Advanced Encryption Standard (AES)

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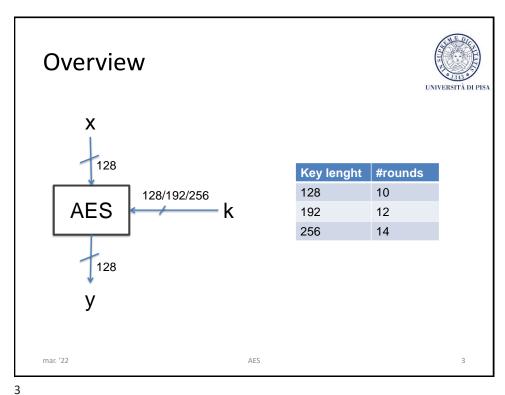
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AES history



- 1997: NIST publishes request for proposal
- 1998: Fifteen proposals
- 1999: NIST chooses five finalists
 - Mars, RC6, Rijndel, Serpent, Twofish
- 2000: NIST choses Rijndael as AES
 - Key sizes: 128, 192, 256
 - the longer, the more secure but the slower
 - Block size: 128 bits
- 2003: NSA allows AES in classified documents
 - Level SECRET: all key lengths
 - Level TOP SECRET: k = 256, 512
 - Never happened before for a public algorithm

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Introduction



- AES
 - Has rounds
 - Does not have a Feistel network structure
 - Encrypts an entire block in each round
 - DES encrypts half a block => #round_{AES} < #round_{DES}
 - Data path is called «state»

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Round and layers



- Each round but the first has three layers
- Layers
 - Key addition layer
 - Byte substitution layer (S-box) Confusion, Non-linear
 - Diffusion layer Diffusion
 - Two (linear) sublayers:
 - ShiftRows permute data byte-wise
 - MixColumn Mix blocks of four bytes (matrix operation)
 - Galois fields mathematical setting
 - · S-box, MixColumn

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Mathematical setting



- Galois field GF(28)
 - Operations in S-box and MixColumn are performed in this field
 - Elements of GF(2^m) can be represented as a polynomials of degree m – 1 with parameters in GF(2)
 - An A element of GF(28) represents one byte

- A =
$$a_7x^7 + ... + a_1x + a_0$$
 with $a_i \in GF(2) = \{0, 1\}$

- $A = (a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0)$
- · We cannot use integer arithmetic
- · We must use polynomial arithmetic

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Mathematical setting



- · Polynomial arithmetic
 - Addition, subtraction
 - Multiplication
 - · Core operation of MixColumn
 - Reduction, irreducible polynomial (rough equivalent of prime number)
 - A(x) \times B(x) \equiv C(x) mod P(x), with P(x) irreducible polynomial of degree m
 - AES: $P(x) = x^8 + x^4 + x^3 + x^1 + 1$

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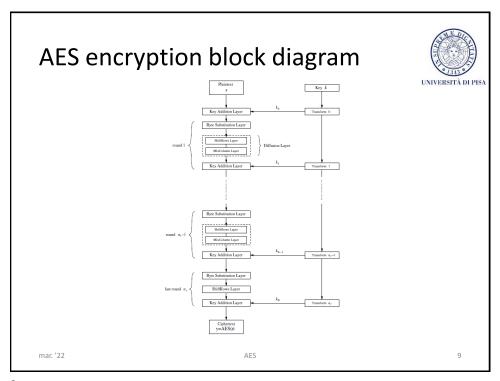
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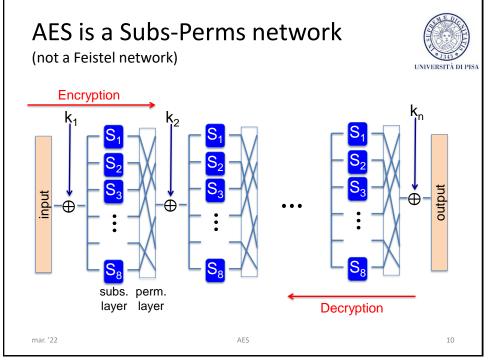
Mathematical setting

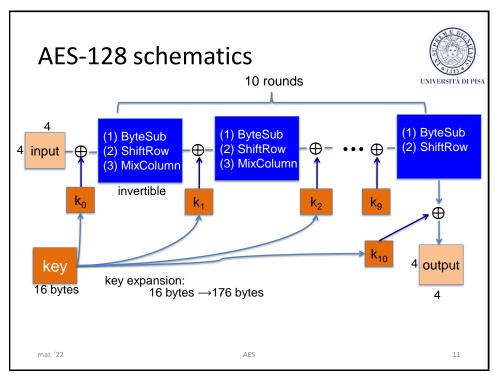


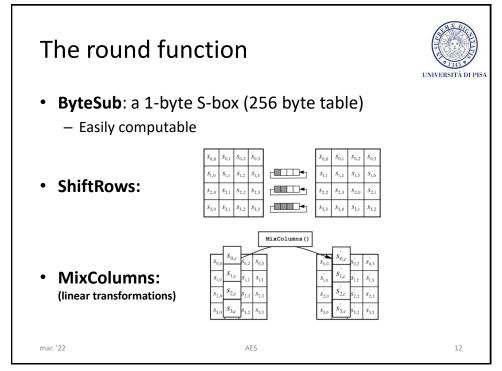
- Polynomial arithmetic
 - Division
 - Core operation of Byte Substitution (S-boxes)
 - $A(x)\cdot A(x)^{-1} \equiv 1 \mod P(x)$
 - In small fields (smaller than 2¹⁶ elements), inverse can be precomputed by lookup tables

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AES Security



- There is currently no analytical attack against AES known to be more efficient than brute force attack
- For more information about AES security see AES Lounge
 - ECRYPT Network of Excellence (FP6)
 - https://www.iaik.tugraz.at/content/research/krypto/aes/

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AES security - best known attacks



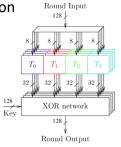
- Best key recovery attack
 - Four times better than exhaustive key search
 - 128-bit key => 126-bit key
- "Related key" attack in AES-256
 - Given 2^{99} pt-ct pairs from four related keys in AES-256, we can recover keys in 2^{99} ($\ll 2^{256}$)
 - · Very large data-/time-complexity
 - · Randomly generated keys cannot be related

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AES Performance (1/2)



- Software implementation
 - Direct implementation is well-suited for 8-bit processors (e.g., smartcard)
 - Processing 1-byte per instruction
 - For 32-/64-bit architecture, T-box optimization
 - Merge all the round functions into one look-up table (but key addition)
 - 4 tables (1 per byte) of 256 entries;
 each entry is 32 bit
 - 1 round, 16 lookups
 - Few hundreds Mbit/s



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AES Implementation (2/2)



- · Hardware implementation
 - AES requires more HW resources than DES
 - · High throughput implementation in ASIC/FPGA
 - Ten Gigabit/s
 - Block cipher is extremely fast compared to
 - · Asymmetric algorithms
 - Compression algorithms
 - · Signal processing algorithms
 - For more information see AES Lounge

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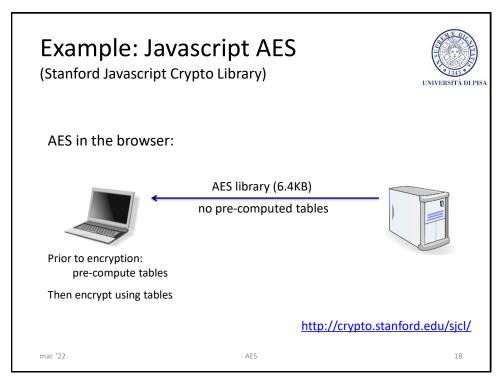
Code size/performance tradeoff



| | Code size | Performance |
|--|-----------|-------------------------------------|
| Pre-compute round functions (24KB or 4 KB) | Largest | Fastest (table lookups and xors) |
| Pre-compute S-box only (256 bytes) | Smaller | Slower |
| No pre-computation | Smallest | Slowest |

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AES in hardware



- AES instructions in Intel Westmere
 - aesenc, aesenclast: do one round of AES
 - 128-bit registers: xmm1 = state, xmm2 = round key
 - aesenc xmm1, xmm2 puts result in xmm1
 - aeskeygenassist performs key expansion
 - Implement AES in ten instructions
 - 9x aesenc + aesenclast
 - Claim 14x speed-up over OpenSSL on the same hw
- Similar instructions for AMD Bulldozer

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