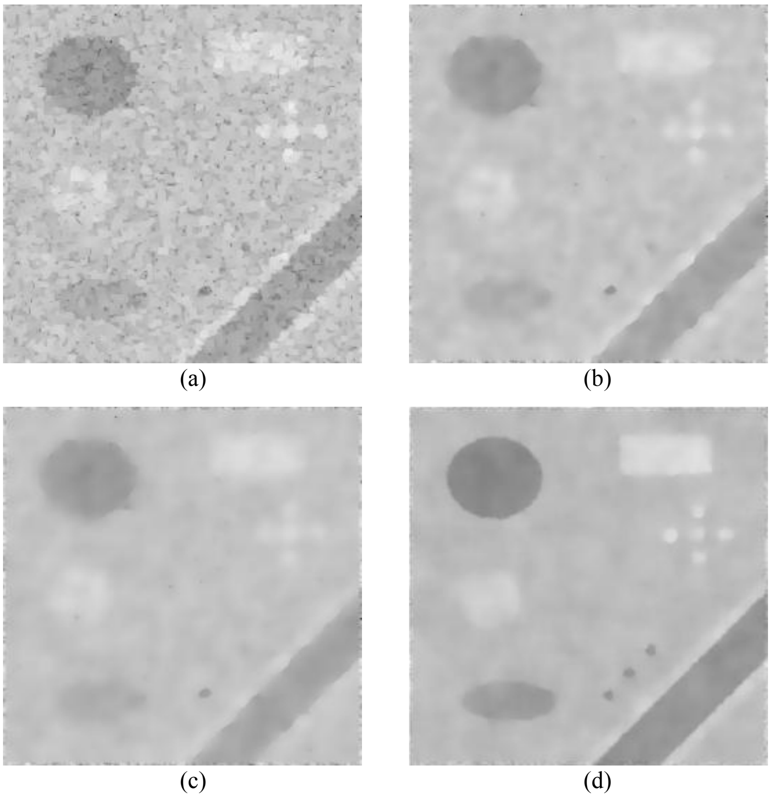


Fig. 3. The filtering results of the proposed method and some other AD methods. (a) The result of P&M AD, TSS = 0.25, 30 iterations. (b) The result of SRAD, TSS = 0.25, 25 iterations. (c) The result of SIND, TSS = 1.5, 5 iterations. (d) The result of the proposed method, TSS = 1.5, 5 iterations

In order to test the performance of the proposed method, two metrics were computed. The first one is mean squared error (MSE), it is defined as,

$$\mathrm{MSE} = \frac{1}{M \times N} \sum_{(i,j)=1}^{M \times N} (\hat{S}(i,j)) - S(i,j))^2 \tag{13}$$

where S and $\hat{S}$ are the reference and filtered images respectively.

The second metric is contrast-to-noise-ratio (CNR), which is sometimes referred as lesion signal-to-noise ratio [11], it is,

$$\mathrm{CNR} = \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} \tag{14}$$

where μ_1 and σ_1^2 are the mean and variance of intensities of pixels in a region of interest (ROI), and μ_2 and σ_2^2 are the mean and variance of intensities of pixels in a background region.

The computed values of MSE are summarized in Table 1. And obtained values of CNR are summarized in Table 2.

Table 1. MSE values on the simulated image

Method	MSE
Noisy	262.56
P&M AD	137.41
SRAD	23.96
SIND	26.55
Proposed method	17.83

Table 2. CNR values on the simulated image

Method	ROI 1	ROI 2	ROI 3	ROI 4
Noisy	3.17	1.90	1.29	1.46
P&M AD	4.19	2.65	1.61	1.92
SRAD	6.21	5.74	3.24	3.16
SIND	5.91	6.34	3.82	3.75
Proposed method	7.61	7.48	4.29	4.87

From the values of MSE and CNR on different diffusion-based filtering methods, we can see that the proposed SR based ND method is much better than some other AD methods.

4.2 Results of In Vivo Experiment

The following filtering results of *in vivo* ultrasound image show that the proposed method can preserve structure details and edges more accurately than other AD methods. The SR method improved original ultrasound B mode image greatly by compounding speckle and enhancing structure and edge through introducing complementary information in data set. In this way, the proposed ND method can get a much better result than other non-image-restored diffusion-based filtering methods.

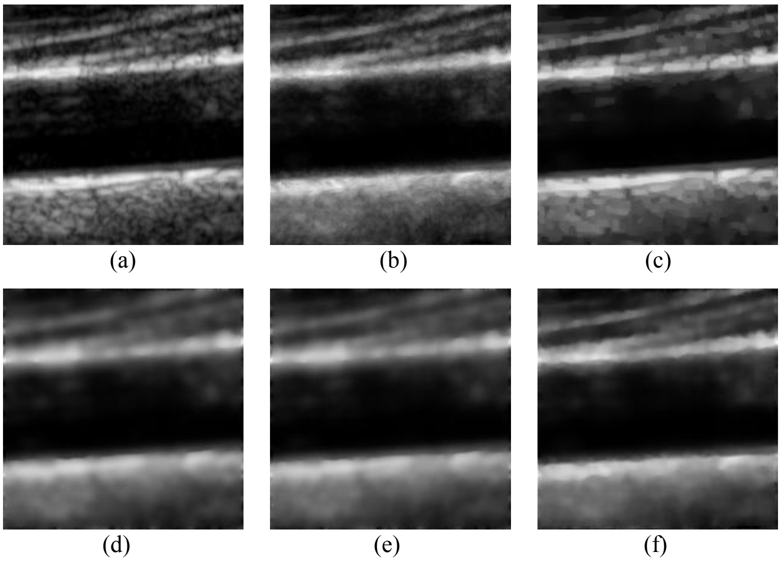


Fig. 4. The results of the proposed method and some other diffusion-based filtering methods on *in vivo* ultrasound image. (a) The original ultrasound image. (b) The result of the SR method. (c) The result of P&M AD, TSS = 0.25, 30 iterations. (d) The result of SRAD, TSS = 0.25, 20 iterations. (e) The result of SIND, TSS = 1.5, 3 iterations. (d) The result of the proposed method, TSS = 1.5, 3 iterations

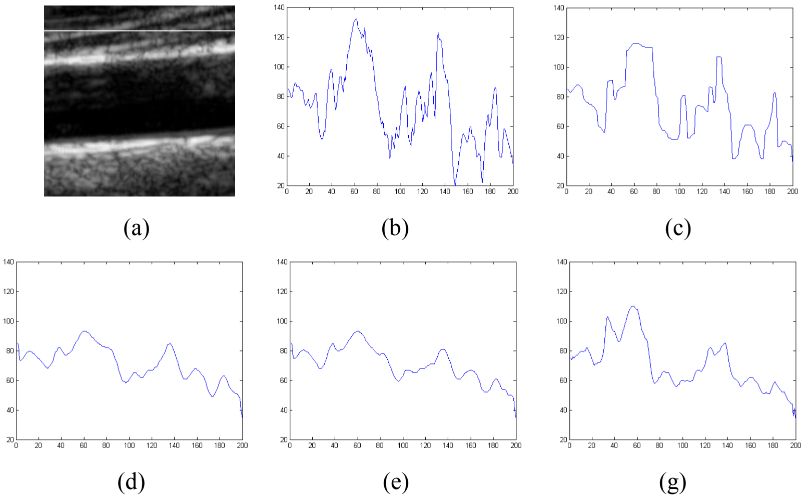


Fig. 5. Profiles along the highlight line in ultrasound image. (a) Original image showing the highlight line. (b) Profiles along the highlight line in the original image (c)-(g) Profiles along the highlight line in the images processed by P&M AD, SRAD, SIND, the proposed method

4 Conclusion

In this paper we present a new SR based nonlinear ND method for speckle reduction and experimental evaluation. From the experimental results we can see that the proposed method can reduce ultrasound speckle noise more exactly than other AD methods. Our new method can also preserve and enhance edge and structure details much better. The reason is that we combine the SR method and the ND method. We firstly improve the ultrasound image quality by the SR method. On the basis of this process, we propose a local coherence ND method to reduce the image noise of the restored image. In this way, we get a much better filtering result than other diffusion-based filtering methods.

To test the proposed method, in the future, we would like to do more experiments in different *in vivo* ultrasound images. Moreover, because the speed of the SR method we used is still not fast enough for real-time application, we would like to explore the possibility to use parallel processing to do speedup. In addition, registration in the SR method is very important, inaccurate registration will result in a poor image restoration. So, improving the registration method in the SR method is another future work.

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