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Block Ciphers

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Block Ciphers

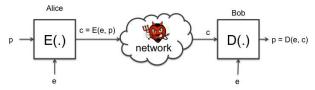
GENERAL CONCEPTS

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Block cipher



 Block ciphers break up the plaintext in blocks of fixed length n bits and encrypt one block at time



- $E_k: \{0,1\}^n \to \{0,1\}^n$ $D_k: \{0,1\}^n \to \{0,1\}^n$
- E is a keyed permutation: E(k, m) = E_k(m)
- $E_{\kappa}(\cdot)$ is a permutation

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Permutation



- E_k is a permutation
 - E_K is efficiently computable
 - Ek is bijective
 - Surjective (or onto)
 - Injective (or one-to-one)
 - E_k^{-1} is efficiently computable

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Examples



Block ciphers

DES n = 64 bits, k = 56 bits
 3DES n = 64 bits, k = 168 bits

- AES n = 128 bits k = 128, 192, 256 bits

Performance (AMD Opteron, 2.2 GHz)

- RC4
- Salsa20/12
- Sosemanuk
- 3DES
- AES-128
126 MB/s
727 MB/s
13 MB/s
109 MB/s

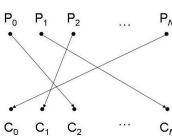
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Random permutations



 $N = 2^n - 1$



- Let Perm_n be the set of all permutations $\pi: \{0,1\}^n \to \{0,1\}^n$
- $|Perm_n| = 2^n!$

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- A true random cipher
 - implements all the permutations in Perm_n
 - $\ \, \text{uniformly selects a permutation} \\ \pi \in \text{Perm}_{\text{n}} \ \, \text{at random}$

A possible random permutation $\boldsymbol{\pi}$

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True Random Cipher



- A True random cipher is perfect
- A true random cipher implements all possible Random permutations (2ⁿ!)
 - Need a uniform random key for each permutation (naming)
 - key size := $\log_2 (2n!) \approx (n 1.44) 2^n$
 - Exponential in the block size!
 - The block size cannot be small in order to avoid a dictionary attack
- A true random cipher cannot be implemented

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Pseudorandom permutations



- Consider a family of permutations parametrized by κ $\in K = \{0, 1\}^k, E_{\kappa} : \{0, 1\}^n \rightarrow \{0, 1\}^n$
- A E_κ is a pseudorandom permutation (PRP) if it is indistinguishable from a uniform random permutation by a limited adversary
- $|\{E_{\kappa}\}| = 2^k << |Perm_n|, with |\kappa| = k$
- A block cipher is a practical instantiation of a PRP

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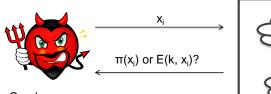
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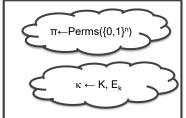
Practical block cipher



 In practice, the encryption function corresponding to a randomly chosen key should appear as a randomly chosen permutation to a limited adversary



- Oracle access
 - · adversary cannot look into the box



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Exhaustive key search



- The attack
 - Given a pair (pt, ct), check whether ct == $E_{ki}(pt)$, i = 0, 1, ..., $2^k 1$
 - Known-plaintext attack
 - Time complexity: O(2k)
- False positives
 - Do you expect that just one key k maps pt into ct?
 - How many keys (false positives) do we expect to map pt into ct?

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– How do you discriminate the good one?

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Exhaustive key search



- False positives
 - Do you expect that just one key k maps pt into ct?
 - How many keys (false positives) do we expect to map pt into ct?
 - How do you discriminate the good one?

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False positives



- Problem: Given (ct, pt) s.t. ct = E_{k*}(pt) for a given k*, determine the number of keys that map pt into ct
- · Solution.
 - Given a certain key k, $P(k) = Pr[E_{k*}(pt) == ct] = 1/2^n$
 - The *expected* number of keys that map pt into ct is $2^k \times 1/2^n = 2^{k-n}$

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False positives



- Example 1 DES with n = 64 and k = 56
 - On average 2⁻⁸ keys map pt into ct
 - One pair (pt, ct) is sufficient for an exhaustive key search
- Example 2 Skipjack with n = 64 and k = 80
 - On average 2¹⁶ keys map pt into ct
 - Two or more plaintext-ciphertext pairs are necessary for an exhaustive key search

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False positives



- Consider now t pairs (pt_i, ct_i), i = 1, 2,..., t
- Given k*, $Pr[E_{k*}(pt_i) = ct_i$, for all $i = 1, 2, ..., t] = 1/2^{tn}$
 - Expected number of keys that map pt_i into ct_i , for all i = 1, 2, ..., t, is $2^k/2^{tn} = 2^{k-tn}$
- Example 3 Skypjack with k = 80, n = 64, t = 2
 - The expected number of keys is = $2^{80-2\times64}$ = 2^{-48}
 - Two pairs are sufficient for an exhaustive key search

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False positives



THEOREM

Given a block cipher with a key length of k bits and a block size of n bits, as well as t plaintext-ciphertext pairs, (pt₁, ct₁),..., (pt_t, ct_t), the expected number of false keys which encrypt all plaintexts to the corresponding ciphertexts is 2^k

FACT

Two input-output pairs are generally enough for exhaustive key search

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EXERCISES

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Exercise 1 - Exhaustive key search



- Exhaustive key search is a known-plaintext attack
- However, the adversary can mount a cyphertext-only attack if (s)he has some knowledge on PT

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Exercise 1 – exhaustive key search



- Assume DES is used to encrypt 64-bit blocks of 8 ASCII chars, with one bit per char serving as parity bit
- How many CT blocks the adversary needs to remove false positives with a probability smaller than ϵ ?

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Exercise 2 - dictionary attack



- Consider E with k and n.
- The adversary has collected D pairs (pt_i, ct_i), i = 1,..., D, with D << 2ⁿ
- Now the adversary reads C newly produced cyphertexts ct*_i, j = 1,..., C.
- Determine the value of C s.t. the Pr[Exists j, j = 1, 2,...
 C, s.t. c*_i is in the dictionary] = P

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Exercise 3 - Rekeying



- An adversary can successfully perform an exhaustive key search in a month.
- Our security policy requires that keys are changed every hour.
- What is the probability P that, in a month, the adversary is able to find any key before it is changed?
 - For simplicity assume that every month is composed of 30 days.
- What if we refresh key every minute?

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Symmetric Encryption

MULTIPLE ENCRYPTION AND KEY WHITENING

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Increasing the Security of Block Ciphers



- DES is a secure cipher
 - No efficient cryptanalys is known
- DES key has become too short
- Can we improve the security of DES?
- · Yes, by means of two techniques
 - Multiple encryption
 - Key whitening

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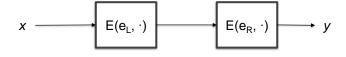
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Two-times Encryption (2E)



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- $y = 2E((e_1, e_R), m) = E(e_R, E(e_1, x))$
 - key size is 2k bits
 - Brute force attack requires 2^{2k} steps
 - 2E is two times slower than E
- Is it really secure?
- Meet-in-the-middle attack



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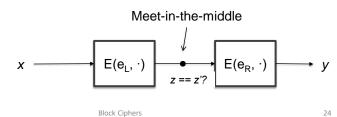
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Meet-in-the-middle attack



- Attack Sketch
 - 1. Build a table T containing $z = E(e_L, x)$ for all possible keys e_L . Keep T sorted according to z.
 - 2. Check whether $z' = D(e_R, y)$ is contained in the table T, for all possible key e_R .
 - 1. If z' in contained in T then (e_L, e_R) maps x into y with e_L s.t. $T[e_L] = z'$.



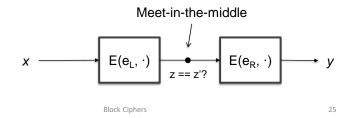
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Meet-in-the-middle attack



- Attack complexity
 - Storage complexity
 - Storage necessary for table $T \approx O(2^k)$
 - Time complexity
 - Time complexity for step 1 + Time complexity for step 2 = Time for building and sorting the table + Time for searching in a sorted table = $k 2^k + k 2^k \approx O(2^k)$



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Two-times DES



- 2DES
 - Time complexity: 2⁵⁶ (doable nowadays!)

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- Space complexity: 2⁵⁶ (lot of space!)
- 2DES brings no advantage

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Triple DES (3DES)



- EDE scheme
 - Standard ANSI X9.17 and ISO 8732
 - $Y = 3E((e_1, e_2, e_3), x) = E(e_1, D(e_2, E(e_3, x)))$
 - If $e_1 = e_2 = e_3$, 3DES becomes DES
 - backward compatibility
 - Key size = 168-bits
 - 3 times slower than DES
 - Simple attack ≈ 2¹¹⁸

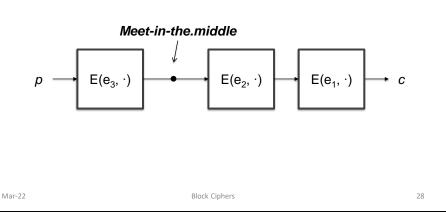
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3DES - meet-in-the-middle attack



- Time = 2¹¹² (undoable!)
- Space = 2⁵⁶ (lot of space!)



False positives for multiple encryption

THEOREM

Given there are r subsequent encyptions with a block cipher with a key lenght of k bits and a block size of n bits, as well as t plaintext-ciphertext pairs, (pt₁, ct₂),..., (pt_t, ct_t), the expected number of false keys which encrypt all plaintext to the corresponsig ciphertext is 2^{rk-tn}

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Limitations of 3DES



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- 3DES resists brute force but
 - It is not efficient regarding software implementation
 - It has a short block size (64 bit)
 - A drawback if you want to make a hash function from 3DES, for example
 - Key lengths of 256+ are necessary to resist quantum computing attack

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Key whitening

- Considerations
 - KW is not a "cure" for weak ciphers
- Applications
 - DESX: a variant of DES
 - AES: uses KW internally
- Performance
 - Negliglible overhead w.r.t. E (Just two XOR's!)

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Key whitening



Ε

Definition 5.3.1 Key whitening for block ciphers

Encryption: $y = e_{k,k_1,k_2}(x) = e_k(x \oplus k_1) \oplus k_2$. **Decryption**: $x = e_{k,k_1,k_2}^{-1}(x) = e_k^{-1}(y \oplus k_2) \oplus k_1$

k2

k1

Attacks

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- Brute-force attack
 - Time complexity: 2^{k+2n} encryption ops
- Meet-in-the-middle:
 - Time complexity 2^{k+n}
 - Storage complexity: 2ⁿ data sets
- The most efficient attack
 - If the adversary can collect 2^m pt-ct pairs, then time complexity becomes 2^{k+n-m}
 - The adversary cannot control m (rekeying)
 - Example: DES (m = 32)
 - Time complexity 288 encryptions (nowadays, out of reach)
 - Storage complexity 2³² pairs = 64 GBytes of data (!!!)

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Symmetric Encryption

ENCRYPTION MODES

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Encryption Modes



- A block cipher encrypts PT in fixed-size *n*-bit blocks
- When the PT len exceeds n bits, there are several modes to use the block cipher
 - Electronic Codebook (ECB)
 - Cipher-block Chaining (CBC)

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Other encryption modes



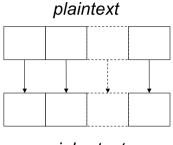
- · Other encryption modes
 - To build a stream cipher out of a block cipher
 - Cipher Feedback mode (CFB)
 - Output Feedback mode (OFB)
 - Counter mode (CTR)
 - Authenticated encryption
 - Galois Counter mode (GCM, CCM, ...)
 - and many others (e.g., CTS, ...)
- Block ciphers are very versatile components

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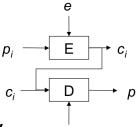
Electronic codebook





ciphertext

 $\forall 1 \le i \le t, c_i \leftarrow E(e, p_i)$ $\forall 1 \le i \le t, p_i \leftarrow D(e, c_i)$



PT blocks are encrypted separately

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Block Ciphers

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ECB - properties



- **PROS**
 - No block synchronization is required
 - No error propagation
 - One or more bits in a single CT block affects decryption of that block only
 - Can be parallelized
- CONS (it is insecure)
 - Identical PT results in identical CT
 - ECB doesn't hide data pattern
 - · ECB allows traffic analysis
 - Blocks are encrypted separately
 - ECB allows block re-ordering and substitution

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ECB doesn't hide data patterns











Plaintext

ECB encrypted

Non-ECB encrypted

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ECB – block attack



- Bank transaction that transfers a customer C's amount of money D from bank B1 to bank B2
 - Bank B1 debits D to C
 - Bank B1 sends the "credit D to C" message to bank B2
 - Upon receiving the message, Bank B2 credits D to C
- Credit message format
 - Src bank: M (12 byte)
 - Rcv banck: R (12 byte)
 - Customer: C (48 byte)
 - Bank account number: N (16 byte)Amount of money: D (8 byte)
- Cipher: n = 64 bit; ECB mode

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ECB – block attack



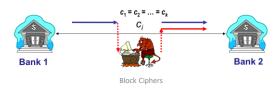
- Mr. Lou Cipher is a client of the banks and wants to make a fraud
- · Attack aim
 - To replay Bank B1's message "credit 100\$ to Lou Cipher" many times
- Attack strategy
 - Lou Cipher activates multiple transfers of 100\$ so that multiple messages "credit 100\$ to Lou Cipher" are sent from B1 to B2
 - The adversary identifies at least one of these messages
 - The adversary replies the message several times

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ECB – block attack



- · The fraud
 - 1. Mr. Lou Cipher performs k equal transfers
 - credit 100\$ to Lou Cipher → c1
 - credit 100\$ to Lou Cipher → c2
 - ...
 - credit 100\$ to Lou Cipher \rightarrow c_k
 - 2. Then, he searches for "his own" CTs, namely k equal CTs!
 - 3. Finally he replies one of these cryptograms (many times)



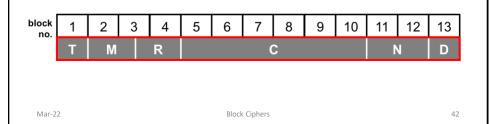
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ECB – block attack



- The message lacks any notion of time so it can be easily replied
- An 8-byte timestamp field T (block #1) is added to the message to prevent replay attacks
- A replied message can now be discarded



ECB – block attack



- However, Mr Lou Cipher can still perform the attack
 - 1. Identify "his own" CTs by inspecting blocks #2-#13
 - 2. Select any his-own-CT
 - 3. Substitute block #1 of his-own-CT with block #1 of any intercepted "fresh" block
 - 4. Replay the resulting CT

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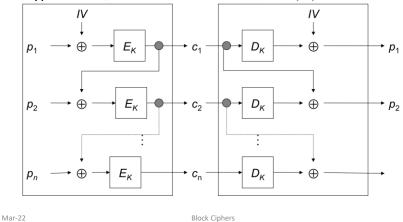
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Cipher block chaining (CBC)



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Encryption: $c_0 \leftarrow IV. \forall 1 \le i \le t, c_i \leftarrow E_k (p_i \oplus c_{i-1})$ Decryption: $c_0 \leftarrow IV. \forall 1 \le i \le t, p_i \leftarrow c_{i-1} \oplus D_k (c_i)$



CBC – properties (\rightarrow)



- · CBC mode is CPA-secure
- Chaining dependencies: c_i depends on p_i and the preceding PT blocks
- Cyphertext expansion is just one block
- CBC-Enc is randomized by using IV (nonce)
 - Identical ciphertext results from the same PT under the same key and IV
- CT-block reordering affects decryption

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CBC – properties



- IV can be sent in the clear but its integrity must be guaranteed
- CBC suffers from Error propagation
 - Bit errors in c_i affect p_i and p_{i+1} (error propagation)
 - CBC is self-synchronizing (error recovery)
 - CBC does not tolerate "lost" bits (framing errors)
- · CBC-dec can be parallelized

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CBC – block attack



- If Bank A chooses a random IV for each wire transfer the attack will not work
- However, if Lou Cipher substitutes blocks #5 #10 and #13, bank B would decrypt account number and deposit amount to random numbers => this is highly undesirable!
- Encryption itself is not sufficient, we need additional mechanisms (MDC, MAC, digsig) to protect integrity

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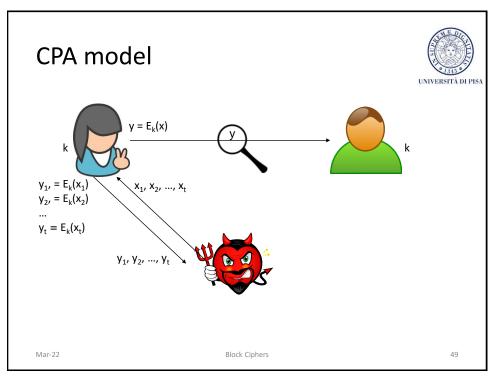
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Chosen-Plaintext Attack (Informal)



- CPA Attack
 - Attacker makes the sender to encrypt $x_1,..., x_t$
 - · The attacker may influence or control encryption
 - The sender encrypts and transmits $y_1 = E_k(m_1)$, ..., $y_t = E_k(m_1)$
 - Later on, the sender encrypts x and transmits $y = E_k(m)$
- CPA-security guarantees that the adversary cannot learn anything about x
- The encryption scheme must be randomized

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MORE ENCRYPTION MODES: OFB,

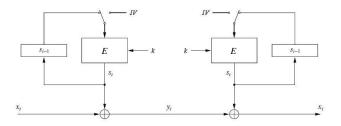
CFB, CTR, CTS

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Output Feedback Mode (OFB)





Let e() be a block cipher of block size b; let x_i , y_i and s_i be bit strings of length b; and IV be a nonce of length b.

Encryption (first block): $s_1 = e_k(IV)$ and $y_1 = s_1 \oplus x_1$

Encryption (general block): $s_i = e_k(s_{i-1})$ and $y_i = s_i \oplus x_i$, $i \ge 2$

Decryption (first block): $s_1 = e_k(IV)$ and $x_1 = s_1 \oplus y_1$

Decryption (general block): $s_i = e_k(s_{i-1})$ and $x_i = s_i \oplus y_i$, $i \ge 2$

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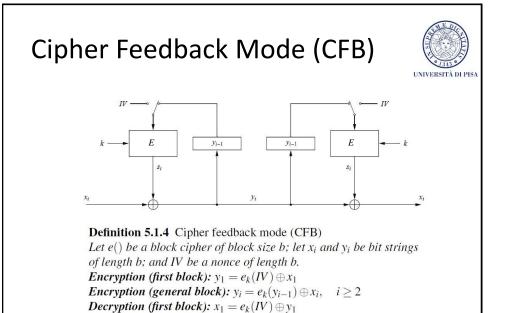
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Output Feedback Mode (OFB)



- OFB builds a stream cipher out of a block cipher
- The key stream is generated block-wise
- OFB is a synchronous stream cipher
- The receiver does not use decryption
- IV should be a nonce and make OFB nondeterministic
- Since OFB is synchronous, pre-computation of key stream blocks is possible

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Cipher Feedback Mode (CFB)



OFB builds a stream cipher out of a block cipher

Decryption (general block): $x_i = e_k(y_{i-1}) \oplus y_i, i \ge 2$

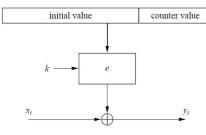
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- CFB is an asynchronous stream cipher as the key stream is also a function of the CT
- Key stream is generated block-wise
- IV is a nonce and makes CFB nondeterministic

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Counter Mode (CTR)





Definition 5.1.5 Counter mode (CTR)

Let e() be a block cipher of block size b, and let x_i and y_i be bit strings of length b. The concatenation of the initialization value IV and the counter CTR_i is denoted by $(IV||CTR_i)$ and is a bit string of length b.

Block Ciphers

Encryption: $y_i = e_k(IV||CTR_i) \oplus x_i, i \ge 1$ Decryption: $x_i = e_k(IV||CTR_i) \oplus y_i, i \ge 1$

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Counter Mode (CTR)



- CTR prevents two-time pad (keystream reuse)
- CTR can be parallelized
- Counter can be a regular counter or a more complex functions, e.g., LFSR
- · Ciphertext expansion is just one block
 - Output y_0 , y_1 , ..., y_t with $y_0 = (IV | ctr_0)$ being the the expansion block
 - IV|ctr₀ does not have to be kept secret
 - Can be transmitted together with ct y_i

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CTR is CPA-secure



- A block cipher is a good approximation of a PRP (PRF), so the sequence E_k(iv|ctr₀+1), ..., E_k(iv|ctr₀+t) is pseudorandom
 - Two-time pad when (iv|ctr₀+i) wraps around → limit to the maximum number of messages you can encrypt
 - Two-time pad when $(iv|ctr_0+i) = (iv'|ctr_0'+j)$ but the probability of this event is exponentially small

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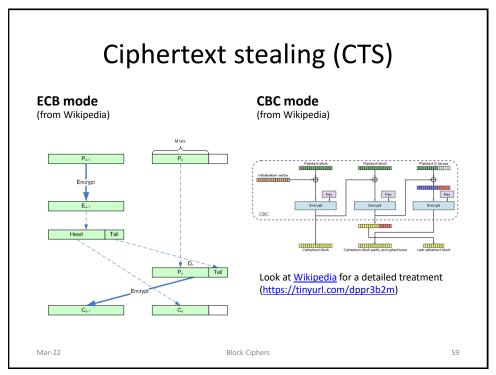
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Ciphertext Stealing (CTS) mode

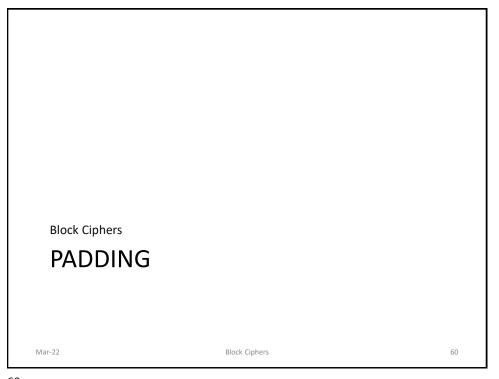


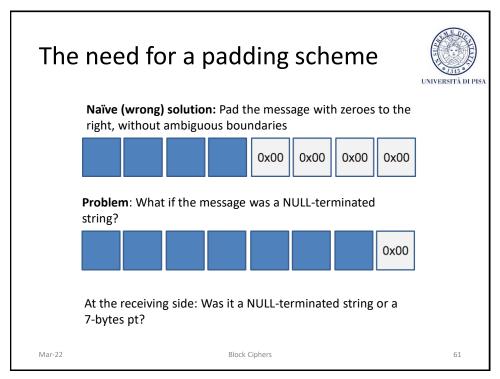
- CTS allows encrypting PT that is not evenly divisible into blocks without resulting in any ciphertext expansion
- sizeof(ciphertext) = sizeof(plaintext)
- CTS operates on the last two blocks
 - A portion of the 2nd-last CT block is stolen to pad the last PT block

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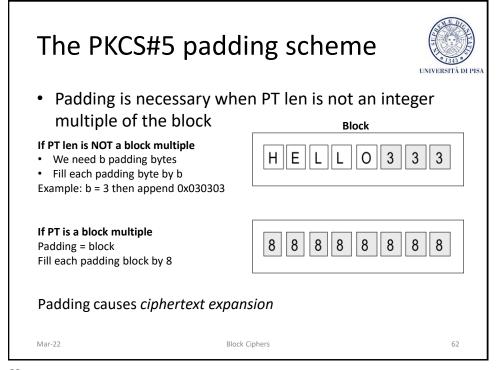


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PKCS #5: encryption



- Let L be the block length (in bytes) of the cipher
- Let b be the # of blocks that need to be appended to the plaintext to get its length a multiple of L
 - $-1 \le b \le L$
- Before encryption
 - Append b (encoded in 1 byte), b times
 - i.e., if b = 3, append 0x030303

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PKCS #5: decryption



- After decryption, say the final byte has value b
 - If b = 0 or b > L, return "error"
 - If the trailing b bytes are not all equal to b, return "error"
 - Strip off the trailing b bytes and output the left as the message

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PADDING ORACLE ATTACK

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Padding Oracle Attack



- The attacker
 - intercepts y and wants to obtain x
 - modifies y into y' and submits to the receiver
- The receiver (the padding oracle)
 - Returns "error", if x' is not properly formatted
- · On padding oracles
 - Frequently present in web applications
 - Error, receiver timing, receiver behaviour,...

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Block Ciphers

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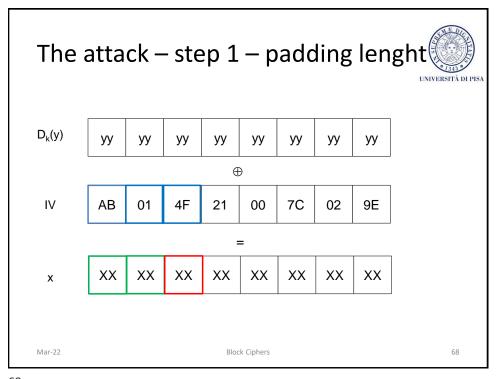
Main idea of the attack

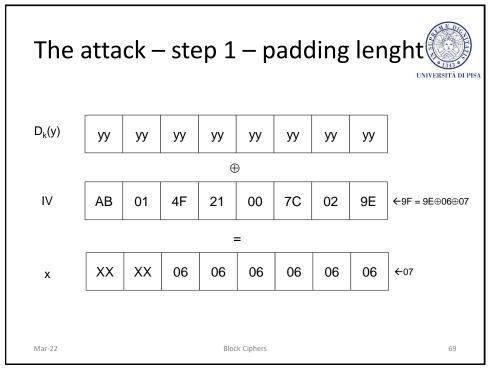


- For simplicity, let the ciphertext be a two-block ciphertext (IV, y), with y = E_k(x)
 - So, at the receiving site, $x = D_k(y) \oplus IV$
- Message x is well formatted (padding)
- · Main intuition
 - If the attacker changes the ith byte of IV, this causes a predictable change (only) to the ith byte of x'

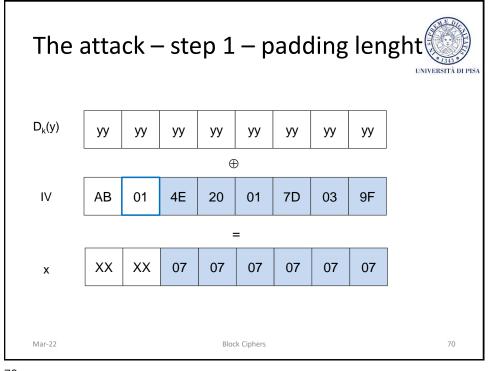
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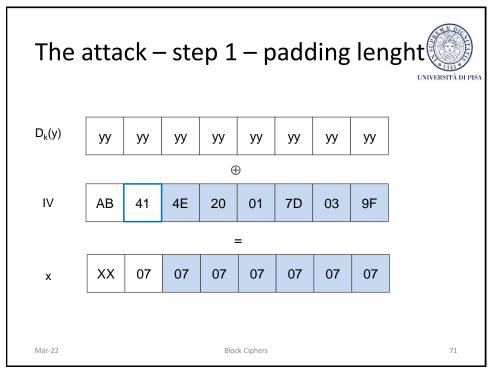
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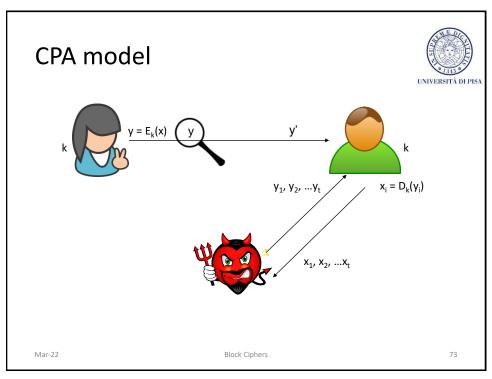
71

Attack complexity



- At most L tries to learn the # of padding bytes
- At most 28 = 256 tries to learn each plaintext byte

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Chosen-ciphertext attack



- Now the attacker becomes active
- The CCA
 - The attacker intercepts $y = E_k(x)$ and modifies it into y'
 - The receiver decrypts y' and returns (the attacker) either x' or some information about x'
 - The adversary can derive either x or some information about x
- · CCA and malleability
 - CCA-security implies non-malleability

Mar-22

Block Ciphers

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CCA-security



- Chosen-ciphertext attacks represent a significant, real-world threat
- Modern encryption schemes are designed to be CCAsecure

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