Assignment 4

Price a lookback put with the binomial tree model. The payoff function of the lookback put is as follows.

$$\text{Payoff}_\tau = \max(S_{\max,\tau} - S_\tau, 0), \text{ where } S_{\max,\tau} = \max S_u, \text{ for } u = 0, \Delta t, 2\Delta t, ..., \tau.$$

- Basic requirement (80 points):

  (i) Implement the binomial tree model to price both European and American lookback puts.

  (ii) Implement the Monte Carlo simulation to price European lookback puts.

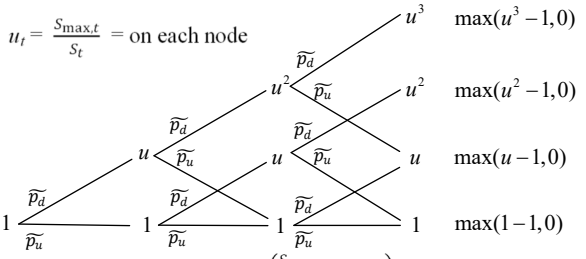
  (Inputs: S_t, r, q, σ, t, T, $S_{\max,t}$, n, number of simulations, number of repetitions. Outputs: Option values for both methods and 95% confidence level for Monte Carlo simulation.)

- Bonus 1 (5 points):

  Based on the same binomial tree framework, devise and implement a quick approach to determine the $S_{\max}$ list for each node and implement your approach to price lookback puts.

- Bonus 2 (10 points):

  Implement the method in Cheuk and Vorst (1997) to price European and American lookback puts.



$$\max(S_{\max,t} - S_t, 0) = S_t \max\left(\frac{S_{\max,t}}{S_t} - 1, 0\right) = S_t \max(u_t - 1, 0)$$

$$\text{EP} = S_0\left[\tilde{E}[e^{-rT}\max(u_T - 1, 0)]\right] \text{ or AP} = S_0\left[\tilde{E}[e^{-r\tau}\max(u_\tau - 1, 0)]\right]$$

※ Note that $\widetilde{p_u}$ and $\widetilde{p_d}$ are not exactly to be the branching probabilities p and $1-p$ in the CRR binomial tree, respectively.

- Reference

Cheuk and Vorst (1997), "Currency lookback options and observation frequency: a binomial approach," *Journal of International Money and Finance* 16, pp. 173–187.