Bauhaus-Universität Weimar

Image Analysis and Object Recognition

Assignment 4

Image filtering in frequency domain
&
Shape recognition using Fourier descriptors

SS 2017

(Course notes for internal use only!)

Comments A 3: Algorithm Outline

- **Input:** binary edge image (from GoG-filtering)
- **Initialize** index vectors
  - $\boldsymbol{\rho_{ind}} = [-\rho_{max}, \dots, \rho_{max}],\ \rho_{max} = \sqrt{n_{rows}^2 + n_{columns}^2}$
  - $\boldsymbol{\theta_{ind}} = [-90, \dots, 89]$
- **Initialize** voting array H
  - $H = zeros(2 \cdot \rho_{max} + 1, 180)$
- **for** each edge point (x, y) in the image

  $\boxed{\theta}$ = gradient orientation at (x, y)

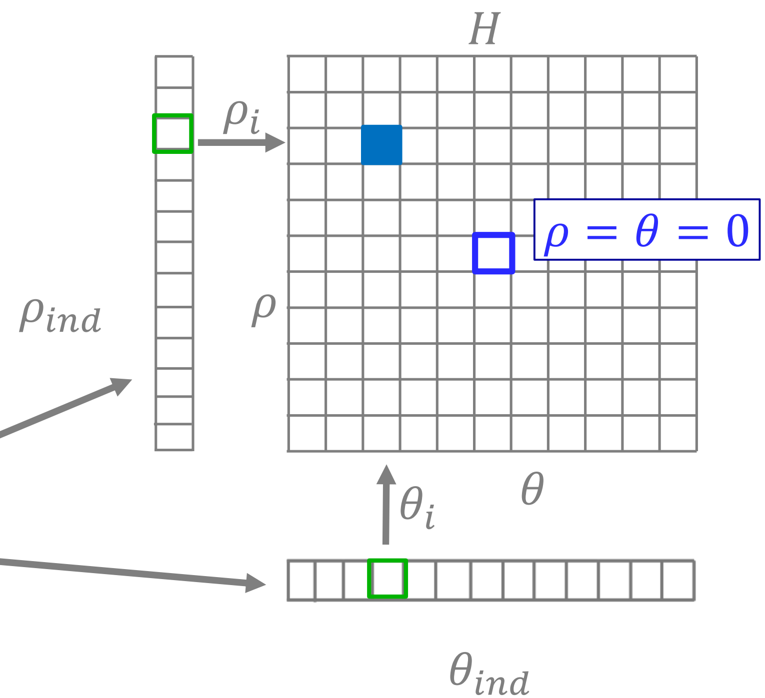
  $\boxed{\rho} = x \cdot cos\theta + y \cdot sin\theta$

  $\theta_i = find(\theta_{ind} == \theta)$

  $\rho_i = find(\rho_{ind} == \rho)$

  $\boxed{H(\rho_i, \theta_i)} = H(\rho_i, \theta_i) + 1$

  end



Comments A 3: Algorithm Outline

- **Input:** binary edge image (from GoG-filtering)
- ~~**Initialize** index vectors~~
  - $\cancel{\boldsymbol{\rho_{ind}} = [-\rho_{max}, \dots, \rho_{max}],\ \rho_{max} = \sqrt{n_{rows}^2 + n_{columns}^2}}$
  - $\cancel{\boldsymbol{\theta_{ind}} = [-90, \dots, 89]}$
- **Initialize** voting array H
  - $H = zeros(2 \cdot \rho_{max} + 1, 180)$
- **for** each edge point (x, y) in the image

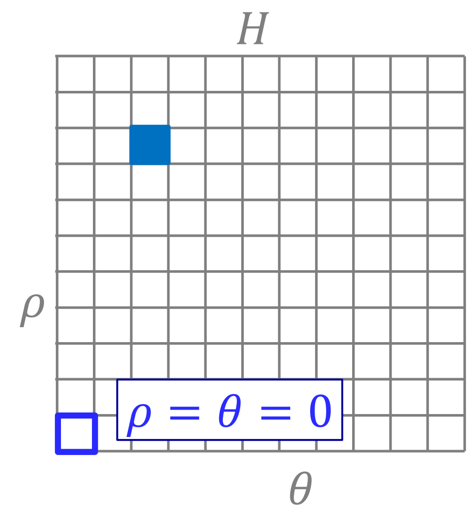
  θ = gradient orientation at (x, y)

  $\rho = x \cdot cos\theta + y \cdot sin\theta$

  $$\boxed{\begin{aligned} \rho_i &= \rho + \rho_{max} + 1 \\ \theta_i &= \theta + 91 \end{aligned}}$$

  $H(\rho_i, \theta_i) = H(\rho_i, \theta_i) + 1$

  end



Comments A 3: Algorithm extension

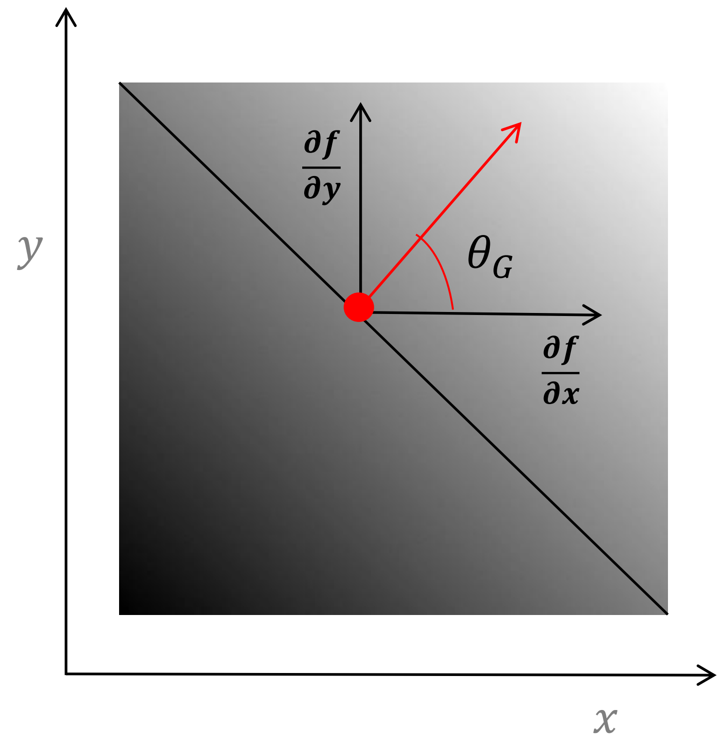
- Use the **gradient direction** of detected edges
  - GoG-filtering → first derivatives in x- and y-direction: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$
  - Gradient direction: $\theta_G = atan\boxed{\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)}$

- Attention using coordinates

$$\rho = x \cdot cos\theta + y \cdot sin\theta$$

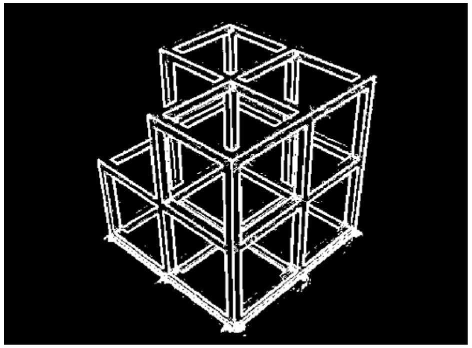
→ x related to columns

→ y related to rows

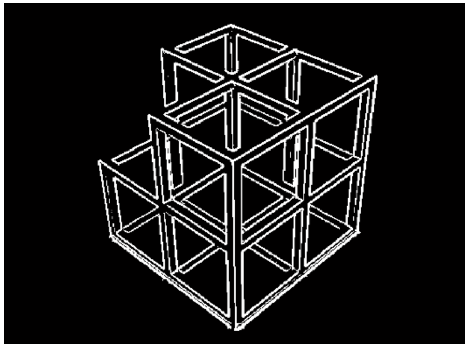


Assignment 3: Quality Dependencies

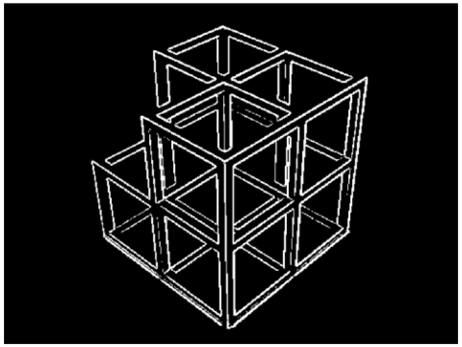
- Thresholding of gradient magnitude image → Line thickness



$t = 0.02$



$t = 0.05$



$t = 0.1$

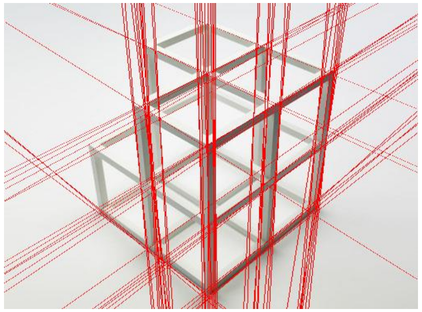
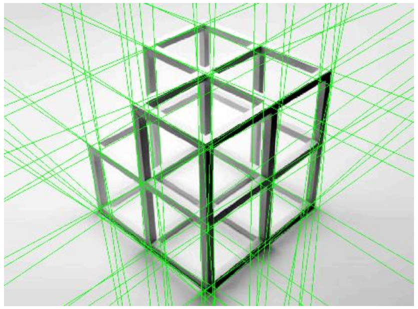
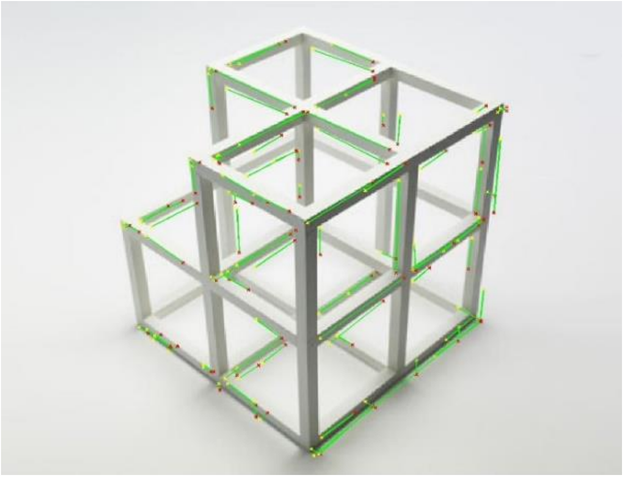
- Threshold for houghpeaks in voting table

```
peaks = houghpeaks(double(H), 40, 'threshold', double(ceil(0.05*max(H(:)))));
```

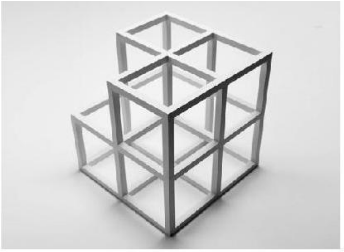
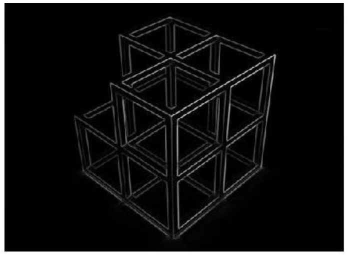
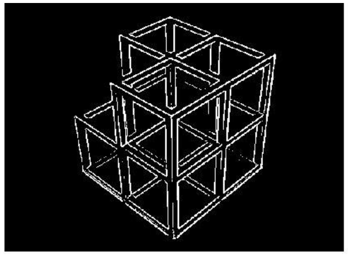
- Using a single value $\rho = x \cdot cos\theta + y \cdot sin\theta$ may introduce inaccuracies
  → vote for a small range of angles around ρ

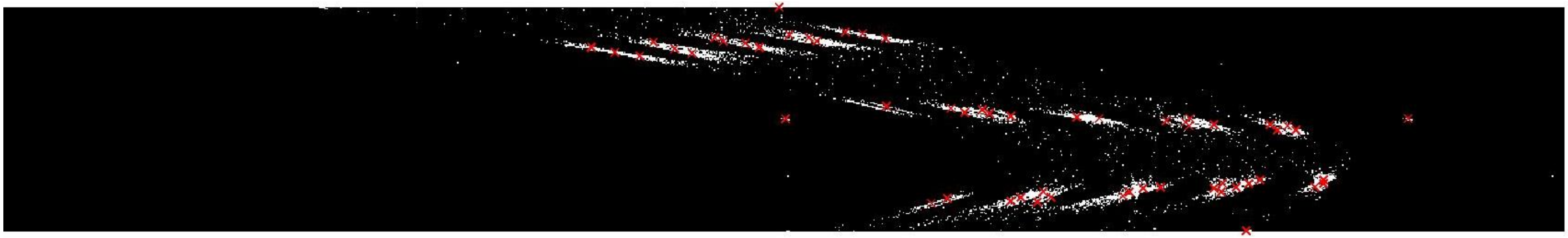
Exercise 3: Results

- Octave: function *houghlines* not available

- Example results



Assignment 4

A: Image filtering in frequency domain

&

B: Shape recognition using Fourier descriptors

Assignment 4

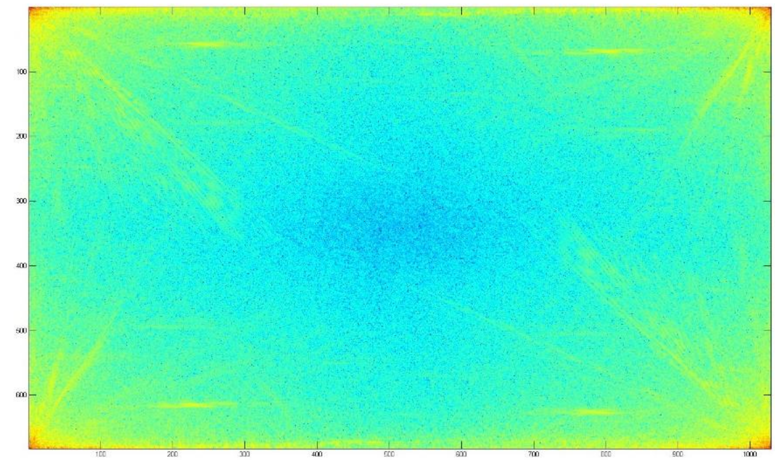
A: Image filtering in frequency domain

Fast Fourier Transform

Input image

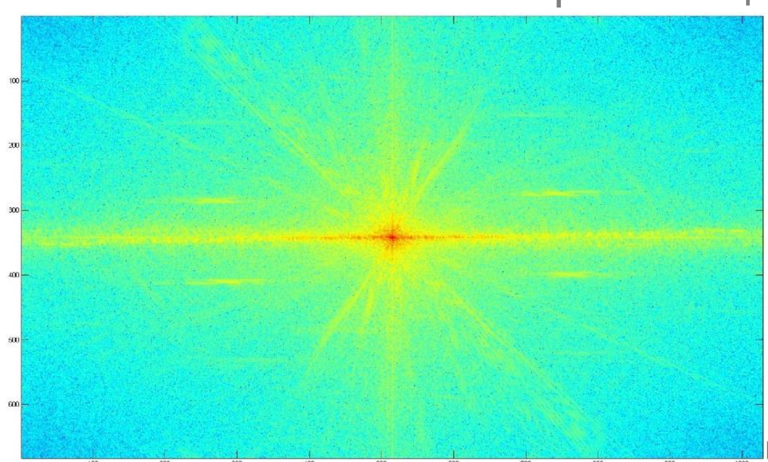
→ *fft2*

Logarithmic scaled spectrum $|F(u,v)|$



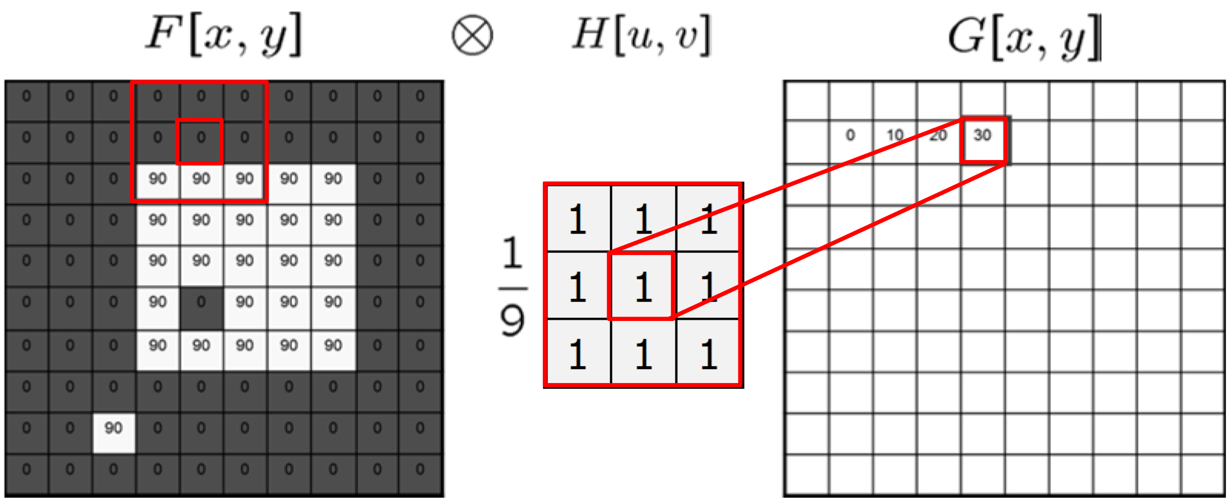
imagesc(log(abs(fft_image)))

Logarithmic scaled centered spectrum $|F(u,v)|$



*imagesc(log(abs(**fftshift**(fft_image))))*

Task A: Image Filtering



- **Cross-correlation:**

  $G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]\, F[i+u, j+v]$

  $G = H \otimes F$

- **Convolution:**

  $G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]\, F[i-u, j-v]$

  $G = H \star F$

  - Can easily be done in frequency-domain
  - **Symmetric filter kernel → Correlation = Convolution**

The Convolution Theorem

$$f(x,y) \star h(x,y) \Leftrightarrow F(u,v) .* H(u,v)$$
$$f(x,y) .* h(x,y) \Leftrightarrow F(u,v) \star H(u,v)$$

where „⇔" indicates a Fourier transform pair

→**Convolution** in spacial domain is equivalent to **multiplication** in frequency domain

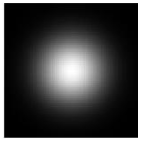
→Efficient, when filter mask is large

Smoothing in Spatial Domain

$f(x,y)$

$\star$

$h(x,y)$



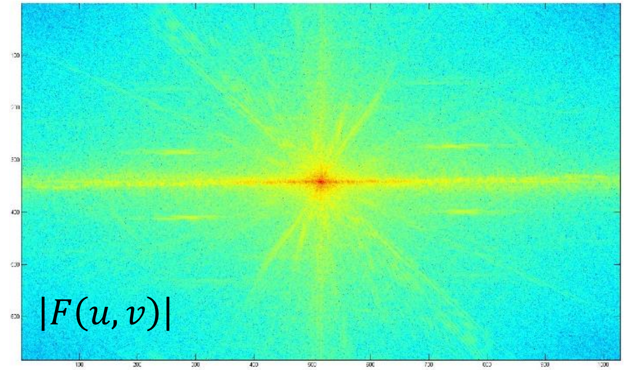
$\Downarrow$

$g(x,y)$

Smoothing in Frequency Domain

$f(x,y)$

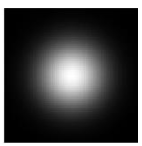
FFT $\Rightarrow$



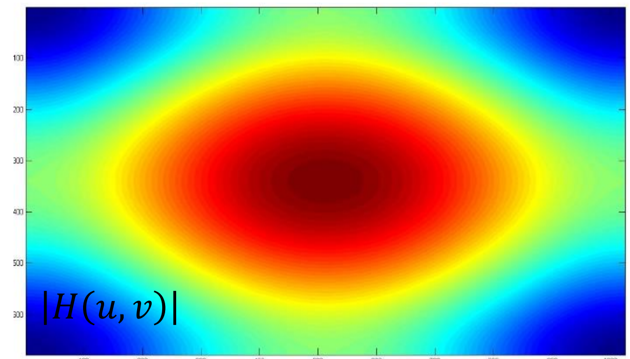
$F(u,v)$

$.*$

$h(x,y)$



Padding + FFT $\Rightarrow$

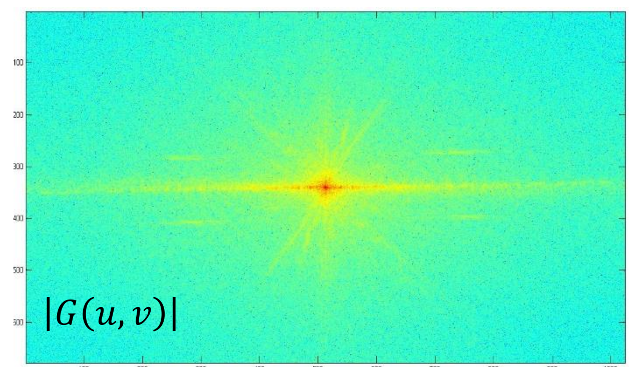


$H(u,v)$

$\Downarrow$

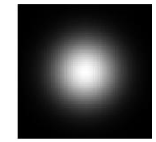
$g(x,y)$

FFT^{-1} $\Leftarrow$



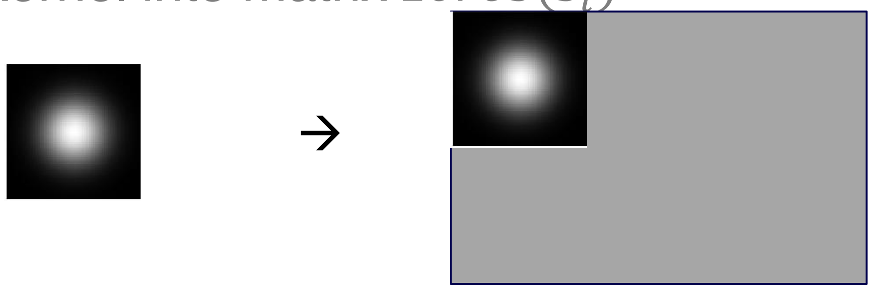
$G(u,v)$

Smoothing in Frequency Domain

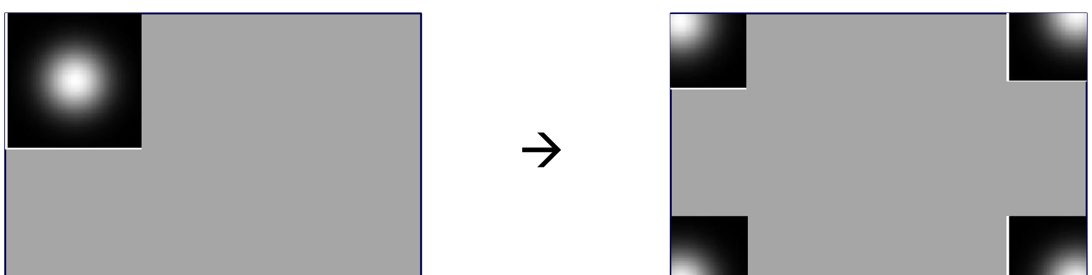
- **Inputs**: image  and filter kernel 

1) Padding of filter → enlarge filter kernel to size s_i of image

- Copy filter kernel into matrix $zeros(s_i)$

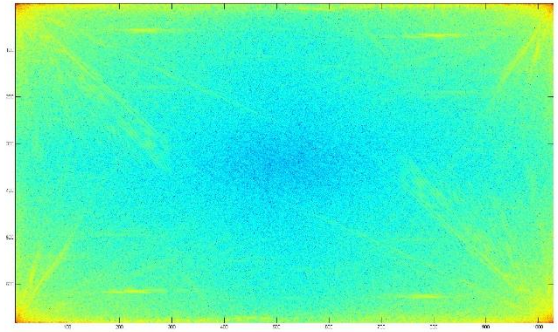


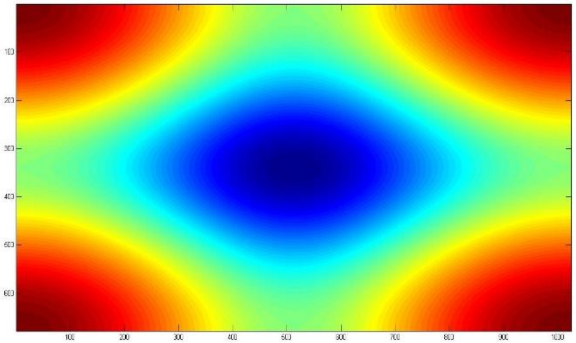
2) Center the filter using function *circshift*



Smoothing in Frequency Domain

3) Transform image and filter kernel to frequency-domain using the function *fft2* (centering not necessary)





4) Multiply these arrays (comlpex values!) element-wise and transform the result to spatial domain using function *ifft2*

Task A: Noise removal

a. Read the input image *taskA.png* and convert it to a grayscale image from data type double with values between 0.0 and 1.0

b. Add Gaussian noise to the image (function `imnoise`, parameters e.g. M=0, V=0.01) and plot the result

c. Convolve the noisy image with a self-made 2d Gaussian filter in the frequency-domain (`circshift`, `fft2`, `ifft2`). Which σ is suitable here? Plot the result → noise removed?

d. Plot the logarithmic centered image spectra of the original image, the noisy image, the Gaussian filter (padding) and the filtered image

Assignment 4

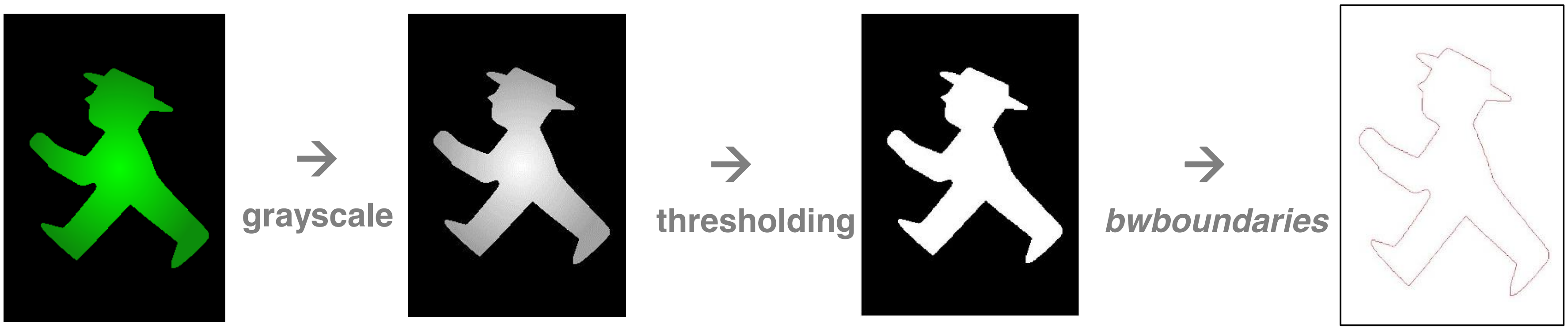
B: Shape recognition using Fourier descriptors

Fourier Descriptors

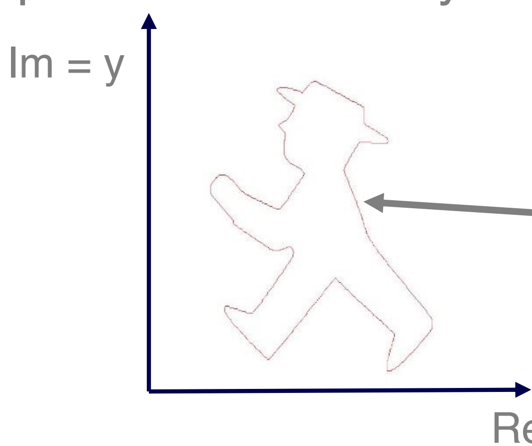
- **Given:** Image which represents a shape of interest
- **Task:** Find this shape in other images automatically

Fourier Descriptors

- **Given**: m points representing the boundary of a **closed** region in the image



- Interprete the boundary coordinates as complex numbers



- Coordinate representation for the ith of m points:

$$x_i + jy_i$$

Fourier Descriptors

Hint: Building the complex vector in Matlab

- Interprete the boundary coordinates (x, y) as complex numbers
- $b = \begin{bmatrix} (y_1, x_1) \\ \vdots \\ (y_m, x_m) \end{bmatrix}$ ($m \times 2$ array: output of *bwboundaries*)

- Building the complex vector D:

$$D = b(:,2) + j * b(:,1);$$

  - Don‘t use ***j*** as variable in your code!

Fourier Descriptors

- Result: Vector D with m complex-valued elements

$$D = [(x_1 + jy_1), \dots, (x_m + jy_m)]^T$$

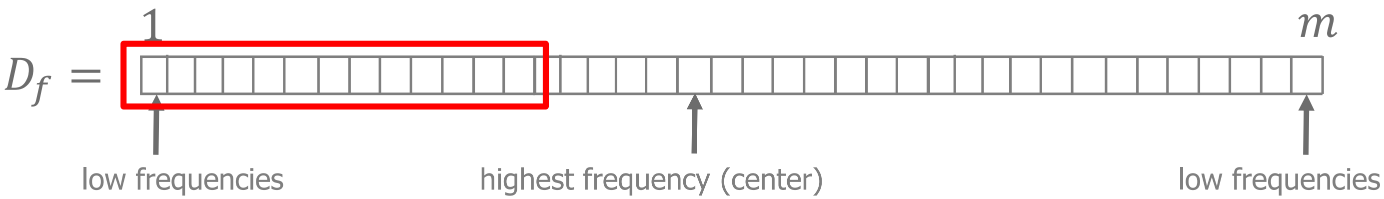
- DFT of D using function *fft* → **Fourier descriptor $\boldsymbol{D_f}$**

- Simple manipulations of D_f in frequency-domain allow…
  - …the representation of a generalized shape
  - …the elimination of dependency of D_f from **position**, **scale** and **orientation**!

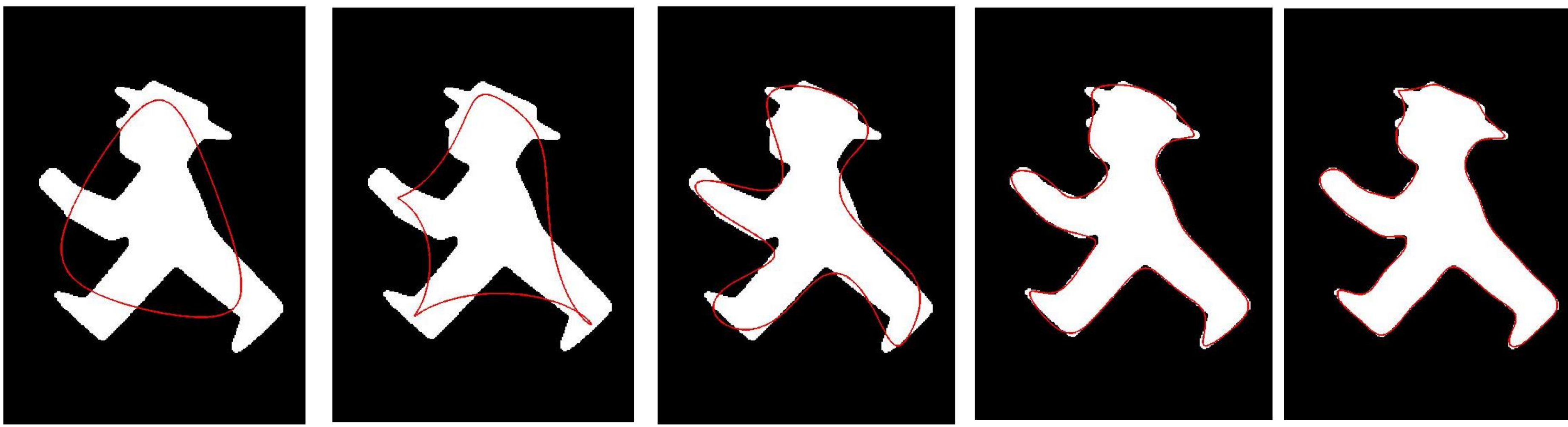
→ **Crucial** for comparison of shapes!

Fourier Descriptor Manipulation

- Number of elements n in $D_f \rightarrow$ generalization



$n = 2$ $n = 4$ $n = 8$ $n = 16$ $n = 24$

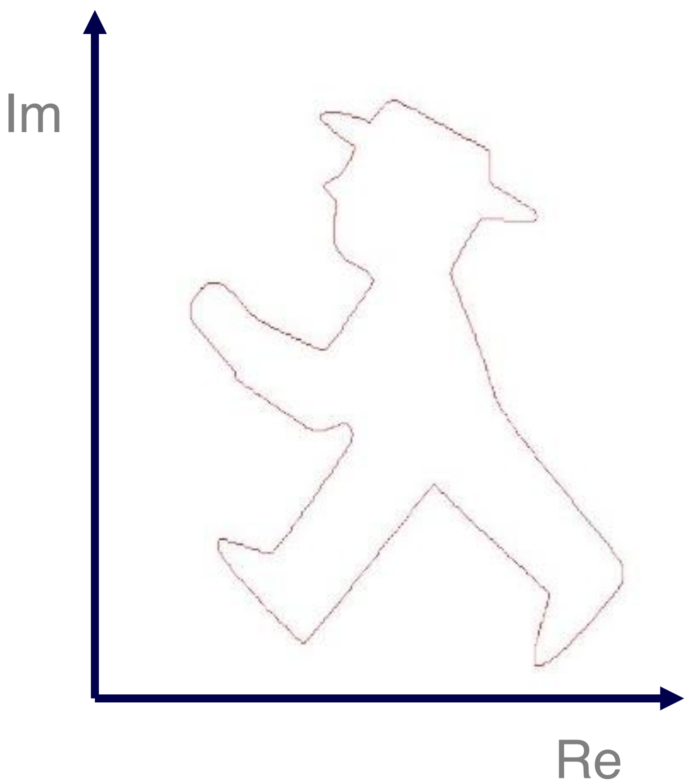


- Reducing elements of D_f: Extract the first $\boxed{n \text{ elements}}$ (low frequency values) of D_f and forget the rest

Translation Invariance

- First Fourier component in D_f = centroid

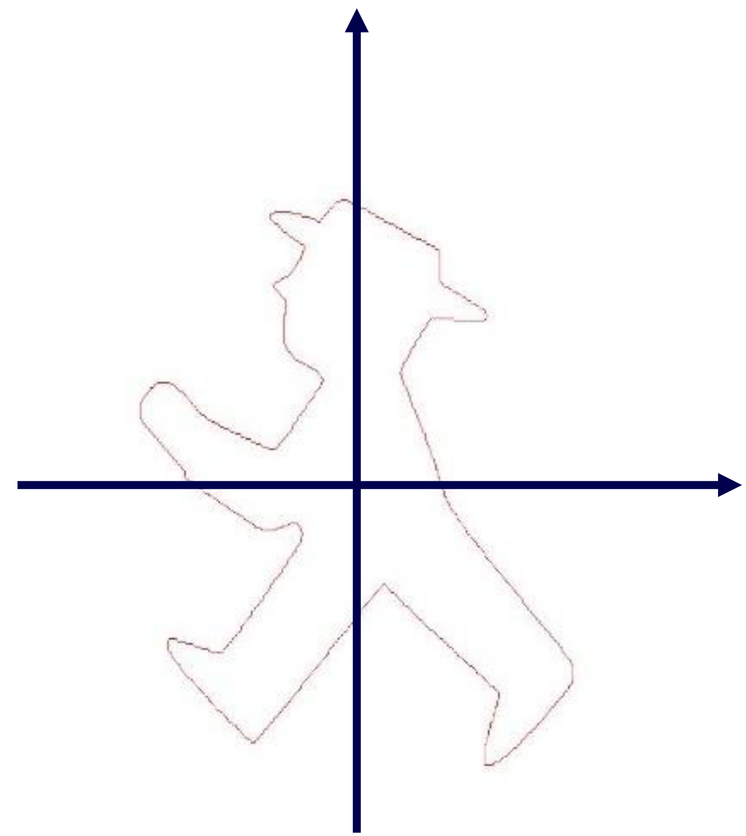
→ Throw away the first element by $D_f = D_f(2:(n+1))$



equal to

$D_f(1) := 0$

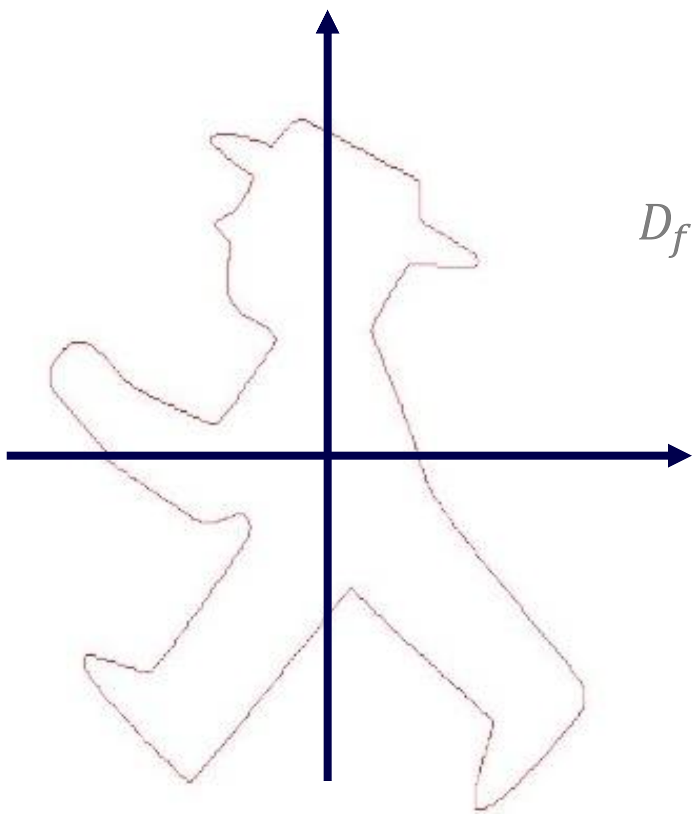
→



Scale Invariance

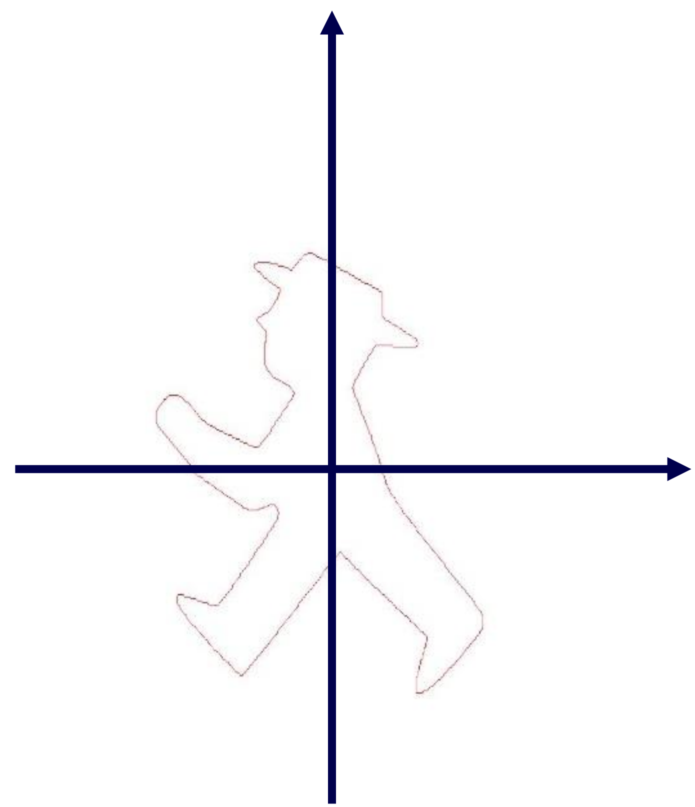
- The second component $D_f(2)$ corresponds to the radius (Corresponds to $D_f(1)$ if we have a translation invariant D_f!!)

→ Set this component to 1 by normalization of D_f



$$D_f := \frac{D_f}{|D_f(2)|}$$

→



Orientation Invariance

- Orientation is encoded in the phase information of D_f

→ Remove phase information by computing the absolute values of D_f:

$$D_f := \left|D_f\right|$$

→ Function `abs` in Matlab

→ The **amplitude spectrum** remains as final descriptor

Comparison of Descriptors

- Comparison of two normalized Descriptors $D_{f,1}$ and $D_{f,2}$

→ Euclidean distance d

$$d = \sqrt{\sum_i \left(D_{f,1}(i) - D_{f,2}(i)\right)^2}$$

→ Matlab: $d = norm(D_{f,1} - D_{f,2});$

→ $D_{f,1}$ and $D_{f,2}$ represent the same shape, if $d < t$

→ e.g. $t = 0.06$

Task B 1/4

a. Read the image *trainingB.png* and convert it to a grayscale image from data type double with values between 0.0 and 1.0

b. Derive a binary mask of the image where 1 represents the object of interest and 0 is background (functions `graythresh` and `im2bw`)

Task B 2/4

c. Build a Fourier-descriptor based on the binary image of b.
   i. Extraction of boundaries of the binary mask: `bwboundaries`
   ii. Use $n = 24$ elements for the descriptor
   iii. Make it invariant against translation, orientation and scale

→ Results:
- → The final descriptor $D_{f,train}$ is a $1 \times n$ vector where the first element is 1.0
- → A 1×1 cell (matlab data type) containing an $m \times 2$ array which represent the m corresponding border pixel coordinates of the found shape (output of `bwboundaries`)

Task B 3/4

d. Apply steps a.-c. on images *test1B.jpg* and *test2B.jpg* in order to identify all potential objects

→ Results for each image:

- → Descriptors: $\mathrm{k} \times n$ array, where k is the number of identified boundaries
- → Boundaries: $\mathrm{k} \times 1$ cell containing k $(m \times 2)$ arrays which represent the corresponding border pixel coordinates of the k found shapes

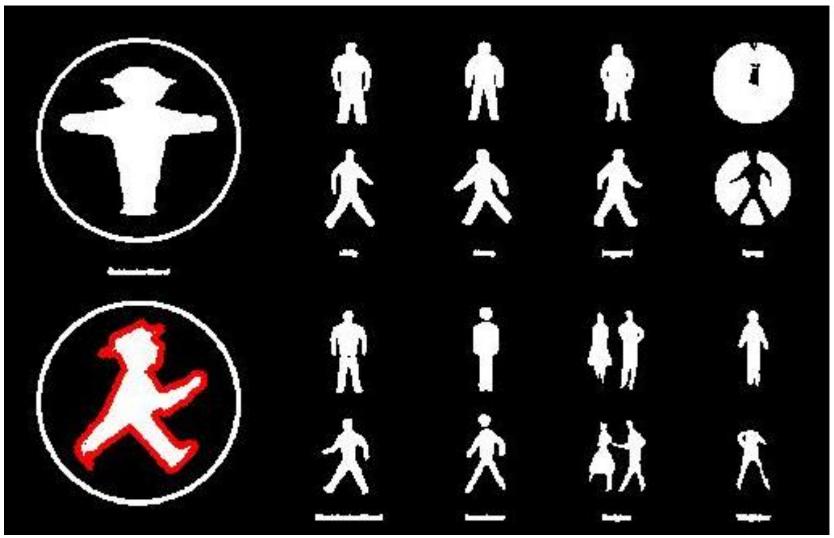
e. Identify the searched object by comparison of Fourier-descriptor $D_{f,train}$ (result of c) with all identified descriptors of the two test images $D_{f,test}$ (result of d). Use the Euclidean distance of the element-wise differences, e.g. if

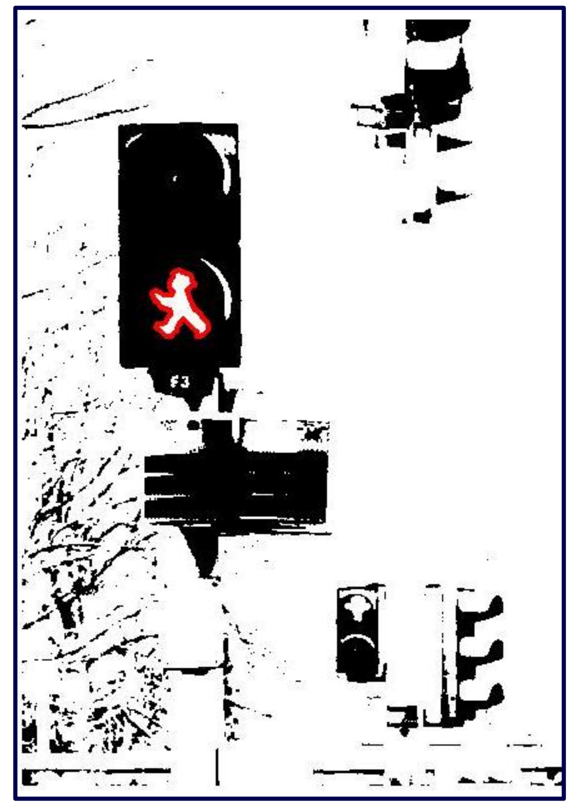
$$norm\left(D_{f,train} - D_{f,test}\right) < 0.06$$

→ $D_{f,test}$ represents the searched object

Task B 4/4

f. Plot the identified boundaries on the masks of the test images in order to validate the results (*imshow, hold on, plot*).





- Use the pixel coordinates of the shapes for plotting (result of *bwboundaries*)

Matlab Cells

Output of `bwboundaries`: $(k \times 1)$ Cell, where k is the number of identified closed boundaries

```
My_Cell =

    [682x2 double]
    [686x2 double]
    [654x2 double]
    [685x2 double]
    [154x2 double]
    [168x2 double]
    [328x2 double]
    [335x2 double]
    [377x2 double]
    [332x2 double]
    [ 52x2 double]
    [333x2 double]
    [350x2 double]
    [288x2 double]
    [ 98x2 double]
    [196x2 double]
    [ 57x2 double]
    [ 41x2 double]
    [ 44x2 double]
    [189x2 double]
    [458x2 double]
    [326x2 double]
    [253x2 double]
    [ 84x2 double]
    [ 74x2 double]
    [244x2 double]
    [289x2 double]
    [209x2 double]
    [239x2 double]
    [ 87x2 double]
    [238x2 double]
    [ 84x2 double]
    [ 58x2 double]
    [ 12x2 double]
    [  3x2 double]
    [216x2 double]
```

- Access the 34th array of boundary coordinates:

```
K>> boundary_points = My_Cell{34}

boundary_points =

    51   886
    50   887
    49   888
    50   888
    51   888
    52   888
    53   888
    54   888
    54   887
    53   887
    52   887
    51   886
```

Thank you!