

# Homework 3

**Due: November 18, 2020 in class**

**Note: No late homework will be accepted.** You may discuss with your classmates but **you may not plagiarize.** You need to turn in **your analysis and also your code** (printout) written in Octave or Matlab.

## Part A. (20%)

A.1 Using the error analysis for the trapezoidal and rectangle rules, show that Simpson's rule for integration over the entire interval is fourth-order accurate.

A.2 Explain why the rectangle and trapezoidal rules can integrate a straight line exactly and the Simpson's rule can integrate a cubic exactly.

## Part B. (20%)

Use the Richardson extrapolation to compute  $f'(1.0)$  and  $f'(5.0)$  to five place accuracy with  $f = (x + 0.5)^{-2}$ . Use the central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

and take the initial grid spacing  $h_0 = 0.5$ . The other grid spacings you may choose, for example, are  $h_1 = 0.5h_0$ ,  $h_2 = 0.5h_1$ ,  $h_3 = 0.5h_2$ ,  $h_4 = 0.5h_3$ , etc. Does  $f'(1.0)$  or  $f'(5.0)$  converge faster? Can you explain the reason?

## Part C. (20%)

We would like to calculate

$$I = \int_0^\pi \sin(x) dx.$$

C.1 Develop a quadrature method for  $x \in [x_j, x_{j+2}]$  based on the cubic spline interpolation.

C.2 Use this method to calculate the integral using 4, 8, 16, 32 intervals. Show the absolute values of error versus grid spacing on a log-log plot. What is the order of accuracy of this method?

## Part D. (20%)

Combine Simpson's rule with the trapezoidal rule with end correction to obtain a more accurate method for the integral of

$$\int_{x_j-h}^{x_j+h} f(x) dx.$$

You may use the values of  $f(x_j - h)$ ,  $f(x_j)$ ,  $f(x_j + h)$ ,  $f'(x_j - h)$ ,  $f'(x_j + h)$ . What is the **order of accuracy** of your scheme? What will be the **global scheme** for

$$\int_a^b f(x)dx$$

based on this method?

## Part E. (20%)

We now compute the following integral (which has the exact value as listed) using different methods.

$$\int_1^8 \frac{\log x}{x} dx = 2.1620386$$

E.1 Use Simpson's rule with nine points (eight panels). What is the **value** of the numerical integral and what is the absolute value of the **error**?

E.2 Use Gauss-Legendre quadrature with nine points. What is the **value** of the numerical integral and what is the absolute value of the **error**? (You may use the subroutine `gauss_leg` provided on CEIBA to compute the points and weights in Gauss-Legendre quadrature.)