Candidate name: ________________________________

Computer Architecture
Quantum Computing test
a.y. 2022/2023
26/06/2023

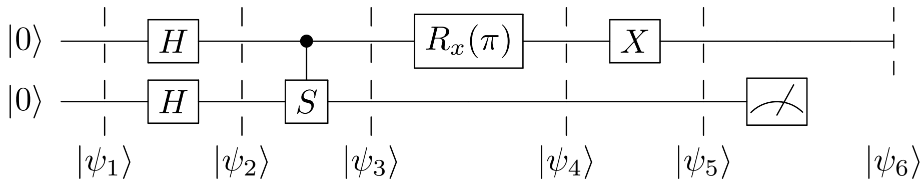

Fig. 1

Exercise 1

Given the quantum circuit in Fig. 1, the candidate shall:

a) Compute all the intermediate 2-qubit states $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$.
b) Write the state $|\psi_5\rangle$ as a column state vector along the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.
c) Compute the measurement probabilities (the measurement is along the standard basis, only the second qubit is measured).
d) Write the collapsed state $|\psi_6\rangle$ of the remaining qubit after both possible measurement outcomes.
e) From what you obtained at point **d)**, can you determine whether the state $|\psi_5\rangle$ was entangled?
f) Regardless of outcome of the previous point, quantify the degree of entanglement within $|\psi_5\rangle$ through an entanglement measure.
g) Now assume that the gates $R_x(\pi)$ and X are aggregated into an equivalent gate U. Compute the unitary matrix that describes U.

Exercise 2 (optional)

The candidate can implement the quantum circuit in Fig. 1 using the Qiskit platform and compare the results obtained analytically with those estimated by the *qasm_simulator*.

Recall that:

$$R_x(\theta) = \begin{bmatrix} \cos(\theta/2) & -i\,\sin(\theta/2) \\ -i\,\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

Solution

a) $|\psi_1\rangle = |00\rangle$
$|\psi_2\rangle = (H \otimes H)|00\rangle = |+\rangle|+\rangle$
$|\psi_3\rangle = CS|\psi_2\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + i|11\rangle)$
$R_x(\pi) = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$
$|\psi_4\rangle = (R_x(\pi) \otimes I)|\psi_3\rangle = \frac{1}{2}(-i|10\rangle - i|11\rangle - i|00\rangle + |01\rangle)$
$|\psi_5\rangle = (X \otimes I)\,|\psi_4\rangle = \frac{1}{2}(-i|00\rangle - i|01\rangle - i|10\rangle + |11\rangle)$

b) $|\psi_5\rangle = \frac{-i}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ i \end{bmatrix}$

c) Pr(0) = Pr(1) = 1/2

d) Post measurement state
Outcome “0” → $-i|+\rangle$
Outcome “1” → $\frac{1}{\sqrt{2}}(-i|0\rangle + |1\rangle)$

e) Yes, it was

f) $|\tilde{\psi}_5\rangle = (Y \otimes Y)|\psi^*{}_5\rangle = \frac{1}{2}\begin{bmatrix} -1 \\ -i \\ -i \\ i \end{bmatrix}$

$|\langle\psi_5|\tilde{\psi}_5\rangle| = |\frac{1}{2}[i \quad i \quad i \quad -1] * \frac{1}{2}\begin{bmatrix} -1 \\ i \\ i \\ -i \end{bmatrix}| = |\frac{1}{4}(-2)| = 1/2$

g) $U = X * R_x(\pi) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} = -i\,I$