Candidate name: ________________________________

Computer Architecture
Quantum Computing test
a.y. 2022/2023
11/09/2023

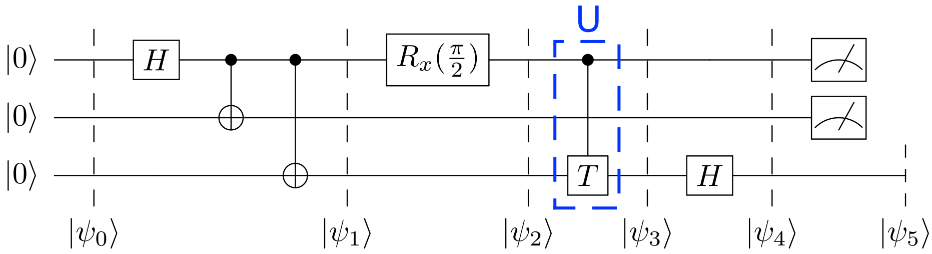

Fig. 1

Exercise 1

Given the quantum circuit in Fig. 1, the candidate shall:

a) Compute all the intermediate 3-qubit states $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle$.
b) Consider the state $|\psi_1\rangle$. Do you remember its name? Has it any property?
c) Write the 3-qubit unitary matrix that represents the U gate highlighted in Fig. 1.
d) Compute the measurement probabilities (the measurement is along the standard basis, only the first two qubits are measured).
e) Write the collapsed state $|\psi_5\rangle$ of the remaining qubit after all possible measurement outcomes.
f) From what you obtained at point **e)**, can you infer whether the state $|\psi_4\rangle$ is entangled? Explain your reasoning in a few words.

Exercise 2 (optional)

The candidate can implement the quantum circuit in Fig. 1 using the Qiskit platform and compare the results obtained analytically with those estimated by the *qasm_simulator*.

Recall that:

$$R_x(\theta) = \begin{bmatrix} \cos(\theta/2) & -i\,\sin(\theta/2) \\ -i\,\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

Solution

a) $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

Given $R_x(\pi/2) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$

$|\psi_2\rangle = \frac{1}{2}(|000\rangle - i|000\rangle - i|011\rangle + |111\rangle)$

$|\psi_3\rangle = \frac{1}{2}(|000\rangle - i|100\rangle - i|011\rangle + e^{i\pi/4}|111\rangle)$

$$|\psi_4\rangle = \frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle - i|100\rangle - i|101\rangle - i|010\rangle + i|011\rangle + e^{i\pi/4}|110\rangle - e^{i\pi/4}|111\rangle)$$

b) GHZ state, maximally entangled

c) $I \otimes T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\pi/4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\pi/4} \end{bmatrix}$,

Then $CT = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{i\pi/4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{i\pi/4} \end{bmatrix}$

d) All four probabilities are ¼

e) "00" $\rightarrow |+\rangle$

"01" $\rightarrow -i|-\rangle$

"10" $\rightarrow -i|+\rangle$

"11" $\rightarrow e^{i\pi/4}|-\rangle$

f) Yes, it is entangled because the state of the remaining qubit varies depending on the measurement outcome.