抽樣、離散型
連續型、抽樣

機率與統計小考，2020/06/12, 14:00~15:00

- 【禁止】:【手機】以及【任何形式的通訊】。可以翻書以及使用 Jupyter and Python
- 需要用 python 計算的題目，必須寫出 python 之指令。
- 下課之後，請把本試題當作作業，於 2020/06/12, 23:59 以前，以 pdf 檔案或 ipynb 於 github 之連結，上傳 Microsoft Team, 機率與統計之教學團隊之【作業】之中。

[1]

若 $Z_0, Z_1, Z_2, \ldots, Z_n$ 為彼此獨立的標準型常態分佈之隨機變數，請說明以下統計量(Statistics)為何種機率分布(要標示機率分布之內部參數值)，並寫出其機率密度函數。

(a) $Z_0 + Z_1$ (b) Z_0^2 (c) $Z_1^2 + Z_2^2$ (d) Z_0/Z_1 (e) $\frac{Z_0}{\sqrt{(Z_1^2+Z_2^2)/2}}$ (f) $\frac{Z_0^2}{(Z_1^2+Z_2^2)/2}$

二項分布 抽樣分布 柯西 卡方 T分布 F分布

[2]

Follow [1]，計算以下機率值 (需寫出所使用之 python 指令；若為查表，需寫出所使用之表格之名稱或其所在教科書之頁碼)。

(a) $P(Z_0 + Z_1 \leq 1) = ?$ (b) $P(Z_0^2 \leq 1) = ?$ (c) $P(Z_1^2 + Z_2^2 \leq 1) = ?$ (d) $P\left(\frac{Z_0}{Z_1} \leq 1\right) = ?$

(e) $P\left(\frac{Z_0}{\sqrt{(Z_1^2+Z_2^2)/2}} \leq 1\right) = ?$ (f) $P\left(\frac{Z_0^2}{(Z_1^2+Z_2^2)/2} \leq 1\right) = ?$

[3]

The concentration (濃度) of an active ingredient in the output of a chemical reaction is strongly influenced by the catalyst (觸媒) that is used in the reaction. It is traditionally believed that when catalyst *A* is used, the population mean concentration should be $\mu_A = 65\%$. Now, a sample of outputs from 25 independent experiments gives the average concentration of $\overline{x_A} = 64\%$.

(a) When the standard deviation is known to be $\sigma_A = 3\%$. Please calculate $P(\overline{X_A} \leq 64\%) = ?$, under the assumption $\mu_A = 65\%$

(b) When **the standard deviation is unknown.** Then the sample standard deviation was calculated out to be $S_A = 3\%$. Please recalculate $P(\overline{X_A} \leq 64\%) = ?$, under the same assumption $\mu_A = 65\%$

(c) Following (b), *Let* $P(\overline{X_A} \leq x) = 0.05$, please find out x = ??

(d) Following (b), *Let* $P(x_1 \leq \overline{X_A} \leq x_2) = 0.90$, please find out $(x_1, x_2) = ?$