Homework #1

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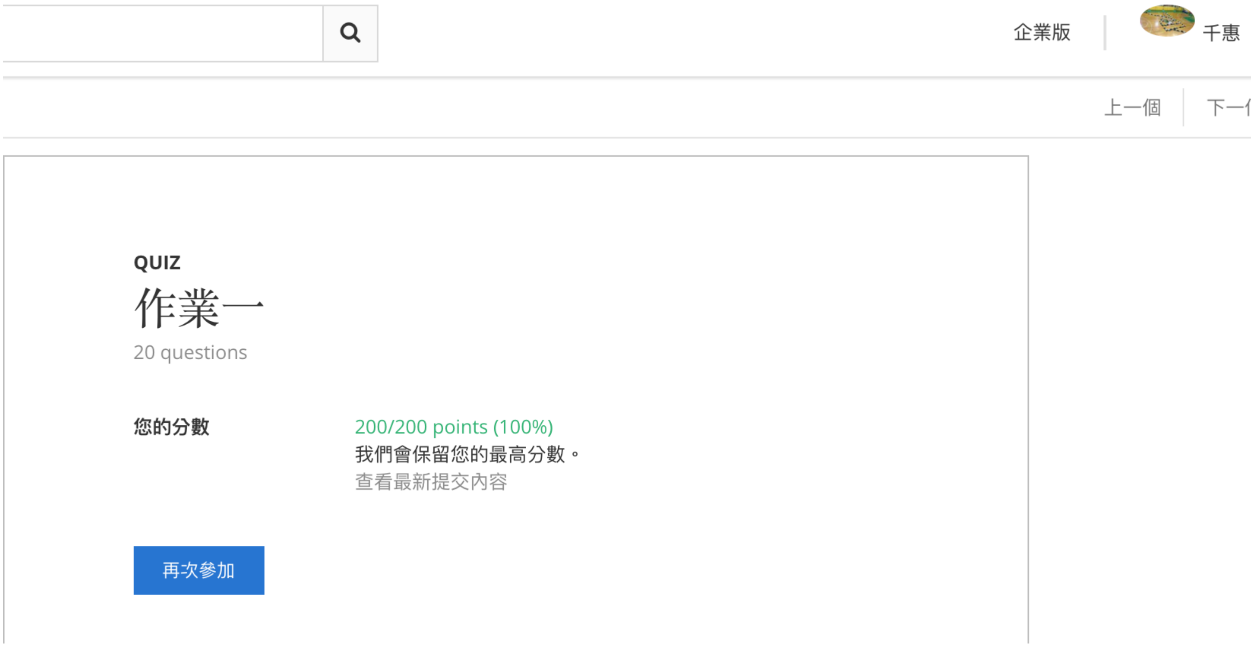
Problem 1

Score: 200 / 200



Problem 2

Application of active learning: **Redesign of census**

Original census: collects "complete information" from "all participants" in the population. All participants are asked to fill out a detailed long form.

Census by active learning:

1. **The algorithm gets a lot of data, but not the labels.**
   Ask all participants in the population to fill out a short form.
2. **Select representative respondents. (A subset of data.)**
   The algorithm learns the task and tells what labels would be most useful at the current state.
3. **Manually label just the data in the subset.**
   Ask the repondents who were selected to fill out a more detailed long form.

Problem 3

$$E_{OTS}(g, f) = \frac{1}{L}\sum_{l=1}^{L} [\![g(x_{N+l}) \neq f(x_{N+l})]\!] = |\{k \mid N+1 \leq k \leq N+L \text{ and k is even}\}|$$

1. When N is an even number, let n_e be the number of even number between 1 and L.

   $E_{OTS}(g, f) = \dfrac{n_e}{L}$

   $n_e = \lfloor \dfrac{L}{2} \rfloor$

   $= \lfloor \dfrac{L}{2} \rfloor + \lfloor \dfrac{N}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$

   $= \lfloor \dfrac{N+L}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$ (Since $\lfloor \dfrac{N}{2} \rfloor \in \mathbb{N}$, $\lfloor \dfrac{L}{2} \rfloor + \lfloor \dfrac{N}{2} \rfloor = \lfloor \dfrac{N+L}{2} \rfloor$)

   $\Rightarrow E_{OTS}(g, f) = \dfrac{\lfloor \frac{N+L}{2} \rfloor - \lfloor \frac{N}{2} \rfloor}{L}$

2. When N is an odd number, let n_o be the number of odd number between 1 and L.

   $E_{OTS}(g, f) = \dfrac{n_o}{L}$

   (a) If L is an even number, $n_o = \dfrac{L}{2} \in \mathbb{N}$

   $n_o = \dfrac{L}{2} + \lfloor \dfrac{N}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$

   $= \lfloor \dfrac{N+L}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$ (Since $\dfrac{L}{2} \in \mathbb{N}$)

   (b) If L is an odd number, $n_o = \dfrac{L+1}{2} \in \mathbb{N}$

   $n_o = \dfrac{L+1}{2} + \lfloor \dfrac{N}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$

   $= \dfrac{L+1}{2} + \lfloor \dfrac{N-1}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$ (Since N is an odd number, $\lfloor \dfrac{N}{2} \rfloor = \lfloor \dfrac{N-1}{2} \rfloor$)

   $= \lfloor \dfrac{L+1+N-1}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$ (Since $\dfrac{L+1}{2} \in \mathbb{N}$)

   $= \lfloor \dfrac{N+L}{2} \rfloor - \lfloor \dfrac{N}{2} \rfloor$

   $\Rightarrow E_{OTS}(g, f) = \dfrac{\lfloor \frac{N+L}{2} \rfloor - \lfloor \frac{N}{2} \rfloor}{L}$

From 1. & 2., $E_{OTS}(g, f) = \dfrac{\lfloor \frac{N+L}{2} \rfloor - \lfloor \frac{N}{2} \rfloor}{L}$

Problem 4

$f : \mathcal{X} \rightarrow \mathcal{Y}$, $\mathcal{X} = \{x_1, x_2, x_3, ..., x_N, x_{N+1}, ...x_{N+L}\}$, $\mathcal{Y} = \{-1, +1\}$

Let $\mathcal{F} = \{f \mid f$ can "generate" $\mathcal{D}$ in a noiseless setting$\}$

$$\Rightarrow \forall f \in \mathcal{F} \begin{cases} (x_i, f(x_i)) \in \mathcal{D} & , 1 \leq i \leq N \\ \\ f(x_i) \in \{-1, +1\} & , N < i \leq N + L \end{cases}$$

$$\Rightarrow |F| = 1^N * |\{-1, +1\}|^{(N+L)-N} = 2^L$$

Problem 5

Let $g_1 = \mathcal{A}_1(\mathcal{D})$ and $g_2 = \mathcal{A}_2(\mathcal{D})$

$$E_{OTS}(g, f) = \frac{1}{L} \sum_{l=1}^{L} [\![g(x_{N+l}) \neq f(x_{N+l})]\!]$$

$\mathcal{F}$ is the set of all f that can generate $\mathcal{D}$ in a noiseless setting. $\mathcal{F} = \{f_1, f_2, ...f_{2^L}\}$

$$\mathbb{E}_f \Big\{ E_{OTS}(g, f) \Big\} = \frac{1}{2^L} \sum_{i=1}^{2^L} E_{OTS}(g, f_i) = \frac{1}{2^L} \frac{1}{L} \sum_{i=1}^{2^L} \sum_{l=1}^{L} [\![g(x_{N+l}) \neq f_i(x_{N+l})]\!]$$

$$\forall N < k \leq L, \begin{cases} \sum_{i=1}^{2^L} [\![f_i(x_k) = 1]\!] = \frac{2^L}{2} = 2^{L-1} \Rightarrow \sum_{i=1}^{2^L} [\![f_i(x_k) \neq -1]\!] = 2^{L-1} \\ \sum_{i=1}^{2^L} [\![f_i(x_k) = -1]\!] = \frac{2^L}{2} = 2^{L-1} \Rightarrow \sum_{i=1}^{2^L} [\![f_i(x_k) \neq 1]\!] = 2^{L-1} \end{cases}$$

$$\Rightarrow \forall N < k \leq L, \sum_{i=1}^{2^L} [\![g(x_k) \neq f_i(x_k)]\!] = \sum_{i=1}^{2^L} \Big([\![1 \neq f(x_k)]\!] or [\![-1 \neq f(x_k)]\!] \Big) = 2^{L-1}$$

$$\Rightarrow \mathbb{E}_f \Big\{ E_{OTS}(g, f) \Big\} = \frac{1}{2^L} \frac{1}{L} \sum_{l=1}^{L} 2^{L-1} = \frac{1}{2^L} * \frac{1}{L} * L * 2^{L-1} = \frac{1}{2}$$

$$\Rightarrow \mathbb{E}_f \Big\{ E_{OTS}(\mathcal{A}_1(\mathcal{D}), f) \Big\} = \mathbb{E}_f \Big\{ E_{OTS}(\mathcal{A}_2(\mathcal{D}), f) \Big\} = \frac{1}{2}$$

Problem 6

If B or D dice is picked, we get green 1's. Otherwise, we get orange 1's.

$\Rightarrow$ For every pick, the probability of getting a green 1's is $\frac{2}{4} = \frac{1}{2}$

$\Rightarrow$ Picking five dices, the probability of getting five green 1's is $(\frac{1}{2})^5 = \frac{1}{32}$

Problem 7

The target is to get "some number" that is purely green when picking 5 dice from the bag.

$\Rightarrow$ Dice A and dice B can't be picked together.

Dice C and dice D can't be picked together either.

1. Picking only one kind of dices. $\Rightarrow P = 4 * (\frac{1}{4})^5 = \frac{1}{256}$

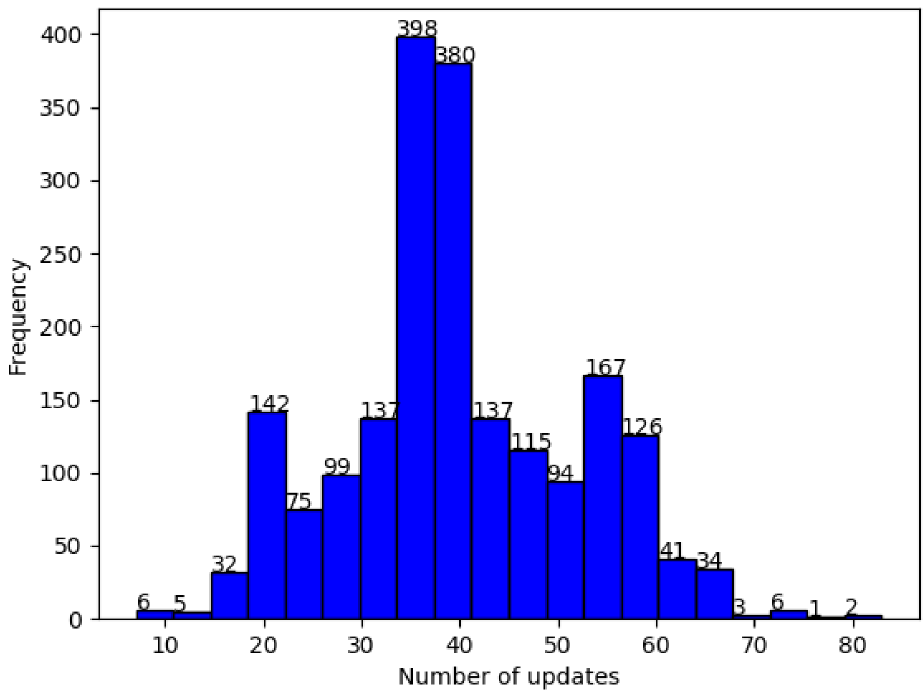
2. Picking two kinds of dices: (A&C) or (A&D) or (B&C) or (B&D)

   $\Rightarrow P = 4 * [(\frac{2}{4})^5 - 2 * (\frac{1}{4})^5] = \frac{30}{256}$

$\Rightarrow$ When picking 5 dice from the bag, the total probability of getting "some number" that is purely green is $\frac{1}{256} + \frac{30}{256} = \frac{31}{256}$

Problem 8

Average number of updates: 39



Problem 9(Bonus)

$R^2 = \max_n ||\mathbf{x}_n||^2$, $\rho = \min_n y_n \frac{\mathbf{w}_f^T}{||\mathbf{w}_f||}\mathbf{x}_n$, $T \leq \frac{R^2}{\rho^2}$

When scaling down all $\mathbf{x}_n$ linearly by a factor of 20,

$$\begin{cases} \max_n ||\mathbf{x'}_n|| = \frac{1}{20}\max_n ||\mathbf{x}_n|| \rightarrow (R')^2 = (\frac{1}{20})^2 R^2 \\ \min_n ||\mathbf{x'}_n|| = \frac{1}{20}\min_n ||\mathbf{x}_n|| \rightarrow \rho' = \frac{1}{20}\rho \end{cases}$$

$\Rightarrow T' \leq \frac{(R')^2}{(\rho')^2} = \frac{(\frac{1}{20}R)^2}{(\frac{1}{20}\rho)^2} = \frac{R^2}{\rho^2}$

$\Rightarrow$ PLA algorithm would not run 20 times faster.