Data Encryption Standard

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Version: 2022-03-16

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Data Encryption Standard

- May 15, 1973
  - National Bureau of Standards (NBS) published a solicitation for cryptosystems in the Federal Register (mildly revolutionary act)
- 1974
  - IBM submitted LUCIFER (n = 64, k = 128)
  - DES was a modification of LUCIFER (n = 64, k = 56, resistant to differential cryptanalysis) under NSA guidance
- March 17, 1975
  - DES was published in the Federal Register

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Data Encryption Standard

- January 15, 1977 (FIPS PUB 46)
  - (called DEA) considered a standard for "unclassified" applications, after much public discussion
  - Reviewed every 5 years, being January 1994 the most recent review
  - Not a standard since 1998
- 1999 (FIPS PUB 46-3)
  - DES recommended for legacy systems
  - 3DES Recommend
  - DES replaced by AES

Confusion and diffusion

- Two primitives for strong ciphers (Shannon 1949)
  - **Diffusion** is an encryption operation where the influence of one PT symbol is spread over many CT symbols with the goal of hiding statistical properties of the PT
    - A simple diffusion element is the bit permutation (DES)
    - AES uses the more advanced MixColumn
  - **Confusion** is an encryption operation where the relationship between key and CT is obscured.
    - A common element to achieve confusion is substitution
    - AES and DES use substitution

A good diffusion property

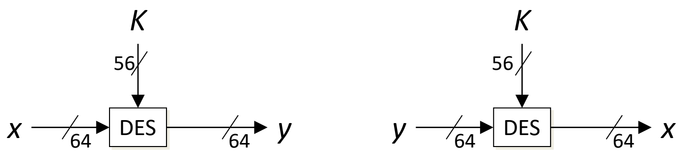
- (Informally) Changing of one bit of PT results on average in the change of half the output bits of the CT, i.e., If PT $\rightarrow$ PT' $\Rightarrow$ CT $\rightarrow$ CT' s.t. CT' looks statistically independent of CT

Confusion and diffusion

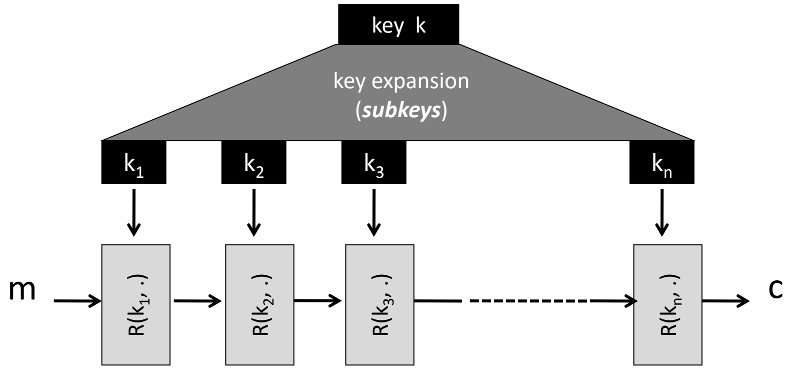
- Confusion only or diffusion only is not secure
  - Shift cipher and Enigma used confusion only
- Confusion and diffusion must be concatenated to build a strong cipher
- *Product ciphers* are composed of *rounds* which concatenate confusion and diffusion

Data Encryption Standard (DES)

- The 56-bit input key K is specified as a 64-bit key
  - 8 bits (bits 8; 16, …, 64) are used as parity bits
  - The key is actually 56-bit long
- Product cipher, 16 rounds



Block Ciphers Built by Iteration

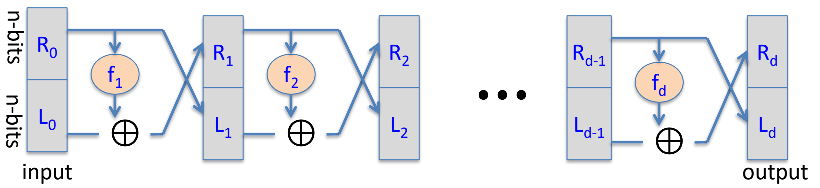


- $R(k_i,\cdot)$ is the round function
- K_i: subkeys, one per round
- DES (r = 16) , 3DES (r=48), AES-128 (n=10)

Feistel Network

Given functions $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Goal: build invertible function $F: \{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$



In symbols: $\begin{cases} L_i = R_{i-1} \\ R_i = L_{i-1} \oplus f_i(R_{i-1}) \end{cases}$

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Round f-function

- Realizes diffusion and confusion
- Can be considered as a pseudorandom generator with two inputs:
  1. Right half of the input R_{i-1}
  2. The round subkey k_i

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Feistel net is invertible

Theorem: for any $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$, Feistel network F: $\{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ is invertible

Proof: *construct inverse*

Given

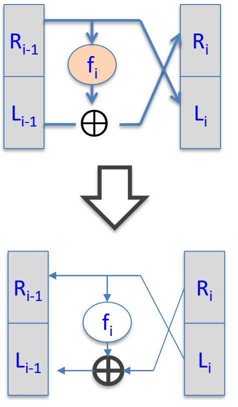
$R_i = L_{i-1} \oplus f_i(R_{i-1})$

$L_i = R_{i-1}$

then

$R_{i-1} = L_i$

$L_{i-1} = R_i \oplus f_i(R_{i-1}) = R_i \oplus f_i(L_i)$



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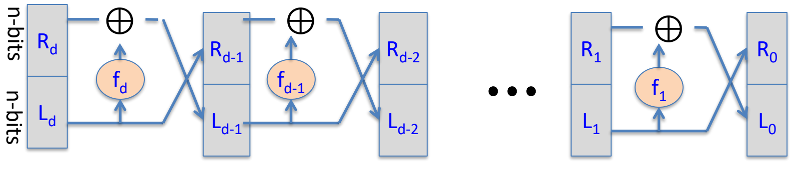
Decryption circuit



- Inversion is basically the same circuit, with $f_1, \ldots, f_d$ applied in reverse order
- FN is a general method for building invertible functions (block ciphers) from arbitrary functions f.

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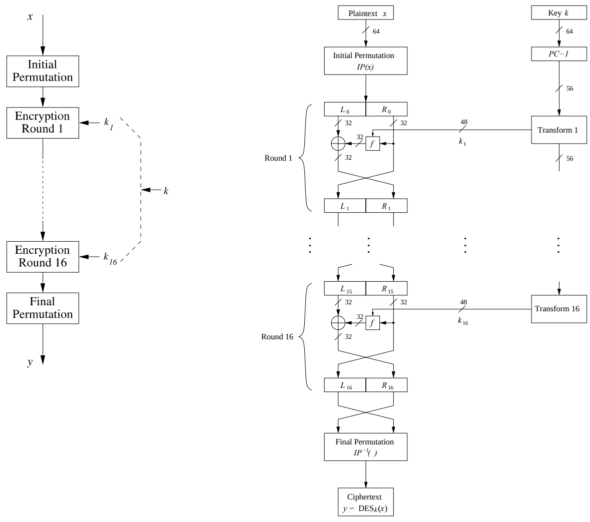
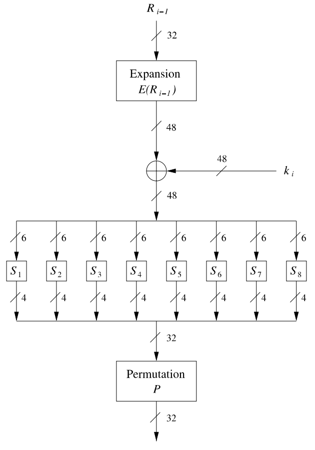
The internal structure of DES



Initial and final permutation

- IP and IP^{-1}
  - Very fast hw implementation
  - Don’t increase DES security
  - Their rationale is not known

The *f*-function



- Expansion box E increases diffusion
- S-boxes provide confusion
- Permutation P increases diffusion
- Avalanche effect
  - Diffusion caused by E, S and P guarantees that every bit at the end of the 5-th round depends on every plaintext bit and key bit

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S-box

- Provide confusion
  - Core of the DES cryptographic strength
  - The motivations behind S-box were never motivated
- Lookup table: $\{0, 1\}^6 \rightarrow \{0, 1\}^4$
  - Larger tables would be better but 4-by-6 tables were close to the maximum size for ICs in the 70s
- The only non-linear element of the system
  - $S(a \oplus b) \neq S(a) \oplus S(b)$
    - If S_i's were linear then DES could be described by a linear system where key bits are the unknowns

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S-box

- Design criteria
  - Notation
    - Let in_i denote the i-th input of s-box S
    - Let out_j denote the j-th output of s-box S
  - Strict avalanche criterion
    - If in_i of S is commuted, then out_j commutes with probability 0.5, for all i, j
  - Bit independence criterion
    - If in_i of S is commuted, then out_j and out_k commute independently, for all i, j, and k

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S-box

- Design criteria (more refined)
  1. Each S-box has six input bits and four output bits.
  2. No single output bit should be too close to a linear combination of the input bits.
  3. If the lowest and the highest bits of the input are fixed and the four middle bits are varied, each of the possible 4-bit output values must occur exactly once.
  4. If two inputs to an S-box differ in exactly one bit, their outputs must differ in at least two bits. [%]

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S-box

4. If two inputs to an S-box differ in the two middle bits, their outputs must differ in at least two bits.
5. If two inputs to an S-box differ in their first two bits and are identical in their last two bits, the two outputs must be different.
6. For any nonzero 6-bit difference between inputs, no more than 8 of the 32 pairs of inputs exhibiting that difference may result in the same output difference.
7. A collision (zero output difference) at the 32-bit output of the eight S-boxes is only possible for three adjacent S-boxes.

S-box

$x \rightarrow \boxed{S_i} \rightarrow y$

$x = b_1 b_2 b_3 b_4 b_5 b_6$

Row → $b_1 b_6$ (outer bits)
Column → $b_2 b_3 b_4 b_5$ (inner bits)

row	column number															
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
S[illegible]																
[0]	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
[1]	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
[2]	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
[3]	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13
S[illegible]																
[0]	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
[1]	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
[2]	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
[3]	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9
S[illegible]																
[0]	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
[1]	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
[2]	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
[3]	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12
S[illegible]																
[0]	7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
[1]	13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
[2]	10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
[3]	3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14
S[illegible]																
[0]	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
[1]	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
[2]	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
[3]	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
S[illegible]																
[0]	12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
[1]	10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
[2]	9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
[3]	4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13
S[illegible]																
[0]	4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
[1]	13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
[2]	1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
[3]	6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12
S[illegible]																
[0]	13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
[1]	1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
[2]	7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
[3]	2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

S-box

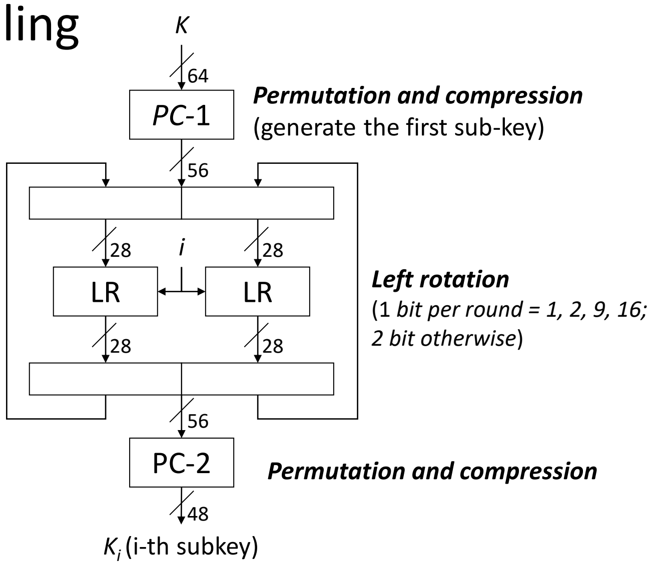
$$S_i: \{0,1\}^6 \longrightarrow \{0,1\}^4$$

$$S_5(011011) \rightarrow 1001$$

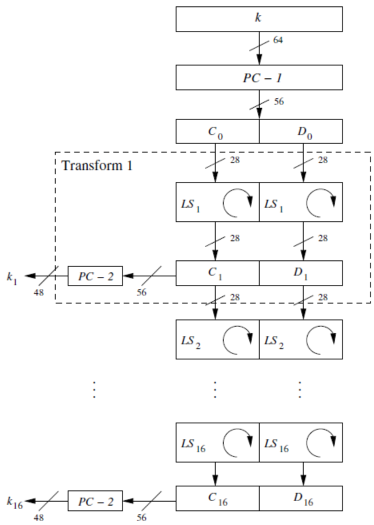
S_5		Middle 4 bits of input															
		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Outer bits	00	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

Key scheduling

- PC1 e PC2 guarantee that, at each round, a different subset of bits is extracted
- Each bit of the key participates to 14 rounds on average



Key scheduling: encryption



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Facts on key schedule

- The key schedule is a method to realize 16 permutations systematically
  - The key schedule is easy to implement in HW
  - The key schedule is such that each of the 56 key bits is used in different rounds
  - Each key bit is used in approximately 14 of the 16 rounds
- Every round key is a selection of 48 permuted bits of the input key
- Total number of rotations: $4 + 12 \times 2 = 28$
  - $C_0 = C_{16}$, $D_0 = D_{16}$ (fundamental for decryption)

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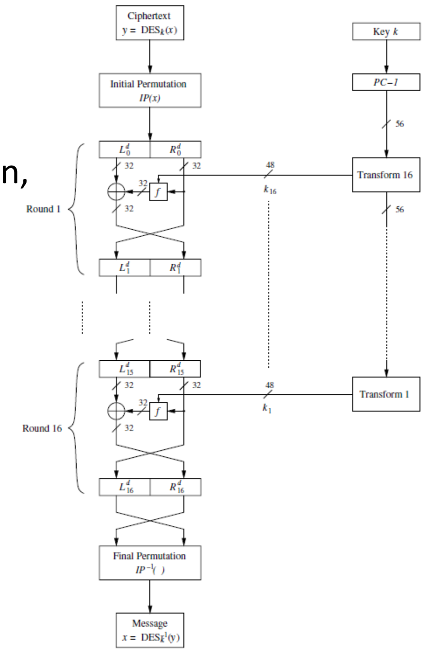
Decryption

- Compared to encryption, only key scheduling is reversed



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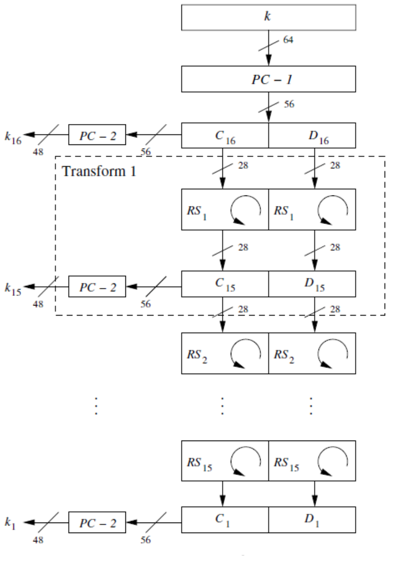
Key scheduling: decryption



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Decryption

- Given k it is easy to reverse the key schedule
  - k_{16} = PC-2(C_{16}, D_{16}) = PC-2(C_0, D_0) = PC-2(PC-1(k))
  - k_{15} = PC-2(C_{15}, D_{15}) = PC-2(RS2(C_{16}), RS2(D_{16})) = PC-2(RS2(C_0), RS2(D_0))
  - …
- Reverse encryption round-by-round
  - Decryption round 1 reverses encryption round 16
  - Decryption round 2 reverses encryption round 15
  - …

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Decryption

- The input of the 1st decryption round is equal to the output of the last encryption
  - $(L^d_0, R^d_0) = IP(Y) = IP(IP^{-1}(R_{16}, L_{16})) = R_{16}, L_{16}$
  - Thus $L^d_0 = R_{16}$ and $R^d_0 = L_{16} = R_{15}$
- The first decryption reverses the last encryption
  - L1d = $R_0^d = L_{16} = R_{15}$
  - $R_1^d = L_0^d \oplus f(R_0^d, k_{16}) = R_{16} \oplus f(L_{16}, k_{16}) = [L_{15} \oplus f(R_{15}, k_{16})] \oplus f(R_{15}, k_{16})] = L_{15}$
  - Iteratively
    - $L_i^d = R_{16-i}$
    - $R_i^d = L_{16-i}$
    - where i = 0, 1, 2,… 16

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Decryption

- After the last decryption round
  - $L_{16}^d = R_0$
  - $R_{16}^d = L_0$
- Finally,
  - $IP^{-1}(R^d_{16}, L^d_{16}) = IP^{-1}(L_0, R_0) = IP^{-1}(IP(x)) = x$

DES in practice

- DES can be efficiently implemented either in hardware or in software
  - Arithmetic operations are
    - exclusive-or
    - E, S-boxes, IP, IP^{-1}, key scheduling can be done in constant time by table-lookup (sw) or by hard-wiring them into a circuit

DES in practice [↓]

- One very important DES application is in banking transactions
  - DES is used to encrypt PINs and account transactions carried out at ATM
  - DES is also used in government organizations and for inter-bank transactions

Empirical properties of DES

Empirically, DES fulfills these requirements:

- Each CT bits depends on all key bits and PT bits
- There are no evident statistical relationships between CT and PT
- The change of one bit in the PT (CT) causes the change of every bit in the CT (PT) with 0.5 probability

Security of DES

- Exhaustive key search or brute force attack
- Analytical attacks
  - Differential Cryptanalysis, Eli Biham and Adi Shamir, 1990
  - Linear Cryptanalysis, Mitsuru Matsui, 1993
  - Effectiveness of these attacks depend on S-boxes
  - Applicable to any block cipher
  - Not practical for DES
    - Require a large number of (CT, PT)s
    - Collecting and storing (PT, CT)s requires large amount of time and memory
    - Attacks recover just one key (key refresh)

Strength of DES

attack method	data complexity(***)		storage complexity	processing complexity
exhaustive precomputation	—	1	2^{56}	1 (table lookup)
exhaustive search	1	—	negligible	2^{55}
linear cryptanalysis(*)	2^{43} (85%)	—	for texts	2^{43}
	2^{38} (10%)	—	for texts	2^{50}
differential cryptanalys(**)	—	2^{47}	for texts	2^{47}
	2^{55}	—	for texts	2^{55}

(*) Mitsuru Matsui, 1993
(**) Eli Biham and Adi Shamir, 1990
(***) First column: known-plaintex; second column: chosen-plaintext

DES challenge (1981)

```
p = "The unknown messages is: XXX …"
          c1       c2       c3
```

- Find k ∈ $\{0,1\}^{56}$ s.t. c_i = DES(k, p_i), i = 1, 2, 3
  - 1997: Internet search – 3 months
  - 1998: EFF machine (Deep Crack) – 3 days (250K$)
  - 1999: combined search – 22 hours
  - 2006: COPACABANA (120 FPGAs) – 7 days (10K$)
- 56-bit ciphers should not be used

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Brute force attack

- In 1977, Diffie & Hellman hypothesized a $ 20 mln dedicated parallel computer able to try 10^6 key per second fnd find a key in 10 hours
- Currently, customary technology allows us to try 10^9 keys per second
- Currently, supercomputer can try 10^{13} keys per second

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Performance of DES

- Software implementation
  - Desktop ÷ smart cards
  - Bit permutation (E, P, IP) are inefficient in sw
  - S-box moderately efficient in sw
  - Optimization through precomputation
  - Throughput: 100 Megabit/s
- Hardware implementation
  - Bit permutation are efficient in hw
  - S-box efficiently implemented in Boolean logic (on average a box requires 100 gates)
  - DES requires less than 3000 gates (fit RFIDs)
  - Optimizations: pipelining, FPGA, ASICS
  - Throughput: 100 Gigabit/s

DES alternatives and variants

- 3DES (Triple encryption)
- DESX (Key whitening)
- AES
  - k = 128, 256, 512; n = 128
  - Finalists: Mars, RC6, Serpent, Twofish
    - Efficient especially in SW
    - Mars, Serpent and Twofish are royalty-free
- PRESENT
  - Lightweight encryption, i.e., low complexity, especially in HW
  - Applications RFID tags and pervasive applications