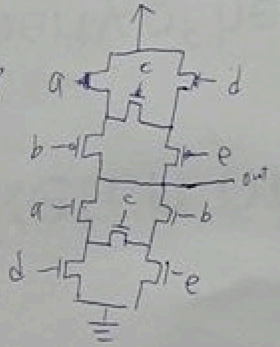
Yes. 1. A factor form is ① conjunction of two factor forms ② disjunction of two factor forms. Use Induction. 1 literal can be implemented with 2 transistors, 1 for PMOS, and 1 for NMOS. Suppose n_1 literal factor form f_1 can be implemented by n_1 PMOS and n_1 NMOS where N- and P- networks implement complementary, f_1 and $\bar{f_1}$, and n_2 literal factor form f_2 can be implemented by n_2 PMOS and n_2 NMOS. The factor form f' ① $f' = f_1 \cdot f_2$ f' can be implemented by $n_1 + n_2$ PMOS, where $\bar{f_1}, \bar{f_2}$ in parallel and $\underbrace{f_1, f_2}_{n_1+n_2 \text{ NMOS}}$ in series. ② $f' = f_1 + f_2$ ~~can be implemented~~ can be implemented in similar way, also with $n_1 + n_2$ PMOS and $n_1 + n_2$ NMOS $\Rightarrow$ f' can be implemented with $2(n_1 + n_2)$ transistor.

2.



total 10 transistors, if $c=0 \Rightarrow out = ad + be$

if $c = 1 \Rightarrow out = (a+b)(d+e)$

$Out = \bar{c}(ad+be) + c(a+b)(d+e) = ad + be + c(bd + ae)$

it can't be represented by 5 literals.

2. Suppose H & R are not unique $\Rightarrow \exists\ H_1, H_2, R_1, R_2$ s.t.

$F = G \cdot H_1 + R_1 \quad F = GH_2 + R_2$ both are weak division

$\Rightarrow GH_1 + R_1 = GH_2 + R_2 \Rightarrow GH_1\bar{R_1}\bar{H_2} + \cancel{R_1\bar{R_1}\bar{H_2}} = GH_2\cancel{\bar{H_2}}\bar{R_1} + R_2\bar{R_1}\bar{H_2}$

$\Rightarrow G(H_1\bar{R_1}\bar{H_2}) = R_2\bar{R_1}\bar{H_2} \Rightarrow \because R_2\bar{R_1}\bar{H_2} \subseteq R_2$ R_2 is not the result of weak division since it ~~[illegible]~~ contains ~~[illegible]~~ some cube can be divided by G $\Rightarrow$ conflict $\nleftarrow$

$aefh + aegh + aei + befh + begh + bei + cdefh + cdegh + cdei$

a: $egh + ei + efh$
- **e**: $gh + i + fh$
  - **h**: $f + g$

b: $efh + egh + ei$
- **e**: $fh + i + gh$
  - **h**: $f + g$

c: $defh + degh + dei$
- **d**: $efh + egh + ei$
  - **e**: $fh + gh + i$
    - **h**: $f + g$

d: $cefh + cegh + cei$ (c)

e: $afh + agh + ai + bfh + bgh + bi + cdfh + cdgh + cdi = (fh + gh + i)\cdot(a + b + cd)$
- **f**: $h(a+b+cd)$
  - **h**: $a+b+cd$
- **g**: $h(a+b+cd)$
  - **h**: $a+b+cd$
- **h**: $(f+g)(a+b+cd)$
- **i**: $a+b+cd$

f: $aeh + beh + cdeh$ (e)

g: $aeh + beh + cdeh$ (e)

h: $aef + aeg + bef + beg + cdef + cdeg$ (e)

i: $ae + be + cde$ (e)

Kernel	Co kernel
~~e(a+b+cd)(fh+gh+i)~~	~~1~~
$f+g$	$aeh,\ beh,\ cdeh$
$h(f+g)+i$	$ae,\ be,\ cde$
$a+b+cd$	$efh,\ egh,\ ei$
$(a+b+cd)(h(f+g)+i)$	e

4. (a)

$$SDC = \sum_{j=1}^{M} (y_j \overline{F_j} + \overline{y_j} F_j)$$

$$= (y_1 \oplus f_1) + (y_2 \oplus f_2) + (y_3 \oplus f_3) + (y_4 \oplus f_4) + (z_1 \oplus f_5) + (z_2 \oplus f_6)$$

$$= y_1' x_1 x_2 + y_1 (x_1' + x_2') + y_2' x_2 + y_2' x_3 + y_2 \overline{x_2} \overline{x_3} + y_3' x_3' + y_3' x_4' + y_3 x_3 x_4$$

$$+ y_4' (y_1' y_2' y_3 + y_1 y_2 y_3') + y_4 (y_1' y_2' y_3 + y_1 y_2 y_3')'$$

$$+ z_1' y_1 + z_1' y_4 + z_1 y_1' y_4' + z_2' y_3 y_4 + z_2 y_3' + z_2 y_4'$$

(b)

$$SDC_4 = (y_1 \oplus f_1) + (y_2 \oplus f_2) + (y_3 \oplus f_3) + (y_4 \oplus f_4)$$

$$= y_1' x_1 x_2 + y_1 x_1' + y_1 x_2' + y_2' x_2 + y_2' x_3 + y_2 x_2' x_3'$$

$$+ y_3' x_3' + y_3' x_4' + y_3 x_3 x_4 + y_4' y_1' y_2' y_3 + y_4' y_1 y_2 y_3' +$$

$$y_1 y_2' y_4 + y_1 y_3 y_4 + y_1' y_2 y_4 + y_2 y_3 y_4 + y_1' y_3' y_4 + y_2' y_3' y_4$$

$$\Rightarrow \forall x_1 x_2 x_3 x_4 y_4 . SDC_4 .$$

(c)

$$ODC_4 = ODC_{45} \cdot ODC_{46} = \neg\left(\frac{\partial f_5}{\partial y_4}\right) \cdot \neg\left(\frac{\partial f_6}{\partial f_4}\right) = y_1 y_3' = x_1 x_2 x_3 x_4$$

5. (a)

$$DC_4 = DC_{45} \cdot DC_{46} = (XDC_{45} + ODC_{45})(XDC_{46} + ODC_{46}) = y_1 x_2 x_3 x_4$$

$$D_4 = \neg ZMG(\neg DC_4)$$

$$= y_1 \overline{y_2} + y_1 \overline{y_3} + \overline{y_2}\,\overline{y_3}$$

(b)

y_2'

6.

a) $$S(x,z) = [(z_1 \equiv x_1x_2 + x_2'x_3') + x_1'x_2'x_3'x_4]$$
$$[(z_2 \equiv x_2'x_3') + x_1x_2x_3'x_4]$$

b) $$I(x, y_4, z) = [(z_1 \equiv x_1x_2) + y_4][z_2 \equiv (x_3' + x_4')y_4]$$

c) $$E(x,y) = (y_1 \equiv x_1x_2)(y_2 \equiv (x_2 + x_3))(y_3 \equiv (x_3x_4)')$$

d) $$CF_4(Y, y_4) = \forall x, z\overline{[E(x,Y) \cdot I(x, y_4, z)\overline{S(x,z)}]}$$

$$= \forall x, z. \neg((y_1 \equiv x_1x_2)(y_2 \equiv (x_2 + x_3))(y_3 \equiv (x_3' + x_4'))(z_1 \equiv x_1x_2 + y_4)$$
$$(z_2 \equiv (x_3' + x_4')y_4) \cdot (\neg[(z_1 \equiv (x_1x_2 + x_2'x_3') + x_1'x_2'x_3'x_4') \cdot$$
$$((z_2 \equiv x_2'x_3') + x_1x_2x_3'x_4)]))$$

e) Yes.

4. (b)

Sol 1.

$$SDC_4 = (y_1 \oplus f_1) + (y_2 \oplus f_2) + (y_3 \oplus f_3)$$

$\Rightarrow$ To make it only depend on $y_1 . y_2 . y_3$

$\Rightarrow \forall x_1 . x_2 . x_3 . x_4 . SDC_4$.

$$(\because \text{Care-Set} = \exists X_1X_2X_3X_4 \overline{SDC_4} \Rightarrow SDC_4 = \overline{\text{Care-Set}} = \overline{\exists X_1X_2X_3X_4 \overline{SDC_4}}$$

$$= \forall X_1X_2X_3X_4 . SDC_4$$

Sol 2 $SDC_4 = (y_1 \oplus f_1) + (y_2 \oplus f_2) + (y_3 \oplus f_3) + (y_4 \oplus f_4)$

$\Rightarrow$ To make it only depend on $y_1 . y_2 . y_3$

$\Rightarrow \underline{\forall X_1 X_2 X_3 X_4 . y_4 . SDC_4}$

① why not $\forall X_1X_2X_3X_4 . \exists y_4 . SDC_4$?

assume : $F_3 = (y_1 \oplus f_1) + (y_2 \oplus f_2) + (y_3 \oplus f_3)$

$SDC_4 = F_3 + y_4 \oplus f_4$

$\forall X \exists y_4 . SDC_4 = \forall X [(F_3 + f_4) + (F_3 + \bar{f_4})] = 1$

② why not. $\exists y_4 \forall X . SDC_4$?

$\exists y_4 \forall X . SDC_4 = \forall X (F_3 + f_4) + \forall X (F_3 + \bar{f_4}) = 1$