Gaming Strategy

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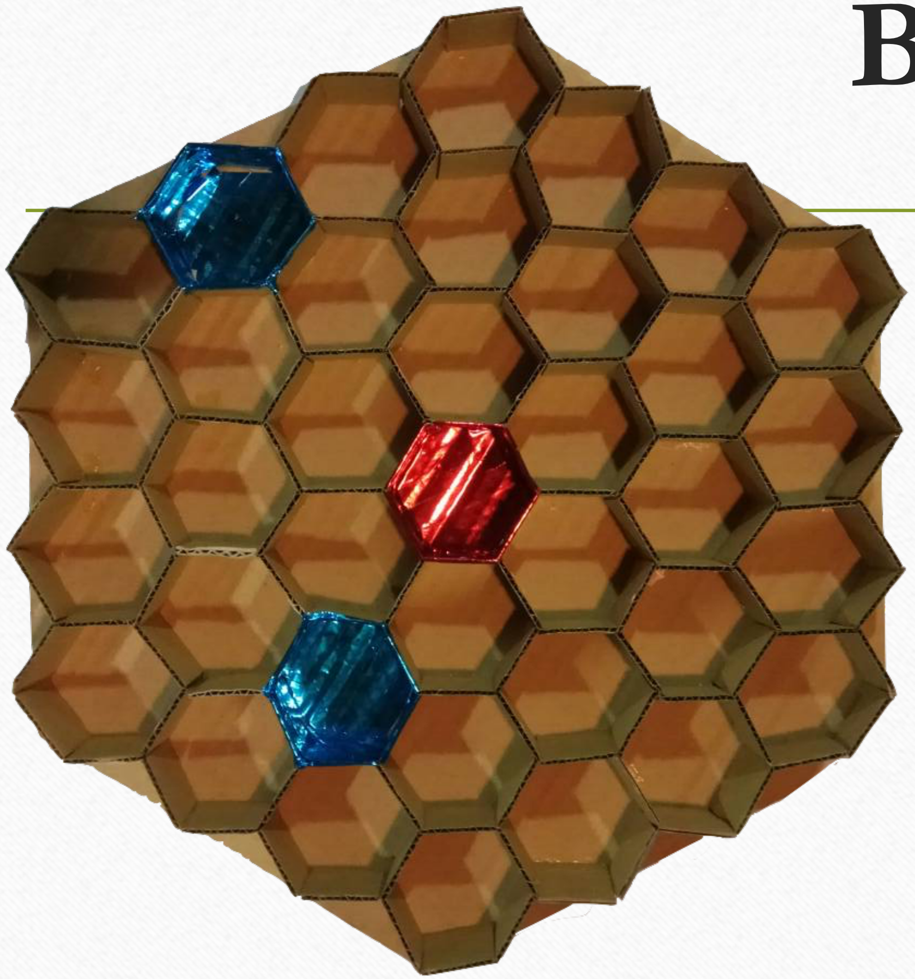
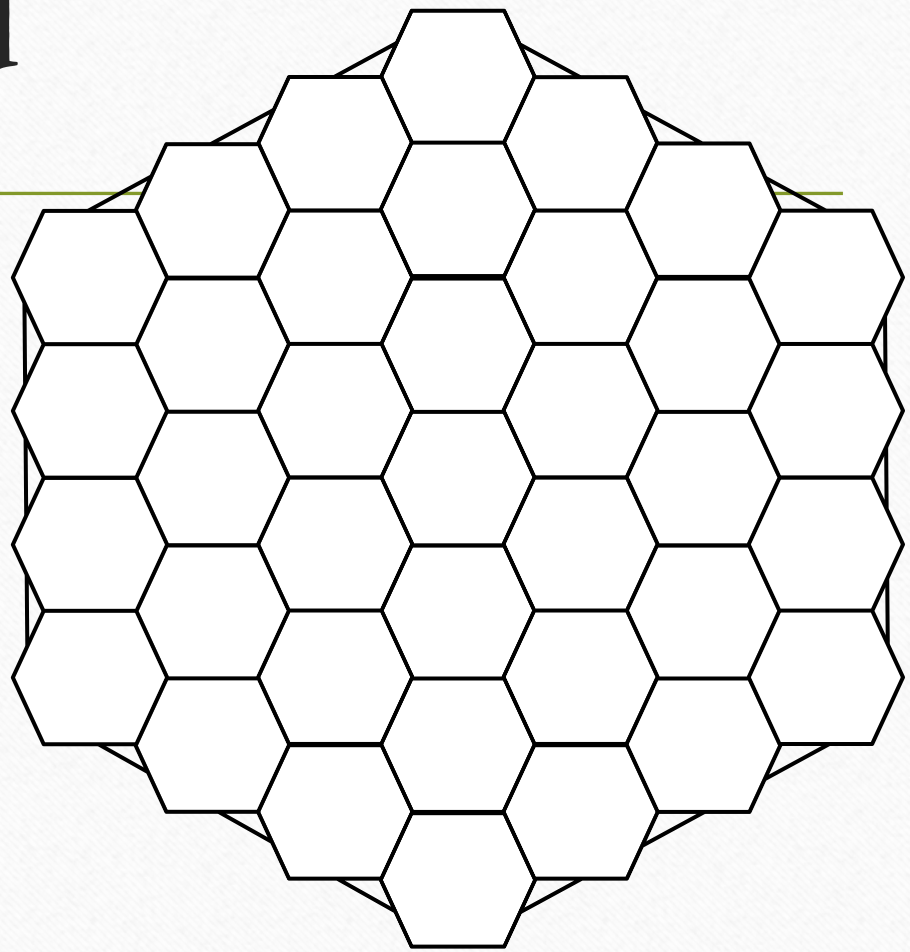
Board





Game Design Credit: OT B04 蔡心瑜

Game Introduction

- Players: 2
- Round: N ($N \geq 10$)
- Two players take turns to throw a piece onto the board. If the player missed the board, he/she cannot re-throw it.
- If the piece is thrown into an occupied grid, the ownership of the grid will change.

Game Design Credit: OT B04 蔡心瑜

Winning Condition

1) Each grid has a weighted score, after N rounds, the player with higher total score wins.
2) After N rounds, the player with highest number of pieces in a same vertical or diagonal row wins.

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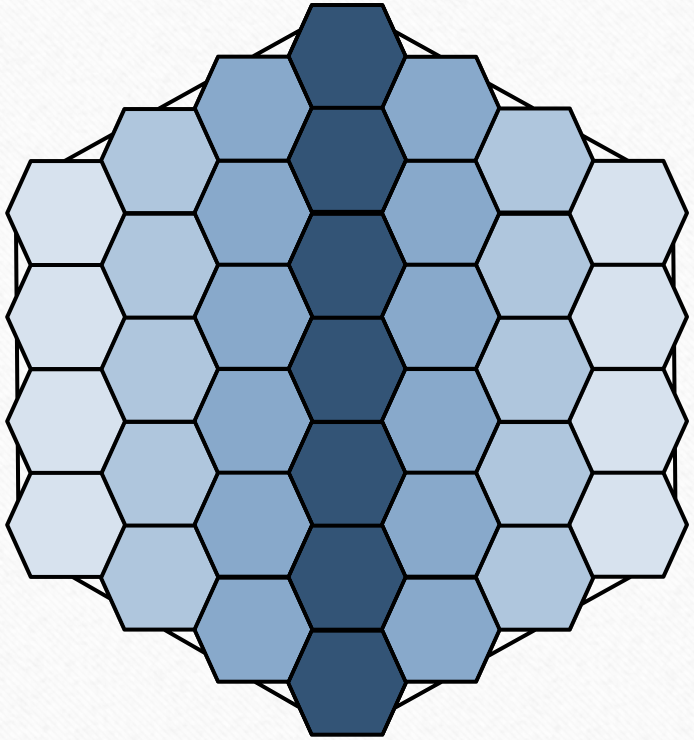
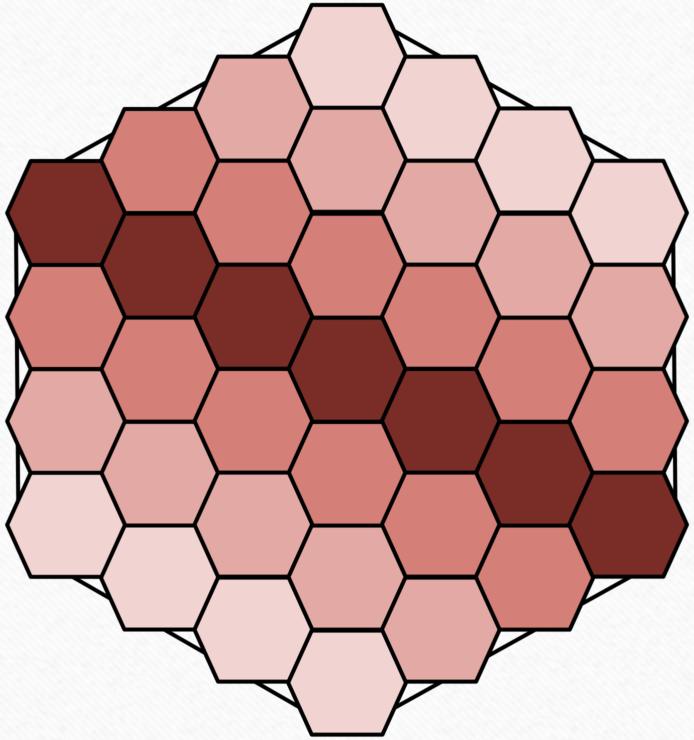
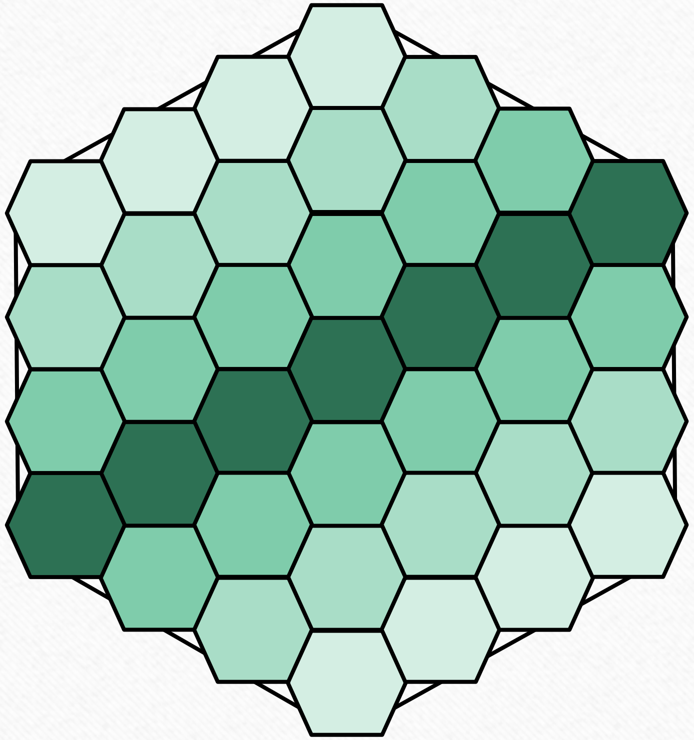
Vertical row

Diagonal row







Coordinates

- Polar coordinates
- Oblique coordinate
- Cartesian coordinates

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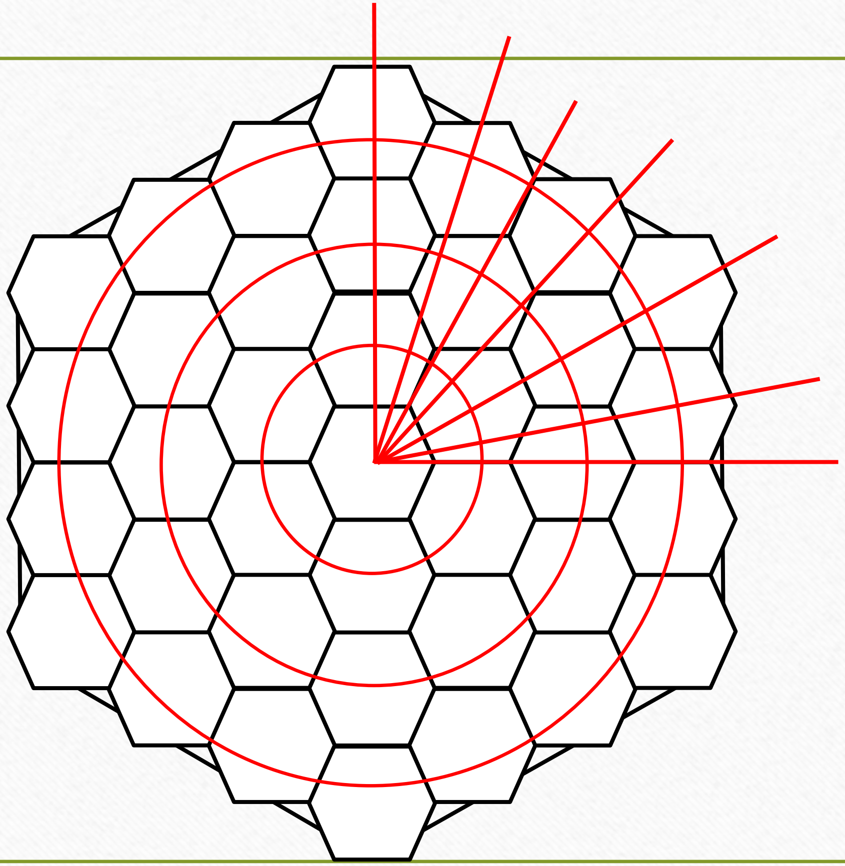
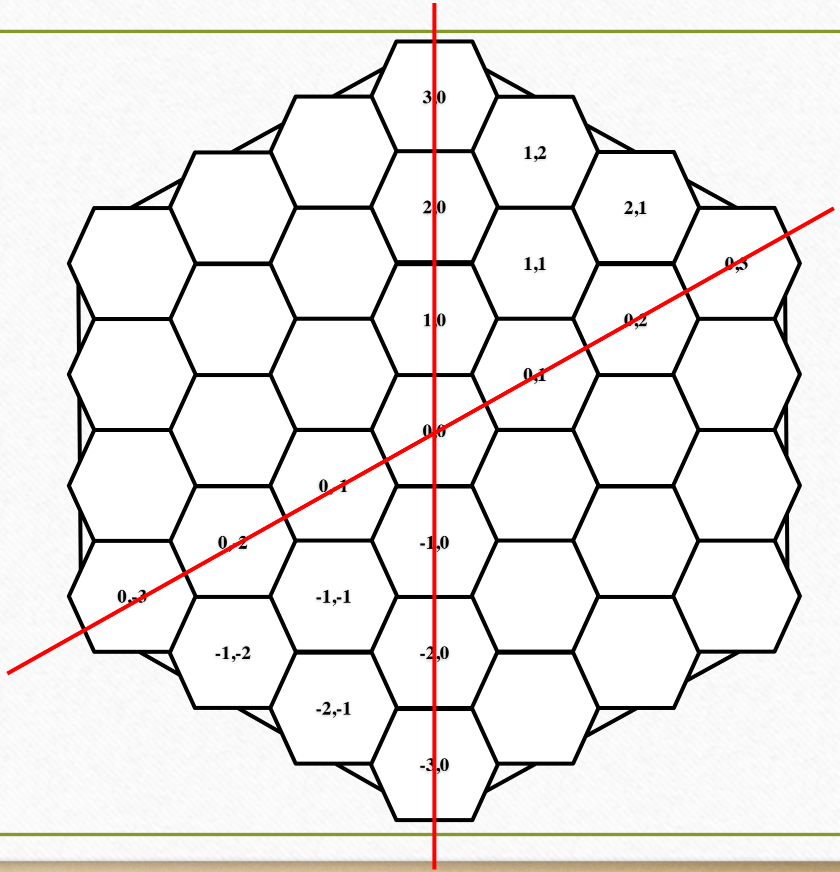
- **Polar coordinates**
- Oblique coordinates
- Cartesian coordinates



Coordinates

- ~~Polar coordinates~~
- **<u>Oblique coordinates</u>**
- Cartesian coordinates



Coordinates

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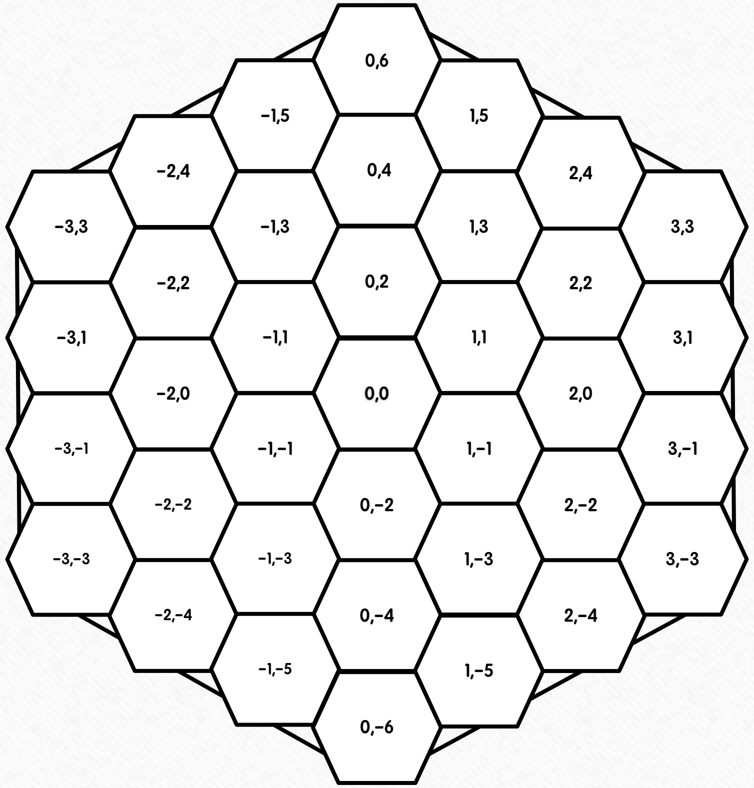
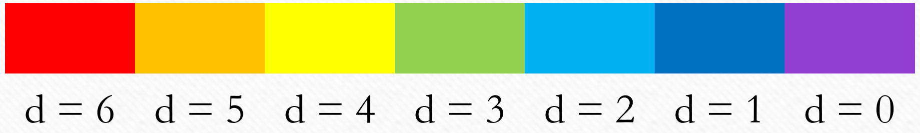
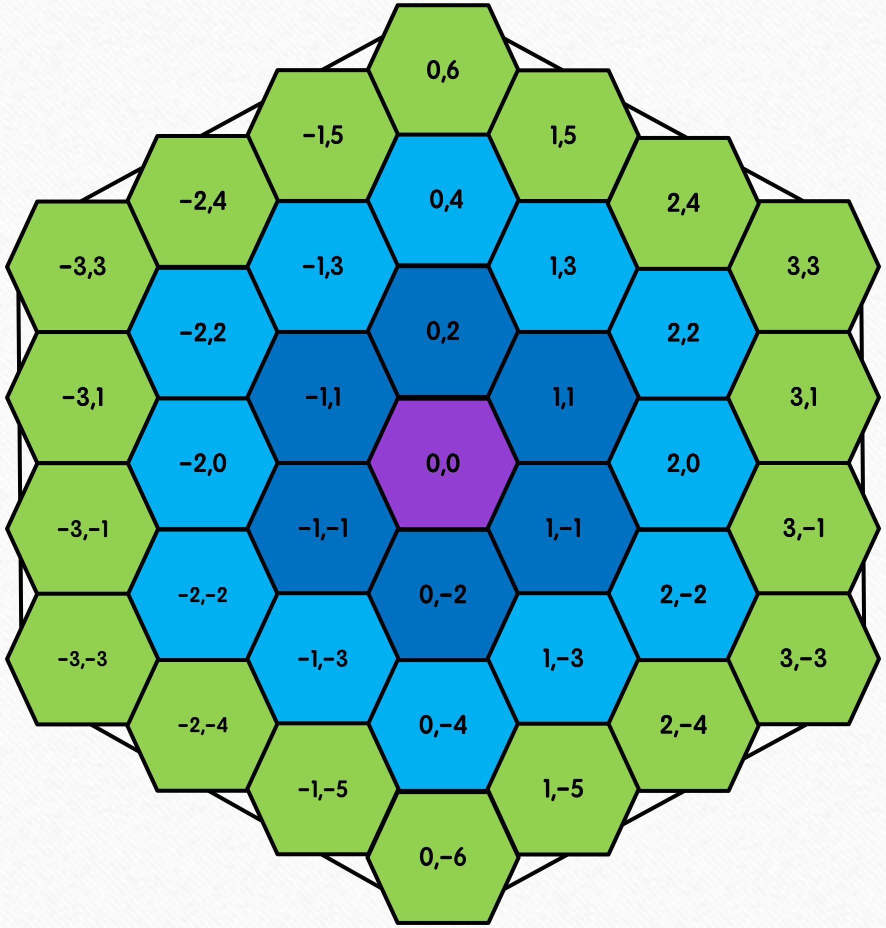
- **Cartesian coordinates**

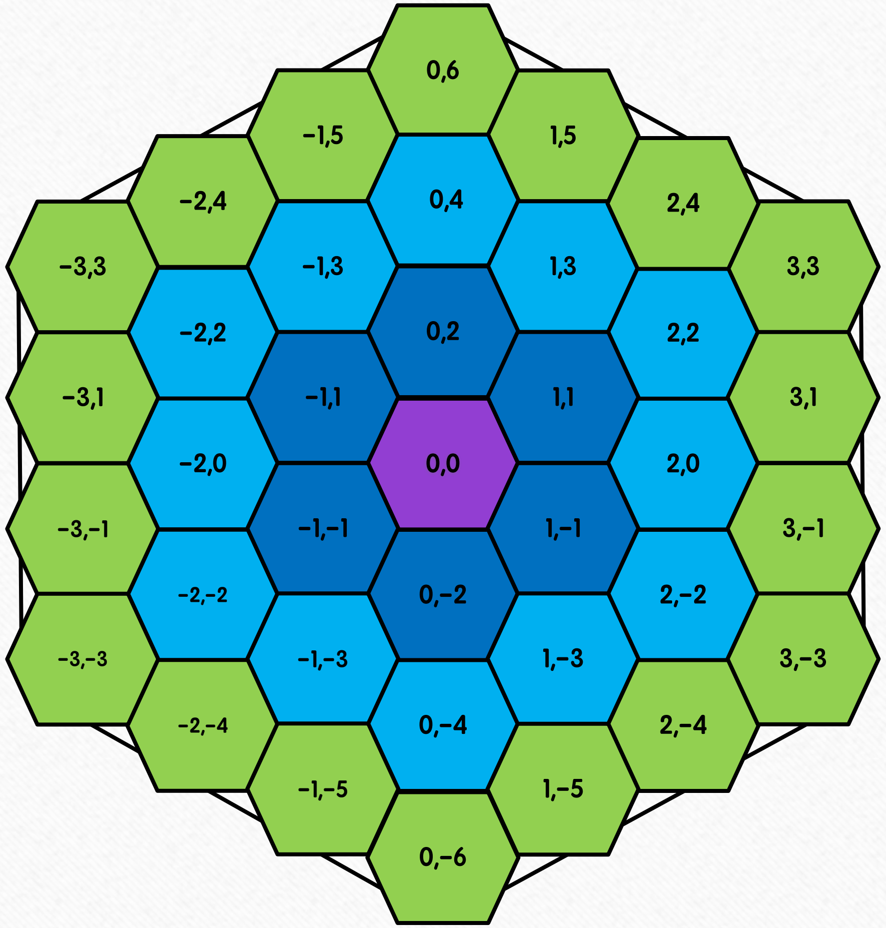


- Take (0,0) as the center.
- Definition of d(x, y):
  the minimal number of grids from the grid (x, y) to the center grid (0,0)

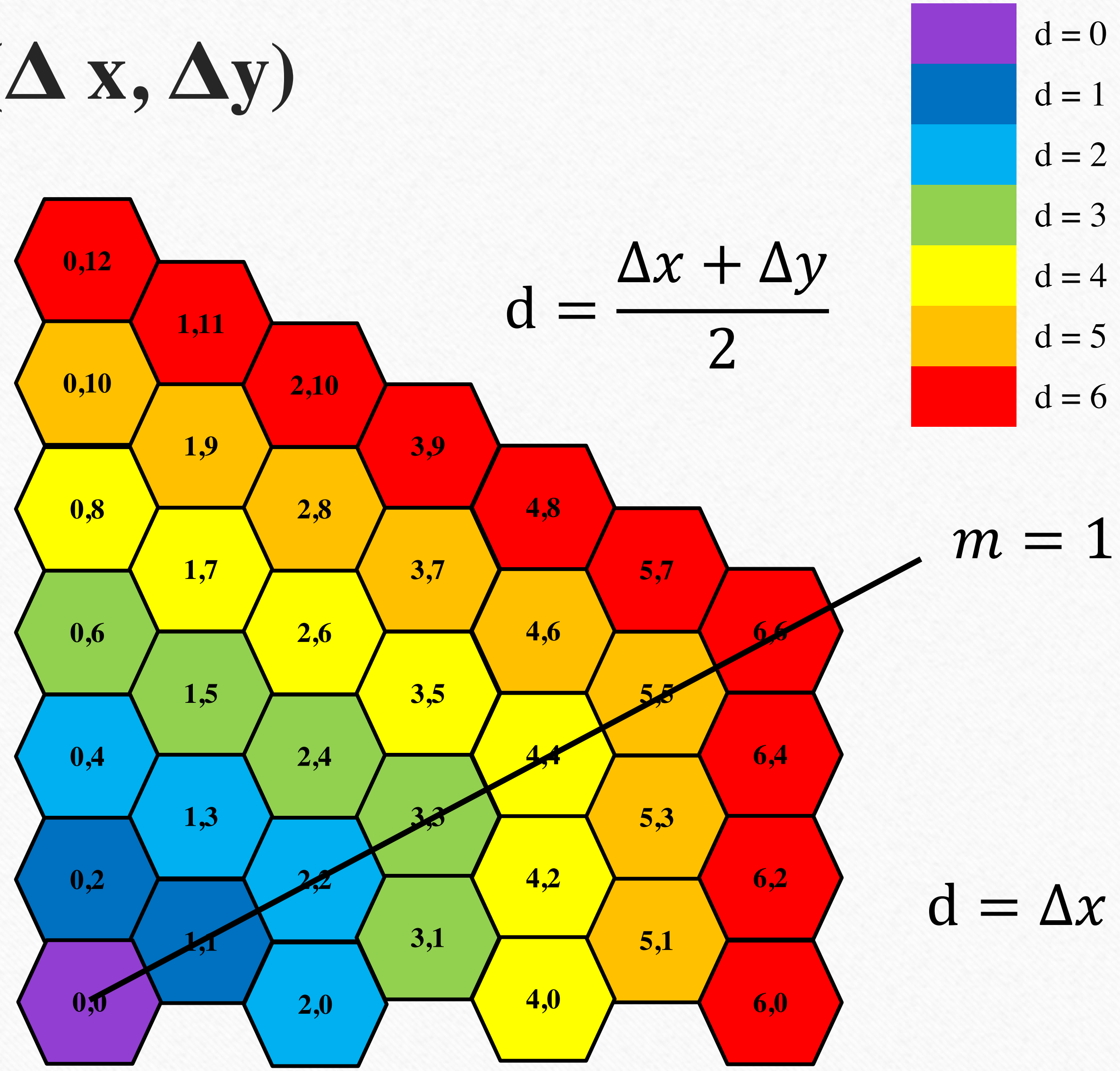


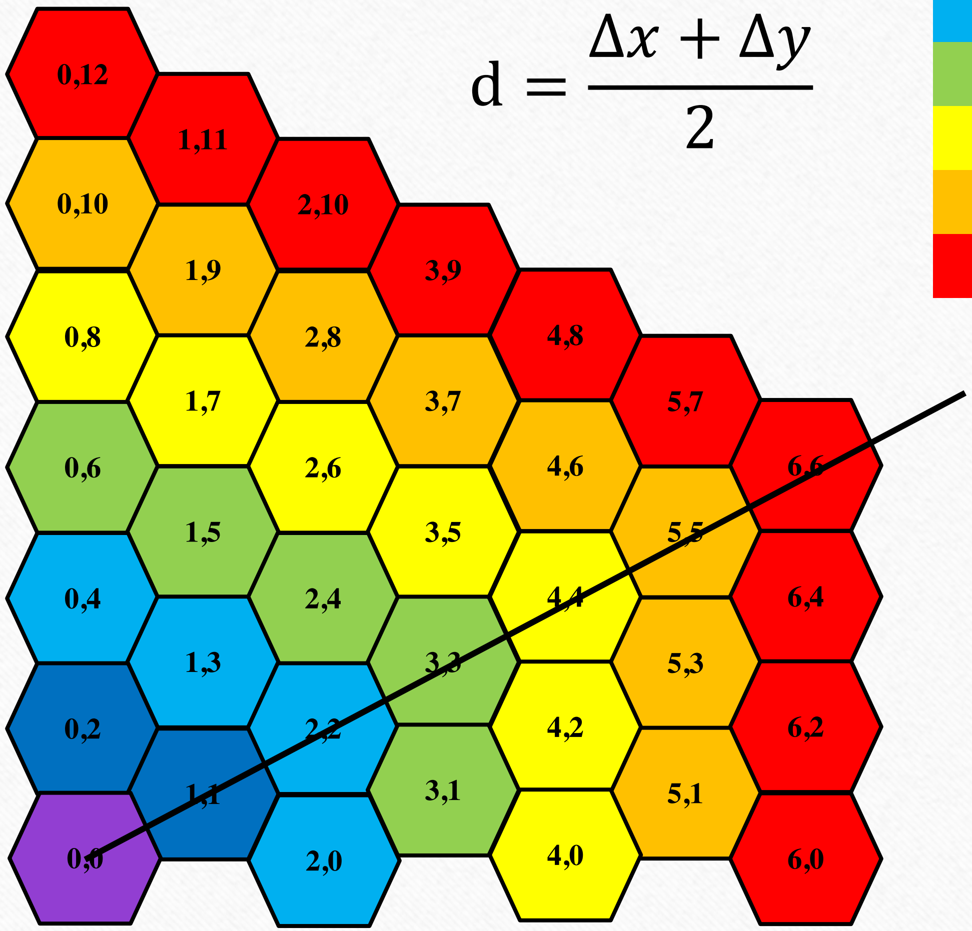


(x, y)

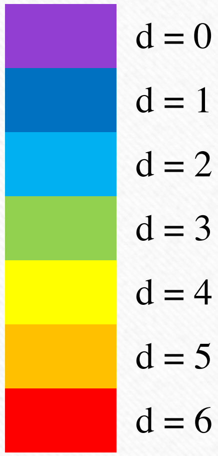

0,6
-1,5
1,5
-2,4
0,4
2,4
-3,3
-1,3
1,3
3,3
-2,2
0,2
2,2
-3,1
-1,1
1,1
3,1
-2,0
0,0
2,0
-3,-1
-1,-1
1,-1
3,-1
-2,-2
0,-2
2,-2
-3,-3
-1,-3
1,-3
3,-3
-2,-4
0,-4
2,-4
-1,-5
1,-5
0,-6

(Δ x, Δy)


d = 0
d = 1
d = 2
d = 3
d = 4
d = 5
d = 6
$d = \frac{\Delta x + \Delta y}{2}$
$m = 1$
$d = \Delta x$
0,12
1,11
0,10
2,10
1,9
3,9
0,8
2,8
4,8
1,7
3,7
5,7
0,6
2,6
4,6
6,6
1,5
3,5
5,5
0,4
2,4
4,4
6,4
1,3
3,3
5,3
0,2
2,2
4,2
6,2
1,1
3,1
5,1
0,0
2,0
4,0
6,0



$$d = \frac{\Delta x + \Delta y}{2}$$



$m = 1$

$d = \Delta x$

Conclusion

$$d = \begin{cases} \dfrac{\Delta x + \Delta y}{2}, & \Delta x \leq \Delta y \\ \Delta x & , otherwise \end{cases}$$

Next?

1) Test for N.D.
2) Adopt Strategies

Test the Model for Normal Distribution

$$\alpha = 0.1$$

$$\chi^2{}_{0.1}(8) = 13.36$$

d	Class	Mid	Frequency (Y)	f*mid	f*(mid-mean)^2
-4	[-31.5,-24.5)	-28	0	0	0
-3	[-24.5,-17.5)	-21	1	-21	441
-2	[-17.5,-10.5)	-14	7	-98	1372
-1	[-10.5,-3.5)	-7	11	-77	539
0	[-3.5,3.5)	0	7	0	0
1	[3.5,10.5)	7	11	77	539
2	[10.5,17.5)	14	7	98	1372
3	[17.5,24.5)	21	1	21	441
4	[24.5,31.5)	28	0	0	0
		sum	45	0	2352

mean	0
variance	53.45454545
stdev	7.31126155

x	z-score	p-value	d	Probability (p)	area	p*area	Adjusted p (p')	Excepted (np')	$\frac{(Y-np')^2}{np'}$
-31.5	-4.30842	0.00001	-4	0.00039439	1231.50432	0.485690264	0.00186359	0.083861563	0.083861563
-24.5	-3.35099	0.00040	-3	0.00794009	923.6282402	7.333691665	0.028139326	1.266269664	0.055990865
-17.5	-2.39357	0.00834	-2	0.06713846	615.7521601	41.34065213	0.158623806	7.13807126	0.002670704
-10.5	-1.43614	0.07548	-1	0.24059005	307.8760801	74.07192186	0.284213469	12.7896061	0.250413497
-3.5	-0.47871	0.31607	0	0.36785758	38.48451001	14.15681864	0.054319618	2.44438283	8.490342655
3.5	0.47871	0.68393	1	0.24059005	307.8760801	74.07192186	0.284213469	12.7896061	0.250413497
10.5	1.43614	0.92452	2	0.06713846	615.7521601	41.34065213	0.158623806	7.13807126	0.002670704
17.5	2.39357	0.99166	3	0.00794009	923.6282402	7.333691665	0.028139326	1.266269664	0.055990865
24.5	3.35099	0.99960	4	0.00039439	1231.50432	0.485690264	0.00186359	0.083861563	0.083861563
31.5	4.30842	0.99999	**sum**	1	6196.006111	260.6207305	1	45	9.276215913

Test the Model for Normal Distribution

- H_0: The model fits normal distribution.
- $\chi^2{}_{0.1}(8) = 13.36$
- $q_8 = \sum_{i=1}^{9} \frac{(Y-np)^2}{np} = 9.2762$
- We can't reject $H_{0.}$
- We believe that it's a normal distribution.

Probability of grids with different d

d	Adjusted p(p')	n	$\frac{p'}{n}$
0	0.054314335	1	0.054314335
1	0.568371647	6	0.094728608
2	0.317216753	12	0.026434729
3	0.056273178	18	0.003126288
4	0.003726818	24	0.000155284
5	9.62846E-05	30	3.20949E-06
6	9.85521E-07	36	2.73756E-08

Strategies

- Given a certain board, the optimal strategy is to
  - optimize player’s score.
  - optimize score difference between player and opponent.

  - considering future conditions (consider opponent’s target grid.)
  - without consideration of future conditions

	Player		Opponent	
	Consider future	Optimization	Consider future	Optimization
1	Yes	Difference	No	Own score
2	Yes	Difference	No	Difference
3	No	Own score	X	X
4	No	Difference	X	X

Formula

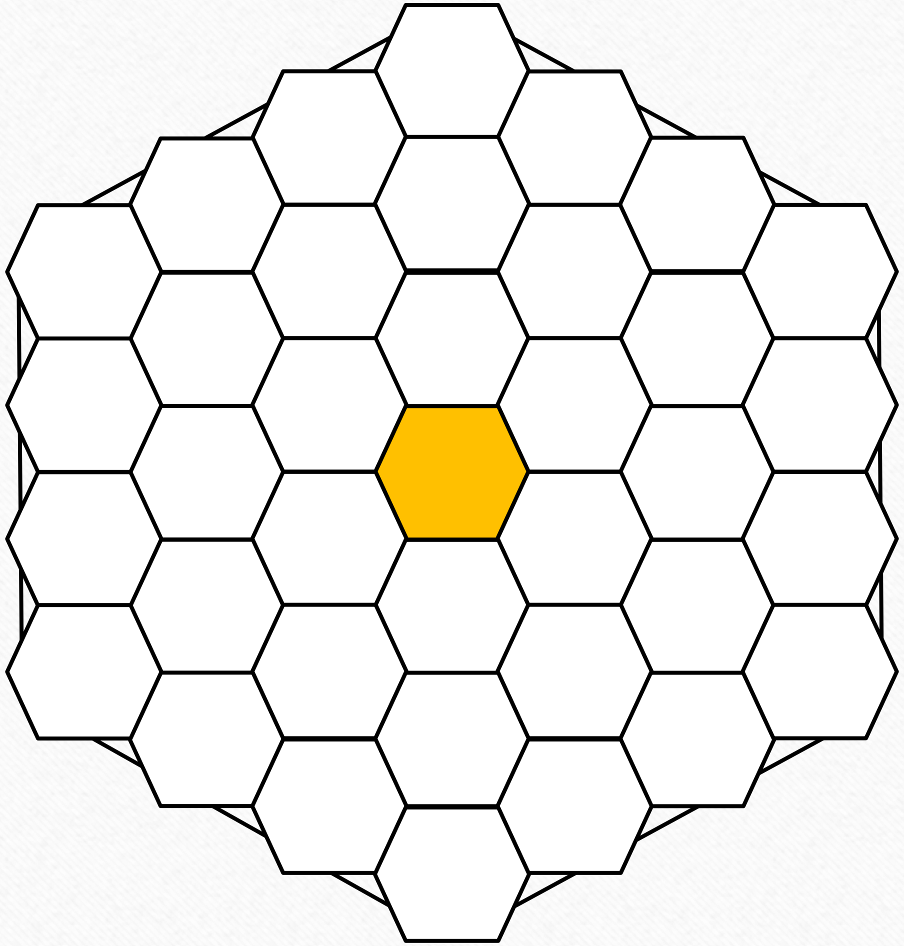
- **Maximize Strategy**

$$AIM_A = MAX\left(\forall X \in grid, \left(\sum_{Y \in grid} prob_{d(X,Y)} * score_{A_{after}}\right) + (1 - prob_{sum}) * score_{A_{miss}}\right)$$

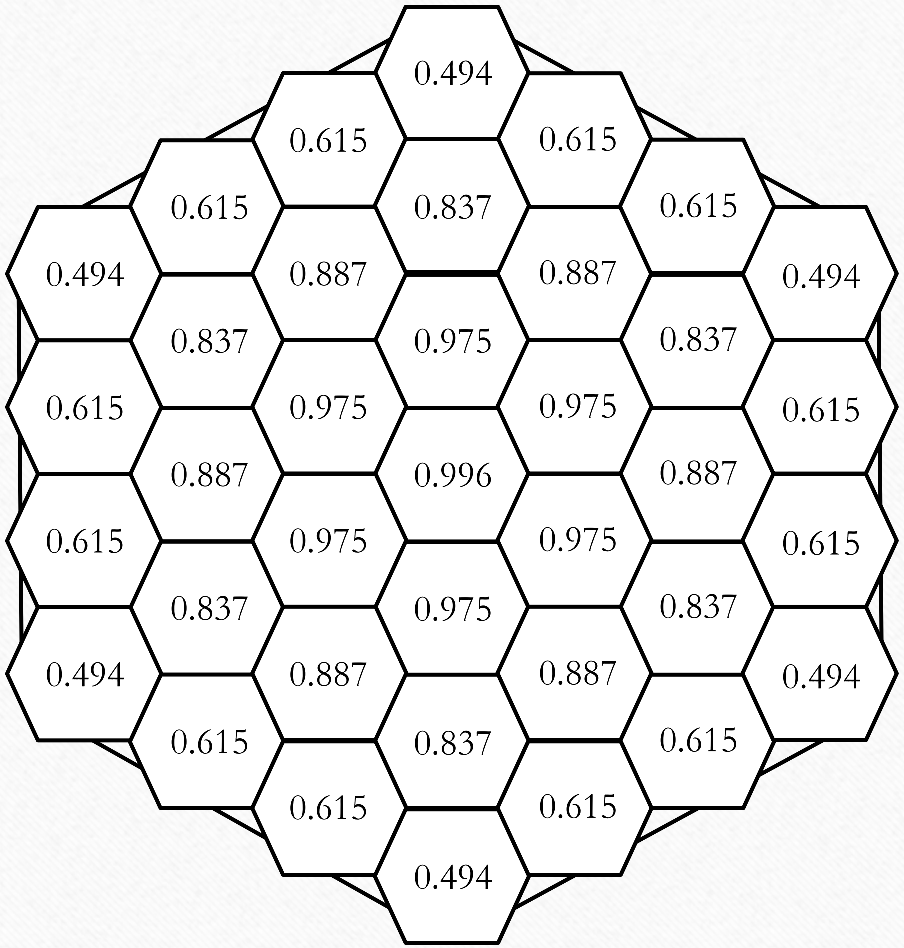
- **Difference Strategy**

$$AIM_A = MAX(\forall X \in grid,$$

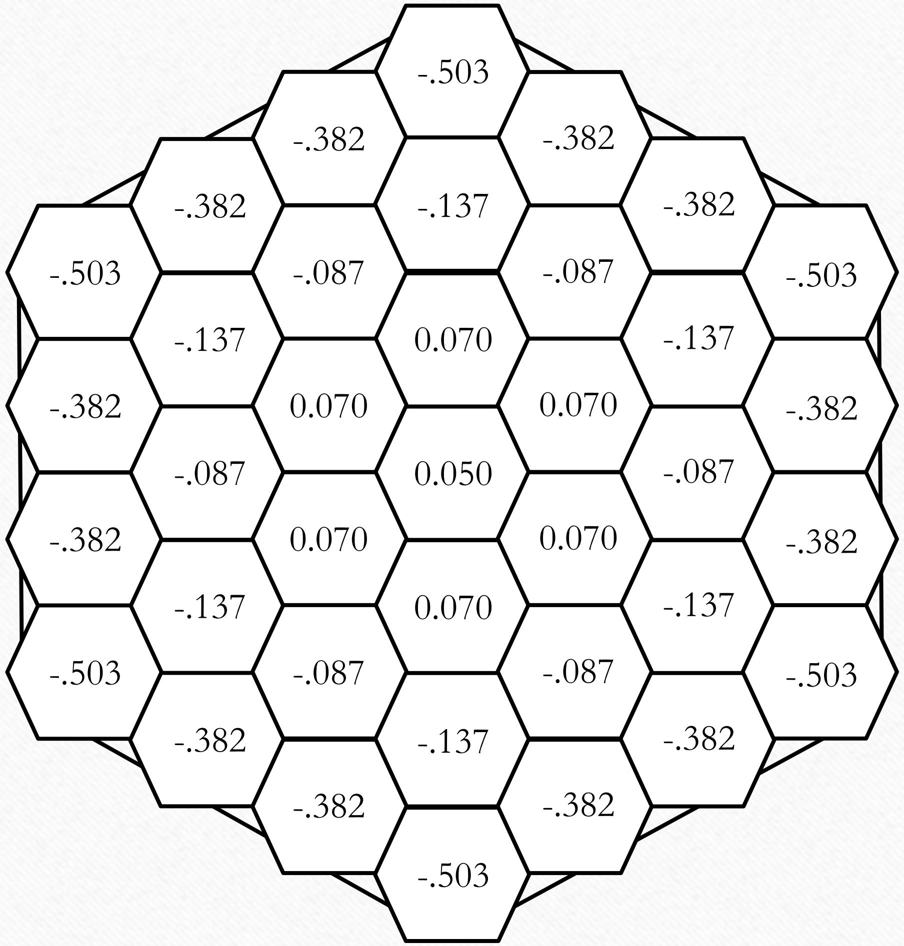
$$\left(\sum_{Y \in grid} prob_{d(X,Y)} * \left(score_{A_{after}} - score_{B_{after}}\right)\right) + (1 - prob_{sum}) * (score_{A_{miss}} - score_{B_{ori}}))$$



Optimize player's score



Optimize score difference



Simulation Result

Only partial data are shown in the slides

Full source code, data and records please check

https://github.com/zolution/Prob2017_Final

Simulation Parameters

- Test_Time = 10000 (Default)
- Round = 10
- Prob = {0.054314335, 0.094728608, 0.026434729, 0.003126288, 0.000155284, 3.20949e-6, 2.73756e-8}
- Player 1 Goes First

Player 1			Player 2														
Future	Self Max	Opp Max	Future	Self Max	Opp Max	1 AVG	2 AVG	2-1AVG	1 WON	1 WON%	1 WON% No Draw	2 WON	2 WON%	2 WON% No Draw	DRAW	DRAW (%)	N
0	0	0	0	0	0	2.3126	2.4303	0.1177	2372	23.72%	42.35%	3229	32.29%	57.65%	4399	43.99%	10000
0	0	0	0	1	0	2.4357	2.4777	0.042	2747	27.47%	47.72%	3010	30.10%	52.28%	4243	42.43%	10000
0	0	0	1	0	1	2.2826	2.3981	0.1155	2333	23.33%	42.14%	3203	32.03%	57.86%	4464	44.64%	10000
0	1	0	0	0	0	2.3485	2.5046	0.1561	2320	23.20%	39.84%	3503	35.03%	60.16%	4177	41.77%	10000
0	1	0	0	1	0	2.4744	2.5588	0.0844	2599	25.99%	44.85%	3196	31.96%	55.15%	4205	42.05%	10000
0	1	0	1	0	1	2.3106	2.4588	0.1482	2324	23.24%	40.37%	3433	34.33%	59.63%	4243	42.43%	10000
1	0	1	0	0	0	2.2839	2.4068	0.1229	2308	23.08%	42.06%	3180	31.80%	57.94%	4512	45.12%	10000
1	0	1	0	1	0	2.3948	2.4444	0.0496	2689	26.89%	47.41%	2983	29.83%	52.59%	4328	43.28%	10000
0	0	0	1	0	0	2.33736	2.44064	0.1033	1851	24.83%	43.39%	2415	32.39%	56.61%	3189	42.78%	7455
0	1	0	1	0	0	2.35266	2.5198	0.1671	1711	22.97%	39.25%	2648	35.55%	60.75%	3090	41.48%	7449
1	0	0	0	0	0	2.35382	2.45105	0.0972	1876	25.23%	43.54%	2433	32.72%	56.46%	3127	42.05%	7436
1	0	0	0	1	0	2.46951	2.49844	0.0289	2080	28.25%	48.67%	2194	29.80%	51.33%	3089	41.95%	7363
1	0	0	1	0	0	2.37567	2.47195	0.0963	908	24.49%	43.30%	1189	32.07%	56.70%	1611	43.45%	3708
1	0	0	1	0	1	2.32429	2.41424	0.09	1206	24.60%	43.63%	1558	31.78%	56.37%	2139	43.63%	4903
1	0	1	1	0	0	2.29965	2.41894	0.1193	1162	23.82%	41.93%	1609	32.98%	58.07%	2108	43.21%	4879
1	0	1	1	0	1	2.24735	2.36953	0.1222	1665	22.63%	41.02%	2394	32.54%	58.98%	3299	44.84%	7358

Claims

- The **second player** has advantage in this game.
- “**Difference Strategy**” is better than “Maximize Strategy”
- Both players tend to adopt “Difference Strategy” (w/o future)
- “**Consider Future**” is often better than “Not Consider Future” when player assume opponent adopts “Difference Strategy”

Player 1			Player 2														
Future	Self Max	Opp Max	Future	Self Max	Opp Max	1 AVG	2 AVG	2-1AVG	1 WON	1 WON%	1 WON% No Draw	2 WON	2 WON%	2 WON% No Draw	DRAW	DRAW%	N
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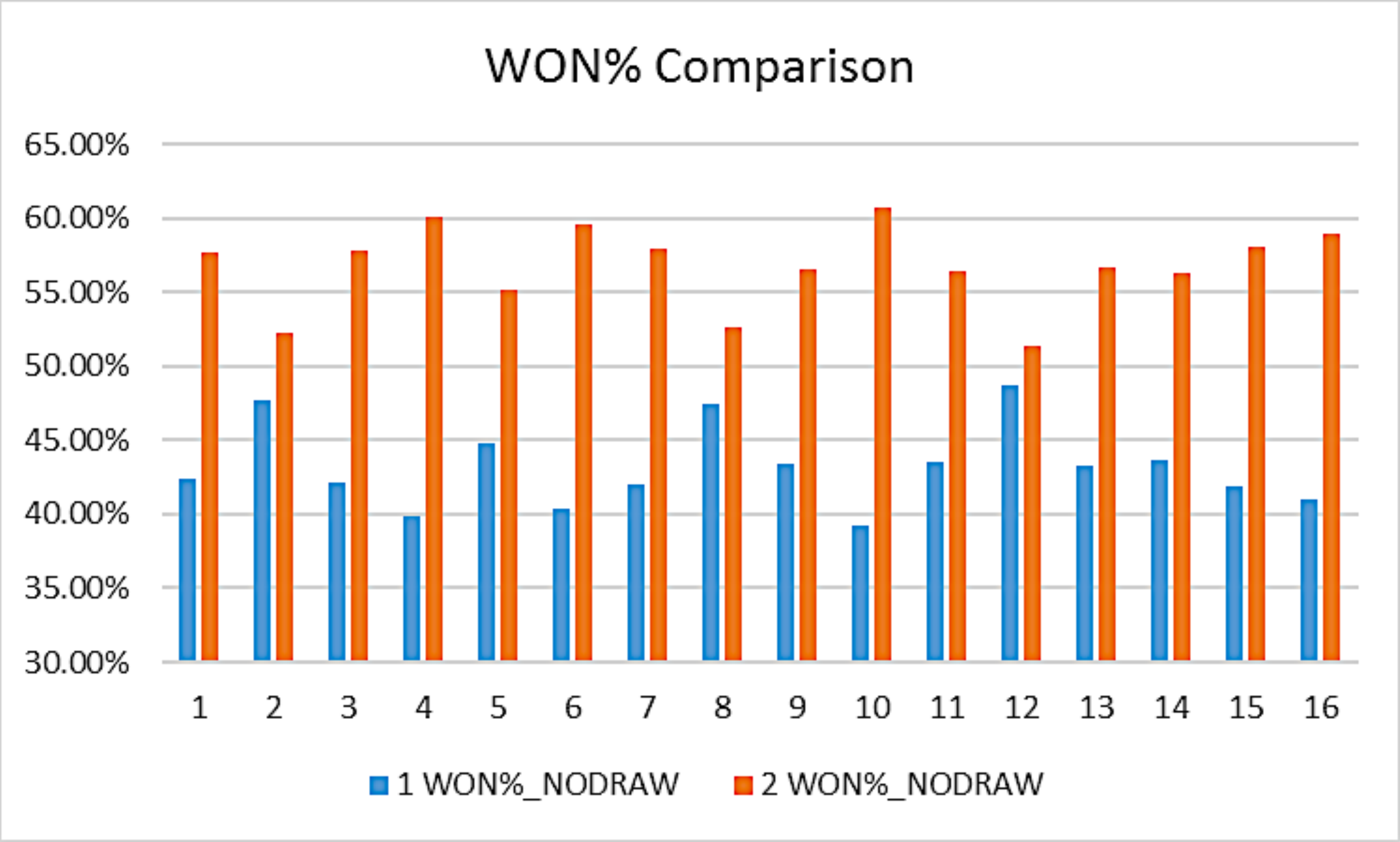
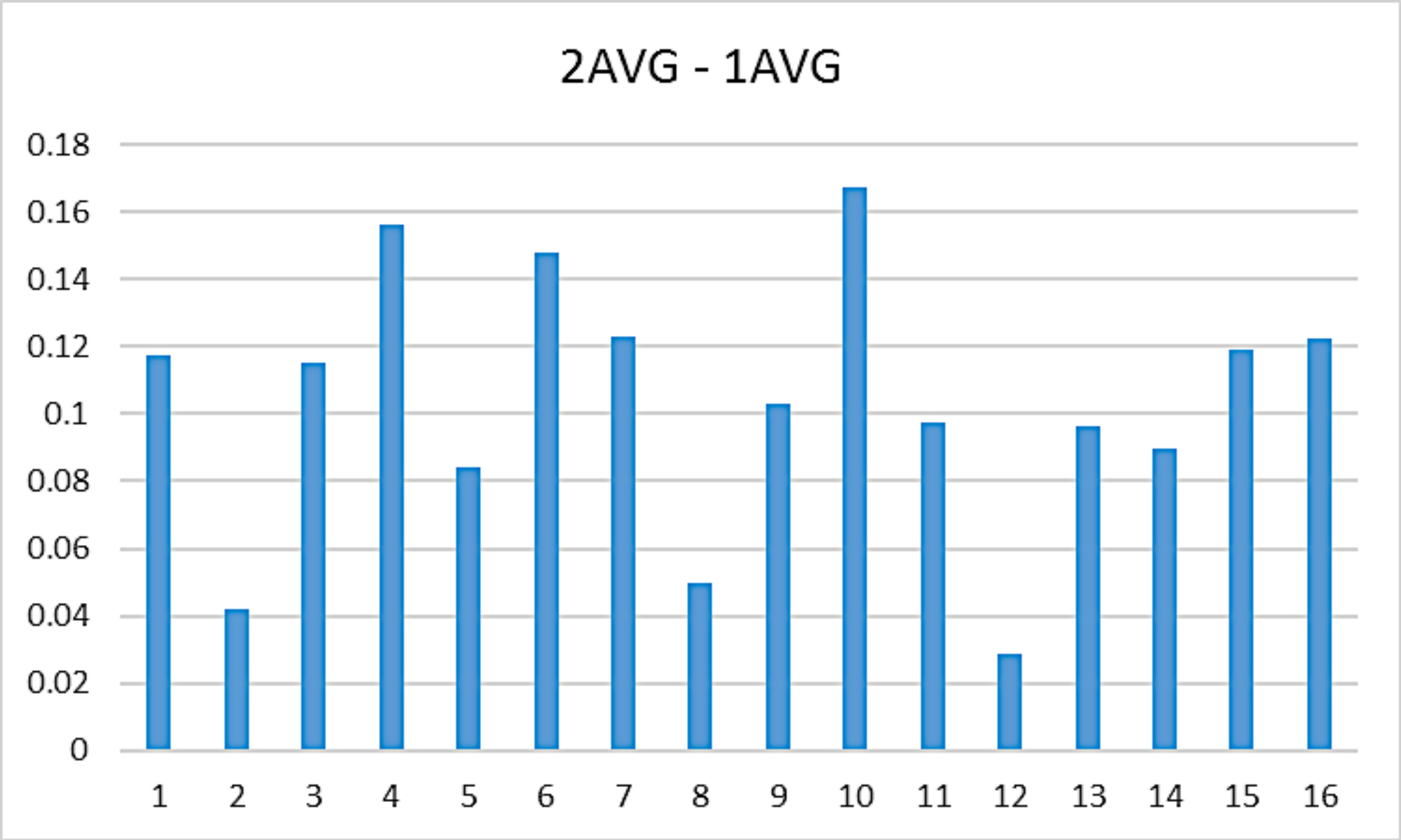
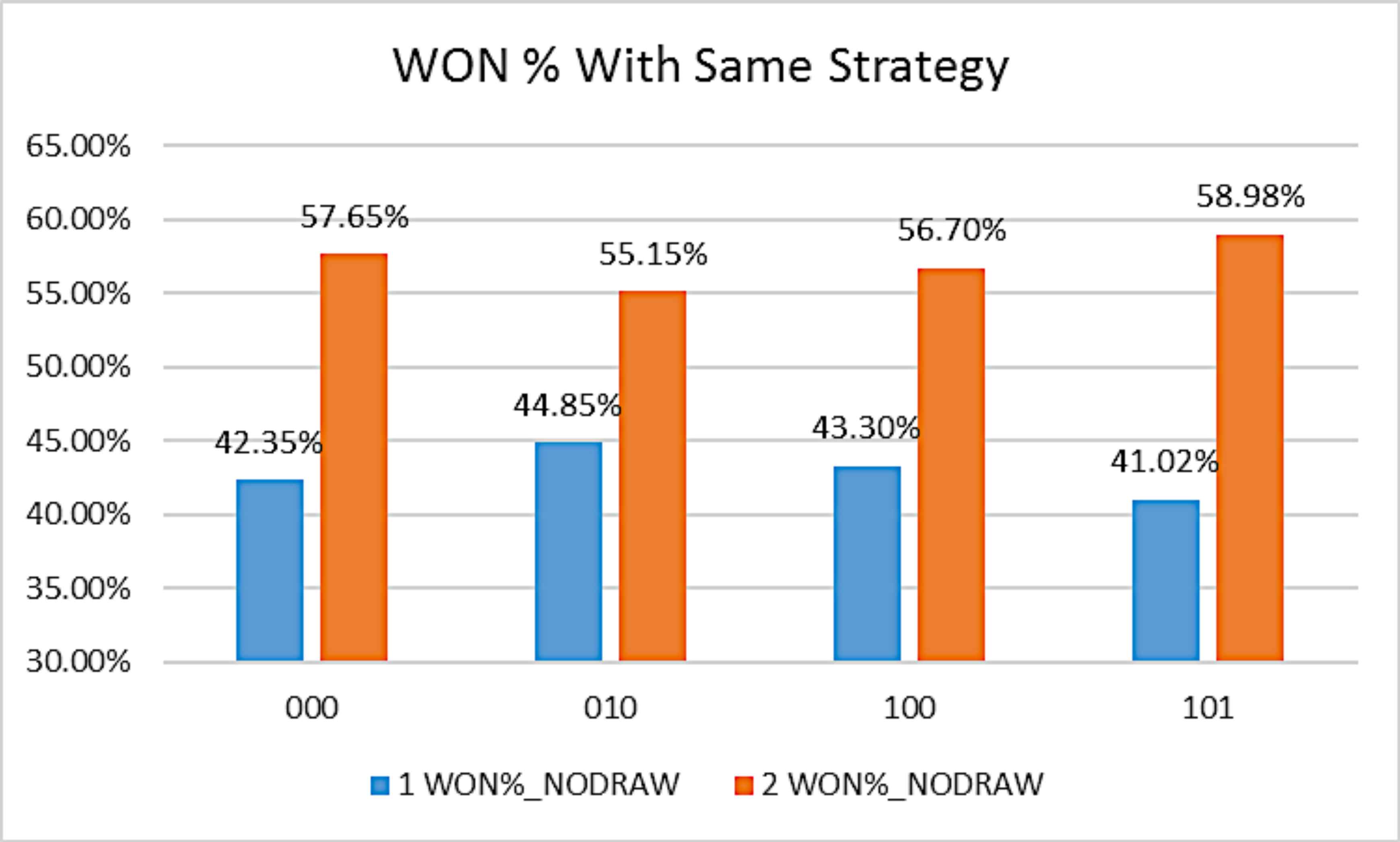
Claim 1

The second player has advantage in this game.

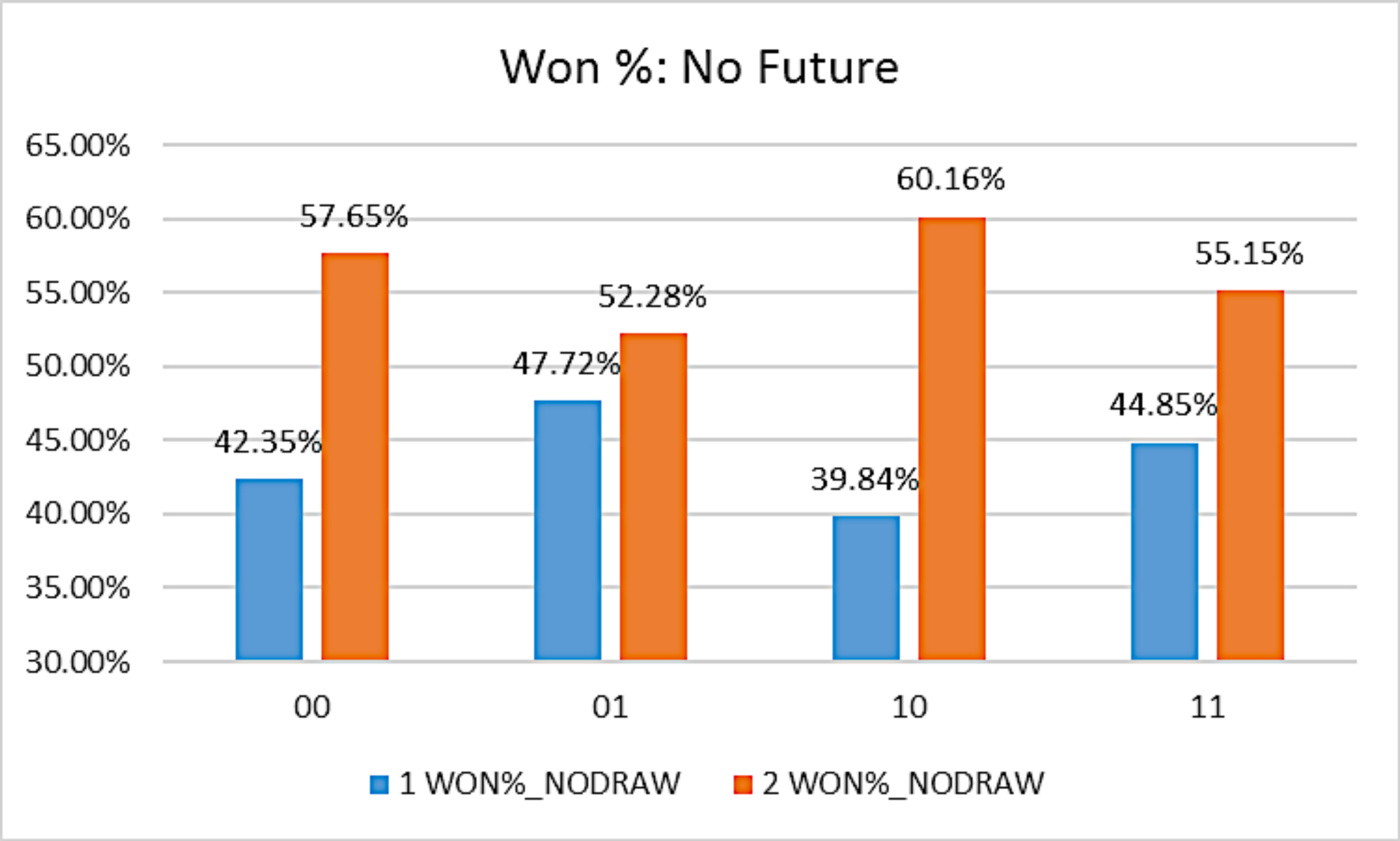

WON% Comparison
65.00%
60.00%
55.00%
50.00%
45.00%
40.00%
35.00%
30.00%
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 WON%_NODRAW
2 WON%_NODRAW

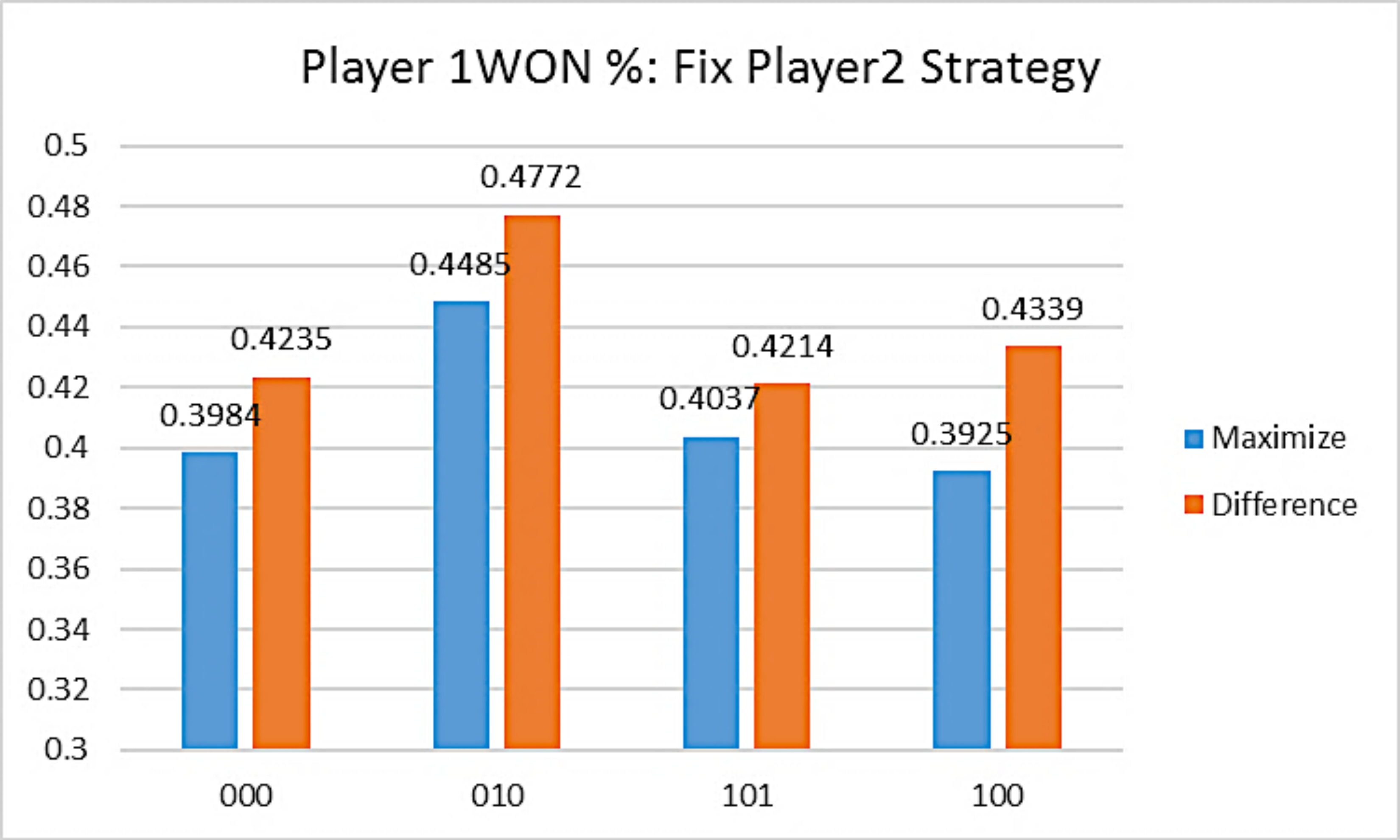

2AVG - 1AVG
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16


WON % With Same Strategy
65.00%
60.00%
55.00%
50.00%
45.00%
40.00%
35.00%
30.00%
57.65%
55.15%
56.70%
58.98%
42.35%
44.85%
43.30%
41.02%
000
010
100
101
1 WON%_NODRAW
2 WON%_NODRAW

Claim 2

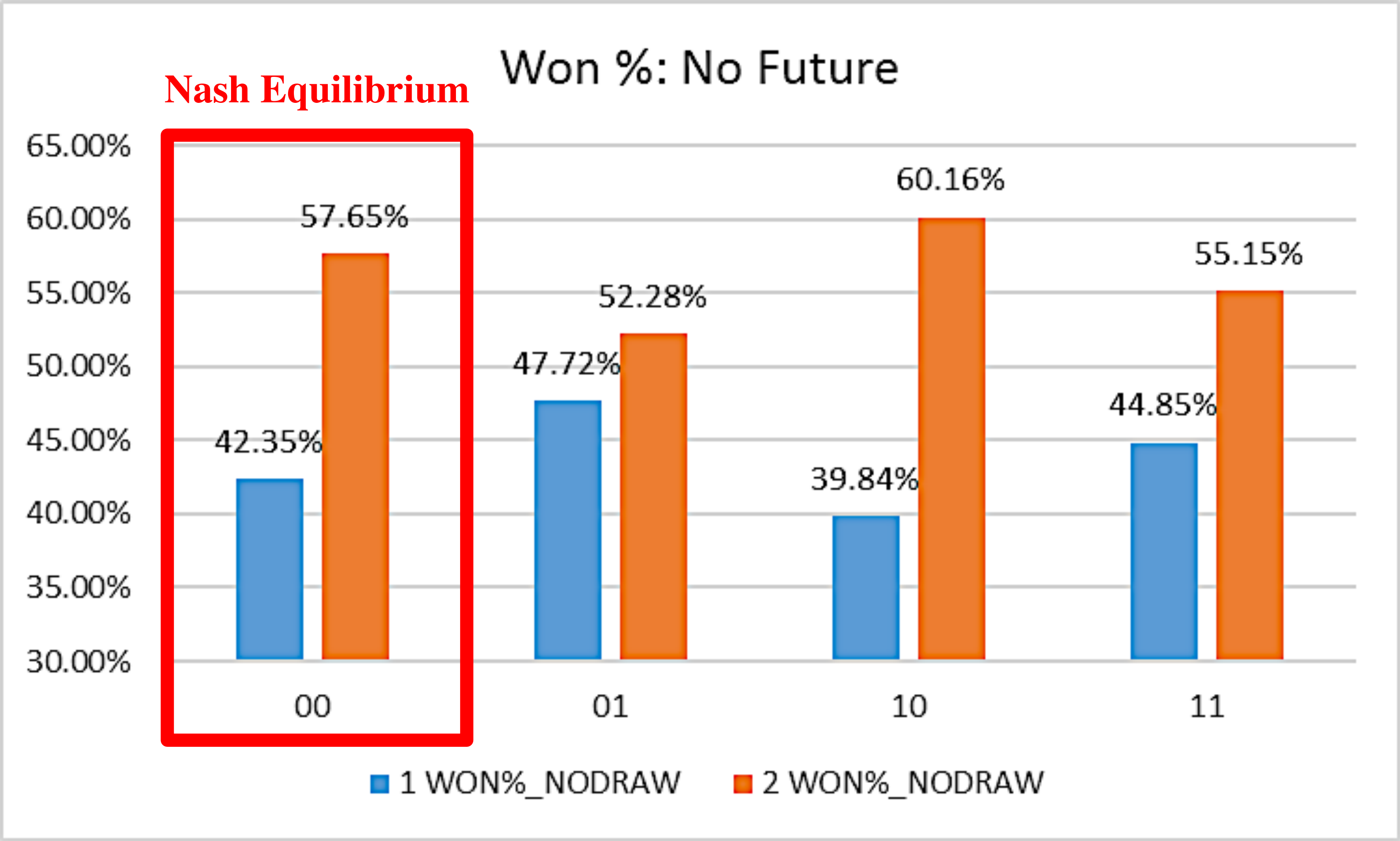
“Difference Strategy” is better than “Maximize Strategy”


Won %: No Future
65.00%
60.00%
55.00%
50.00%
45.00%
40.00%
35.00%
30.00%
57.65%
42.35%
47.72%
52.28%
39.84%
60.16%
44.85%
55.15%
00
01
10
11
1 WON%_NODRAW
2 WON%_NODRAW


Player 1WON %: Fix Player2 Strategy
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.3984
0.4235
0.4485
0.4772
0.4037
0.4214
0.3925
0.4339
000
010
101
100
Maximize
Difference

Claim 3

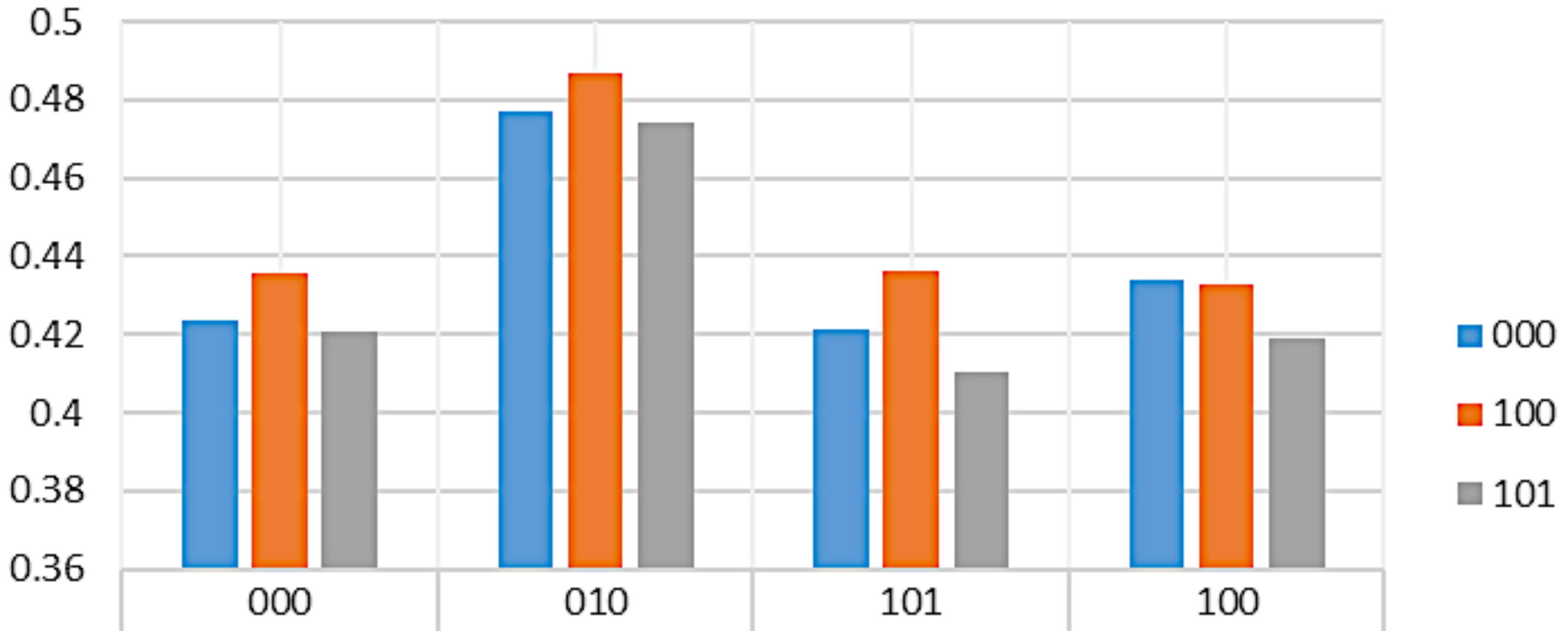
Both players tend to adopt “Difference Strategy” (w/o future)


Won %: No Future
Nash Equilibrium
65.00%
60.00%
55.00%
50.00%
45.00%
40.00%
35.00%
30.00%
57.65%
42.35%
52.28%
47.72%
60.16%
39.84%
55.15%
44.85%
00
01
10
11
1 WON%_NODRAW
2 WON%_NODRAW

Claim 4

“Consider Future” is often better than “Not Consider Future” when player assume opponent adopts “Difference Strategy”

Player 1 Won %: Consider Future



	000	010	101	100
000	0.4235	0.4772	0.4214	0.4339
100	0.4354	0.4867	0.4363	0.433
101	0.4206	0.4741	0.4102	0.4193

Conclusions

- Throwing the pieces satisfies a normal distribution.
  (There’s no enough evidence to show that it is not N.D.)
- This game is not fair: player 2 has advantage.
- Adopting a better strategy can result in higher winning rate.
- This game has a Nash equivalent strategy (w/o future).

Future Work

- Research on different winning conditions and rules.
- Parallelize the simulating process.
- How to solve the imbalance between player 1 and 2?

Thank you!