Logic Synthesis & Verification, Fall 2017

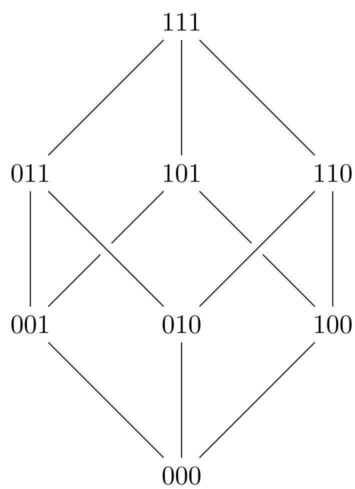
National Taiwan University

Problem Set 1

Due on 2017/10/2 before lecture

1 [Boolean Algebra Definition]

(a) (14%) Given a five-tuple $(\mathbb{B}, \text{join}, \text{meet}, 000, 111)$, where $\mathbb{B} = \{000, 001, 010, 011, 100, 101, 110, 111\}$, and the binary operators join and meet return the least common upper bound and the greatest common lower bound, respectively, with respect to the following lattice. For instance, $\text{join}(001, 100) = 101$ and $\text{meet}(011, 010) = 010$.



Show that the five tuple forms a Boolean algebra by verifying the satisfaction of the five postulates of Boolean algebra, and by establishing an isomorphism to the algebra of classes.

(b) (6%) Let $\mathbb{B} = \{0, 1\}$, $\underline{0} = 0$, $\underline{1} = 1$, and binary operators $+$ and $\cdot$ be AND and OR, respectively. Does the five-tuple form a Boolean algebra? If yes, show the algebra of classes that is isomorphic to the five-tuple. If not, which postulates of Boolean algebra are violated?

2 [Boolean Algebra Properties]

(18%) Prove the following equalities using ONLY the five postulates of Boolean algebra (or other properties that you have proven using the postulates). Please specify clearly which postulate is applied in each step of your derivation.

(a) $a + (a \cdot b) = a$
(b) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
(c) $a \cdot b + a' \cdot c + b \cdot c = a \cdot b + a' \cdot c$

3 [Relation over Boolean Algebra]

(20%) Define the relation $\leq$ in a Boolean algebra with carrier $\mathbb{B}$ as follows

$$a \leq b \text{ if and only if } a \cdot b' = 0$$

for all $a, b \in \mathbb{B}$, where b' is the inverse element of b. Prove that the following properties hold for all $a, b, c \in \mathbb{B}$:

(a) $a \cdot b \leq a \leq a + c$
(b) $a \leq b$ and $a \leq c$ if and only if $a \leq b \cdot c$

4 [Boolean Functions]

(20%) Let g and h be single-variable Boolean functions. For each of the following cases, express $f(0)$ and $f(1)$ as simplified formulas involving $g(0), g(1), h(0)$, and $h(1)$.

(a) $f(x) = g(h(x))$
(b) $f(x) = g(g'(x))$

5 Alternative Views on Boolean Functions

(12%) Given a three-variable Boolean function f_1 over $\mathbb{B} = \{0, 1\}$ with

$$f_1 = (1)x'y'z' + (0)x'y'z + (1)x'yz' + (0)x'yz + (1)xy'z' + (1)xy'z + (0)xyz' + (1)xyz,$$

define a two-variable Boolean function $f_2(y, z)$ taking values over $\{0, 1, x, x'\}$ as an alternative representation of f_1.

6 [Minterm Canonical Form]

(10%) Prove the theorem of minterm canonical form using Boole's expansion theorem.