Large-Scale and Multi-Structured Databases

Graph Databases

Recap of Graph Theory

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DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

IN SUPREMÆ DIGNITATIS 1343

UNIVERSITÀ DI PISA

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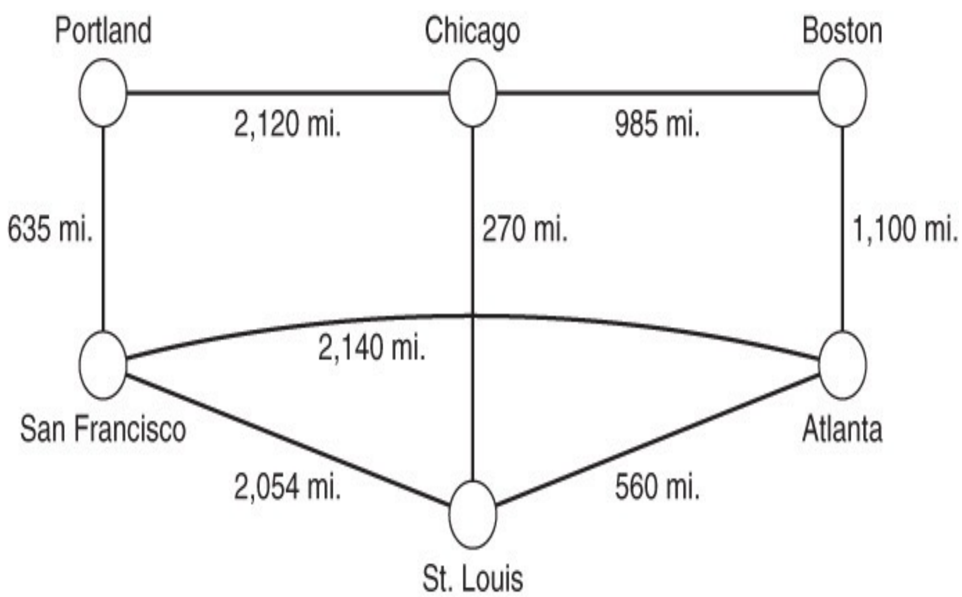
Vertex

In the figure, we show ***instances*** of an ***entity (city)*** represented by a set of vertices. We are in the context of ***highways networks***.

The ***properties*** that the vertices, namely the cities, may have are:

- Population
- Coordinates
- Name
- Geographic Region

Edges: Relationships between Vertices



The ***properties*** that the edge in the highways network, may have are:

- Distance
- Speed limit
- Number of lanes

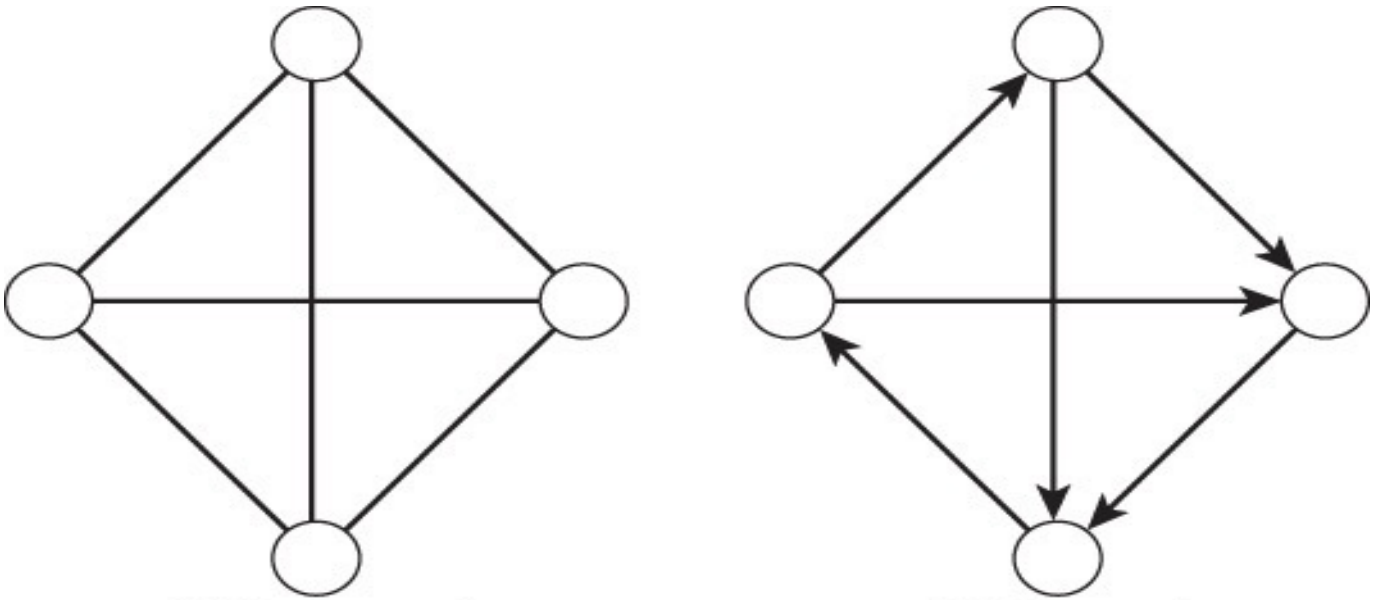
A Weight is a commonly used property of edge.

It represents some value about the ***strength*** of the relationship (cost, distance, etc.)

Undirected and Directed Graphs

Directed edges have a ***direction***, whereas undirected edge does not have a direction.

The ***presence*** or not of a ***direction*** in edges depends on ***specific applications***.



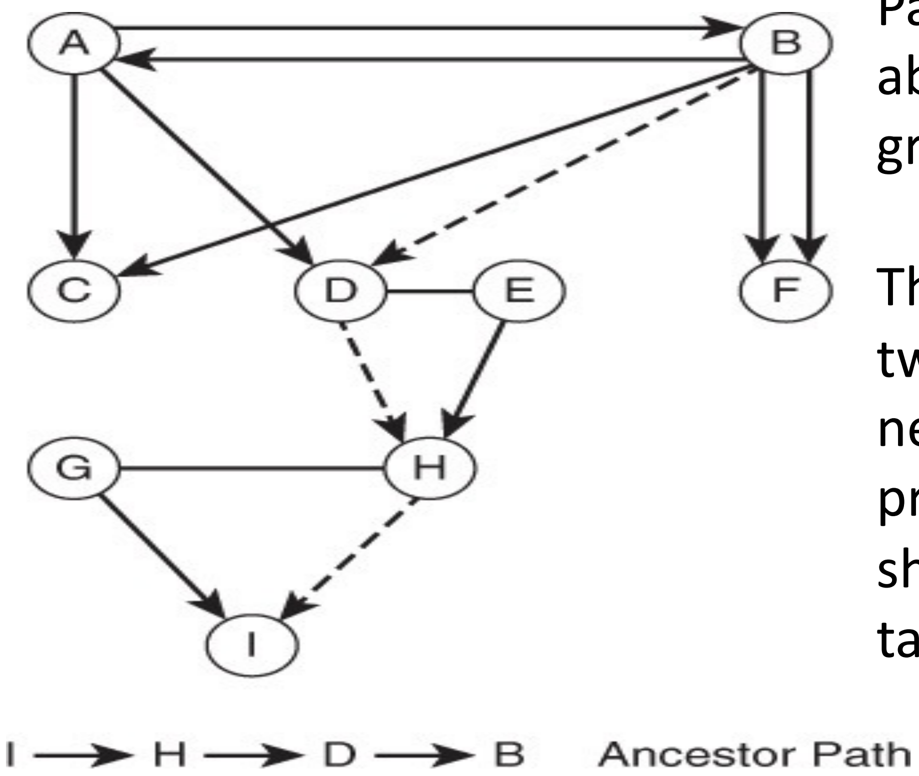
(a) Undirected (b) Directed

Example: Highways Network Graphs

Example: Family Relation Graphs

Path in a Graph

A ***path*** through a graph is a ***set of vertices*** along with the ***edges*** between those vertices.



Paths allow us to ***capture*** information about the ***relationships*** of vertices in a graph.

There may be ***multiple paths*** between two vertices. In this case (highways networks, for instance), often the problem ***of finding the best*** (the shortest, the cheapest, etc.) path is taken into consideration.

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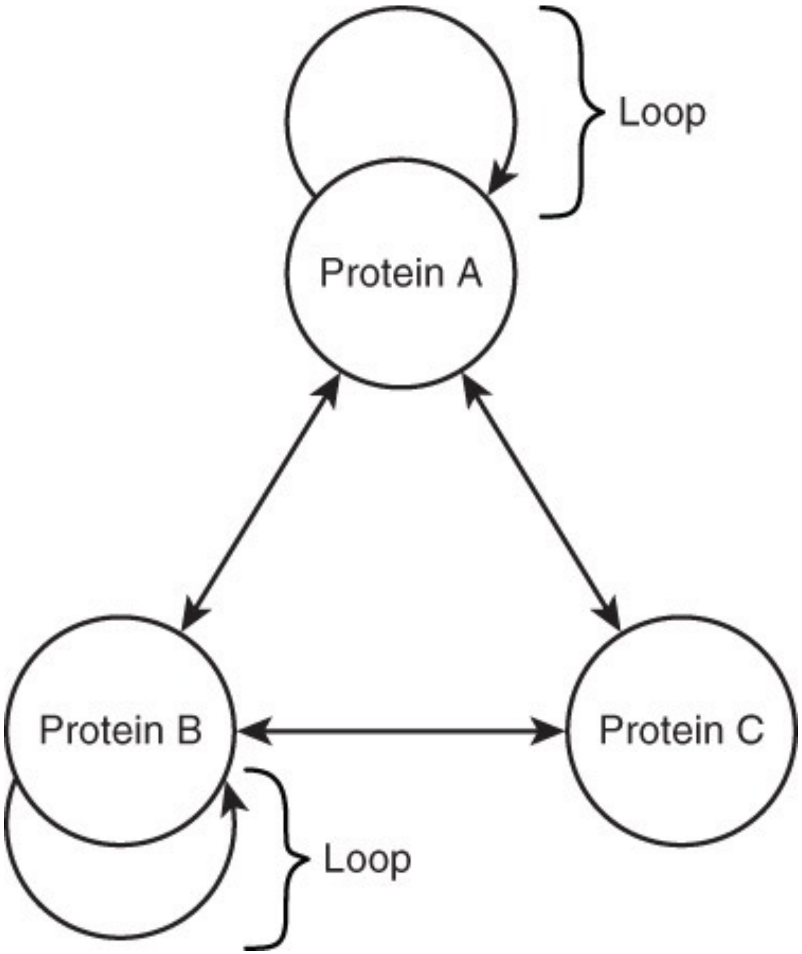
Loops in Graphs

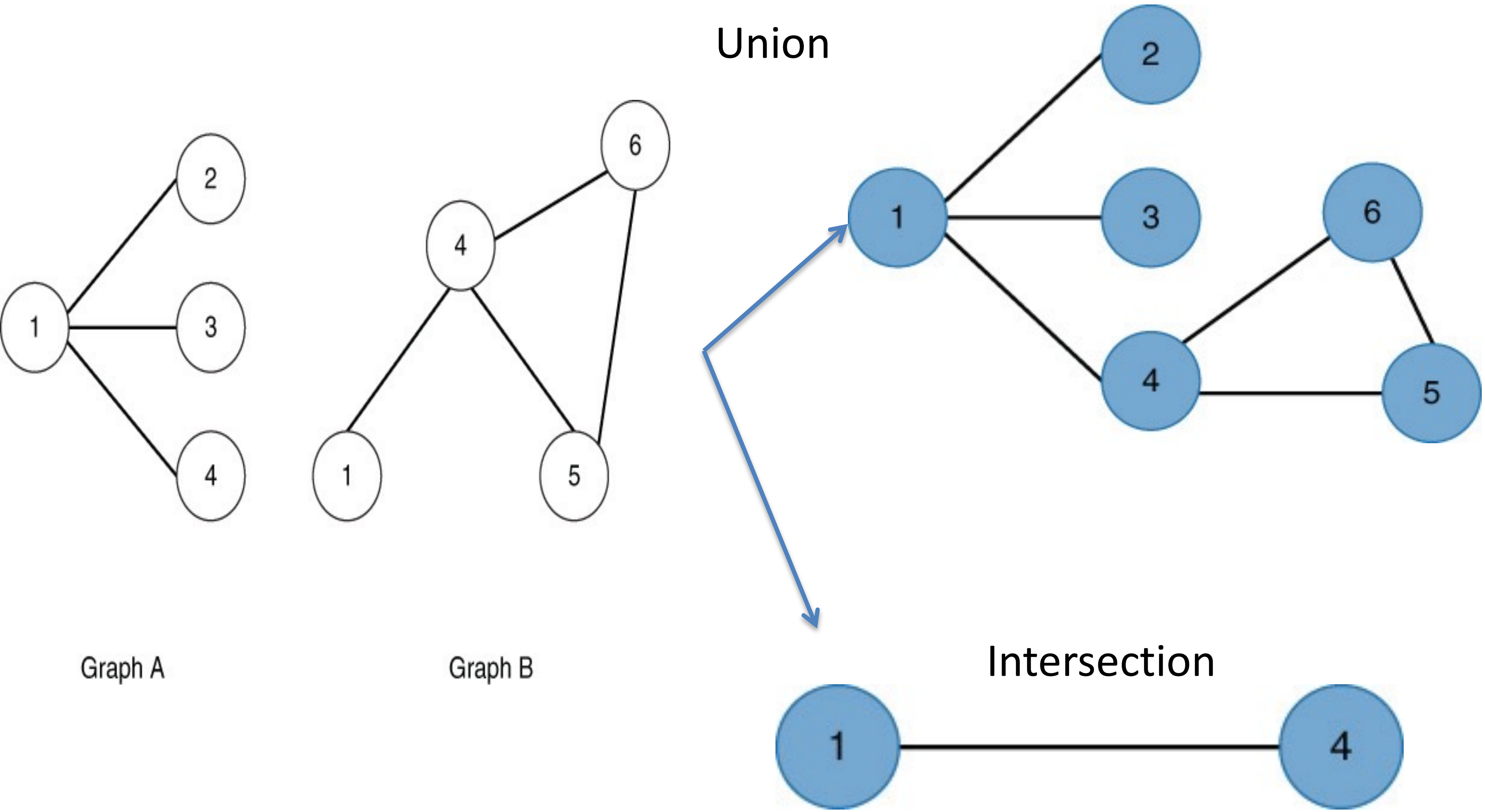


A loops represents a ***relation*** that an entities has with ***itself***.

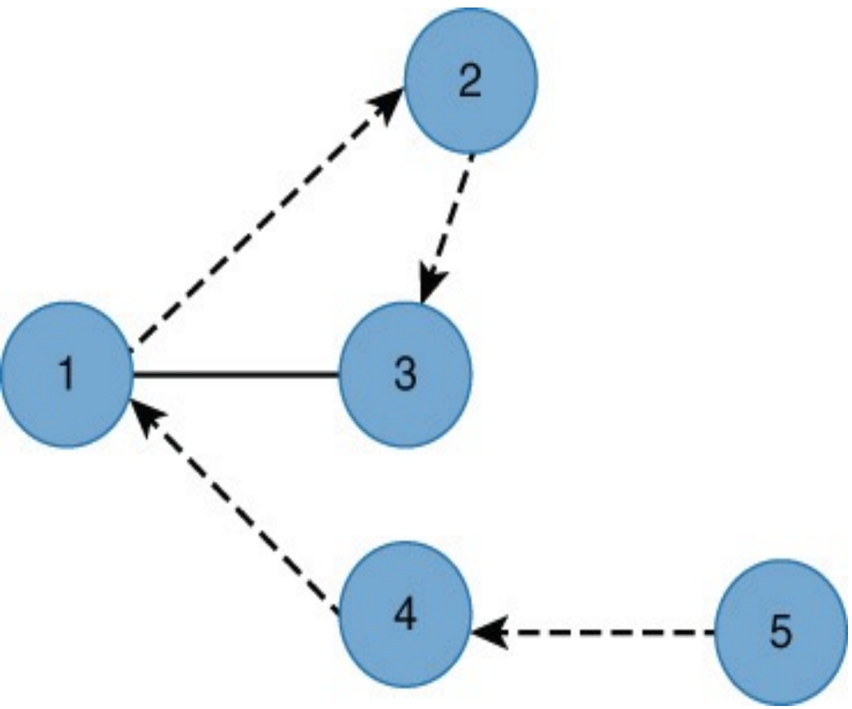
Proteins, for example, may interacts with proteins of the same type.

The presence of loop may ***not have sense*** in some specific domains, such as family tree graphs.

Union and Intersection of Graphs



Graph Traversal

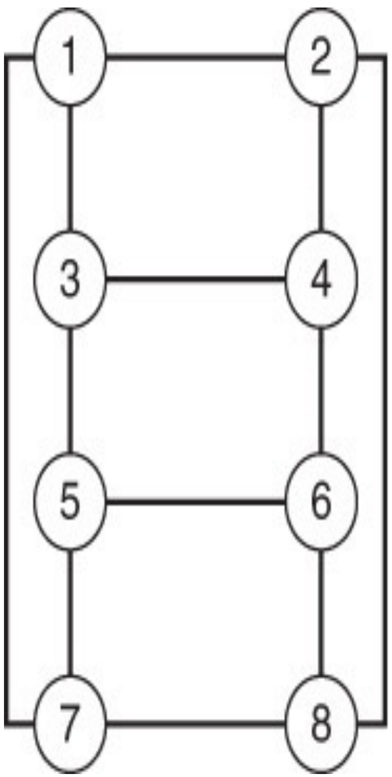


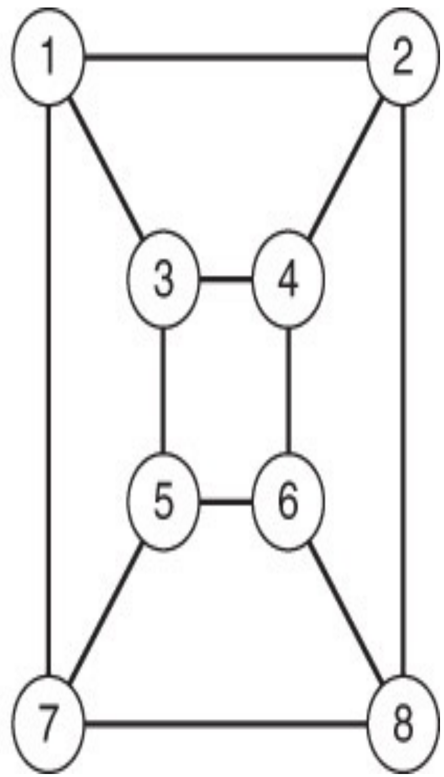
Graph traversal is the process of ***visiting all vertices*** in a graph.

The purpose of this is usually to either ***set*** or ***read*** some property value in a graph.

The traversal of the graph is often carried out in a particular way, such us ***minimizing the cost*** of the path.

Isomorphism





Two graphs are ***isomorphic*** if:

- ***for each vertex*** in the first graph, there is a ***corresponding vertex*** in the other graph

- ***for each edge*** between a pair of vertices in the first graph, there is a ***corresponding edge*** between the corresponding vertices of the other graph.

Detecting isomorphism in graphs or sub graphs allows us to identify ***useful patterns***.

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Order and Size of Graphs

Order: the number of vertices

Size: the number of edges

These two measures are very important to evaluate time and memory occupancy required to handle specific operations.

Example: to find the largest subset of people in a social network that know each other.

The higher the order and the size of the graph, the harder will be to solve the query!

Degree and Closeness of a Vertex

Degree: is the number of edges linked to a vertex. It measures the ***importance*** of any given vertex in a graph.

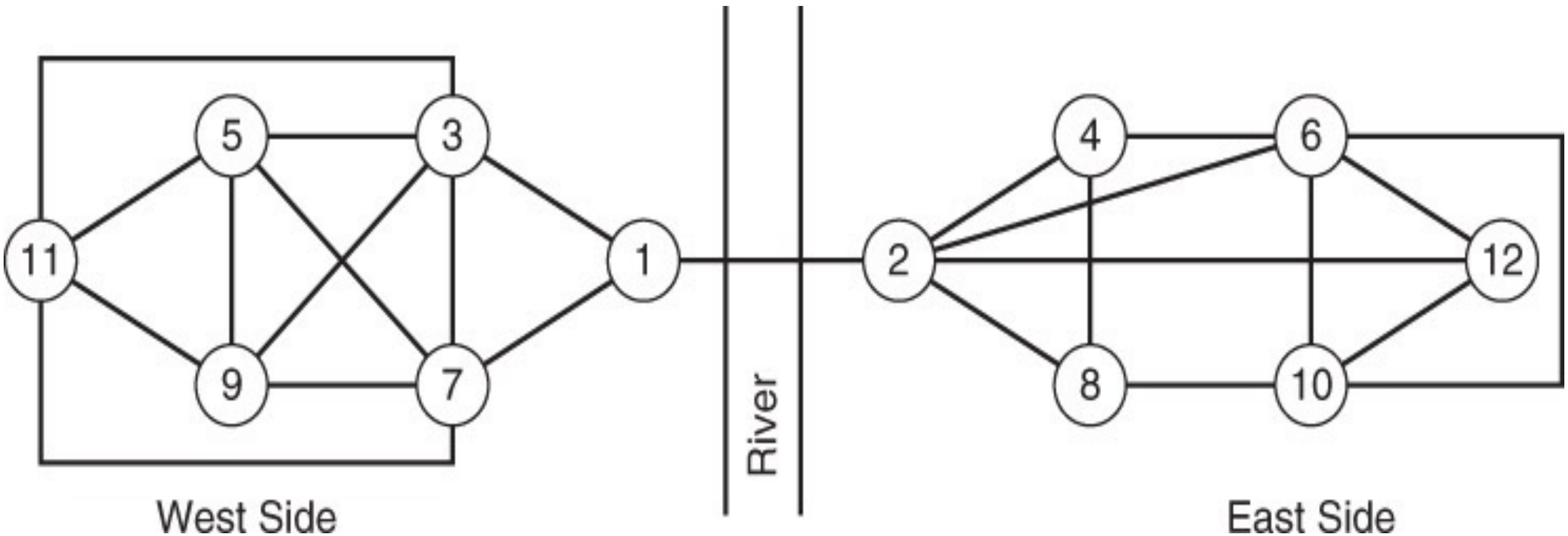
Example: Consider a person with many family members and friends that he/she sees regularly (high degree vertex). What if that person contracts a contagious disease?

Closeness: is the measure of how far the vertex is from all others in the graph**.** It can be calculated, in a fully connected graph, as the ***reciprocal of the sum*** of the length of the ***shortest paths*** between the node and all other nodes in the graph.

It is useful to evaluate ***the speed of information spreading in a network,*** starting from a specific node**.**

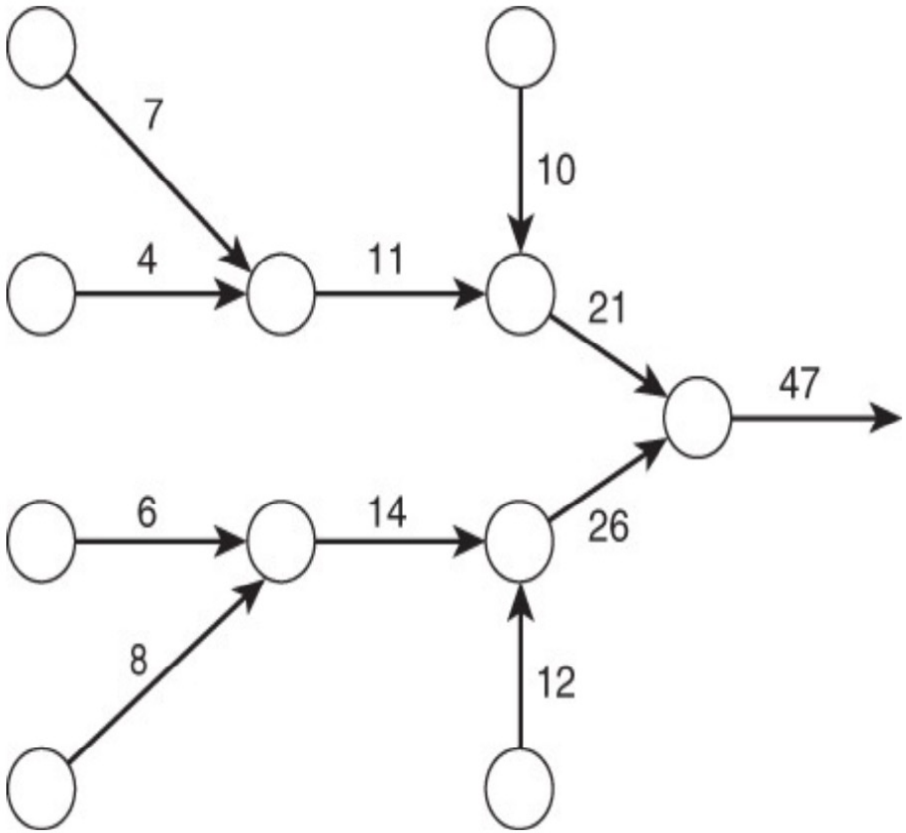
Betweenness of a Vertex

Betweenness is a measure of how much of a ***bottleneck*** a given vertex is.



In the example above, both vertices 1 and 2 will have ***high betweenness*** levels. Indeed, they form a bottleneck in the graph and if one of them were removed, it would leave the graph ***disconnected***.

Flow Networks



A flow network is a ***direct graph***.

It adopts ***capacitated*** edges.

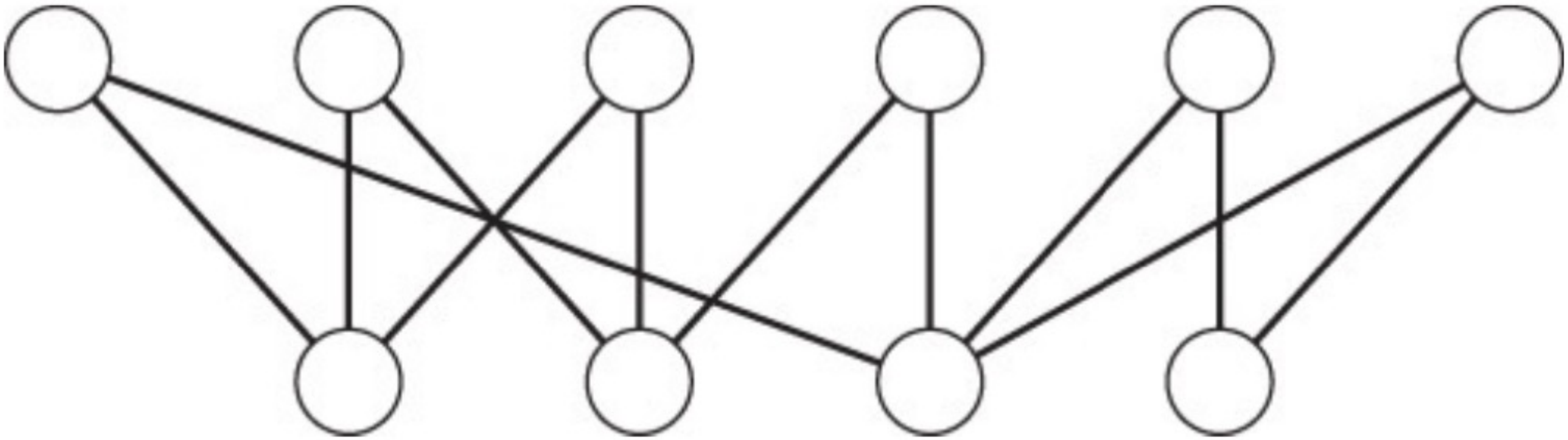
Each vertex has a set of ***incoming*** and ***outcoming*** edges.

In each vertex the sum of the capacities of incoming edges ***cannot be greater*** than the sum of the capacities of outcoming edges.

The unique exception to the previous rule regards the ***source*** and the ***sink*** nodes.

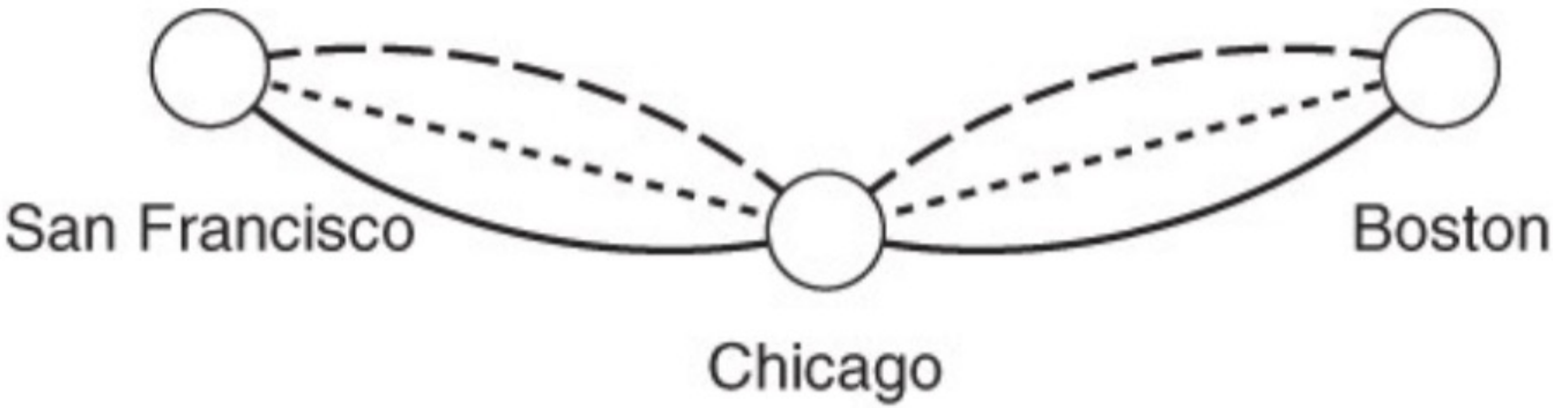
Applications: road systems, transportation networks, networks of storms drains

Bipartite Graph



- There are two ***distinct sets*** of vertices.
- Each vertex is connected only with vertices of ***the other set***.
- It can be used for modeling relationships between students and teachers.

Multigraph



- It is characterized by the presence of ***multiple edges*** between vertices.
- In the example above, each edge may represents a ***different way*** (by plane, by truck, etc.) to shipping item between the cities.
- Each edge can be equipped with ***different sets of properties***.

Suggested Readings

Chapter 13 of the book "*Dan Sullivan, NoSQL For Mere Mortals, Addison-Wesley, 2015*".

Images

If not specified, the images shown in this lecture have been extracted from:

"Dan Sullivan, NoSQL For Mere Mortals, Addison-Wesley, 2015"