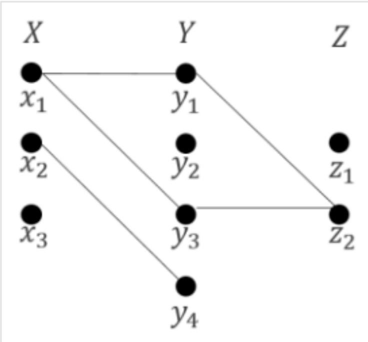
Example 1: Max min composition

We will try to understand max min composition with multiple examples. Let R = { (x_1, y_1), (x_1, y_3), (x_2, y_4) } and S = { (y_1, z_2), (y_3, z_2) } . let us find the Max-Min composition of these relations.

As we know, the representation of crisp relation could take multiple forms. The above relation R and S we can represent as,

Sagittal representation:



Matrix representation:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

The final composition of relation would look something like this,

$$T = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

Composition of relations R and S

Let us see how to fill the cells of the composition matrix T

$\chi_T(x_1, z_1)$ = max(min($\chi_R(x_1, y_1)$, $\chi_S(y_1, z_1)$), min($\chi_R(x_1, y_2)$, $\chi_S(y_2, z_1)$), min($\chi_R(x_1, y_3)$, $\chi_S(y_3, z_1)$), min($\chi_R(x_1, y_4)$, $\chi_S(y_4, z_1)$))

$\chi_T(x_1, z_1)$ = max(min(1, 0),min(0, 0),min(1, 0),min(0, 0))

$\chi_T(x_1, z_1)$ = max(0,0,0,0) = 0

Similarly,

$\chi_T(x_1, z_2) = \max(\min(1, 1), \min(0, 0), \min(1, 1), \min(0, 0)) = \max(1, 0, 1, 0) = 1$

$\chi_T(x_2, z_1) = \max(\min(0, 0), \min(0, 0), \min(0, 0), \min(1, 0)) = \max(0, 0, 0, 0) = 0$

$\chi_T(x_2, z_2) = \max(\min(0, 1), \min(0, 0), \min(0, 1), \min(1, 0)) = \max(0, 0, 0, 0) = 0$

$\chi_T(x_3, z_1) = \max(\min(0, 0), \min(0, 0), \min(0, 0), \min(0, 0)) = \max(0, 0, 0, 0) = 0$

$\chi_T(x_3, z_2) = \max(\min(0, 1), \min(0, 0), \min(0, 1), \min(0, 0)) = \max(0, 0, 0, 0) = 0$

Thus, the composition of relation R and S would be,

$$T = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

Composition of relation R and S