

Candidate name: \_\_\_\_\_

Computer Architecture  
Quantum Computing test  
a.y. 2022/2023  
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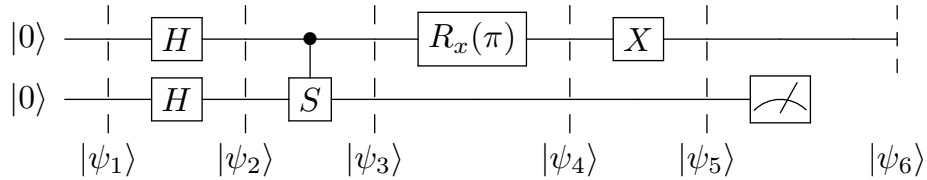


Fig. 1

## Exercise 1

Given the quantum circuit in Fig. 1, the candidate shall:

- Compute all the intermediate 2-qubit states  $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$ .
- Write the state  $|\psi_5\rangle$  as a column state vector along the standard basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ .
- Compute the measurement probabilities (the measurement is along the standard basis, only the second qubit is measured).
- Write the collapsed state  $|\psi_6\rangle$  of the remaining qubit after both possible measurement outcomes.
- From what you obtained at point **d)**, can you infer whether the state  $|\psi_5\rangle$  is entangled? Why?
- Regardless of the answer to the previous point, quantify the degree of entanglement within  $|\psi_5\rangle$  through the entanglement measure  $|\langle\psi_5|\tilde{\psi}_5\rangle|$ .
- Now assume that the gates  $R_x(\pi)$  and  $X$  are aggregated into an equivalent gate  $U$ . Compute the unitary matrix that describes  $U$ .

## Exercise 2 (optional)

The candidate can implement the quantum circuit in Fig. 1 using the Qiskit platform and compare the results obtained analytically with those estimated by the *qasm\_simulator*.

Recall that:

$$R_x(\theta) = \begin{bmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$|\tilde{\psi}\rangle = (Y \otimes Y) |\psi\rangle$$

## Solution

$$\begin{aligned}
 \text{a) } |\psi_1\rangle &= |00\rangle \\
 |\psi_2\rangle &= (H \otimes H)|00\rangle = |+\rangle|+\rangle \\
 |\psi_3\rangle &= CS|\psi_2\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + i|11\rangle) \\
 R_x(\pi) &= \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \\
 |\psi_4\rangle &= (R_x(\pi) \otimes I)|\psi_3\rangle = \frac{1}{2}(-i|10\rangle - i|11\rangle - i|00\rangle + |01\rangle) \\
 |\psi_5\rangle &= (X \otimes I)|\psi_4\rangle = \frac{1}{2}(-i|00\rangle - i|01\rangle - i|10\rangle + |11\rangle)
 \end{aligned}$$

$$\text{b) } |\psi_5\rangle = \frac{-i}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ i \end{bmatrix}$$

$$\text{c) } \Pr(0) = \Pr(1) = 1/2$$

d) Post measurement state

Outcome “0”  $\rightarrow -i|+\rangle$

Outcome “1”  $\rightarrow \frac{1}{\sqrt{2}}(-i|0\rangle + |1\rangle)$

e) Yes, it is entangled.

$$\text{f) } |\tilde{\psi}_5\rangle = (Y \otimes Y)|\psi_5\rangle = \frac{1}{2} \begin{bmatrix} -1 \\ i \\ i \\ -i \end{bmatrix}$$

$$|\langle\psi_5|\tilde{\psi}_5\rangle| = \left| \frac{1}{2} \begin{bmatrix} i & i & i & 1 \end{bmatrix} * \frac{1}{2} \begin{bmatrix} -1 \\ i \\ i \\ -i \end{bmatrix} \right| = \left| \frac{1}{2}(-1 - i) \right| = 1/\sqrt{2}$$

$$\text{g) } U = X * R_x(\pi) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} = -i I$$