數值方法 hw2

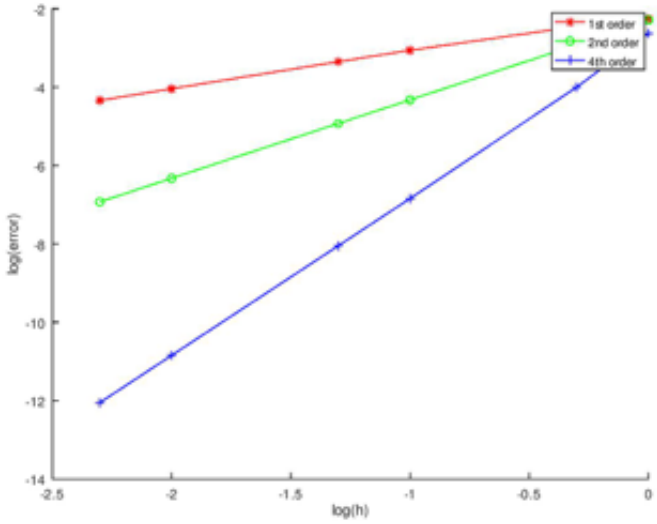
A.2

```
h = [1, 0.5, 0.1, 0.05, 0.01, 0.005]
function f=f(H)
  f = sin((4+H))/(4+H)^3;
endfunction
grad = (cos(4)*4^3 - 3 * sin(4)*4^2)/4^6
grad_1H = []; grad_2H = []; grad_4H = [];
% forward
for i = 1:size(h)(2)
  grad_1H = [grad_1H, (f(h(i))-f(0)) / h(i)];
endfor
% second-order
for i = 1:size(h)(2)
  grad_2H = [grad_2H, (f(h(i))-f(-h(i)))/(h(i)*2)];
endfor
% fourth-order central
for i = 1:size(h)(2)
  grad_4H = [grad_4H, (-f(h(i)*2)+8*f(h(i))-8*f(-h(i))+f(-h(i)*2))/(12*
endfor
error1h = log10(abs(grad - grad_1H))
error2h = log10(abs(grad - grad_2H))
error4h = log10(abs(grad - grad_4H))
x = log10(h)
% plotting part
hold on
plot(x, error1h, "r-*")
plot(x, error2h, "g-o")
plot(x, error4h, "b-+")
hold off
xlabel("log(h)");
ylabel("log(error)");
legend("1st order","2nd order","4th order");
legend('Location','southeast');
print("-djpeg",["A_2",'.jpg'])
```

	First-order f′(xj)	log(abs(error))
@h=1.000	0.0041536	-2.2598
@h=0.500	0.0021954	-2.451
@h=0.100	-0.00047643	-3.0615
@h=0.050	-0.00089912	-3.3514
@h=0.010	-0.0012535	-4.0414
@h=0.005	-0.0012988	-4.3413

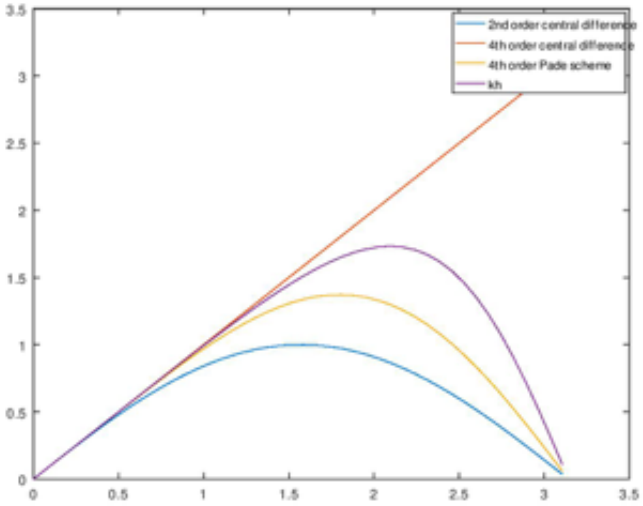
	Second-order f′(xj)	log(abs(error))
@h=1.000	-0.006449	-2.292
@h=0.500	-0.0025458	-2.9203
@h=0.100	-0.0013916	-4.3262
@h=0.050	-0.0013562	-4.9285
@h=0.010	-0.0013449	-6.3266
@h=0.005	-0.0013445	-6.9286

	Fourth-order f′(xj)	log(abs(error))
@h=1.000	0.00098094	-2.6335
@h=0.500	-0.0012448	-4.0015
@h=0.100	-0.0013443	-6.8434
@h=0.050	-0.0013444	-8.0489
@h=0.010	-0.0013444	-10.845
@h=0.005	-0.0013444	-12.049



B.2

```
L = 13
N = 87
h = L / N
n = [0 : 1 : N/2]
k = 2*pi / L*n
kh = 2*pi / N*n
kh_second_order = sin(kh)
kh_fourth_order = (-sin(4*pi*n/N)+8*sin(2*pi*n/N))/6
kh_Pade_scheme = 3*sin(kh)./(2+cos(kh))
% plotting part
kh_true = kh
plot(kh,kh_second_order)
hold on
plot(kh, kh_true)
plot(kh, kh_fourth_order)
plot(kh, kh_Pade_scheme)
hold off
xlabel('kh');
ylabel('k´');
legend('2nd order central difference', '4th order central difference',
'4th order Pade scheme', 'kh');
print("-djpeg",["B_2",'.jpg'])
```



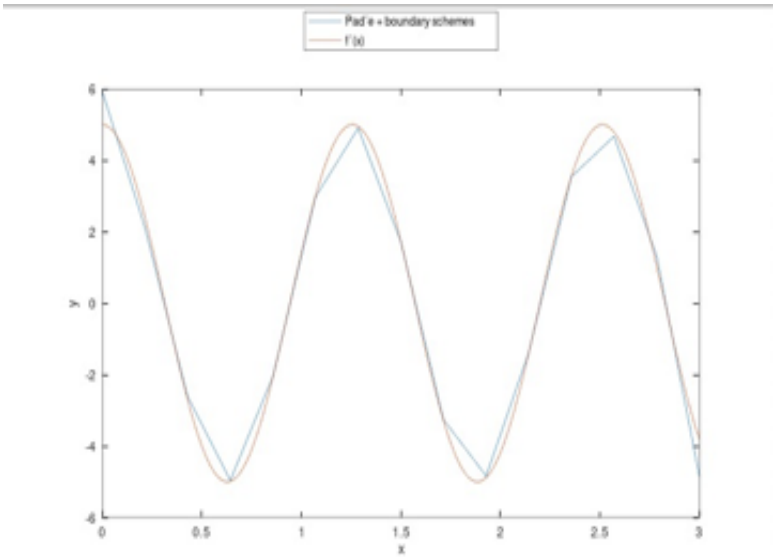
C.2

```
shape = 15
matc=zeros(shape,shape)
matans=zeros(shape,shape)
matc(1,1)=1
matc(1,2)=2
for i=2:shape-1
for j=1:shape
if(j==i-1)
matc(i,j)=1
elseif(j==i)
matc(i,j)=4
elseif(j==i+1)
matc(i,j)=1
endif
end
end
matc(shape,shape-1)=2
matc(shape,shape)=1
matans(1,1)=-5/2
matans(1,2)=2
matans(1,3)=1/2
for i =2:shape-1
for j=1:shape
if(j==i+1)
matans(i,j)=3
elseif(j==i-1)
matans(i,j)=-3
endif
end
end
```

```
matans(shape,shape)=5/2
matans(shape,shape-1)=-2
matans(shape,shape-2)=-1/2
x=linspace(0,3,shape)
x=x(1:shape)
mat_sin=linspace(0,3,shape)
mat_sin=mat_sin(1:shape)
mat_sin=sin(5*mat_sin)
matans=mat_sin.*matans
mat_ans=matc\matans
mat_sum=sum(mat_ans,2)/(x(2)-x(1))
x_real=[0:0.01:3]
mat_realsin=5*cos(5*x_real)
plot(x,mat_sum)
hold
plot(x_real,mat_realsin)
xlabel('x');
ylabel('y');
legend('Pad´e + boundary schemes','f´(x)');
legend('Location','northoutside');
print("-djpeg",["C_2",'.jpg'])
```

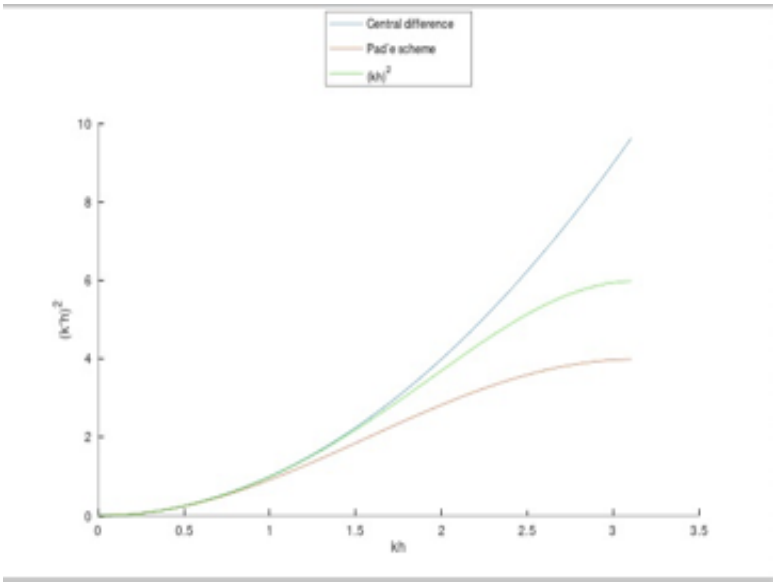
f′(x0), f′(x1), …, f′(x14) are

	1
1	5.9252
2	2.1151
3	-2.6147
4	-4.9636
5	-2.0469
6	2.9785
7	4.9061
8	1.7187
9	-3.2601
10	-4.8391
11	-1.3819
12	3.5432
13	4.6726
14	1.3155
15	-4.8443



D.4

```
L=13
N=87
h=L/N
n=[0:1:N/2]
k=2*pi/L*n
kh=2*pi/N*n
kh_true=kh
kh_D1=(2-2*cos(kh))
kh_D2=(2*cos(kh)-2)./(-1/12*2*cos(kh)-10/12)
plot(kh,kh_true.^2)
hold
plot(kh,kh_D1)
plot(kh,kh_D2,"g")
xlabel('kh');
ylabel('(k´h)^2');
legend('Central difference','Pad´e scheme','(kh)^2');
legend('Location','northoutside');
print("-djpeg",["D_4",'.jpg'])
```



tags: 数値方法