Lecture 3: Static Analysis Principles – Abstract Syntax Trees (AST)

Passive Testing Techniques for Communication Protocols

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ACKNOWLEDGMENTS

Some content was taken from: Saman Amarasinghe, and Martin Rinard. 6.035 Computer Language Engineering, Spring 2010. (Massachusetts Institute of Technology: MIT OpenCourseWare), http://ocw.mit.edu. License: Creative Commons BY-NC-SA

OUTLINE

FROM CFG TO "TREES"

GRAMMAR AMBIGUITIES

ABSTRACT SYNTAX TREES

REMEMBER THE CFG DERIVATION?

Remember the CFG derivation?

Grammar

- $op = +|-|/|*$
- $int = [0-9]^+$
- $opar = ($
- $cpar =)$

1. $Start \mapsto Expr$
2. $Expr \mapsto Expr\ op\ Expr$
3. $Expr \mapsto int$
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Example derivation

- $Start$
- $Expr$ (1)
- $Expr\ op\ Expr$ (2)
- $int\ op\ Expr$ (3)
- $int\ op\ opar\ Expr\ cpar$ (4)
- $int\ op\ opar\ Expr\ op\ Expr\ cpar$ (2)
- $int\ op\ opar\ int\ op\ Expr\ cpar$ (3)
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- 10 / (7 − 5)

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It has a natural tree-like structure!

PARSE TREES

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- Formally, a directed graph $T =< V, A >$, where V is a set of vertices (nodes), A is a set of ordered arcs formed by a pairs $(v_1, v_2) \in V \times V$, in which each two vertices are connected by a unique simple path (tree-like structure).

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- Terminal symbols are leafs
- Non-terminal symbols are intermediate vertices in the tree
- Initial symbol is the tree root (therefore a rooted tree)
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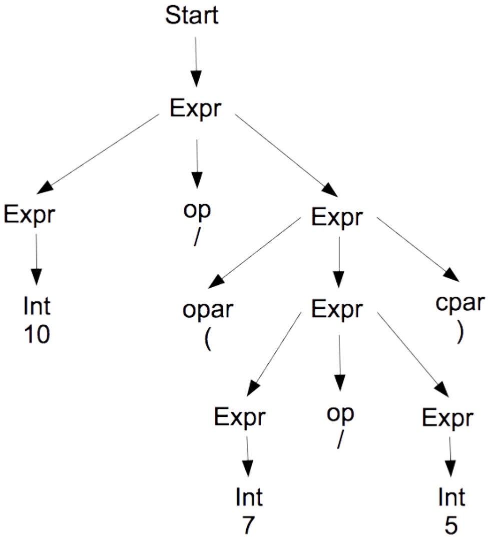
Let's take a look at one…

Parse Trees (cont.)

$10/(7-5)$ parse tree

PARSE TREES (CONT.)

10/(7 – 5) parse tree



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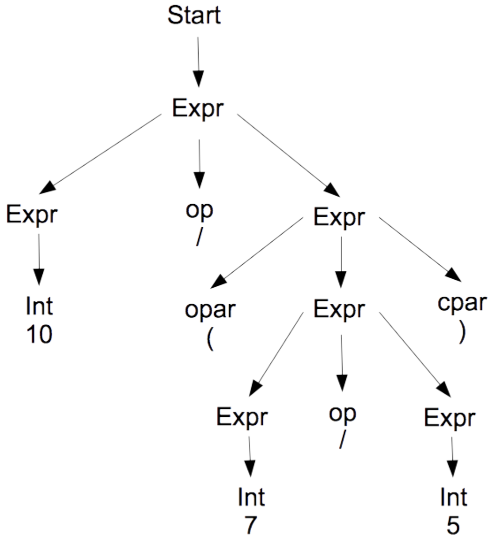

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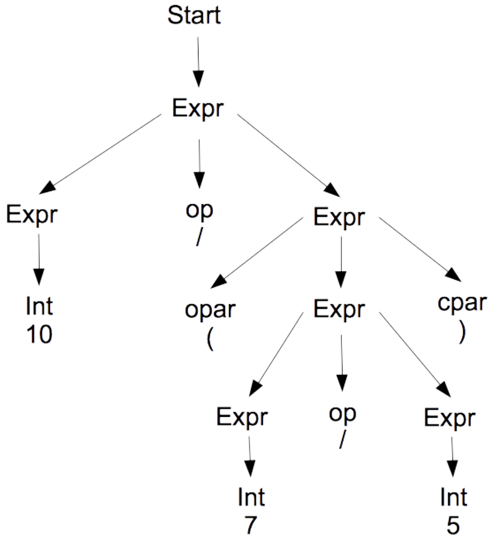
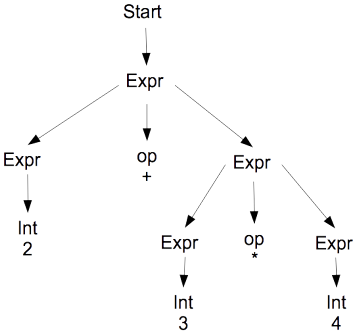


1. *Start* $\mapsto$ *Expr*
2. *Expr* $\mapsto$ *Expr op Expr*
3. *Expr* $\mapsto$ *int*
4. *Expr* $\mapsto$ *opar Expr cpar*

Please, derive the parse tree for $2 + 3 * 4$

PARSE TREES (CONT. CONT.)

$2 + 3 * 4$ parse tree



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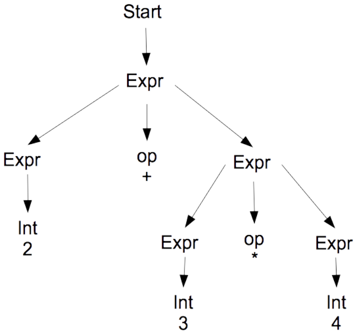
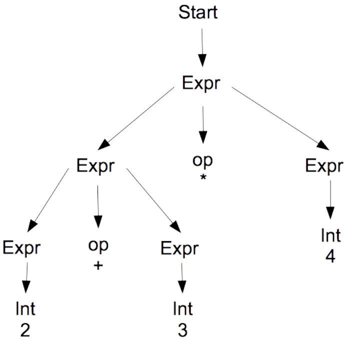


$2 + 3 * 4$ parse tree!



Grammar Ambiguities
(and parse trees)

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Solution?

GRAMMAR MODIFICATIONS (HACKING)

Recursiveness...

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Associativity

- Try to avoid left and right recursion for the same production rules, leave only one, for instance, only right recursiveness ↦ **right associativity**

Grammar modifications (hacking) (cont.)

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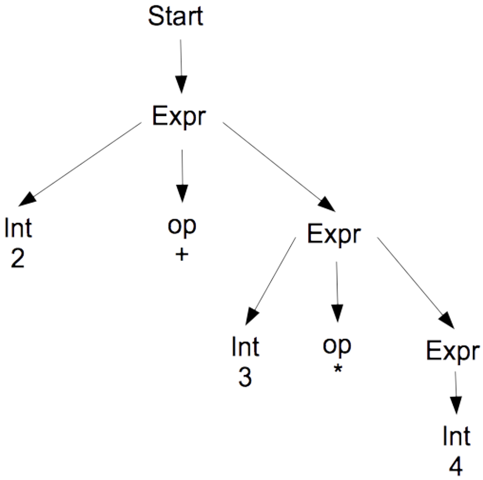

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- Higher precedence↦inner production rule (bind first)

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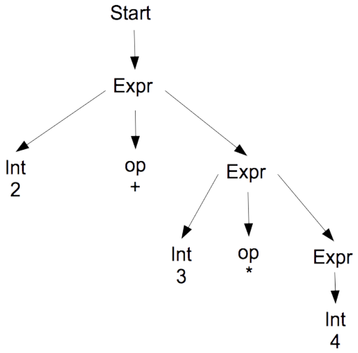
  - **Homework**

CONCRETE PARSE TREES. . .

Old $2 + 3 * 4$ parse tree



Concrete Parse trees. . .

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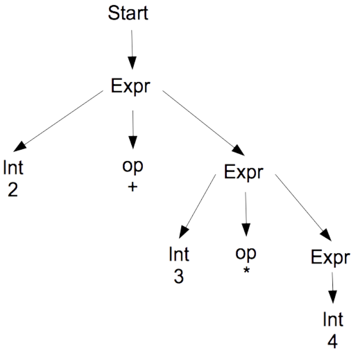

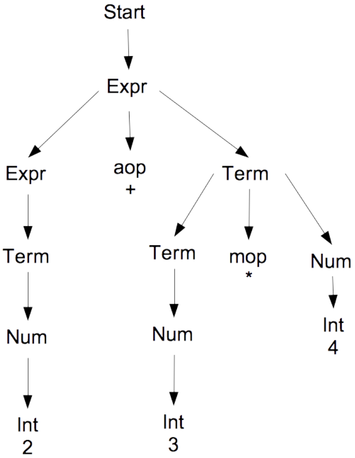

Abstract Syntax Trees (AST)

The desired structure

PARSING...

A Parser (PDA)

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AST vs Concrete Parse Tree

- Concrete used to parse unambiguously
- Concrete too complex, we need simpler, we need an AST

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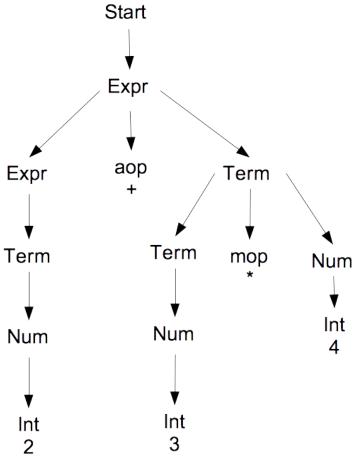
How to obtain it?

- Start with ambiguous grammar (probably)
- Hack the grammar to obtain concrete parse tree
- Remove undesired symbols and obtain the AST

Concrete Parse tree ↦ AST

CONCRETE PARSE TREE ↦ AST

Concrete parse tree for
$2 + 3 * 4$



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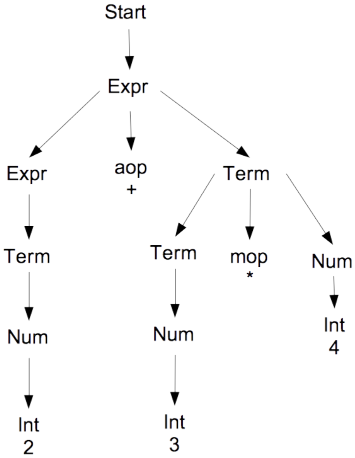
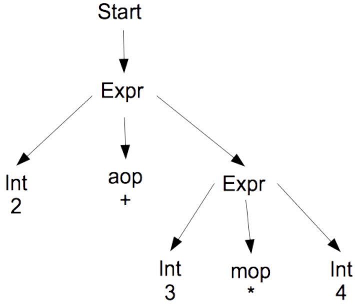


AST for $2 + 3 * 4$



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- Until today, YACC is very alive. . .
- See you on Wednesday, we'll talk about some Static Analysis now that we know on which structure to perform it (please note that: this theory applies to much more than just Static Analysis)