# Decomposed Gaussian Process Regression

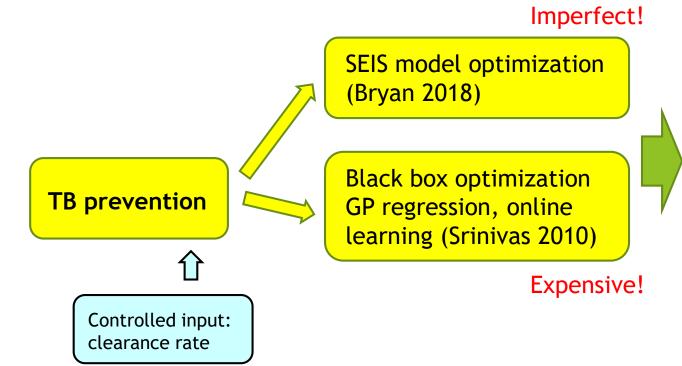
#### What we want to sell

- If we can decompose a continuous function (an outcome of Gaussian process)  $f(x) = g_1(x)f_1(x) + g_2(x)f_2(x) + \cdots + g_k(x)f_k(x)$
- Then the approximation based on the realization of each individual  $f_i(x)$  is better and also more efficient than the approximation only based on the entire f(x)
  - ► This result is correct and can be applied to both online learning and active learning.

#### Theorem

- ► Theorem 1 (not restricted to online learning)
  - Approximating the target function via individual subfunctions would give a better approximation (lower variance).
- Theorem 2
  - ▶ If we apply the GP-UCB algorithm to approximate the individual subfunctions, the whole algorithm is also a no-regret algorithm.
  - We have the same regret order as directly approximating the entire function. But empirically our method beats the naïve one.

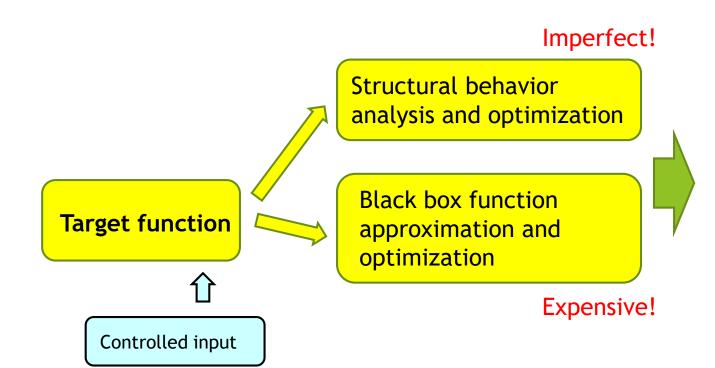
# Story



#### Hybrid method

Use the structure of SEIS and the method of GP-UCB (in online learning).

## Story



#### Hybrid method

Based on the structural behavior, we can use the non-structural method to correct the model.

## Story

- The policy (clearance rate) is in groups-level. So we should try to approximate the function in groups-level, which is our  $f_i(x)$ .
- This leads to a

#### Online Learning

- Problem statement (modified):
  - ▶ We want to minimize the total infected population in the following 100 years.
  - We assume the disease behalves roughly like the SEIS population model (see the next slide) but not perfectly. So we add correction term to capture the error.
  - ▶ We can change the policy yearly (or monthly) based on the observations.

### **SEIS Population Model**

- Let  $I_i^t$  be the infected population in group i (age i) in year t.
- ▶ Similar to all  $S_i^t$  (healthy population) and  $E_i^t$  (latent population).
- In SEIS model, with given parameters activation rate  $\alpha$  and transmitting matrix  $\beta$ :

$$S_{i+1}^{t+1} = S_i^t (1 - \mu_i) \left( 1 - \sum_k \beta_{ik} \frac{I_k^t}{S_k^t + E_k^t + I_k^t} \right) + I_i^t (1 - d_i) \nu_i$$

$$E_{i+1}^{t+1} = E_i^t (1 - \mu_i)(1 - \alpha_i) + S_i^t (1 - \mu_i) \sum_k \beta_{ik} \frac{I_k^t}{S_k^t + E_k^t + I_k^t}$$

$$I_{i+1}^{t+1} = I_i^t (1 - d_i)(1 - \nu_i) + E_i^t (1 - \mu_i)\alpha_i$$

### **SEIS Population Model**

Assuming there is still a small error and uncertainty in the SEIS model:

$$S_{i+1}^{t+1} = S_i^t (1 - \mu_i) \left( 1 - \sum_k \beta_{ik} \frac{I_k^t}{S_k^t + E_k^t + I_k^t} \right) + I_i^t (1 - d_i) \nu_i + f_i^S(\nu, S^t, E^t, I^t)$$

$$E_{i+1}^{t+1} = E_i^t (1 - \mu_i)(1 - \alpha_i) + S_i^t (1 - \mu_i) \sum_k \beta_{ik} \frac{I_k^t}{S_k^t + E_k^t + I_k^t} + f_i^E(\mathbf{v}, \mathbf{S^t}, \mathbf{E^t}, \mathbf{I^t})$$

$$I_{i+1}^{t+1} = I_i^t (1 - d_i)(1 - \nu_i) + E_i^t (1 - \mu_i)\alpha_i + f_i^I(\nu, S^t, E^t, I^t)$$

- For simplification, we assume these error correction terms are independent to *S*, *E*, *I* and also time-invariant.
- So we only have  $f_i^{\S}(\nu), f_i^{E}(\nu), f_i^{I}(\nu)$

### **Terminology**

- ▶ 1. We decide a policy  $\nu$  and run one year of treatment with the SEIS model to predict the outcome.
- ▶ 2. Then we observe the **individual** unknown error function  $f_i^S(v, S^t, E^t, I^t)$ ,  $f_i^E(v, S^t, E^t, I^t)$ ,  $f_i^I(v, S^t, E^t, I^t)$  and learn the error terms.
- ▶ 3. Based on these outcomes, we can decide another policy and run another test.
- We are comparing to another terminology:
  - ▶ 1. We do not decompose the disease by age group but just look at the total infected population and run policy.
  - ▶ 2'. We observe the **entire** unknown error function (here it would be  $\sum_i f_i^I(v, S^t, E^t, I^t)$ ) and learn the error terms.

#### Some Assumption

- We assume the error terms are time-invariant.
  - $\triangleright$  i.e. every year, we decide a new policy, the error functions  $f_i^S$ ,  $f_i^E$ ,  $f_i^E$  are the same.
- We need to care about the cumulative effect:
  - ► The regret in the first year will transmit (it will directly increase the infected population in the second year and thus incur a higher infected population in the following years.)

#### Some Assumption

- ► The whole thing is actually the same as previous except we change the treatment period from 25 years to 1 year.
- And we are aiming to minimizing over the following 100 years (i.e. run 100 trials and learn from them to continuously update the policy)

### Active Learning (todo)

- ► To the best of my knowledge, active learning usually involves exploring the most informative point.
  - i.e. the policy with highest information gain  $I(y_T; f_T) = H(y_T) H(y_T|f_T)$ , which can be written down analytically.
- In our decomposed Gaussian process regression, we can imitate it and do the similar step. At lease this decomposed method is always better than approximating the entire one.
  - So it will also lead to a faster active learning.