# Wang-Mendel Method

***Beatrice Lazzerini***
Department of Information Engineering
Faculty of Engineering
University of Pisa
ITALY

1

## Generating fuzzy rules from numerical data

- Assume we are given a set of *n* desired input-output data pairs:

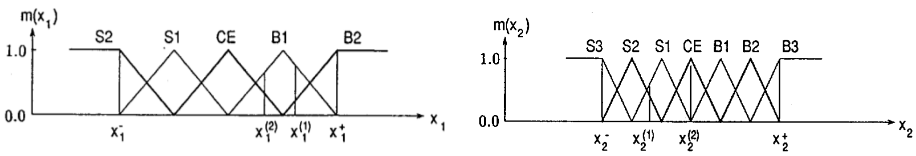
$$\left(x_1^{(1)}, x_2^{(1)}; y^{(1)}\right),\ \left(x_1^{(2)}, x_2^{(2)}; y^{(2)}\right),\ \ldots,\ \left(x_1^{(n)}, x_2^{(n)}; y^{(n)}\right)$$

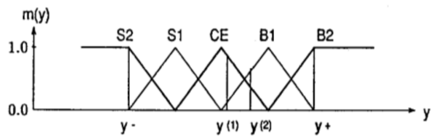
where $x_1$ and $x_2$ are inputs, and $y$ is the output

2

## Step 1. Divide the input and output spaces into fuzzy regions

Usually equidistant overlapping triangular or trapezoidal membership functions are used




**CE = Center**
**S1 = Small 1**
**B1 = Big 1**
**...**



## Step 2. Generate fuzzy rules from given data pairs

- Determine the membership degree of each variable to each fuzzy set
- Assign a given variable to the region with maximum degree
- Obtain one rule from each input-output data pair

$$(x_1^{(1)}, x_2^{(1)}; y^{(1)}) \Rightarrow [x_1^{(1)}(0.8 \text{ in } B1, \text{ max}), x_2^{(1)}(0.7 \text{ in } S1, \text{ max}); y^{(1)}(0.9 \text{ in } CE, \text{ max})] \Rightarrow \text{Rule 1:}$$

$$\text{IF } x_1 \text{ is } B1 \text{ and } x_2 \text{ is } S1, \text{ THEN } y \text{ is } CE$$

$$(x_1^{(2)}, x_2^{(2)}; y^{(2)}) \Rightarrow [x_1^{(2)}(0.6 \text{ in } B1, \text{ max}), x_2^{(2)}(1 \text{ in } CE, \text{ max}); y^{(2)}(0.7 \text{ in} B1, \text{ max})] \Rightarrow \text{Rule 2:}$$

$$\text{IF } x_1 \text{ is } B1 \text{ and } x_2 \text{ is } CE, \text{ THEN } y \text{ is } B1$$

## Step 3. Assign a degree to each rule

- As each data pair generates one rule there may be *conflicting* rules, i.e., rules with the same IF part but different THEN parts. To solve this problem we assign a degree (weight) to each rule (representing the importance of the rule) and accept only the rule from a conflict group with maximum degree
- E.g., *product strategy*: the degree of a rule is the product of the degrees of its components and, possibly, the degree of the data pair that generates this rule

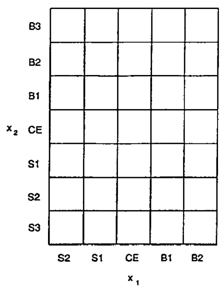
$$D(\text{Rule1}) = m_{B1}(x_1) m_{S1}(x_2) m_{CE}(y) m^{(1)}$$

where $m^{(1)}$ may represent, e.g., the expert's belief of the usefulness of the data pair



## Step 4. Create a combined fuzzy rule base

A combined fuzzy rule base is built from rules generated from numerical data and linguistic rules (a linguistic rule has a degree assigned by the human expert)




In the figure we see the form of the fuzzy rule base

- If there is more than one rule in a box, use the rule with maximum degree
- If a rule is an "and" rule, it fills only one box;
- if a rule is an "or" rule, it fills all the boxes in the rows or columns corresponding to the regions of the IF part

(e.g., the rule "IF x1 is S1 or X2 is CE THEN Y is B2" fills the 7 boxes in column S1 and the 5 boxes in row CE with B2)

## Step 5. Determine a mapping based on the combined fuzzy rule base

This step determines a mapping from input space to output space based on the combined fuzzy rule base using a defuzzifying procedure

- Wang and Mendel designed their algorithm to create fuzzy systems for function approximation
- They proved that this algorithm can create fuzzy rule bases that can approximate any real continuous function over a compact set to arbitrary accuracy