# Logic Synthesis & Verification, Fall 2017

**National Taiwan University**

## Problem Set 3

Due on 2017/11/3 18:00. Hand in your solution in the instructor's mailbox.

## 1 [Generalized Cofactor]

(12%) Prove or disprove the following equalities.

(a) (6%) $co(co(f,g),h) = co(co(f,h),g)$
(b) (6%) $f = g \cdot co(f,g) + \neg g \cdot co(f,\neg g)$

## 2 [Operation on Cube Lists]

(6%) Consider the following orthogonal cube list.

$$\begin{pmatrix} 0\ 1\ -\ -\ 1\ 1\ 0 \\ -\ 0\ -\ 1\ 0\ -\ 0 \\ -\ 0\ 0\ 1\ 0\ 1\ - \end{pmatrix}$$

Add the cube $(0-01--0)$ to the above list with orthogonality being maintained.

## 3 [Special Functions]

(24%) Prove or disprove the following statements.

(a) (8%) If function $f(a,b,c) = f(\neg b,a,c)$ and $f(a,b,c) = f(a,\neg c,b)$, then $f(a,b,c) = f(\neg c,b,a)$.
(b) (8%) $f(a,b,c) = f(a,\neg c,\neg b)$ if and only if $f_{bc} = f_{\neg b\neg c}$.
(c) (8%) Every prime implicant of a unate function is an essential prime implicant.

## 4 [Threshold and Unate Functions]

(24%)

**Definition 1.** *A* threshold function $f$ *over Boolean variables* $x_1, x_2, \ldots, x_n$ *is defined by*

$$g = \begin{cases} 1, & \text{if } \sum_{i=1}^{n} w_i x_i \geq T, \\ 0, & \text{otherwise,} \end{cases}$$

*for some constants* $w_i$*'s and* $T$ *in* $\mathbb{R}$.

(a) (8%) Show that a threshold function must be a unate function.
(b) (8%) Show that a unate function is not necessarily a threshold function.
(c) (8%) Show that any Boolean function can be expressed in terms of a two-level circuit composed by logic gates of threshold functions.

## 5 [Unate Recursive Paradigm]

(18%) Let $f$ be a function with the following cover

$$F = \begin{pmatrix} 1 & 0 & - & 1 & - \\ - & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & - & - \\ 0 & 0 & - & 0 & 1 \\ 1 & - & 1 & - & 1 \\ 0 & - & 0 & 1 & 1 \end{pmatrix}.$$

(a) (6%) Test whether $f$ is a tautology by applying the unate recursive paradigm on $F$ with unate reduction.
(b) (6%) Complement $f$ by applying the unate recursive paradigm on $F$.
(c) (6%) Generate the set of prime implicants of $f$ by applying the unate recursive paradigm on $F$.

## 6 [Quine-McCluskey]

(16%) Given two incompletely specified functions $f$ and $g$ over variables $a, b, c, d$, let $f$ be of onset minterms

$$\{0010, 0100, 0101, 1010, 1110\}$$

and don't care set minterms

$$\{0001, 1101\},$$

and $g$ be of onset minterms

$$\{0100, 0101, 0110, 0111, 1000, 1010, 1100, 1111\}$$

and don't care set minterms

$$\{1110\}.$$

Apply the Quine-McCluskey procedure to minimize the multi-output cover with the following steps.

(a) (4%) Derive all prime implicants for the multi-output cover by pairwise minterm merging.
(b) (4%) Build the Boolean matrix for column covering.
(c) (4%) Simplify the Boolean matrix to its cyclic core.
(d) (4%) Compute the minimum column covering and obtain the minimum multi-output cover.