Deep Learning for Computer Vision HW1 Report

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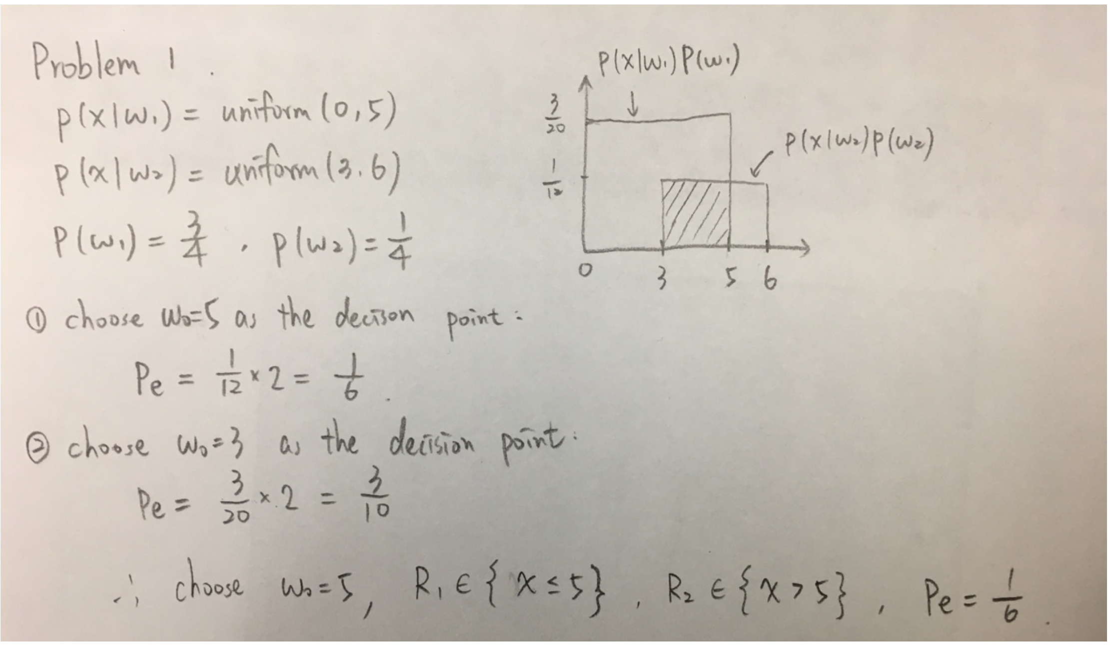
Problem 1   Bayes Decision Rule

For a 2-class problem based on an observable **x**, in order to determine the decision region, by the decision rule mentioned in class, we have:

$$\omega^* = \underset{i}{\operatorname{argmax}}\, P(\omega_i|x)$$

Meanwhile, we should minimize the error rate $P_e = \min\,(P(\omega_1|x), P(\omega_2|x))$,

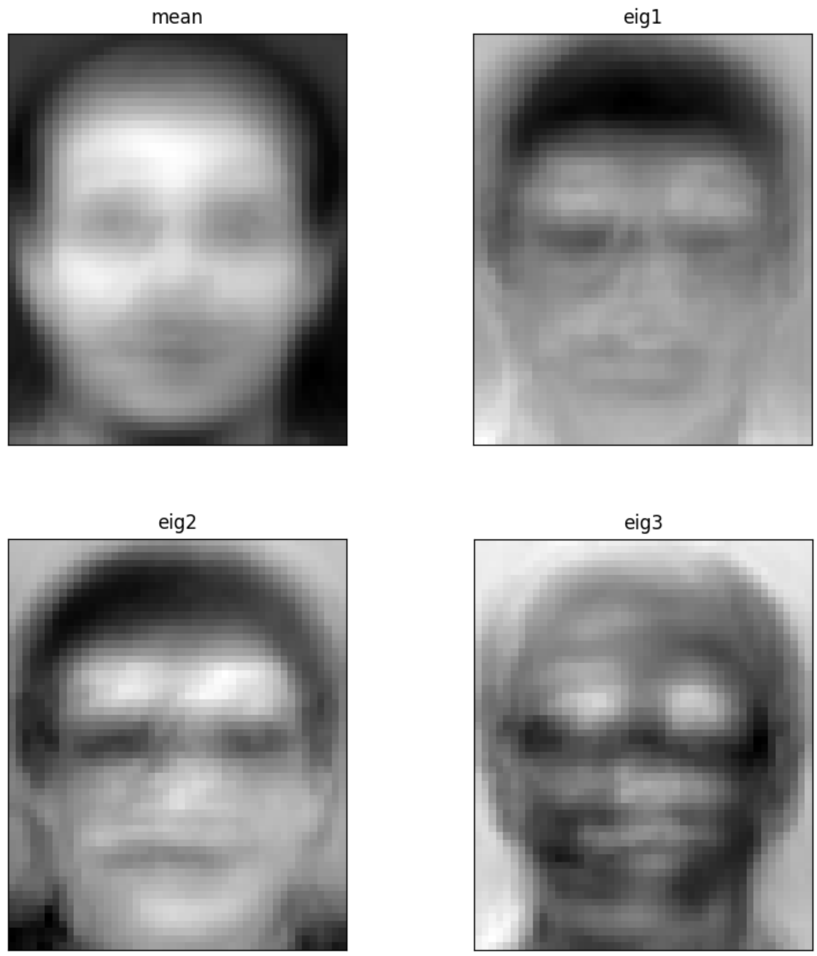
where $P(\omega_i|x) = \frac{P(x|\omega_i)\cdot P(\omega_i)}{P(x)}$



$$P_e = \frac{1}{6}$$

Problem 2 PCA and KNN Classification

(a) Plot the mean face and the first three eigenfaces



(b) Plot the reconstructed images, with the corresponding MSE values.

n = 50, MSE = 213.263227244

n = 100, MSE = 81.9517802869

n = 239, MSE = 2.89831416384e-27

(c) Apply k-nearest neighbors classifier to recognize test set images, determine the best k and n values by 3-fold cross-validation. k = 1, 3, 5 and n = 3, 50, 159

3-fold cross validation, validation set as first (1,2), (3,4), (5,6) pic of each class.

(1,2) :

```
k=1, n=  3, score=0.56250
k=1, n= 50, score=0.92500
k=1, n=159, score=0.92500
k=3, n=  3, score=0.46250
k=3, n= 50, score=0.82500
k=3, n=159, score=0.82500
k=5, n=  3, score=0.38750
k=5, n= 50, score=0.72500
k=5, n=159, score=0.72500
```

(3,4) :

```
k=1, n=  3, score=0.71250
k=1, n= 50, score=0.92500
k=1, n=159, score=0.93750
k=3, n=  3, score=0.61250
k=3, n= 50, score=0.86250
k=3, n=159, score=0.88750
k=5, n=  3, score=0.52500
k=5, n= 50, score=0.80000
k=5, n=159, score=0.78750
```

(5,6) :

```
k=1, n=  3, score=0.76250
k=1, n= 50, score=0.92500
k=1, n=159, score=0.93750
k=3, n=  3, score=0.62500
k=3, n= 50, score=0.87500
k=3, n=159, score=0.88750
k=5, n=  3, score=0.65000
k=5, n= 50, score=0.76250
k=5, n=159, score=0.75000
```

As the results shown, (k=1, n=159) is the best choice for the three cases.

So, we choose hyperparameters k=1, n=159 to test on the testing set

Recognition rate on testing set : <u>0.94375</u>

Problem 3   Determine the first eigenvector

Using the Power Iteration method!

Problem 3.

Let A be the $d \times d$ symmetric matrix. Assume $\underline{v_1}, \underline{v_2}, \cdots, \underline{v_d}$ are the associated eigenvectors, which are linearly independent and form a basis for $\mathbb{R}^d$

Then we choose an initial approximate $\underline{x_0}$, we know that

$$\underline{x_0} = c_1\underline{v_1} + c_2\underline{v_2} + \cdots + c_d\underline{v_d} \quad , \quad c_1, c_2 \ldots c_d \text{ are constants}, \ c_1 \neq 0$$

Next, create a sequence $\{x_k\}$ in the rule of $\underline{x_{k+1}} = \dfrac{A x_k}{\|Ax_k\|}$.

$$\begin{aligned}
\underline{x_1} = A\underline{x_0} &= c_1(A\underline{v_1}) + c_2(A\underline{v_2}) + \cdots + c_d(A\underline{v_d}) \\
&= c_1\lambda_1\underline{v_1} + c_2\lambda_2\underline{v_2} + \cdots + c_d\lambda_d\underline{v_d} \\
\underline{x_2} = A\underline{x_1} &= A^2\underline{x_0} \\
&\vdots \\
x_k = A^k\underline{x_0} &= \lambda_1^k\left[c_1\underline{v_1} + c_2\left(\frac{\lambda_2}{\lambda_1}\right)^k\underline{v_2} + \cdots + c_d\left(\frac{\lambda_d}{\lambda_1}\right)^k\underline{v_d}\right]
\end{aligned}$$

For each $j \geq 2$, $\left|\frac{\lambda_j}{\lambda_1}\right| < 1$, so $\left|\frac{\lambda_j}{\lambda_1}\right|^k \to 0$ as $k \to \infty$.

$$\Rightarrow \lim_{k\to\infty} x_k = A^k\underline{x_0} = \lambda_1^k c_1\underline{v_1} \quad (c_1 \neq 0)$$

Since any nonzero constant times an eigenvector is still an eigenvector with the same eigenvalue, thus $A^k\underline{x_0}$ approaches a multiple of the dominant eigenvector of A.

Reference:

https://en.wikipedia.org/wiki/Power_iteration

http://zoro.ee.ncku.edu.tw/na/res/09-power_method.pdf

https://ccjou.wordpress.com/2010/11/02/power-遞迴法/

http://ergodic.ugr.es/cphys/LECCIONES/FORTRAN/power_method.pdf