數值方法 hw4

B.1 B.2

code

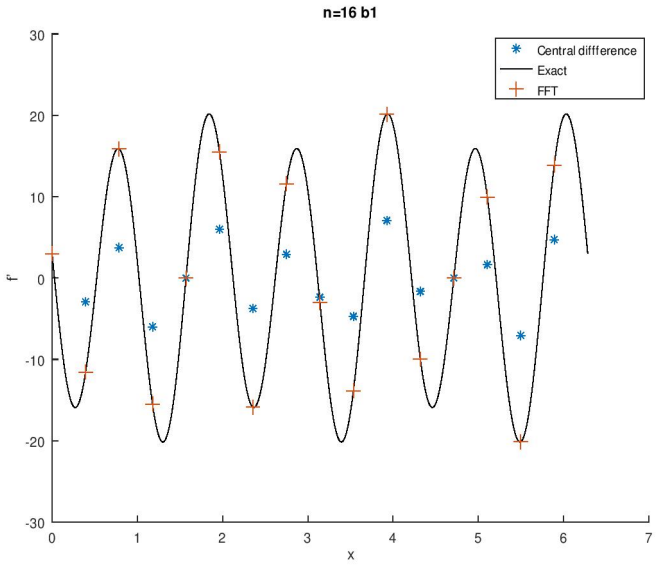
```
clear all
clc
function f=central(x,h)
   h
   L=size(x)(2);
   f=(x(1,3:L)-x(1,1:L-2))/(2*h);
endfunction
function f1=f(x);
  f1=sin(3*x)+3*cos(6*x);
endfunction
function f1=p(x);
  f1=x*6-x.^2;
endfunction
function f1=realdf(x);
  f1=3*cos(3*x)-18*sin(6*x);
endfunction
function f1=realdf2(x);
  f1=6-2*x;
endfunction
function re=remove_boundry(x)
  L=size(x)(2);
  re=x(1,2:L-1);
endfunction
function re=remove_boundry_s(x)
  L=size(x)(2);
  re=x(1,1:L-1);
endfunction
realx=linspace(0,2*pi,1387);

n16=linspace(0,2*pi,17);
n32=linspace(0,2*pi,33);
x16=remove_boundry(n16);
x32=remove_boundry(n32);

yc16=central(f(n16),n16(1,2)-n16(1,1));
yc32=central(f(n32),n32(1,2)-n32(1,1));
```

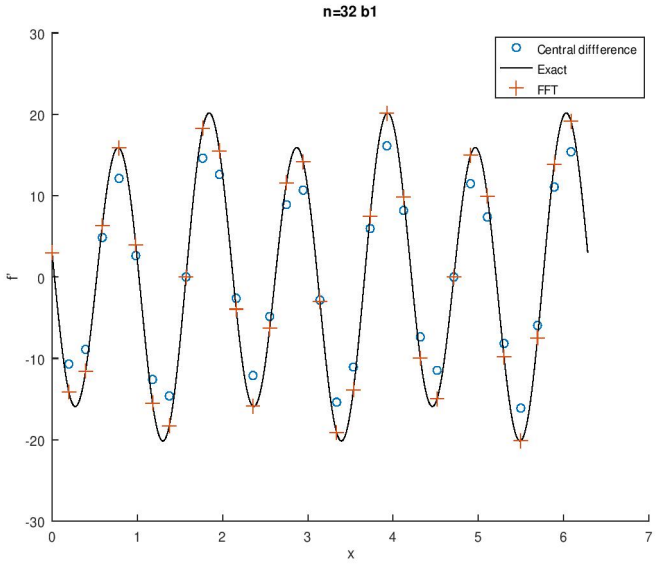
```
fx16=remove_boundry_s(n16);
fx32=remove_boundry_s(n32);
y16=fft(f(fx16));
y32=fft(f(fx32));
yf16=real(ifft((y16.*[[0:1:7],[0],[-7:1:-1]])*1i));
yf32=real(ifft((y32.*[[0:1:15],[0],[-15:1:-1]])*1i));
yreal=realdf(realx);

## B1
figure(1)
hold on
plot(x16,yc16,"*")
plot(realx,yreal,"k")
plot(fx16,yf16,"+","markersize",10)
title("n=16 b1")
legend("Central diffference","Exact","FFT")
legend('Location','northeast')
xlabel("x")
ylabel("f'")
hold off
print("-djpeg",["B1_1",'.jpg'])

figure(2)
hold on
title("n=32 b1")
plot(x32,yc32,"o")
plot(realx,yreal,"k")
plot(fx32,yf32,"+","markersize",10)
legend("Central diffference","Exact","FFT")
legend('Location','northeast')
xlabel("x")
ylabel("f'")
hold off
print("-djpeg",["B1_2",'.jpg'])

## B2
yc16=central(p(n16),n16(1,2)-n16(1,1));
yc32=central(p(n32),n32(1,2)-n32(1,1));
fx16=remove_boundry_s(n16);
fx32=remove_boundry_s(n32);
y16=fft(p(fx16));
y32=fft(p(fx32));
yf16=real(ifft((y16.*[[0:1:7],[0],[-7:1:-1]])*1i));
yf32=real(ifft((y32.*[[0:1:15],[0],[-15:1:-1]])*1i));
yreal=realdf2(realx);

figure(3)
hold on
```

```
plot(x16,yc16,"o")
plot(realx,yreal,"k")
plot(fx16,yf16,"+","markersize",10)
title("n=16 b2")
legend("Central diffference","Exact","FFT")
legend('Location','northeast')
xlabel("x")
ylabel("f'")
hold off
print("-djpeg",["B2_1",'.jpg'])

figure(4)
hold on
plot(x32,yc32,"o")
plot(realx,yreal,"k")
plot(fx32,yf32,"+","markersize",10)
title("n=32 b2")
legend("Central diffference","Exact","FFT")
legend('Location','northeast')
xlabel("x")
ylabel("f'")
hold off
print("-djpeg",["B2_2",'.jpg'])
```

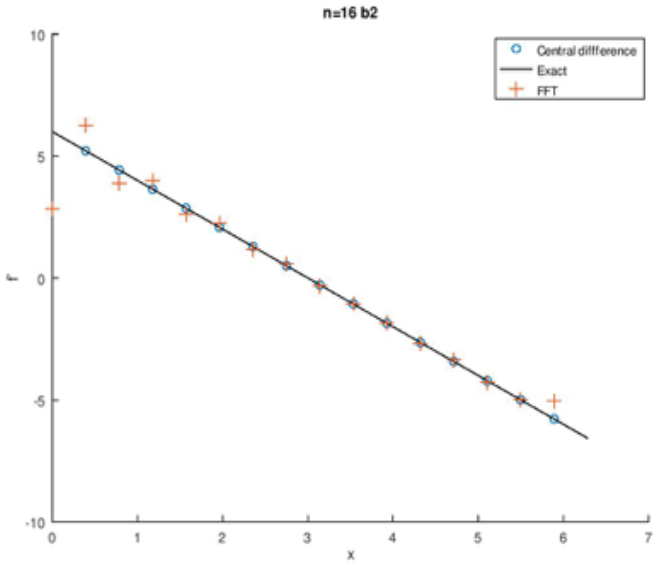
result

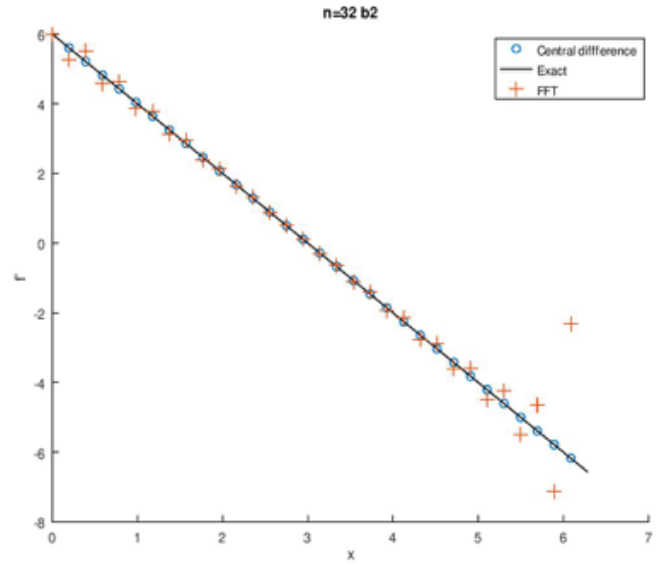
B1





B2





B1 的 f 微分後為週期性函數，FFT is better。

B2 的 f 微分後是線性，central difference has no error, it is better。

C.1

code

```
function f1=f(x);
  f1=sin(2*x)+0.1*sin(15*x);
endfunction
function f1=g(x);
  f1=sin(2*x)+0.1*cos(15*x);
endfunction
function re=remove_boundry_s(x)
  L=size(x)(2);
  re=x(1,1:L-1);
endfunction
axisx=linspace(0,2*pi,1387);
n32=linspace(0,2*pi,33);
x32=remove_boundry_s(n32);
yf=f(x32);
yg=g(x32);
h = yf.*yg
hhat=fft(h)
```

result

```
yf =

 Columns 1 through 11:

   0.00000   0.40219   0.66884   0.97944   0.92929   1.00703   0.61472   0.48076  -0.10000  -0.28460  -0.79949

 Columns 12 through 22:

  -0.84073  -1.07071  -0.86832  -0.74538  -0.36317   0.00000   0.36317   0.74538   0.86832   1.07071   0.84073

 Columns 23 through 32:

   0.79949   0.28460   0.10000  -0.48076  -0.61472  -1.00703  -0.92929  -0.97944  -0.66884  -0.40219

yg =

 Columns 1 through 9:

   1.0000e-01   2.8460e-01   7.9949e-01   8.4073e-01   1.0707e+00   8.6832e-01   7.4538e-01   3.6317e-01  -1.4702e-16

 Columns 10 through 18:

  -3.6317e-01  -7.4538e-01  -8.6832e-01  -1.0707e+00  -8.4073e-01  -7.9949e-01  -2.8460e-01  -1.0000e-01   4.8076e-01

 Columns 19 through 27:

   6.1472e-01   1.0070e+00   9.2929e-01   9.7944e-01   6.6884e-01   4.0219e-01   4.6530e-16  -4.0219e-01  -6.6884e-01

 Columns 28 through 32:

  -9.7944e-01  -9.2929e-01  -1.0070e+00  -6.1472e-01  -4.8076e-01
```

```
h =
 Columns 1 through 12:
   0.00000   0.11447   0.53473   0.82344   0.99500   0.87442   0.45820   0.17460   0.00000   0.10336   0.59592   0.73003
 Columns 13 through 24:
   1.14642   0.73003   0.59592   0.10336  -0.00000   0.17460   0.45820   0.87442   0.99500   0.82344   0.53473   0.11447
 Columns 25 through 32:
   0.00000   0.19336   0.41115   0.98632   0.86358   0.98632   0.41115   0.19336
hhat =
 Columns 1 through 5:
   16.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.08000i    0.00000 -  0.00000i   -8.00000 +  0.00000i
 Columns 6 through 10:
   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i
 Columns 11 through 15:
   -0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i    0.80000 +  0.80000i   -0.00000 +  0.00000i
 Columns 16 through 20:
   -0.80000 +  0.80000i    0.00000 +  0.00000i   -0.80000 -  0.80000i   -0.00000 -  0.00000i    0.80000 -  0.80000i
 Columns 21 through 25:
   -0.00000 +  0.00000i    0.00000 -  0.00000i   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i
 Columns 26 through 30:
   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i   -8.00000 -  0.00000i    0.00000 +  0.00000i
 Columns 31 and 32:
   -0.00000 -  0.08000i    0.00000 -  0.00000i
```

C.2

```
function f1=f(x);
  f1=sin(2*x)+0.1*sin(15*x);
endfunction
function f1=g(x);
  f1=sin(2*x)+0.1*cos(15*x);
endfunction
function re=remove_boundry_s(x)
  L=size(x)(2);
  re=x(1,1:L-1);
endfunction
axisx=linspace(0,2*pi,1387);
n32=linspace(0,2*pi,33);
x32=remove_boundry_s(n32);
yf=f(x32);
yg=g(x32);
fk=fft(yf);
gk=fft(yg);
fkAsc=[fk,fk](1,17:48);
gkAsc=[gk,gk](1,17:48);
Hk = zeros(1,32);
for k = -16:15
for m = -16:15
if(k-m>=-16 && k-m <= 15)
Hk(k+17) = Hk(k+17)+fkAsc(m+17)*gkAsc(k-m+17);
end
end
end
```

result

```
fkAsc =

 Columns 1 through 5:

    0.00000 +  0.00000i   -0.00000 +  1.60000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 6 through 10:

   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 11 through 15:

    0.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 + 16.00000i

 Columns 16 through 20:

   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 - 16.00000i    0.00000 -  0.00000i

 Columns 21 through 25:

    0.00000 -  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 26 through 30:

    0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 31 and 32:

   -0.00000 +  0.00000i   -0.00000 -  1.60000i

gkAsc =

 Columns 1 through 5:

    0.00000 +  0.00000i    1.60000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 6 through 10:

   -0.00000 -  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 11 through 15:

    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 + 16.00000i

 Columns 16 through 20:

   -0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i   -0.00000 - 16.00000i    0.00000 -  0.00000i

 Columns 21 through 25:

   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i    0.00000 -  0.00000i

 Columns 26 through 30:

    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 +  0.00000i    0.00000 -  0.00000i    0.00000 +  0.00000i

 Columns 31 and 32:

   -0.00000 +  0.00000i    1.60000 -  0.00000i

Hk =

 Columns 1 through 6:

    0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   25.60000 - 25.60000i   -0.00000 +  0.00000i    0.00000 -  0.00000i

 Columns 7 through 12:

   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i

 Columns 13 through 18:

 -256.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 -  0.00000i  512.00000 +  0.00000i    0.00000 +  0.00000i

 Columns 19 through 24:

   -0.00000 +  0.00000i    0.00000 -  0.00000i -256.00000 +  0.00000i   -0.00000 -  0.00000i    0.00000 +  0.00000i   -0.00000 +  0.00000i

 Columns 25 through 30:

   -0.00000 -  0.00000i    0.00000 -  0.00000i   -0.00000 +  0.00000i    0.00000 +  0.00000i   -0.00000 -  0.00000i   25.60000 + 25.60000i

 Columns 31 and 32:

   -0.00000 -  0.00000i    0.00000 -  0.00000i
```

- Ek = Hk, from results of C1 we find Hk is not the original function, hence Ek does not represent E(x) correctly

- Ek and Hk has value at k=±2,±5, which can represent the real function, but the function we get does not. We can find that these are aliasing error.

- When N=32, E(x) cannot represent the about 30th 17th term. While hk can represent, hence, hk is most correct.

D.1 D.2

code

```
clear all
clc

function re=u(x)
  re=4*(x.^2-x.^4).*exp(-x/2)
endfunction
function re=firstd(x)
  re=4*(exp(-x/2).*(2*x-4*x.^3)-1/2*exp(-x/2).*(x.^2-x.^4))
endfunction
function re=secd(x)
  re=-47*exp(-x/2).*x.^2   -exp(-x/2).*x.^4+16*exp(-x/2).*x.^3-8*exp(-x
endfunction
d=[]
n=8
x=[0:1:n-1]
x=cos(x/(n-1)*pi)
#x=[1,0.809,0.309,-0.309,-0.809,-1]
N=n-1
cj=1
ck=1
for j =1:n
  cj=1
  if(j==1 || j==n)
    cj=2
  endif
  for k =1:n
    ck=1

    if(k==1 || k==n)
      ck=2
    endif
    if(j!=k)
      d(j,k)=cj*(-1)^(j+k-2)/(ck*(x(1,j)-x(1,k)))
    elseif (j==k && j==1)
      d(j,k)=(2*N^2+1)/6
```

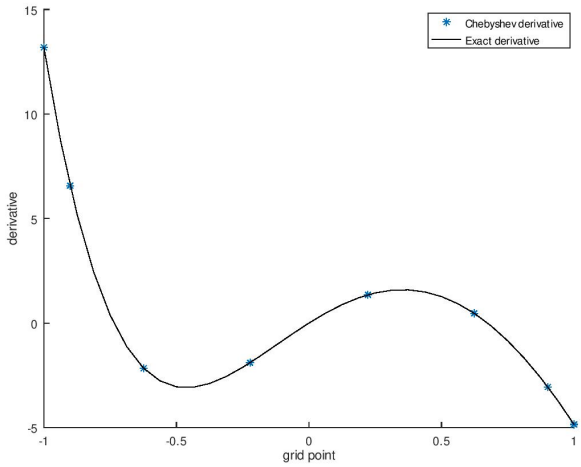
```
    elseif (j==k && j==n)
      d(j,k)=-(2*N^2+1)/6
    else
      d(j,k)=-x(1,j)/(2*(1-x(1,j)^2))
    endif
  end
end
u1=u(x)
t1=rot90(u1,-1)
yd=d*rot90(u1,-1)

figure(1)
hold on
plot(x,yd,"*")
xreal=linspace(-1,1,33)
fd=firstd(xreal)
plot(xreal,fd,"k")
legend("Exact derivative","Chebyshev derivative")
legend('Location', "northeast")
xlabel("grid point");
ylabel("derivative");
hold off
print("-djpeg",["D1_5",'.jpg'])

D = d*d
figure(2)
ydd=d*(d*t1)
hold on
plot(x,ydd,"*")
plot(xreal,secd(xreal))
legend("Exact derivative","Chebyshev derivative")
legend('Location', "northeast")
xlabel("grid point");
ylabel("derivative");
hold off
print("-djpeg",["D2_5",'.jpg'])
```

result

- D1

```
x =

   1.00000   0.90097   0.62349   0.22252  -0.22252  -0.62349  -0.90097  -1.00000

u1 =

   0.00000   0.38957   0.69591   0.16843   0.21041   1.29818   0.95911   0.00000
```

```
d =                                                                                 yd =

   16.50000  -20.19567    5.31194   -2.57242    1.63596   -1.23191    1.05210   -0.50000      -4.85015
    5.04892   -2.39295   -3.60388    1.47395   -0.89008    0.65597   -0.55496    0.26302      -3.05993
   -1.32799    3.60388   -0.51000   -2.49396    1.18202   -0.80194    0.65597   -0.30798       0.46577
    0.64310   -1.47395    2.49396   -0.11706   -2.24698    1.18202   -0.89008    0.40899       1.34965
   -0.40899    0.89008   -1.18202    2.24698    0.11706   -2.49396    1.47395   -0.64310      -1.89666
    0.30798   -0.65597    0.80194   -1.18202    2.49396    0.51000   -3.60388    1.32799      -2.16625
   -0.26302    0.55496   -0.65597    0.89008   -1.47395    3.60388    2.39295   -5.04892       6.57305
    0.50000   -1.05210    1.23191   -1.63596    2.57242   -5.31194   20.19567  -16.50000      13.18722
```



- D2

```
x =

   1.00000   0.90097   0.62349   0.22252  -0.22252  -0.62349  -0.90097  -1.00000

u1 =

   0.00000   0.38957   0.69591   0.16843   0.21041   1.29818   0.95911   0.00000
```

```
D =                                                                                 ydd =

  160.00000 -258.59203  147.07734  -78.27242   51.31043  -39.13555   33.61223  -16.00000     -19.3404
   77.80268 -113.20778   43.22360  -11.39925    5.84434   -4.00000    3.27193   -1.53553     -16.7671
   -5.69963   22.29986  -28.85180   14.98352   -4.00000    2.10419   -1.52969    0.69354      -8.4445
    1.50377   -4.00000   11.85580  -17.94043   10.62388   -3.07106    1.79288   -0.76484       3.6388
   -0.76484    1.79288   -3.07106   10.62388  -17.94043   11.85580   -4.00000    1.50377       8.1303
    0.69354   -1.52969    2.10419   -4.00000   14.98352  -28.85180   22.29986   -5.69963     -12.7193
   -1.53553    3.27193   -4.00000    5.84434  -11.39925   43.22360 -113.20778   77.80268     -55.3902
  -16.00000   33.61223  -39.13555   51.31043  -78.27242  147.07734 -258.59203  160.00000     -79.0538
```