Communication systems

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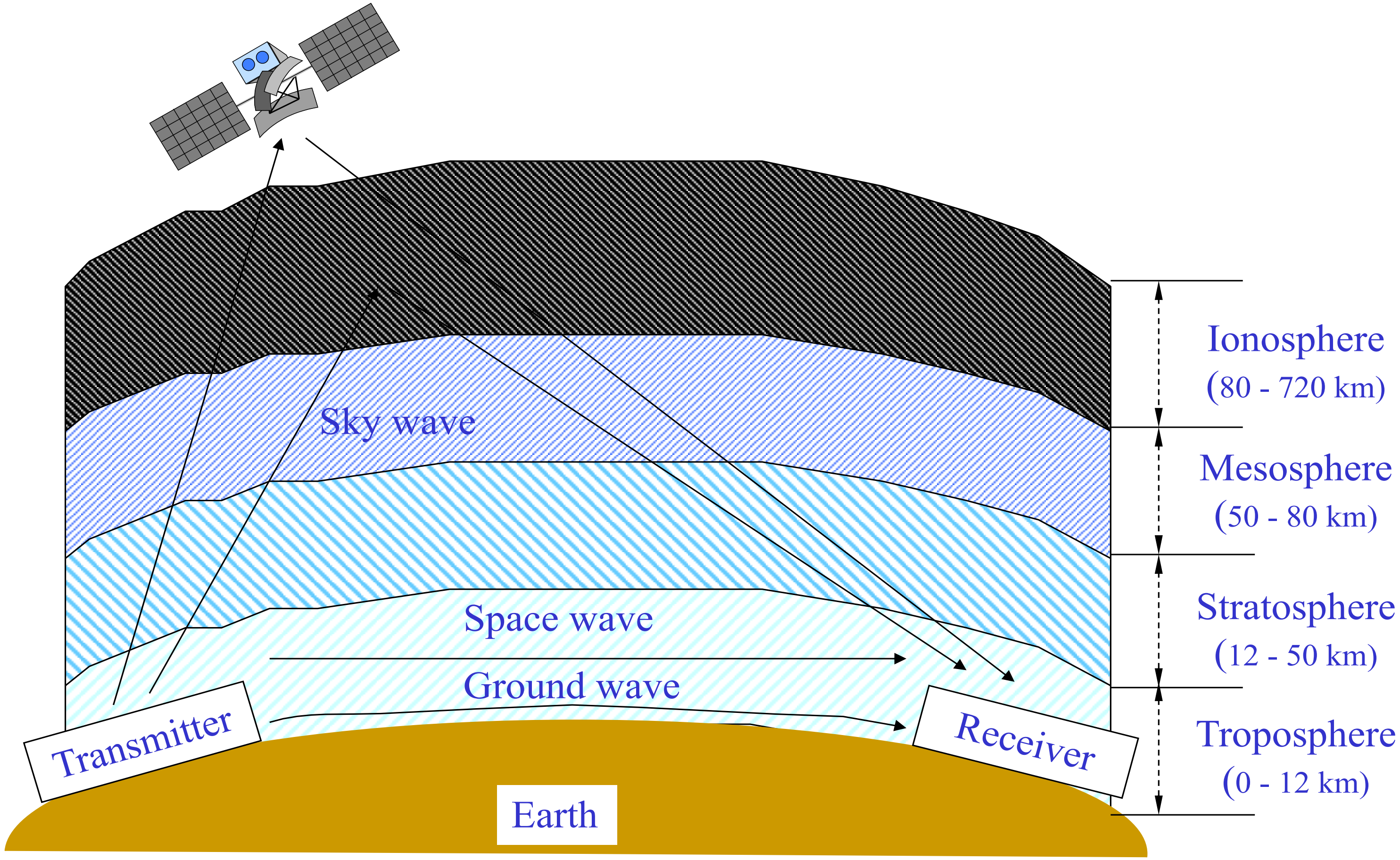
ELECTRONICS AND COMMUNICATIONS SYSTEMS

COMPUTER ENGINEERING

Wireless propagation channel

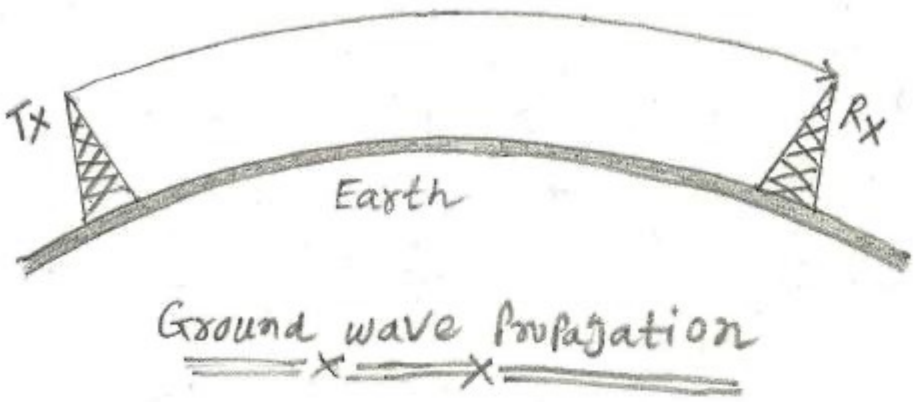
Signal propagation in the air

Classification Band	Initials	Frequency Range	Characteristics
Extremely low	ELF	< 300 Hz	Ground wave
Infra low	ILF	300 Hz - 3 kHz	
Very low	VLF	3 kHz - 30 kHz	
Low	LF	30 kHz - 300 kHz	
Medium	MF	300 kHz - 3 MHz	Ground/Sky wave
High	HF	3 MHz - 30 MHz	Sky wave
Very high	VHF	30 MHz - 300 MHz	Space wave
Ultra high	UHF	300 MHz - 3 GHz	
Super high	SHF	3 GHz - 30 GHz	



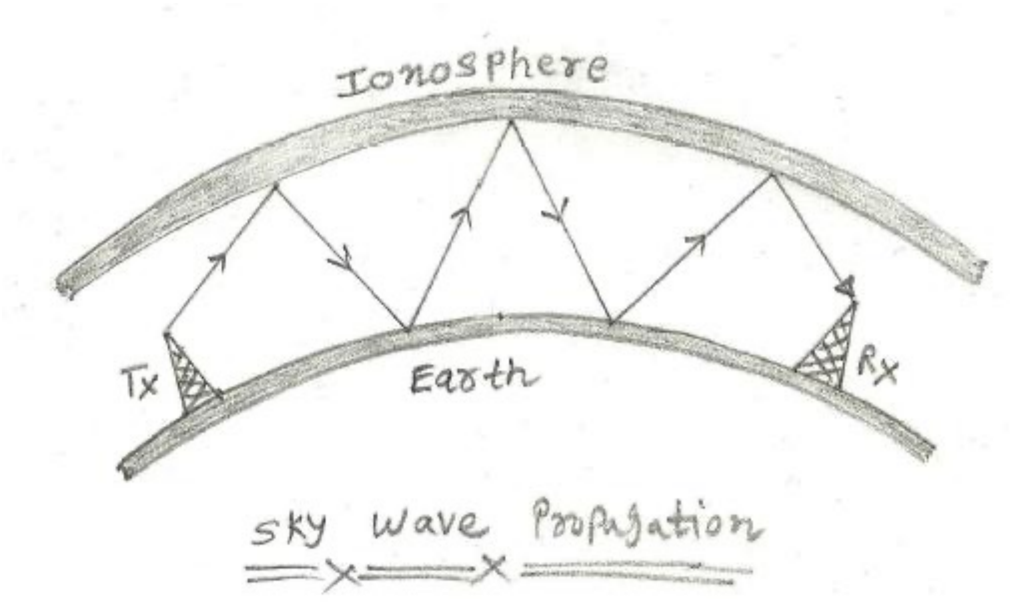
Ground wave propagation

- The wave propagates following earth’s curvature reaching receivers beyond the horizon (in certain cases hundreds of km).
- Valid for frequencies below 2 MHz, LF-MF bands



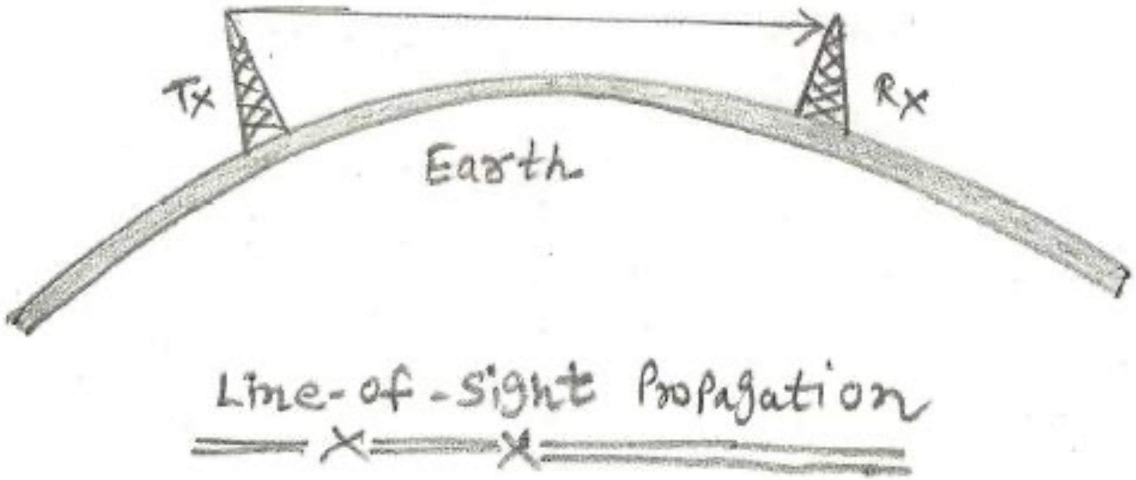
Sky wave propagation

- For a certain ranges of frequency (around 10 MHz) the ionosphere acts as a mirror and reflects the signals back to the earth.
- Bouncing between earth and the ionosphere the signal can propagate up to a few thousands of km.
- Valid mainly for HF.



Space wave propagation

- For frequencies larger than 30 MHz, the main form of propagation is line-of-sight.
- The received signal is composed by the direct component plus the paths reflected by nearby obstacles.
- The larger the frequency the larger is the propagation attenuation.



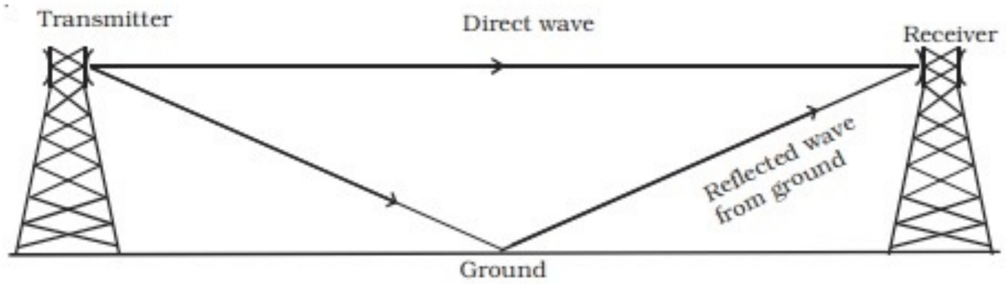
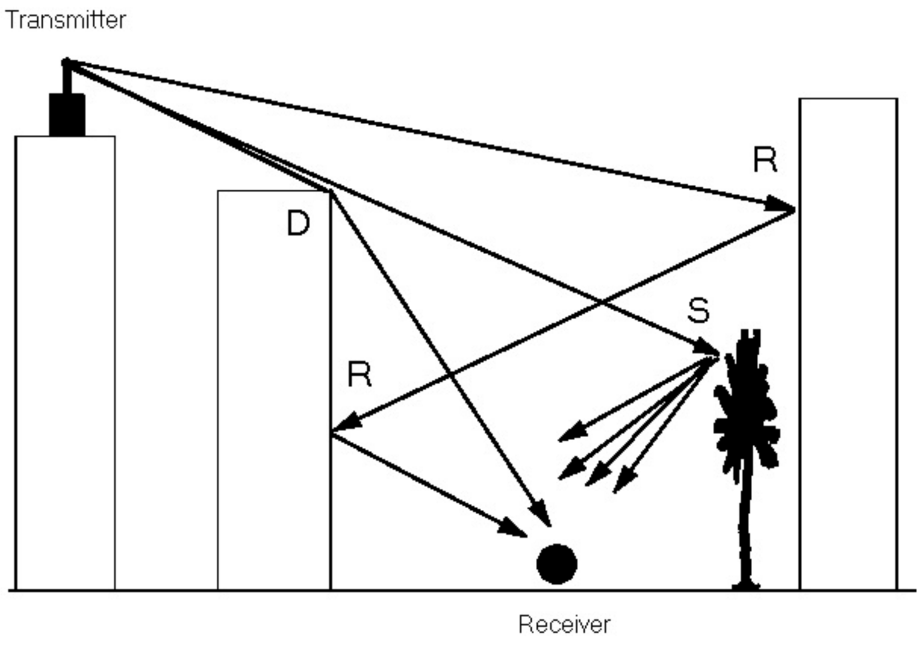


Fig Space wave propagation

The wireless propagation channel (space wave)

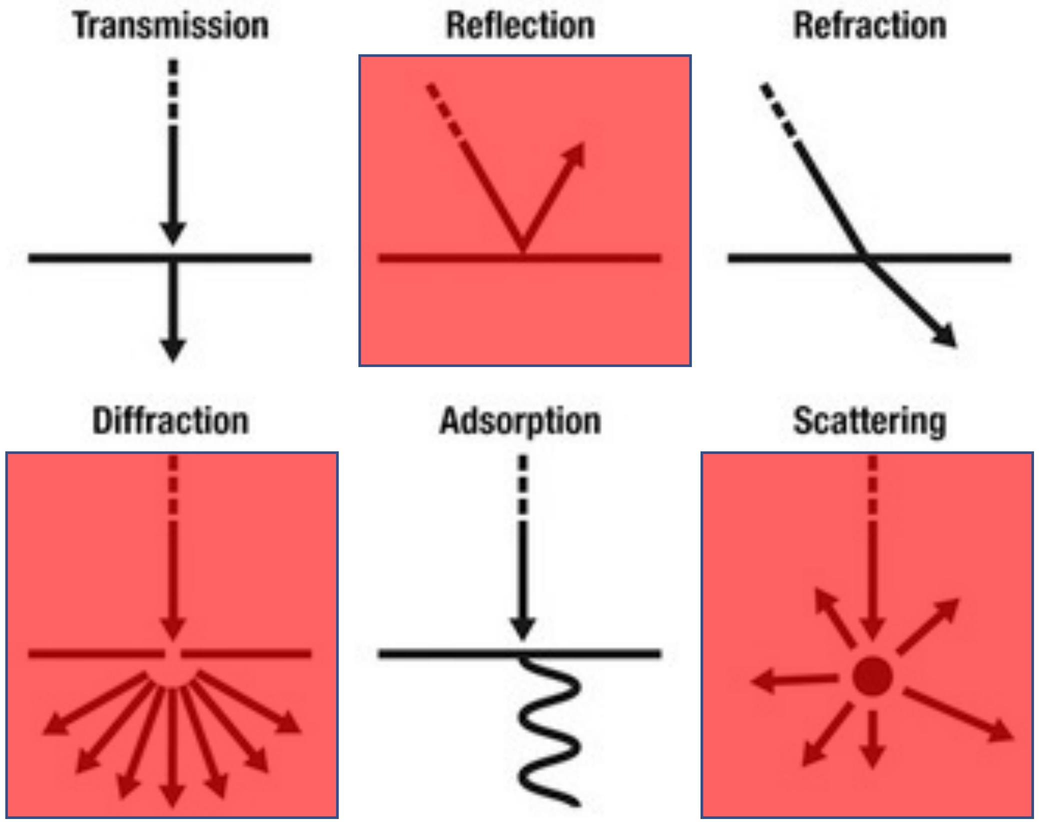
- Because, mobile services are mostly in the bandwidth 30MHz-30 GHz, *spacewave* is the most important wave propagation mechanism we need to consider.
- The main physical phenomena are: reflection, diffraction, scattering.
- The effect of the combination of these propagation phenomena can be summarized into *large-scale* and *small-scale* fading.



Reflection (R), diffraction (D) and scattering (S).

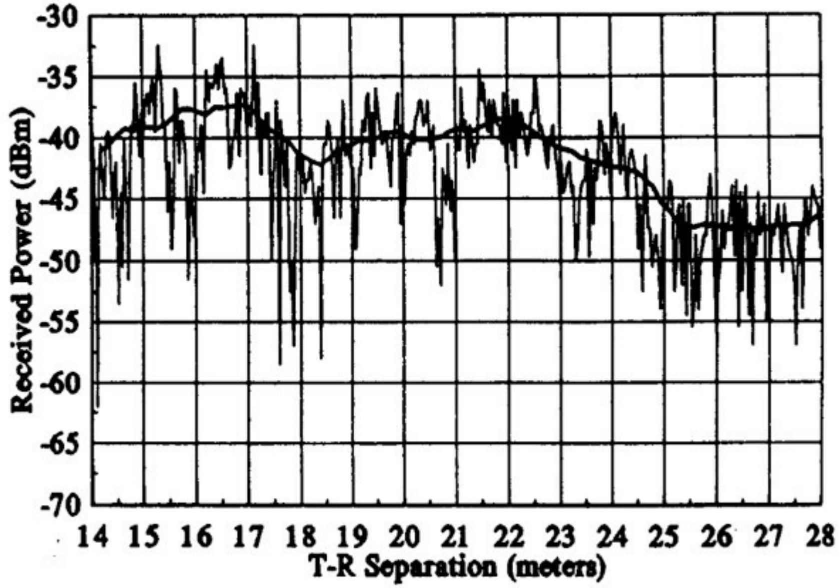
Propagation phenomena

- Three major propagation mechanisms:
  - **Reflection**
    Signal impinges on very large (w.r.t. to signal wavelength) objects. When a wave meets a boundary, it can be either reflected or transmitted.
  - **Diffraction**
    Signal is obstructed by objects that have sharp irregularities. Diffraction depends on the size of the object relative to the wavelength of the wave.
  - **Scattering**
    Propagation medium populated by small (wrt to signal wavelength) objects or rough surfaces (e.g. foliage, street signs).



Large-scale fading

- Large-scale fading: propagation models that characterize average signal strengths over Tx-Rx separation distance.
- Accounts for averaged received power, changes over distances $\approx$ 1 m.
- Large-scale fading can be modelled as the combination of *path-loss* and *shadowing*.



Large-scale fading: path-loss

- Path-loss models simplify Maxwell's equations.
- Models vary in complexity and accuracy but, in general, mean power falloff w.r.t. the tx-rx distance d is proportional to d^2 in free space and to d^n in other environments.
- Considering only path-loss, the average received signal power is

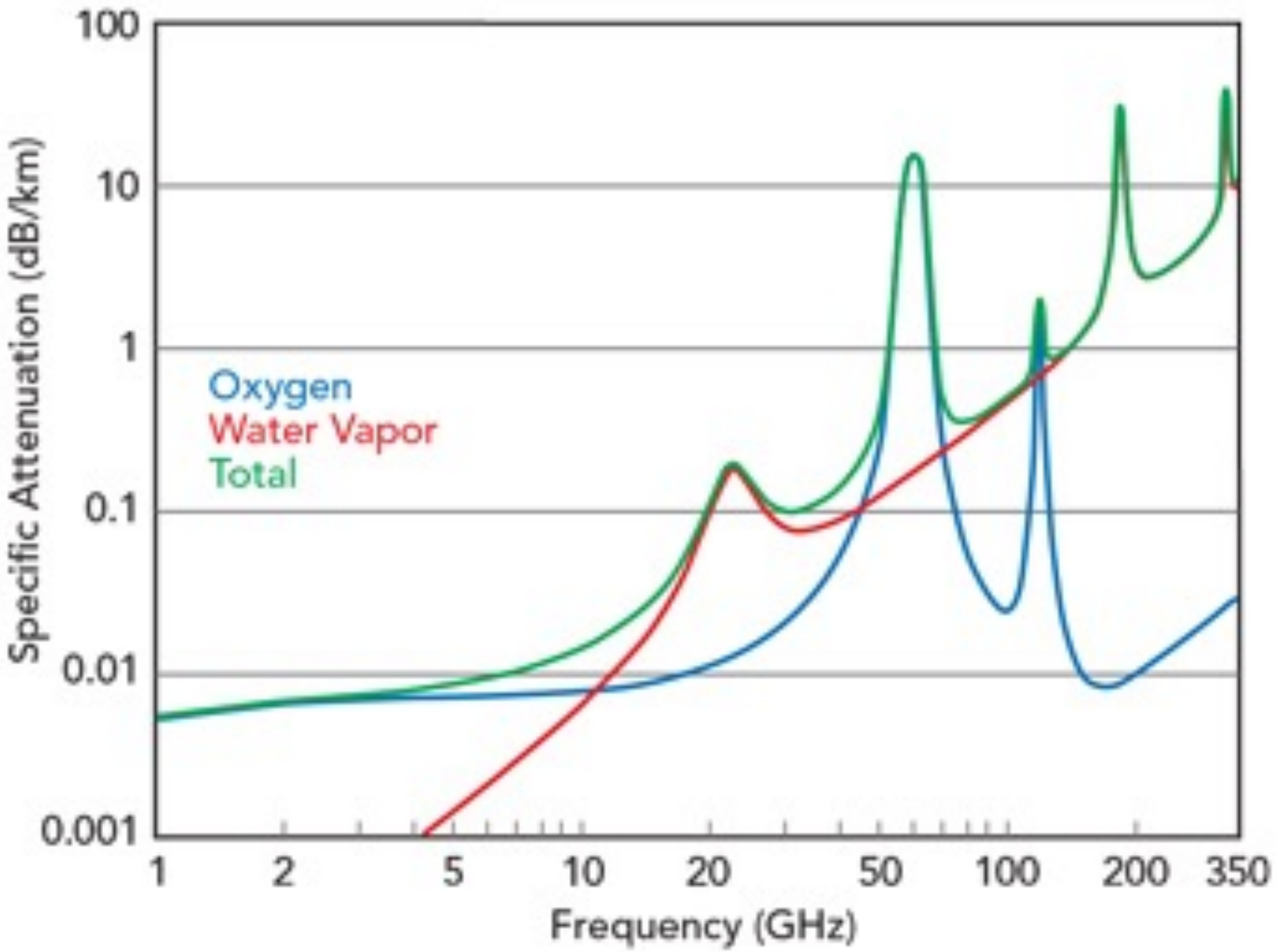
$$P_{Rx} \propto P_{Tx} \underbrace{\Gamma(f_0, d_0) \left(\frac{d_0}{d}\right)^{n}}_{\text{path-loss}} \qquad d > d_0$$

- *Near field term* $\Gamma(f_0, d_0) \approx \left(\frac{\lambda}{4\pi d_0}\right)^2$

Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 – 3.5
Urban area cellular (obstructed)	3 – 5
In-building line-of-sight	1.6 – 1.8
Obstructed in-building	4 – 6
Obstructed in-factories	2 – 3

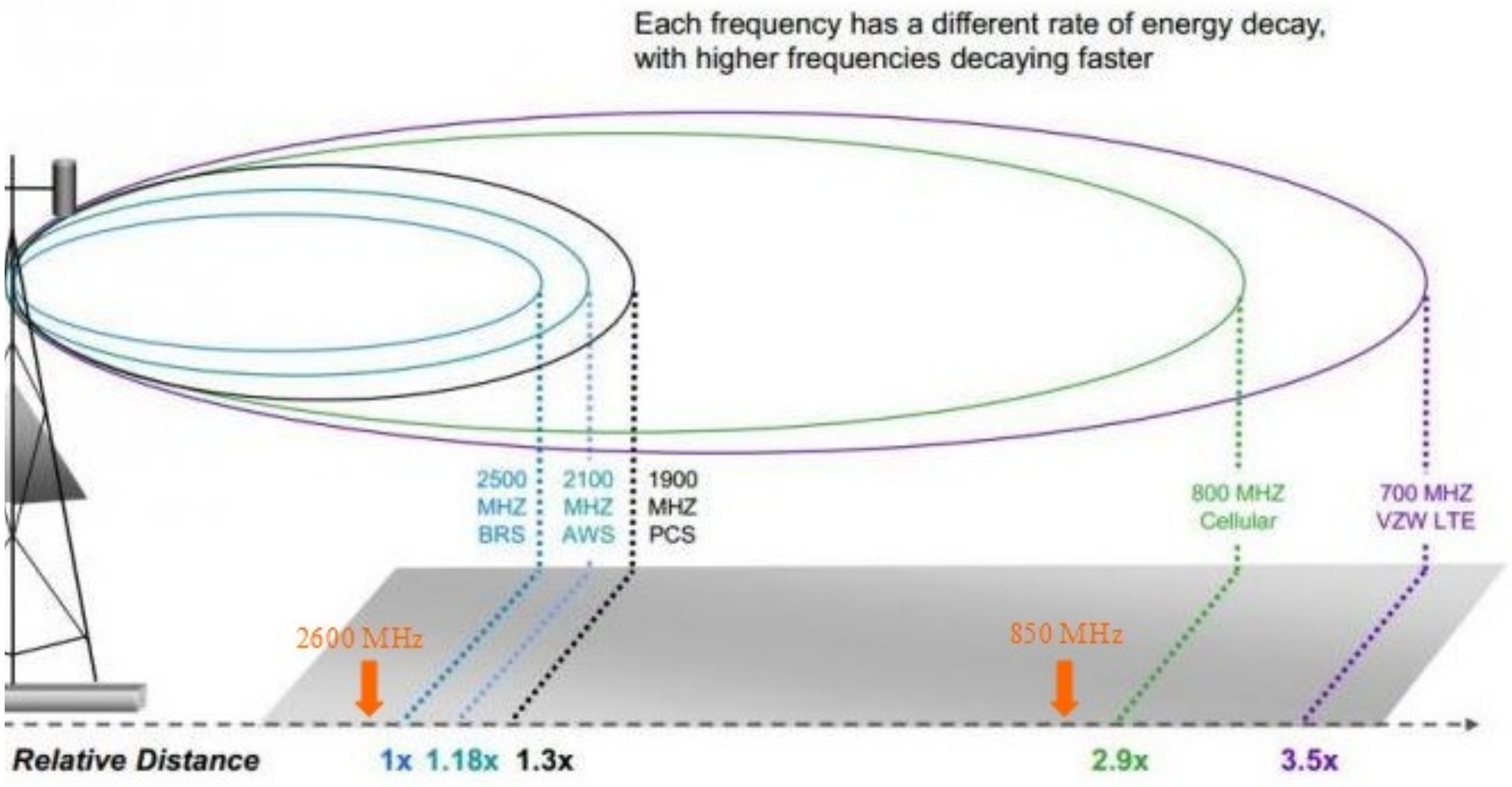
Large-scale fading: attenuation due to frequency

- At frequencies $f_c < 6$ GHz ($\lambda > 50$ mm) the channel attenuation depends on the carrier frequency with a square law.
- At larger frequencies the channel attenuation depends on other physical phenomena such as oxygen and water vapor absorption.
- Mmwave channel is very much attenuated!



Path-loss and cell size

- At cell borders path-loss attenuation may exceed 100 dB.
- The larger the carrier frequency the larger is the attenuation and the smaller is the cell radius.
- Large cells, designed for *coverage*, use low carrier frequencies, small or very small cells, designed to boost *capacity*, use mmwave frequencies.



Large-scale fading: shadowing

- Two points with the same distance d from the transmitter have theoretically the same path-loss, nevertheless their average attenuation may still greatly differ.
- *Shadowing* accounts for the random variations of the average channel attenuation.
- Shadowing A_S is a random variable *log-normally distributed* with parameters $\mu = 0$ and σ_S, expressed in dB. The pdf in dB of A_S is

$$p(A_S) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{A_S^2}{2\sigma_S^2}}$$

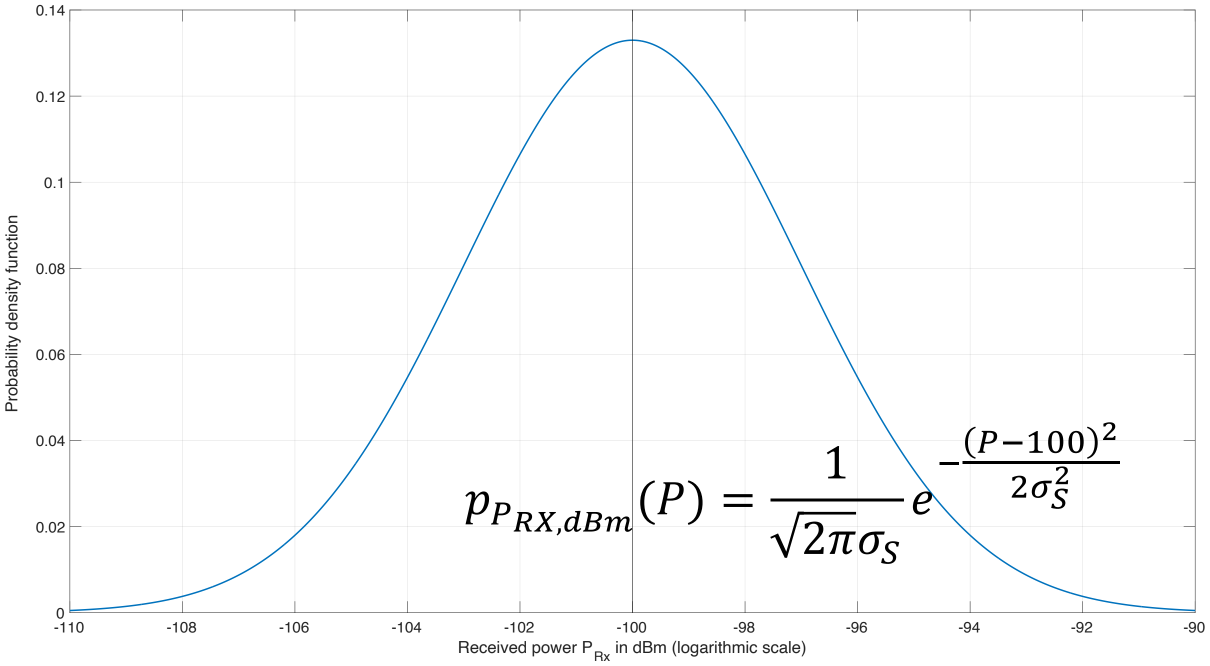
where σ_S is the standard deviation in decibels (typical values 0-9 dB)

Large-scale fading: shadowing

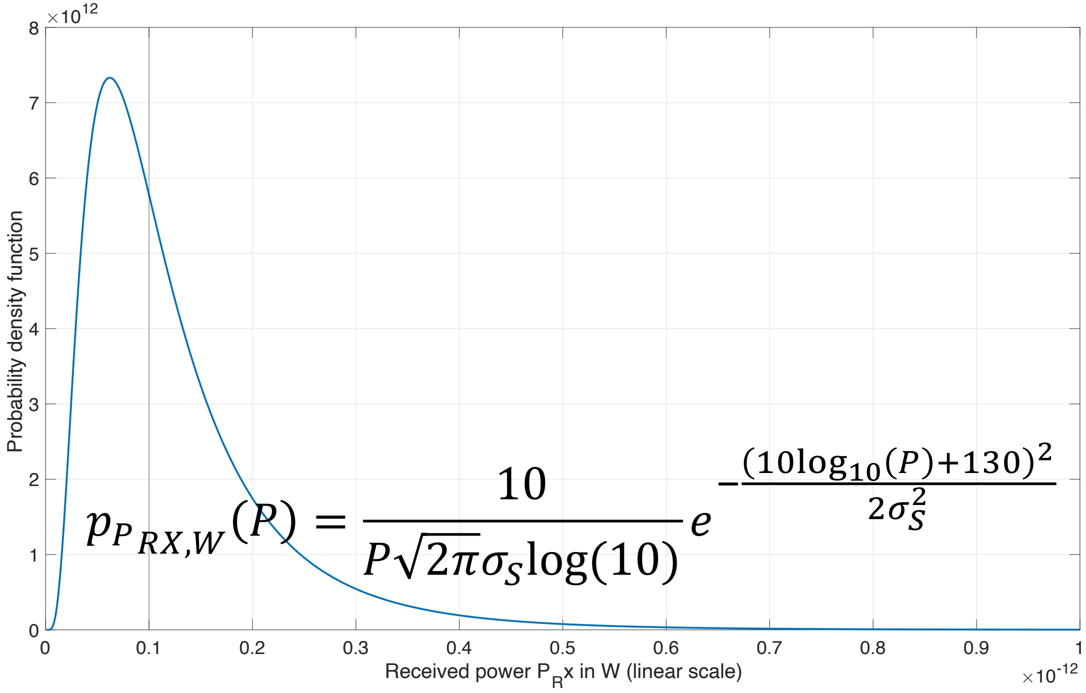
- Let's consider a channel with path-loss and shadowing ($\sigma_S = 3\ \mathrm{dB}$) only. The received power P_{RX} is

$$P_{RX} = P_{TX} A_{PL} A_S$$

- Assume that $P_{TX} A_{PL} = -100\ \mathrm{dBm} = -130\ \mathrm{dBW} = 10^{-13}\ \mathrm{W}$.
- Because of shadowing, P_{RX} is a random variable and its probability density function in dBm and linear scale is



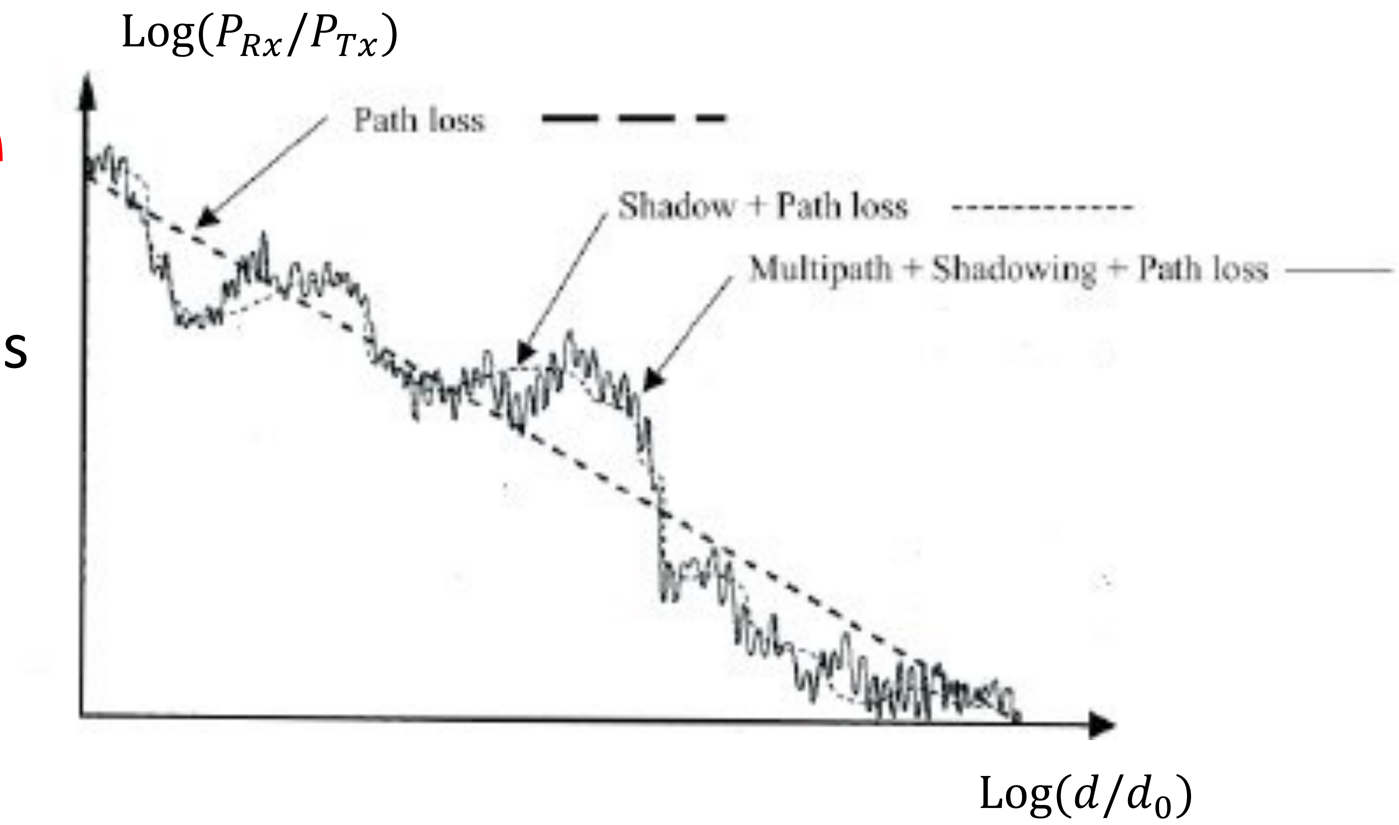
$$p_{P_{RX,dBm}}(P) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{(P-100)^2}{2\sigma_S^2}}$$

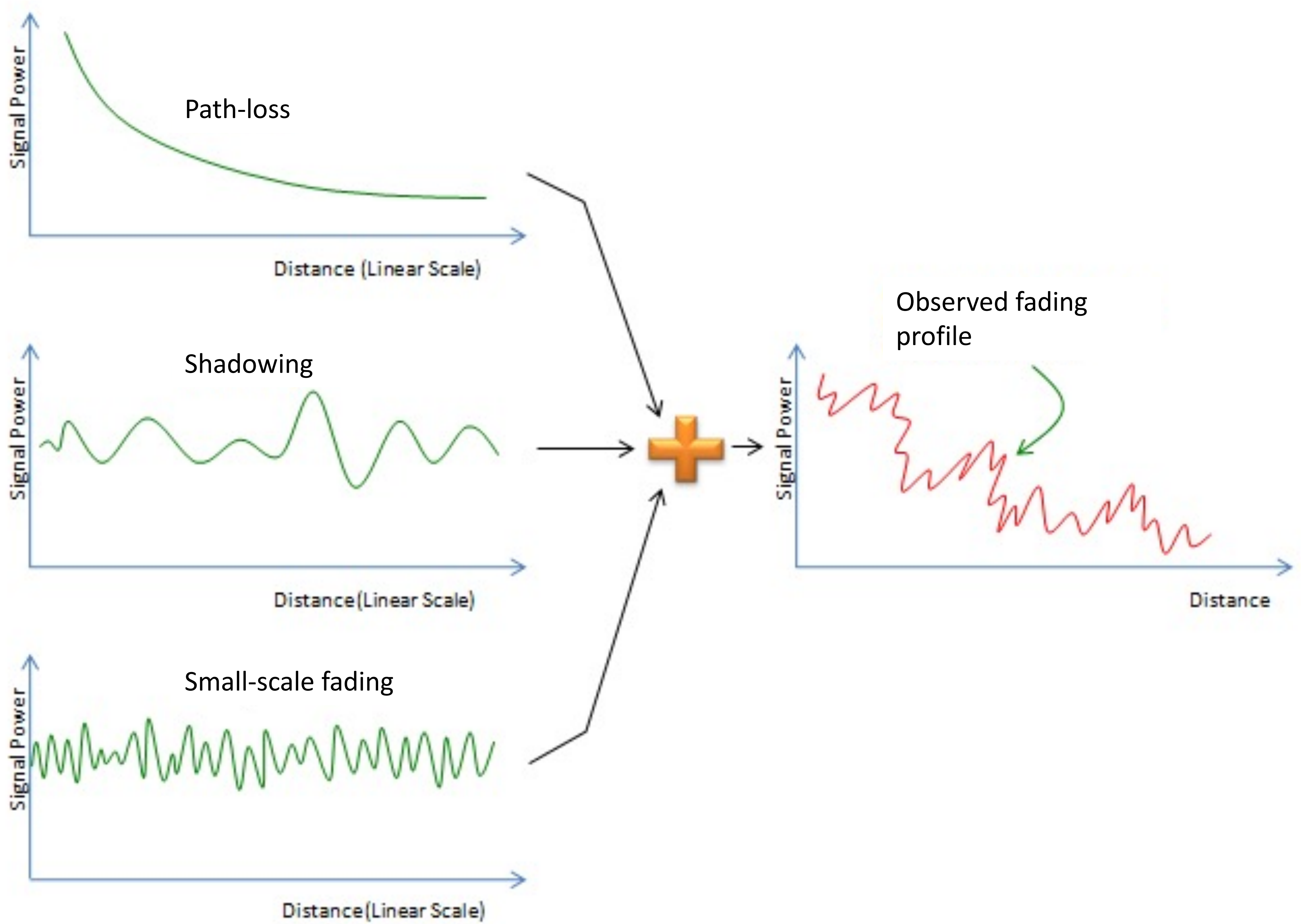


$$p_{P_{RX,W}}(P) = \frac{10}{P\sqrt{2\pi}\sigma_S \log(10)} e^{-\frac{(10\log_{10}(P)+130)^2}{2\sigma_S^2}}$$

Large-scale fading

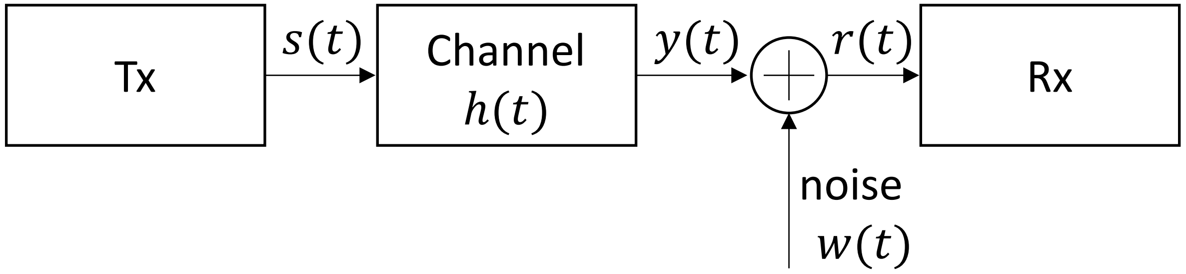
- The received power in dB is computed as
  $$P_{Rx}[dBm] = P_{Tx}[dBm] + \overbrace{A_{PL}\,[dB] + A_S[dB]}^{A_{LS}} + A_{SS}[dB]$$
- A_{PL} is deterministic and depends on the distance d.
- A_S is a log-normally distributed random variable.
- A_{SS} is the attenuation due to small scale fading, which fluctuates rapidly with the distance.




Path-loss
Signal Power
Distance (Linear Scale)
Shadowing
Signal Power
Distance(Linear Scale)
Small-scale fading
Signal Power
Distance(Linear Scale)
Observed fading profile
Signal Power
Distance

Propagation channel: small scale fading

- The propagation channel can be modeled as a LTI filter.

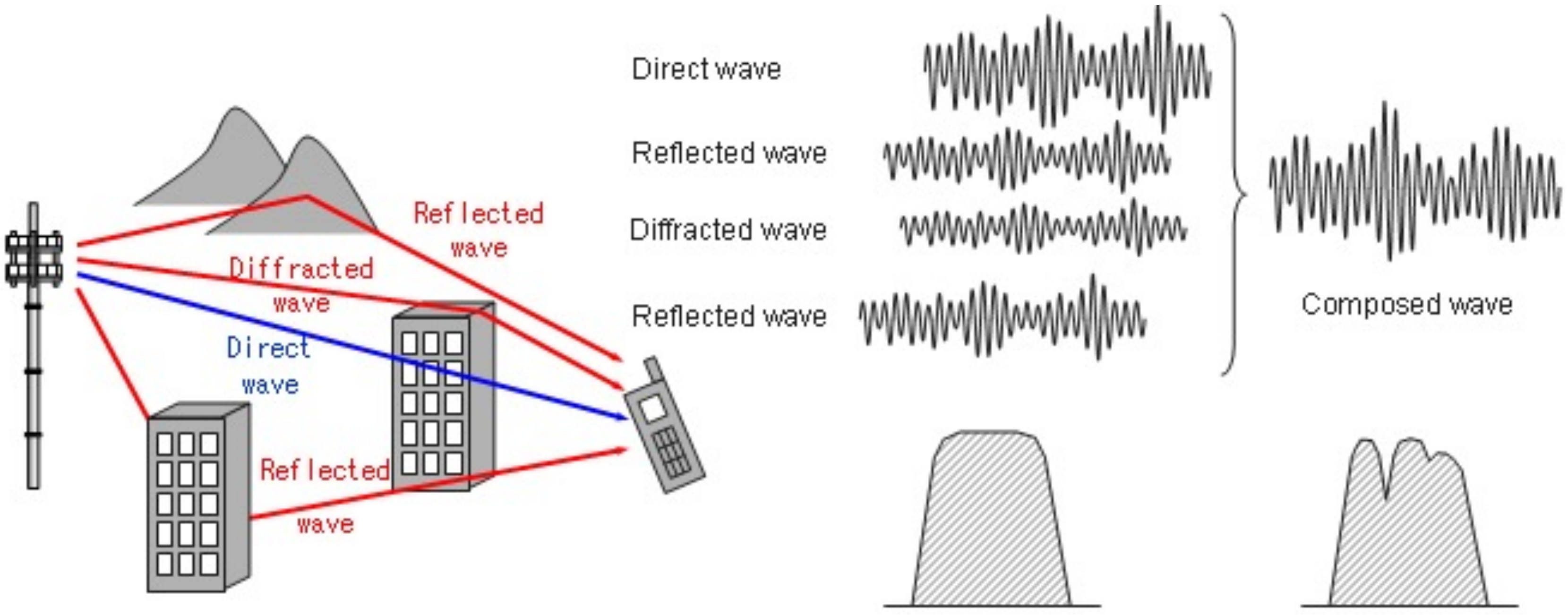


- The filter impulse response $h(t)$ depends on the *small-scale fading* characteristics.

$$y(t) = s(t) \otimes h(t)$$

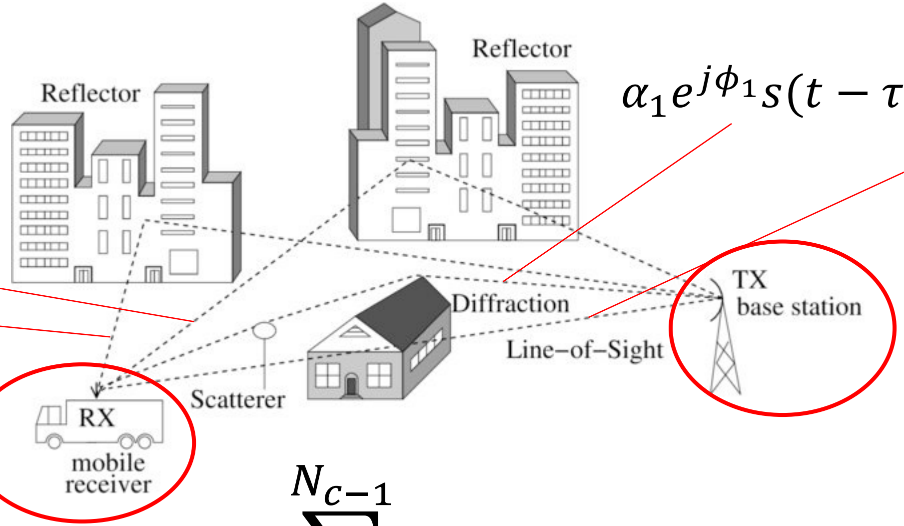
Small-scale fading

- Small-scale fading: accounts for the random variations of the instantaneous received power over distances of the order of a wavelength.
- Because of the various propagation phenomena, a large number of waves, each carrying a replica of the transmitted signal, arrives at the receiver.



Small-scale fading

- Neglecting the noise, the complex envelope of the signal at the receiver is the sum of delayed replicas of the transmitted signal $s(t)$, each with its own delay phase and attenuation, i.e.



$\alpha_1 e^{j\phi_1} s(t-\tau_1)$

$\alpha_0 e^{j\phi_0} s(t-\tau_0)$

$\alpha_2 e^{j\phi_2} s(t-\tau_2)$

$\alpha_3 e^{j\phi_3} s(t-\tau_3)$

$y(t)$

$s(t)$

$$y(t) = A_{LS} \sum_{\ell=0}^{N_{c-1}} \alpha_\ell e^{j\phi_\ell} s(t-\tau_\ell)$$

Small-scale fading: Rayleigh distribution

- Considering that a τ-delayed replica can be seen as the convolution of $s(t)$ with $\delta(t-\tau)$, the impulse response of the channel can be represented as

Multipath

$$h(t) = A_{LS} \sum_{\ell=0}^{N_c-1} \alpha_\ell e^{j\phi_\ell} \delta(t-\tau_\ell)$$

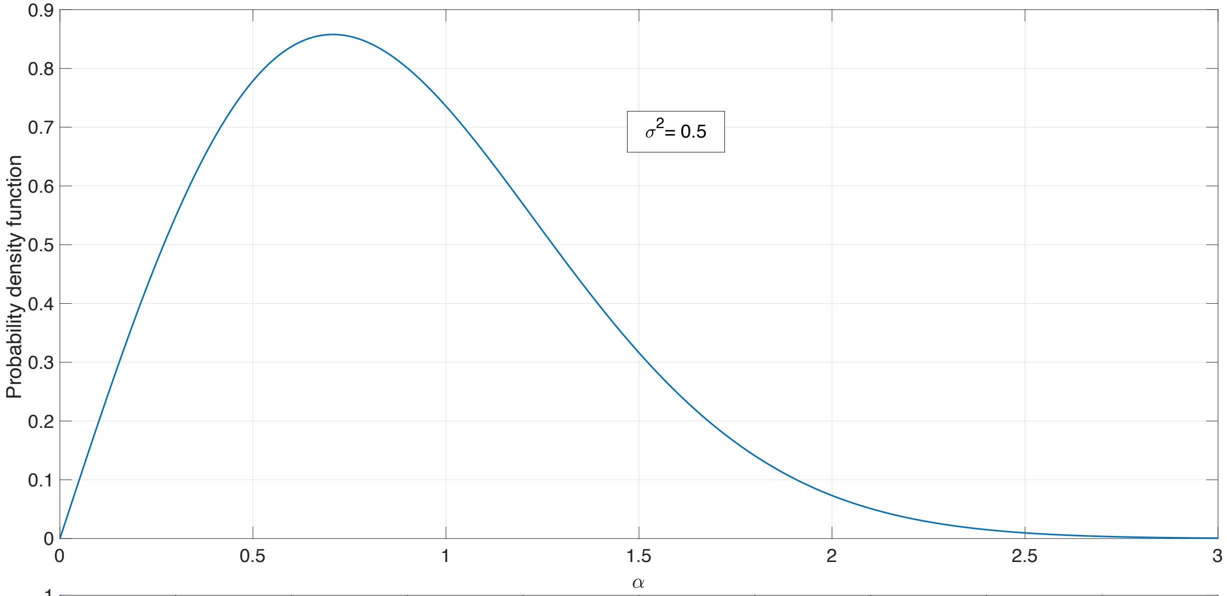
Path-loss and shadowing

  - The path gains α_ℓ are modeled as random variables. In most cases, they follow a Rayleigh distribution.
  - The path phases ϕ_ℓ are modelled as uniformly distributed variables in the interval $[0,2\pi]$.

Channel gain characterization

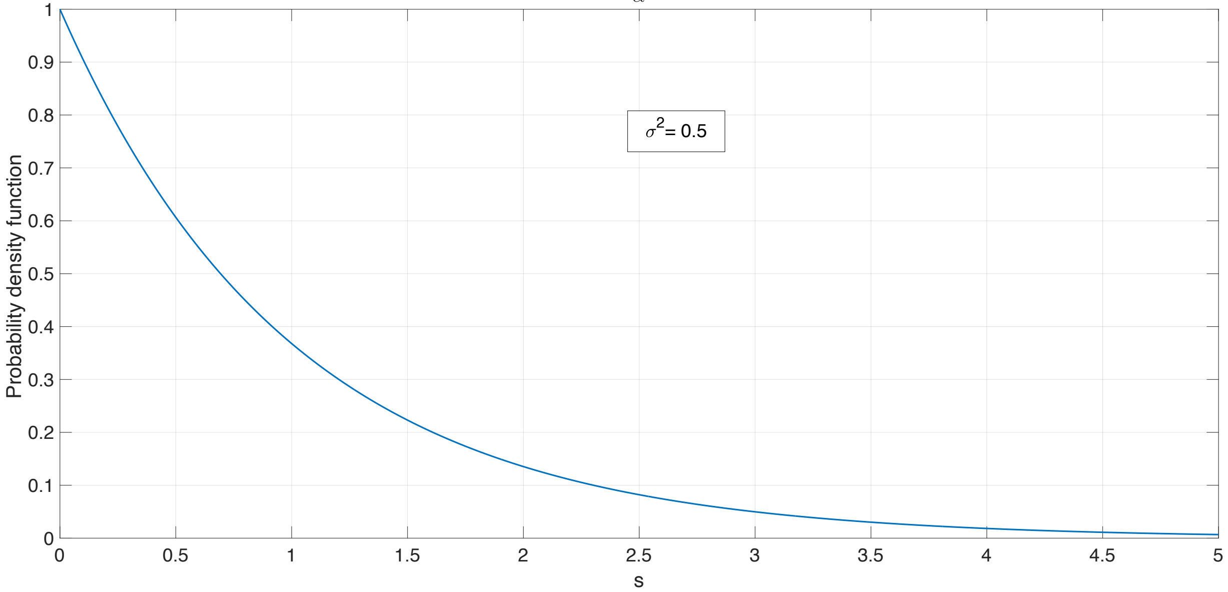
- The distribution for channel amplitude α is *Rayleigh*

$$p(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases}$$



- The distribution for channel power $s = \alpha^2$ is *exponential*

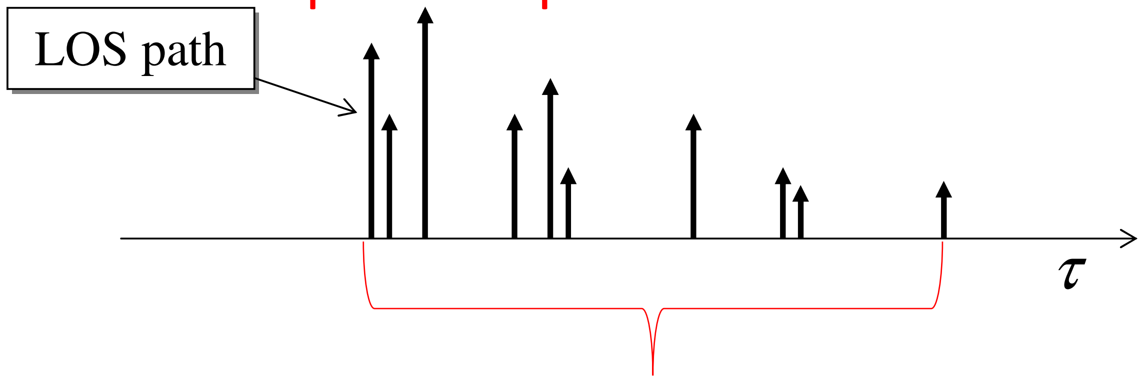
$$p(s) = \begin{cases} \frac{1}{2\sigma^2} e^{-\frac{s}{2\sigma^2}} & s \geq 0 \\ 0 & s < 0 \end{cases}$$



Small-scale fading: multipath channel

- The various signal's replicas may interfere with each other and generate inter-symbol (ISI) interference.

Impulse response of the channel



Time dispersion

Small-scale fading: multipath channel

- Neglecting the noise, the output of the receive filter is

$$x(t) = \sum_i c_i g(t - iT)$$

where $g(t) = g_T(t) \otimes h(t) \otimes g_R(t) = g_N(t) \otimes h(t)$ so that it is

$$g(t) = \sum_{\ell=0}^{N_c-1} \alpha_\ell e^{j\phi_l} g_N(t - \tau_\ell)\,.$$

- In the multipath channel, the decision variable $x(m)$ is

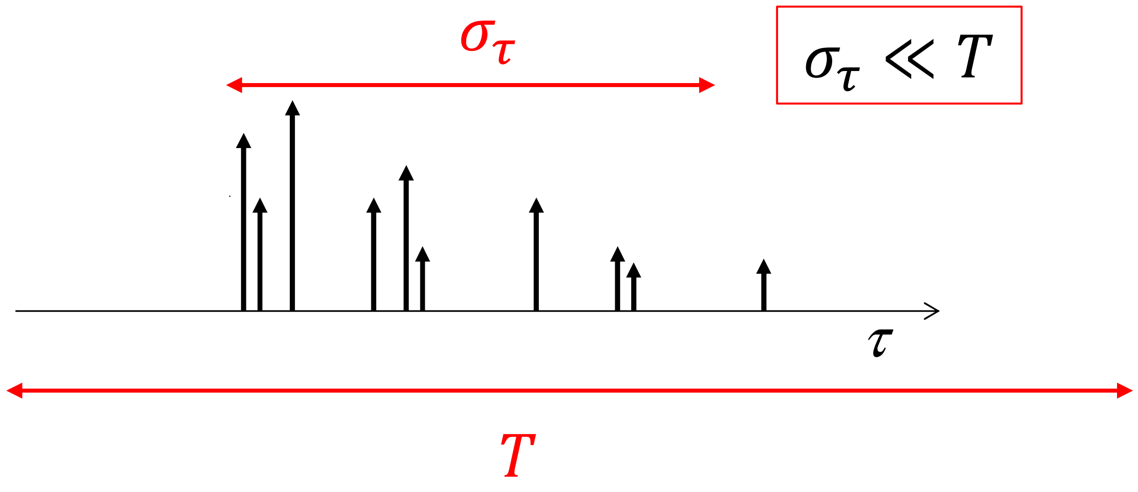
$$x(m) = x(t)\Big|_{t=mT} = \sum_k c_{m-k} g(kT) = c_m g(0) + \sum_{k,k\neq 0} c_{m-k} g(kT)$$

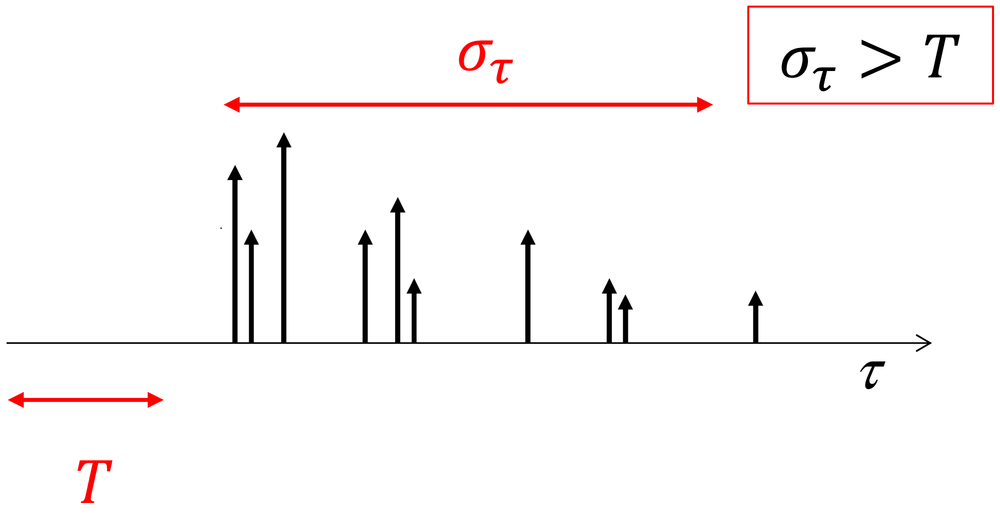
where it is

$$g(kT) = g(t)\Big|_{t=kT} = \sum_{\ell=0}^{N_c-1} \alpha_\ell e^{j\phi_l} g_N(kT - \tau_\ell)$$

Delay spread

- The channel's time dispersion is measured by the *delay spread* σ_τ
  - If the delay spread is smaller than the symbol time T, $\sigma_\tau \ll T$, there is only one resolvable path and the channel is *flat* fading.
  - If $\sigma_\tau > T$, there are more than one resolvable path and the channel is *multipath*. The varies replicas of the received signal interfere with each other



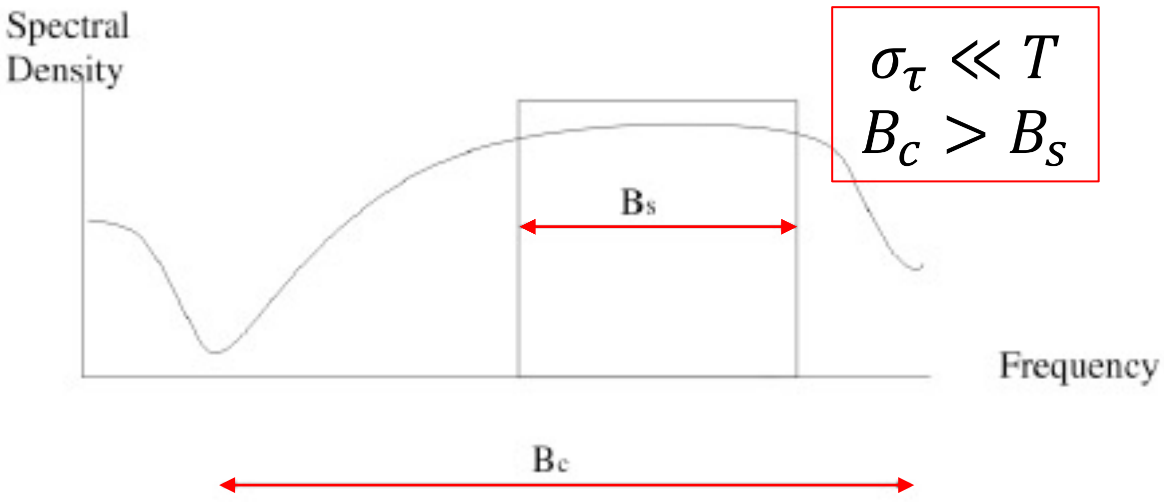


Coherence bandwidth

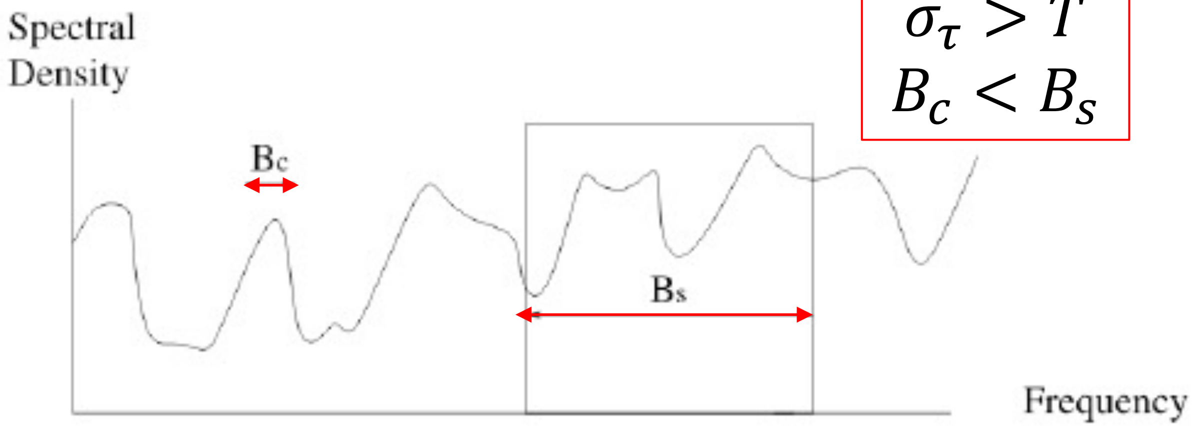
- The channel *coherence bandwidth* B_c is the frequency interval over which the channel's frequency response is approximately constant.

$$B_c \approx \frac{1}{5\sigma_\tau}$$

  - When $\sigma_\tau < T$, it is $B_c > B_s$, the channel is flat.
  - When $\sigma_\tau > T$, it is $B_c < B_s$, the channel is frequency selective (multipath).



$\sigma_\tau \ll T$
$B_c > B_s$



$\sigma_\tau > T$
$B_c < B_s$

Channel’s delay spread

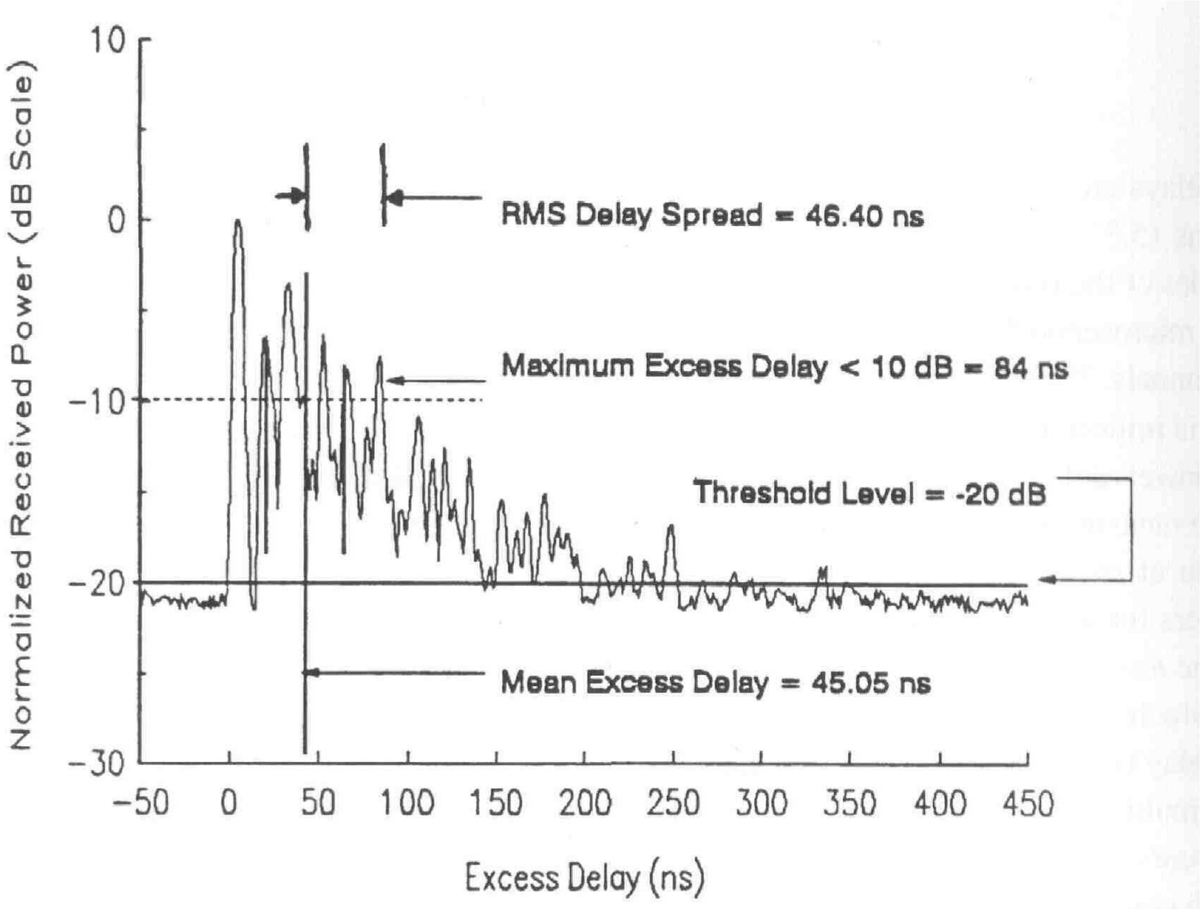
- To compute the delay statistics one should integrate over the density function of the delays.
- Too difficult, a practical method consist in weighting the delay of each path by the coefficients $0 < \alpha_\ell^2 \backslash \sum_{\ell=0}^{L-1} \alpha_\ell^2 < 1$, which are equivalent to empirical mass probabilities.
- *Mean excess delay*

$$\bar{\tau} = E\{\tau\} = \int_0^{+\infty} \tau\, pdf(\tau) d\tau \approx \sum_{\ell=0}^{N_c-1} \frac{\alpha_\ell^2}{\sum_{\ell=0}^{L-1} \alpha_\ell^2} \tau_\ell\, ;$$

- *RMS delay spread*

$$\sigma_\tau^2 = E\{(\tau - \bar{\tau})^2\} \approx \sqrt{\overline{\tau^2} - \bar{\tau}^2}, \overline{\tau^2} = \sum_{\ell=0}^{N_c-1} \frac{\alpha_\ell^2}{\sum_{\ell=0}^{L-1} \alpha_\ell^2} \tau_\ell^2$$

RMS delay spread



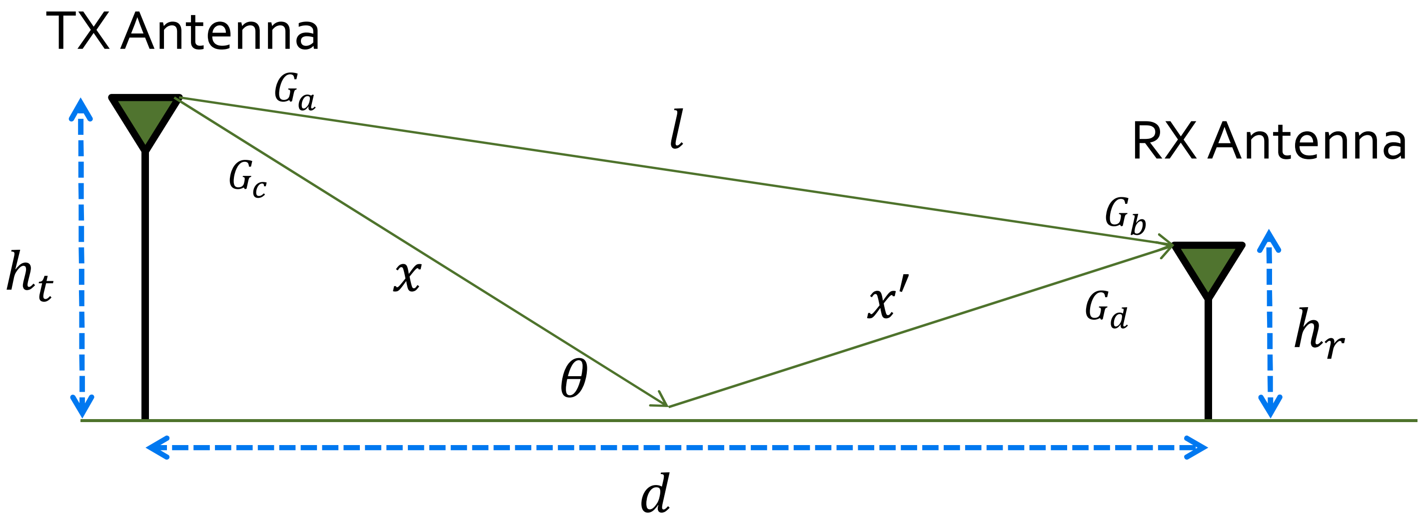
Typical values of RMS delay spread

- Measurements depend on signal frequency and environment.
- Typical values of delay spread are 0.2μs (rural area), 0.5μs (suburban area), 3-8μs (urban area), <2 μs (urban microcell) and 50-300ns (indoor picocell)
- A delay spread of 5μs corresponds to a coherence bandwidth

$$B_c \approx \frac{1}{5}\frac{1}{5 \cdot 10^{-6}} = 40\text{kHz!}$$

Environment	RMS delay spread (σ_τ)	Notes
Urban	1300 ns (3500 ns max)	NYC
LTE ETU	Up to 5 μs	Averaged typical case
Suburban	1960-2110 ns	Averaged extreme case
Indoor	10-50 ns	Office building
Indoor	70-94 ns (1470 ns max)	Office building

Example: the two-ray channel model



$$r_{2-ray}(t) = Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_a G_b}\,\tilde{g}(t)\exp\left(-\frac{j2\pi l}{\lambda}\right)}{l} + \frac{R\sqrt{G_c G_d}\,\overbrace{\tilde{g}(t-\tau)}^{\text{Delayed since x+x' is longer. } \tau = (x+x'-l)/c}\exp\left(-\frac{j2\pi(x+x')}{\lambda}\right)}{x+x'}\right]\exp(j2\pi f_c t)\right\}$$

R: ground reflection coefficient (phase and amplitude change)

Two-path channel, $h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - \tau)$

- Channel parameters

$\alpha_1 = 1, \alpha_2 = 0.9, \tau = \frac{T}{10}$

- Delay spread

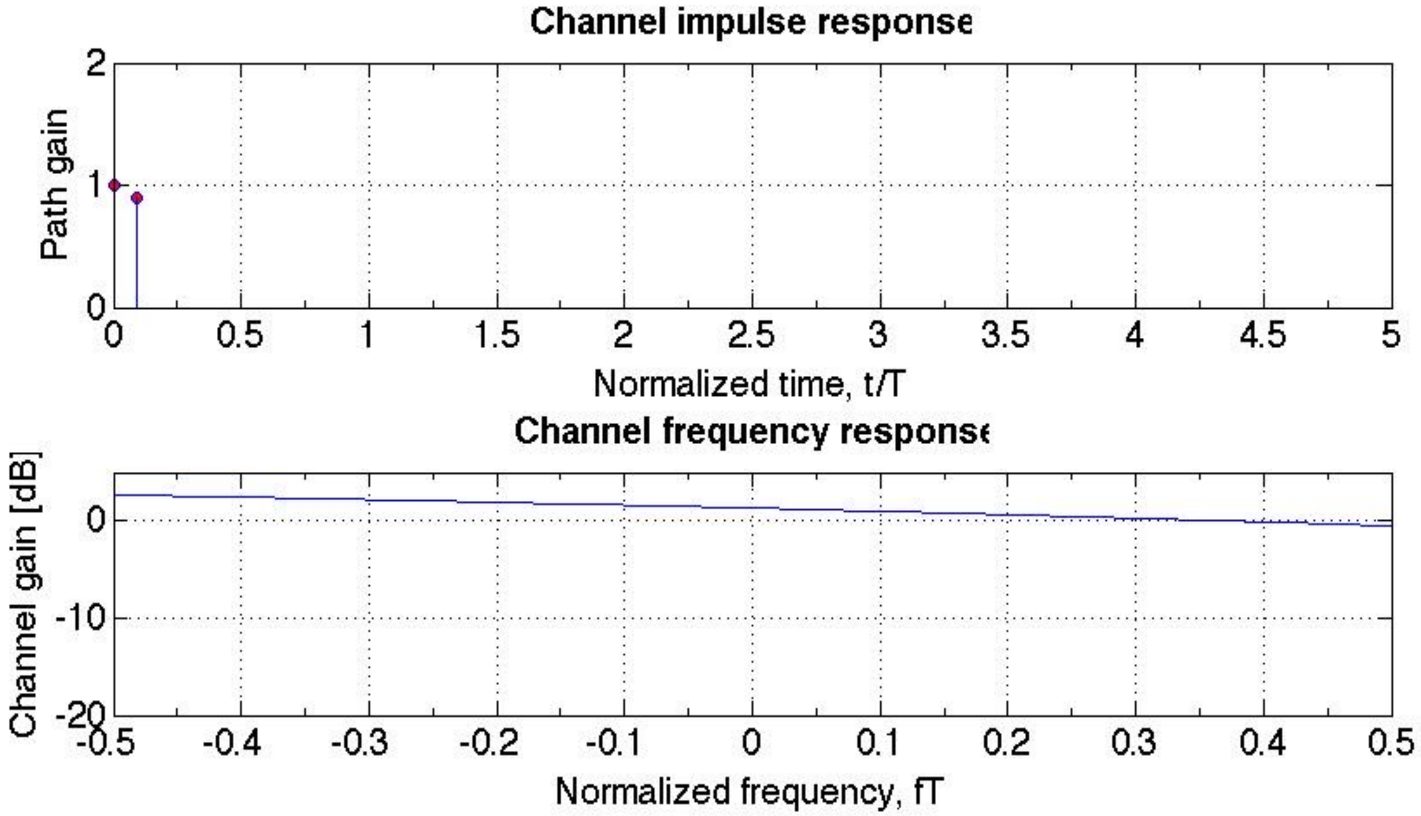
$$p_1 = \frac{1}{1 + 0.9\text{^}2} = \frac{1}{1.81} \approx 0.55$$

$$p_2 = \frac{0.81}{1.81} \approx 0.45$$

$$\bar{\tau} = 0.045T$$

$$\bar{\tau}^2 = 0.45 \cdot \frac{T^2}{100} = 0.0045T^2$$

$$\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} = 0.05T$$



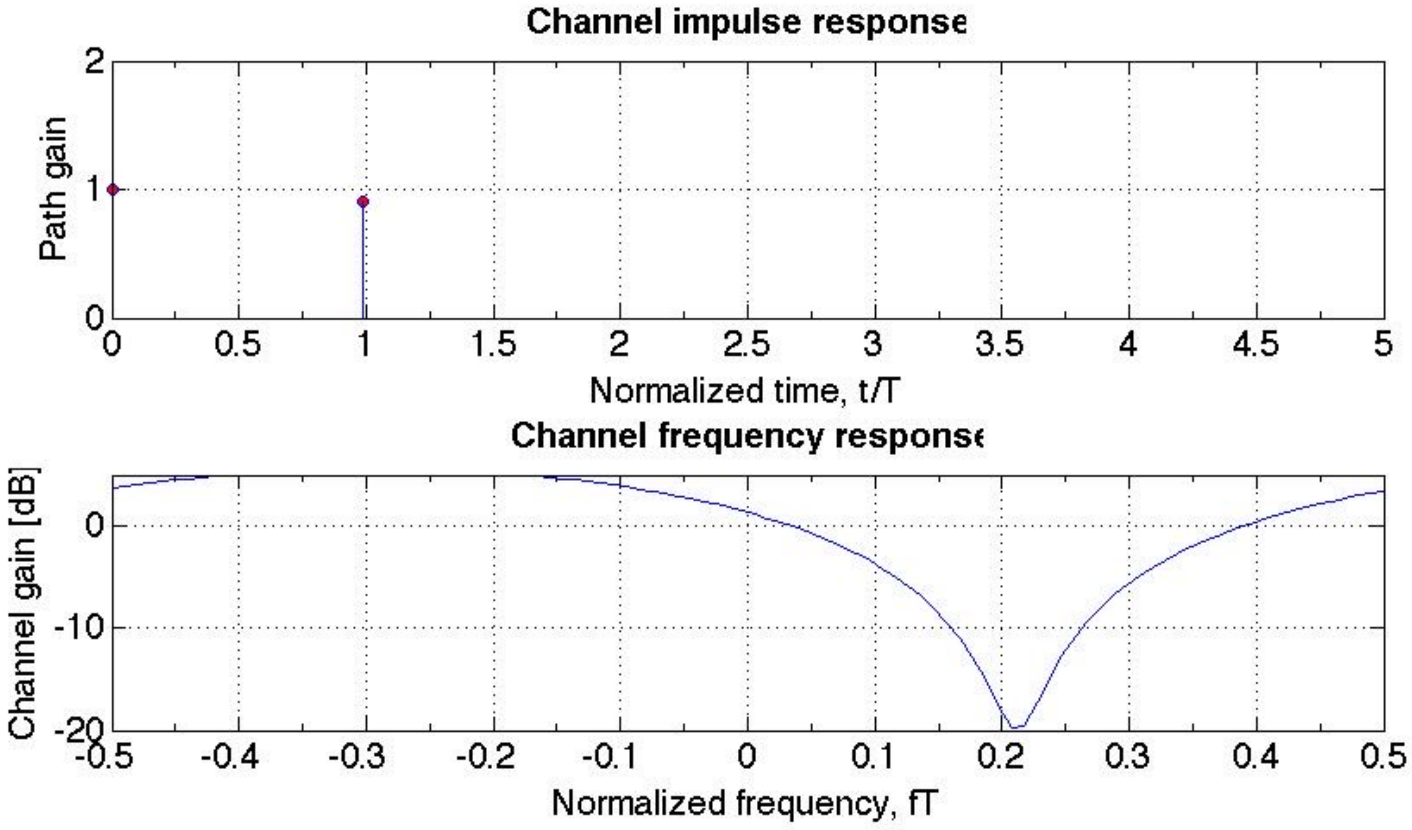
Two-path channel, $h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau)$

- Channel parameters

$\alpha_1 = 1, \alpha_2 = 0.9, \tau = T$

- Delay spread

$$p_1 \approx 0.55$$
$$p_2 \approx 0.45$$
$$\bar{\tau} = 0.45T$$
$$\bar{\tau}^2 = 0.45T^2$$
$$\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} = 0.5T$$



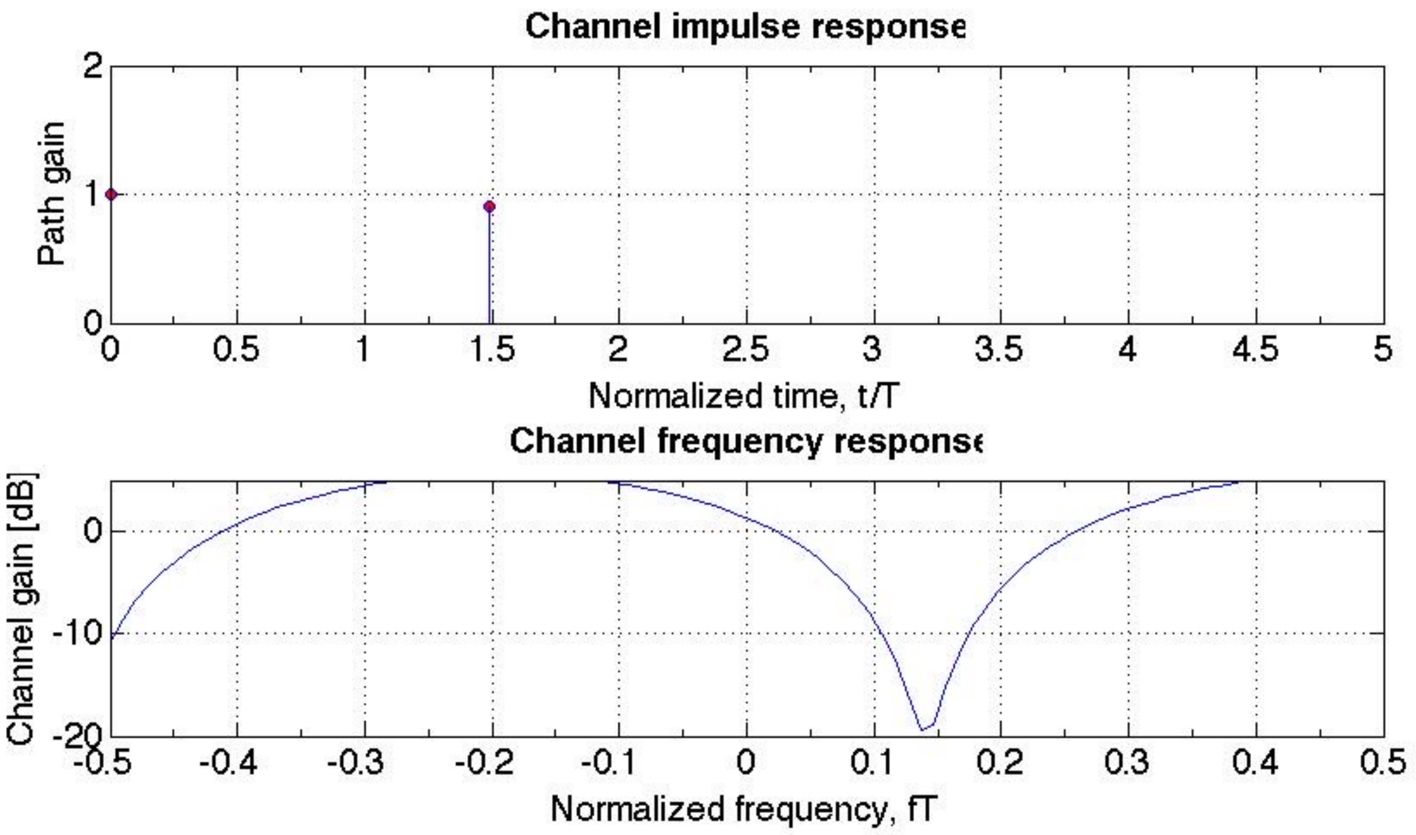
Two-path channel, $h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - \tau)$

- Channel parameters

$\alpha_1 = 1, \alpha_2 = 0.9, \tau = 1.5T$

- Delay spread

$$p_1 \approx 0.55$$
$$p_2 \approx 0.45$$
$$\bar{\tau} \approx 0.7T$$
$$\bar{\tau}^2 = T^2$$
$$\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} \approx 0.6T$$



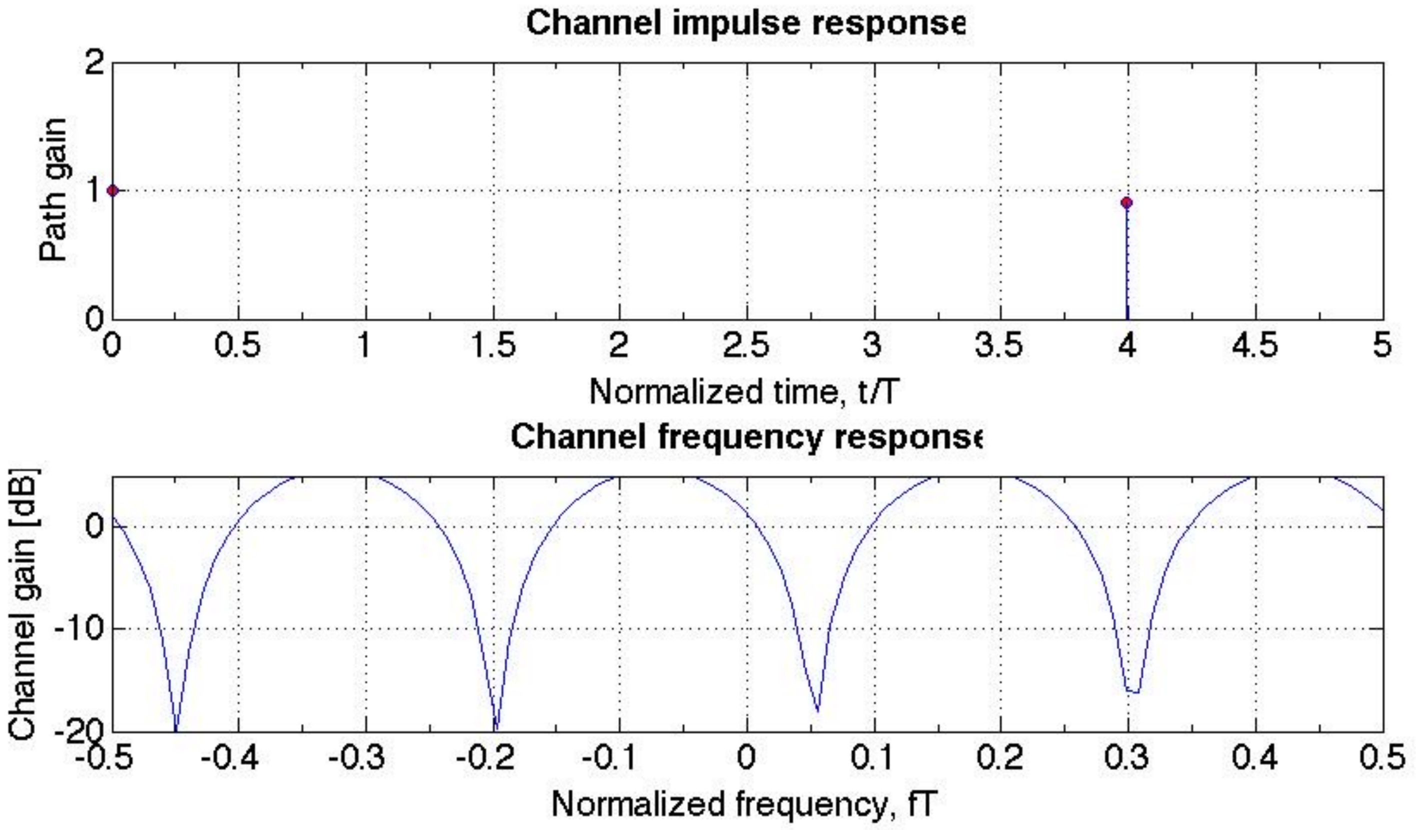
Two-path channel, $h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - \tau)$

- Channel parameters

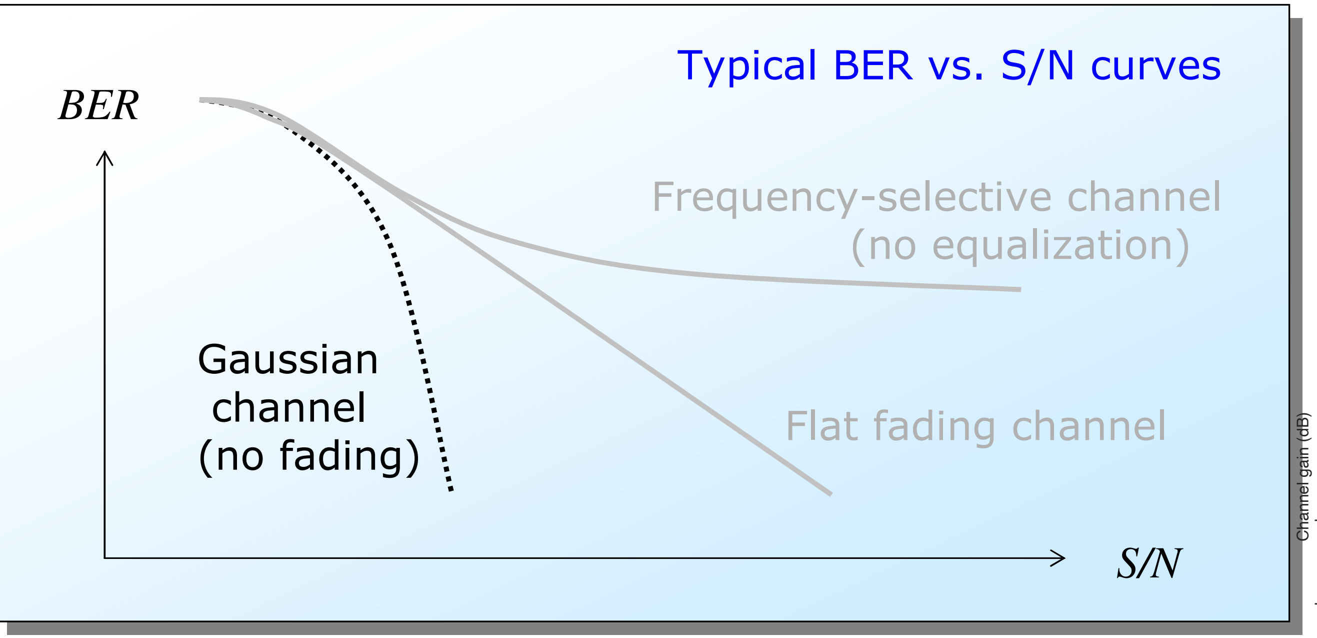
$\alpha_1 = 1, \alpha_2 = 0.9, \tau = 4T$

- Delay spread

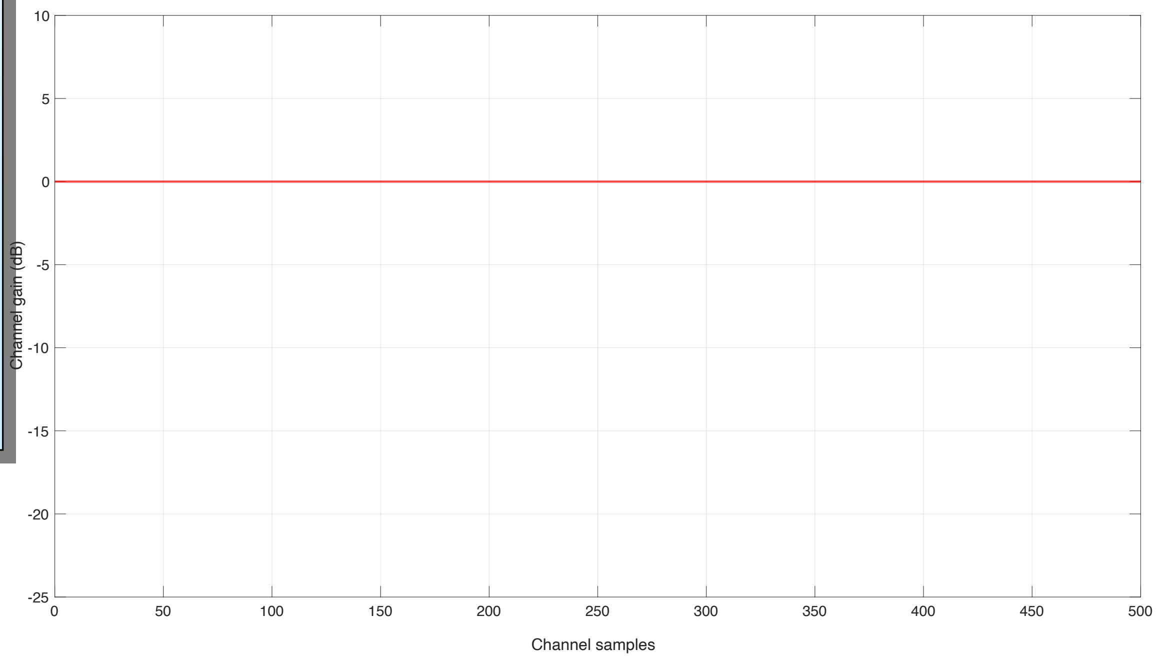
$$p_1 \approx 0.55$$
$$p_2 \approx 0.45$$
$$\bar{\tau} = 1.8T$$
$$\bar{\tau}^2 = 7.2T^2$$
$$\sigma_\tau = T\sqrt{7.2 - (1.8)^2} \approx 2.3T$$



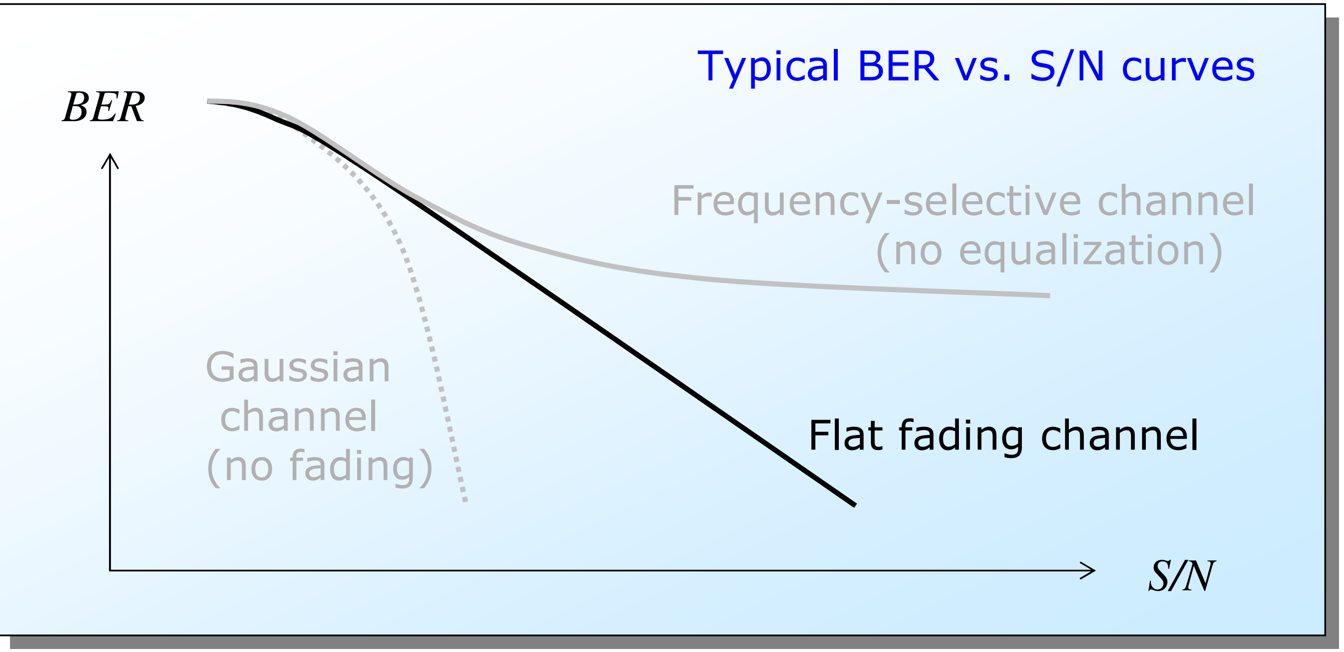
Flat fading channel: BER on AWGN



- With an AWGN channel, the decision variable is
$$x(m) = c_m + n(m)$$



BER on flat Rayleigh fading channel

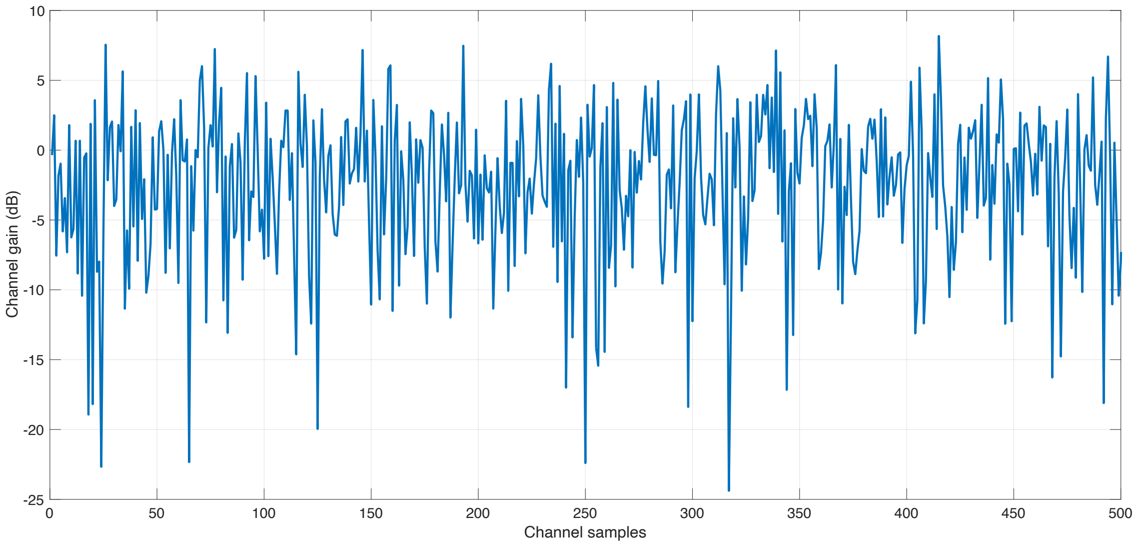


- With a flat fading channel, the decision variable is

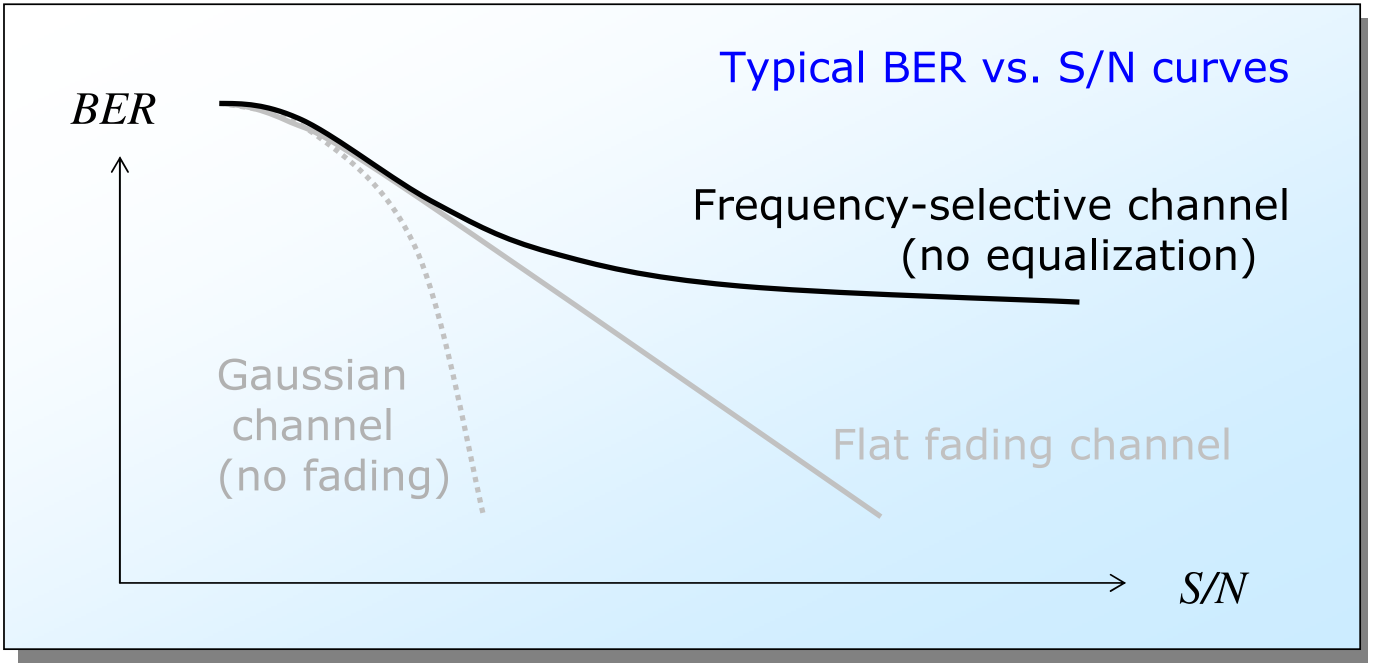
$$x(m) = \alpha c_m + n(m)$$

- The mean error probability is obtained by averaging it over the channel

$$P_e = \int_0^{+\infty} P(e|\alpha)p(\alpha)d\alpha$$



BER on multipath Rayleigh fading channel



- With a frequency selective channel, the decision variable is

$$x(m) = g(0)c_m + \underbrace{\sum_{k,k\neq 0} g(kT)c_{m-k}}_{\text{ISI}} + n(m)$$

- If no countermeasures are taken, the error probability has an irreducible error-floor.

Time-varying channel

- If the mobile receiver is in movement, the gains and the phase of the various paths of the channel vary in time

$$h(t,\tau) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} \delta(\tau - \tau_\ell)$$

- The received signal is

$$y(t) = A_{LS} \sum_{\ell=0}^{N_{C-1}} \alpha_\ell(t) e^{j\phi_\ell(t)} s(t - \tau_\ell)$$

- The channel gains and phases change much faster than the large scale fading A_{LS} and the delays τ_ℓ.

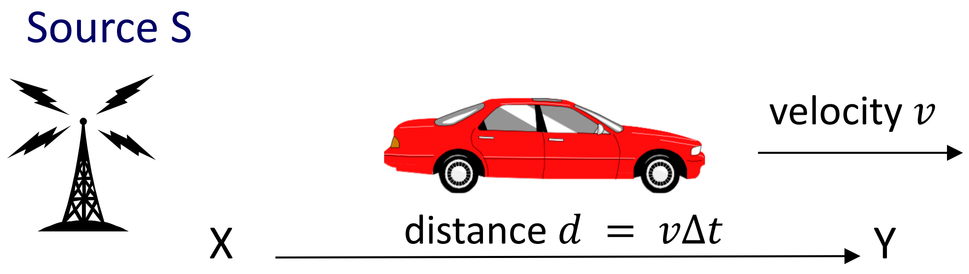
Doppler shift

- Consider a mobile user moving at constant velocity v between points X and Y, while the source S transmits a sinusoidal signal $s(t) = \sin(2\pi f_c t)$.
- The difference in path lengths travelled from source S to the mobile points X and Y is $d = vt$, which the signal takes the time $\Delta\tau = vt/c$ to travel.
- If the received signal at point X at time t is

$$y_X(t) = \sin(2\pi f_c t),$$

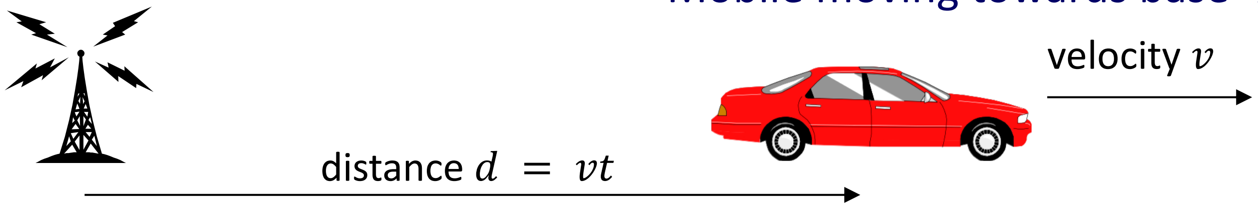
the received signal at point Y at time t is

$$y_Y(t) = \sin(2\,\pi f_c(t - \Delta\tau)) = \sin(2\pi f_c t - f_c vt/c) = \sin(2\pi(f_c - f_c v/c)t)$$

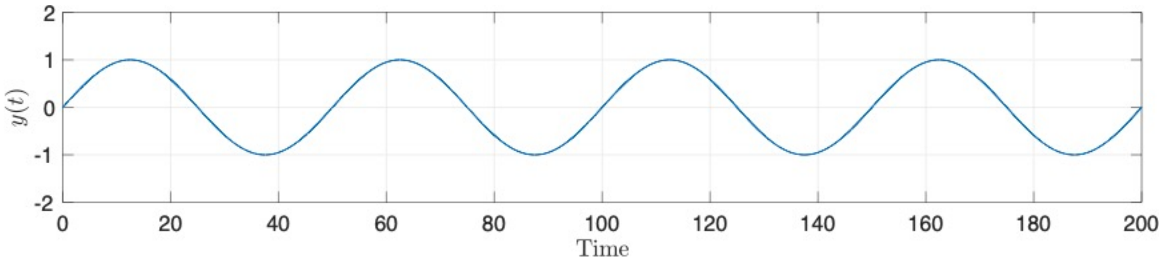


Doppler shift

Mobile moving away from base ➔ $v > 0$, Doppler shift < 0
Mobile moving towards base ➔ $v < 0$, Doppler shift > 0

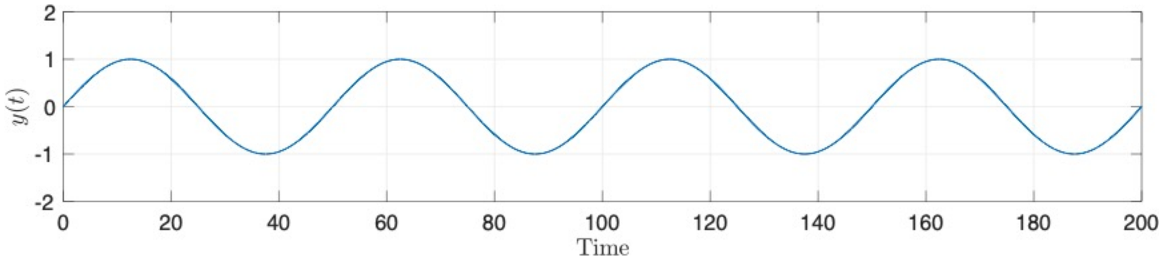


Received signal $y(t)$ at point X



Propagation delay $\Delta\tau$

Received signal $y(t)$ at point Y



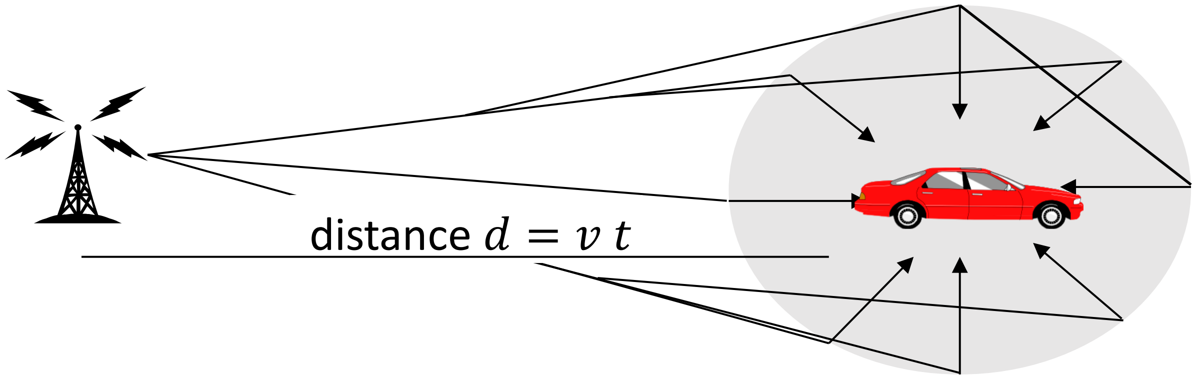
$$y_Y(t) = \sin 2\pi f_c\,(t - vt/c) = \sin 2\pi(f_c - f_c v/c)t$$

received frequency

Doppler shift $f_d = -f_c v/c$

Scattering: Doppler Spectrum

- In fading channels many signal replicas arrive at the receiver with different angles. The effect is a *Doppler spread* rather than a single shift.

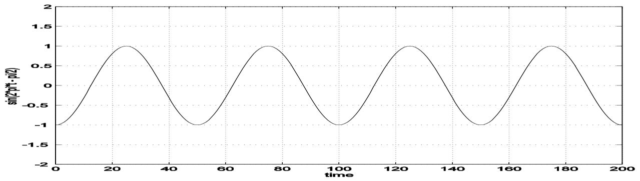


- The received signal is the sum of all scattered waves and is not anymore deterministic but is modeled as a *stochastic process* described by its *autocorrelation* and *power spectral density*.
  - Doppler shift for each path depends on angle θ, each path has a shift $f_c\,\frac{v\cos\theta}{c}$.
  - A typical assumption is that the received energy is the same from all directions (uniform scattering).

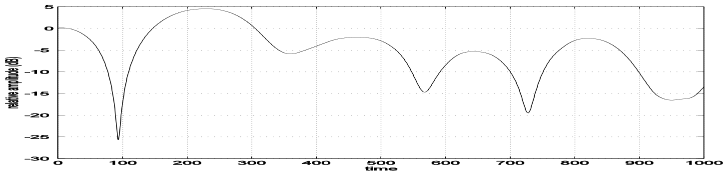
Jakes' Doppler spectrum for a sinusoidal tone

- The spectral broadening caused by the receiver movement is called *Doppler spread.*

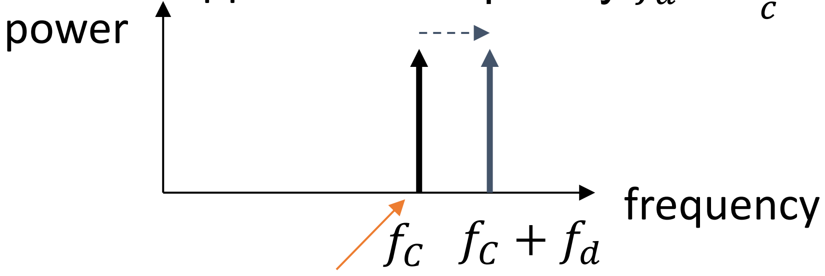
Transmitted signal $s(t)$



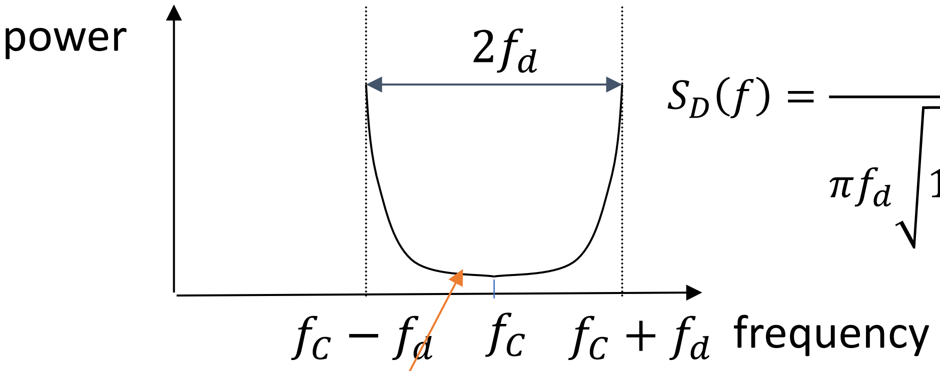
Received signal $y(t)$



Doppler shift frequency $f_d = \frac{f_c v}{c} = \frac{v}{\lambda}$



frequency of $s(t)$



$$S_D(f) = \frac{1}{\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}}$$

Jakes' Doppler spectrum

Jakes' Doppler spectrum

- The effect of mobility is a *broadening* of the signal spectrum.
  - Neglecting the noise, in the case of uniform scattering, if the transmitted signal is a sinusoid, i.e. $s(t) = \sin(2\pi f_c t)$, the received signal $y(t)$ is a stochastic process that takes the form $y(t) = a(t)\sin(2\pi f_c t)$ with

$$S_y(f) = \frac{1}{\pi f_d}\left(\sqrt{1 - \left(\frac{f - f_c}{f_d}\right)^2}\right)^{-1} = S_s(f) \otimes S_D(f)$$

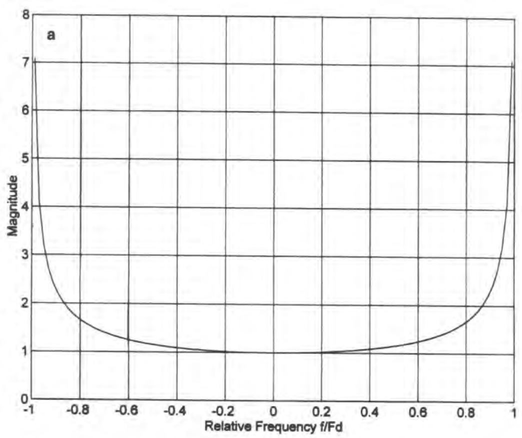
  - In the same way it can be shown that if $s(t)$ is a PAM- (QAM-)modulated signal, the power spectral density of the received signal $y(t) = a(t)s(t)$ is obtained as the convolution of the signal power spectral density $S_s(f)$ and the Jakes Doppler spectrum $S_D(f)$, i.e.

$$S_y(f) = S_s(f) \otimes S_D(f)$$

Time varying channel

- The Doppler spectrum $S_D(f) = \frac{1}{\pi f_d}\left(\sqrt{1-\left(\frac{f}{f_d}\right)^2}\right)^{-1}$ is the power spectral density of the time varying channel.
- In the time domain $\rho(t) = J_0(2\pi f_d t) \leftrightharpoons S_D(f)$ is the autocorrelation function of the channel.
- $J_0(2\pi x) \approx 0$ for $x = \frac{1}{2} \Longrightarrow$ The channel can be assumed uncorrelated for $f_d T_c = \frac{1}{2}$.
- The *coherence time* of the channel is

$$T_c = \frac{1}{2f_d} = \frac{1}{2}\frac{c}{f_c v}$$



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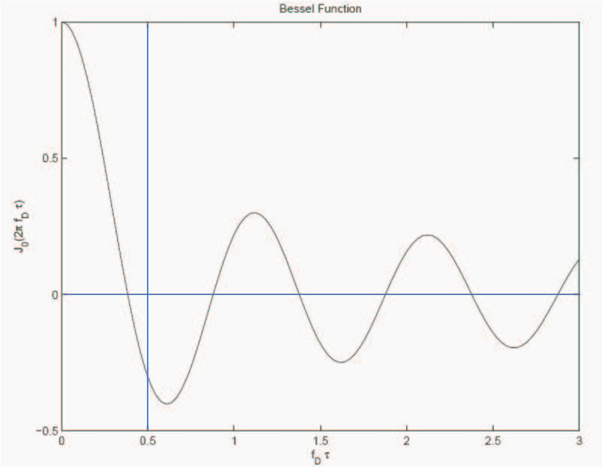


$J_0(x)$ is the 0th-order Bessel function of the first kind

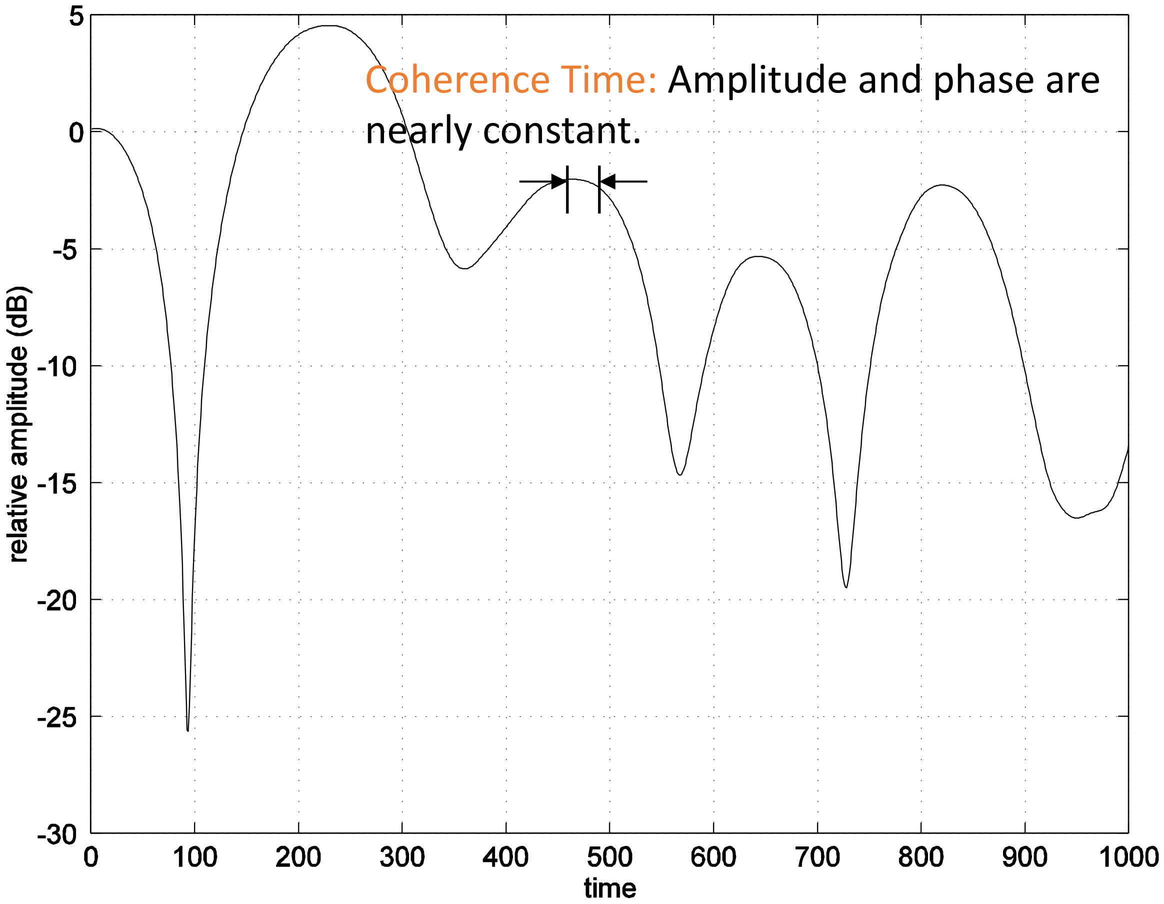
Channel Coherence Time

- The *channel coherence time* T_c is defined as the time interval over which the channel can be approximated as constant.

$$T_c = \frac{1}{2f_d}$$

- In terms of distance, it is

$$d_c = vT_c = \frac{1}{2}v\frac{c}{f_c v} = \frac{\lambda}{2}$$



Doppler spectrum

- Doppler spread is a measure of the *spectral broadening* caused by motion.
  - If the baseband signal bandwidth $B_s \gg f_d$ then the effect of Doppler spread is negligible at the receiver and the channel is *slow fading*.
  - If $B_s < f_d$ then the channel is *fast fading* and the Doppler spread severely distorts the received signal, which often results in an irreducible BER and synchronization problems.
- Similar considerations can be made in terms of symbol duration
  - A channel is *slow fading* if $T_c > T$.
  - A channel is said to be *fast fading* if $T_c < T$.

Fading channel example

- Consider a transmission at $f_c = 2.1$ GHz in a suburban aerea (delay spread $\sigma_\tau = 2\ \mu s$) to a user moving at a speed 90 km/h $\Longrightarrow v = 25$ m/s. The signal bandwidth is $B_S = 2$ MHz $\Longrightarrow$ the symbol time can be approximated as $T \sim \frac{1}{B_S} = 500$ ns.
- The Doppler spread is

$$f_d = \frac{f_c v}{c} = \frac{2.1 \cdot 10^9 \cdot 25}{3 \cdot 10^8} = 175 \text{ Hz} \Longrightarrow T_c = \frac{1}{2 f_d} \sim 3 \text{ ms.}$$

- The channel coherence bandwidth is $B_c = \frac{1}{5\sigma_\tau}$ =100 kHz.
- The channel is *slow* ($B_s \gg f_d$ or $T \ll T_c$) and *frequency-selective* ($B_s > B_c$ or $T < \sigma_\tau$).

Small-scale fading recap

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Small-scale fading recap