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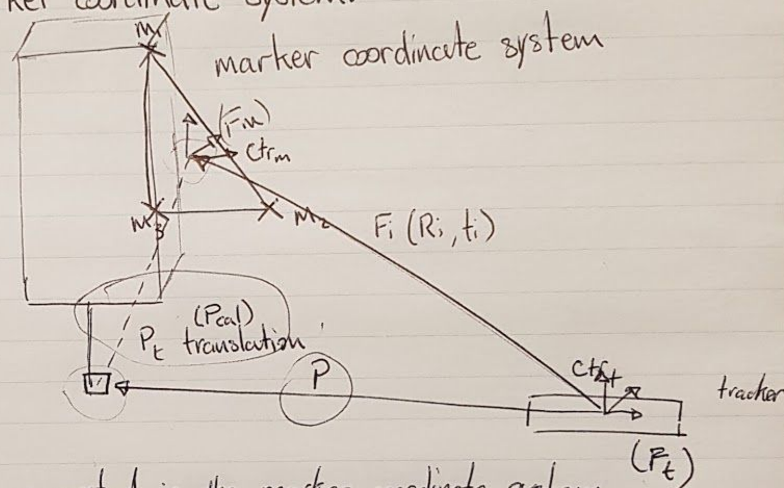
% Assignment 3: Calibration of a Tracked Surgical Drill

Drill Bit Tracking

Since we know T_m (position of the drill tip) and V_m (direction vector of the drill axis) in the F_m (marker) coordinate system

we want to determine P_t translation between the tip and marker coordinate system.



P_{cal} Vector is a constant in the marker coordinate system.
P is the tooltip in the tracker coordinate system
M_1, M_2 and M_3 are the tracker points.

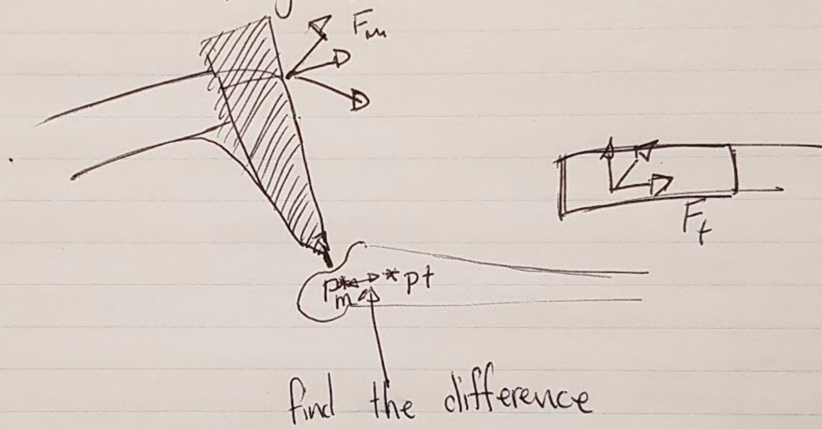
$F_i(R_i, t_i)$ takes the P_{cal} vector to the pivot point P
$F_i * P_{cal} = P$
we want to rotate by R_i, then apply a translation by t_i

$R_i * P_{cal} + t = P$

Knowing this, we can apply R_i and t to T_m and V_m to compute T_t and V_t.

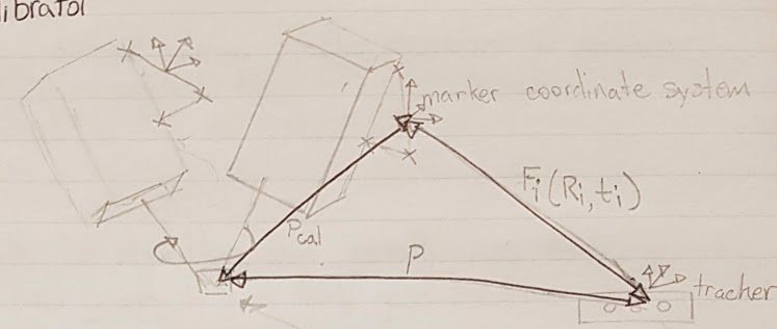
Targeting Error

To determine the targeting error (distance which the drill held in operating position will miss the P_t target point) we must determin P_m (point in marker space) and compare it to P_t (point in tracker space) and find the difference (the translation to get from P_m to P_t).



Drill tip Calibrator

1.



To calibrate the drill tip, the tip is set on a hard surface and becomes the pivot point

The dill is then pivoted such that the markers on the drill go in a spherical shape.

This will result in 3 spheres (m_1, m_2, m_3)

Using the three concentric frames, we can find the average of the three circles which will give us the pivot point in the tracker frame (drill tip)

we then transform the pivot point to the marker frame for each pose $P_{cal_i} = R_i^{-1}(P - t_1)$

? Then we take the average of ($P_{cal_1}, \ldots, P_{cal_n}$) to get P_{cal_i}

Note we need to check and remove outliers after

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Drill tip Calibrator (continued)

2. to detect if the enforcement of the constraint failed (piviotin in one point) we can look for outlier points which are outside the sphere fit. (as all points should be equal distance from the center, m_1, m_2, m_3 should be equal distance from their respective drill point center prior to average center.

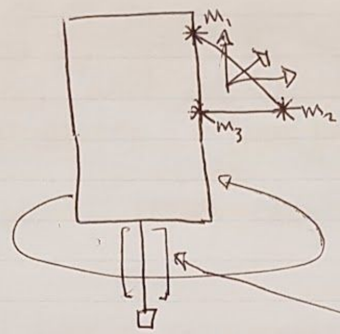
   if a point has a different radius (does not lie on the sphere fit), then it is possible the tip has shifted during the pivoting (calibration step).

3. to detect if there was excessive random error in the tracking of markers, we would see a variance of radii (markers in and above the sphere fit)

   we could set a threshold of variance which is allowed between all (m_1 .... m_i) marker's radius.

Drill axis calibrator

1.



to calibrate the drill axis, the drill is clamped down such that the drill is limited to 2 ~~[illegible]~~ freedoms of rotation.

We then rotate the drill (markers) which will result in three circles (m_1 m_2 m_3).

we then find the normal by using the points from the markers to define a plane.

we then average out the normals.

we then get the orthonormal coordinate system to get the transformation. We can get the marker axis values (and average them) to get v_m

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Drill axis Calibrator (continued)

2. you can detect if the enforcement restraint (clamping ~~??~~ the drill down to two freedoms of rotation) by looking for marker points (M_1, M_2, M_3) which are outside their circles made from rotating (different radius than other points from the center. This could indicate the drill was not properly clamped down.

3. To detect if there is excessive random error in tracking of the markers we would see a variance in the radii of the points (from the markers) from the center. This would cause points to not fall on the circle created when spinning the drill. We could set a threshold of varience, an if the threshold was exceeded we can determine if there was excessive random error.

We can also check to see if the points are off axis of the plane of points generated by spinning the drill, as it should only rotate the points in one elevation as it should only have 2 ~~deo~~ freedoms of rotation.

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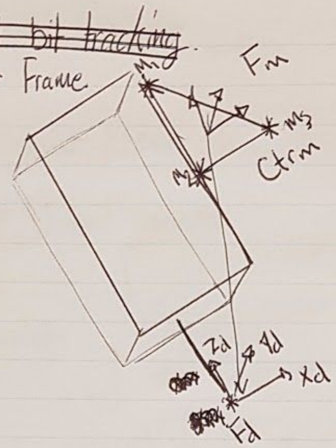
~~Drill bit tracking~~

Compute Marker Frame.



$M_1 = Cd\,(5, 0, 26)$
$M_2 = Ad\,(5, 0, 20)$
$M_3 = Bd\,(11, 0, 20)$

Compute Ground Truth

To compute the ground truths, we find the orthogonal basis of the three points $M_1 = Cd\ (5,0,26)$, $M_2 = Ad(5,0,20)$, $M_3 = Bd\ (11,0,20)$

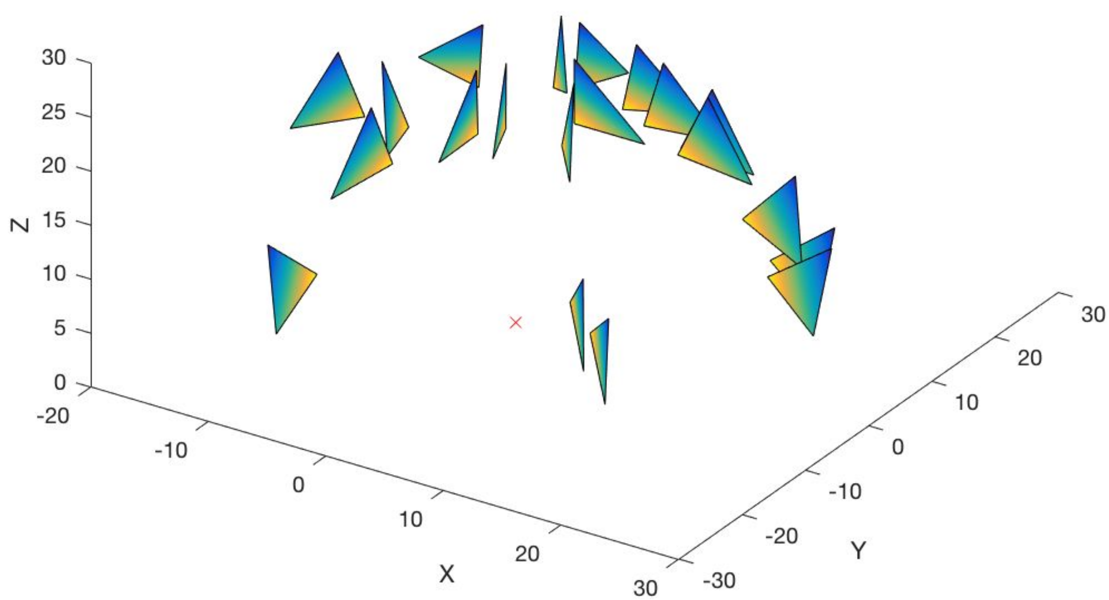
$$\begin{bmatrix} -7 \\ 0 \\ -21 \end{bmatrix}$$ is the orthogonal basis of the three points

Pg

Sudo Code for Drill_Tip_Calibration_Robustness_Test.m

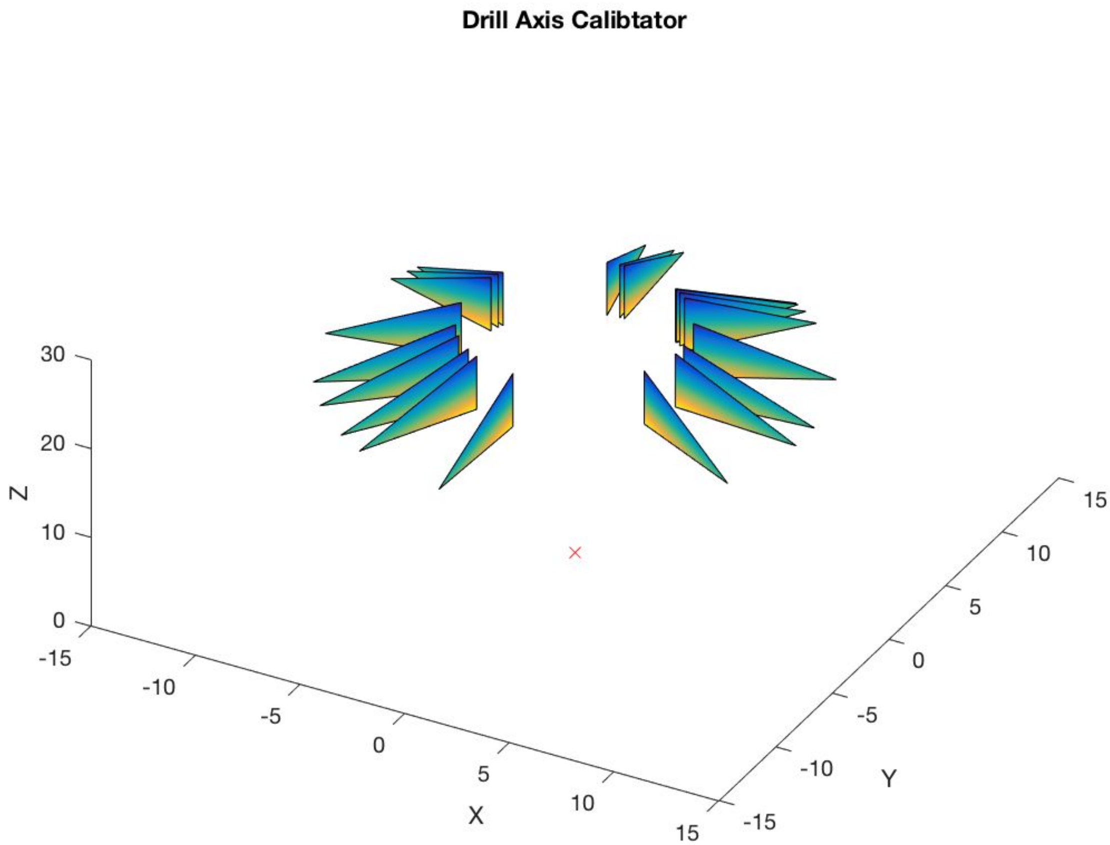
```
We want to run this 20 times

Aa = Ideal points
Ba =
Ca =
% Simulate
[A, B, C] = Drill_Tip_Simulator (n, Aa, Ba, Ca, [0;0;0])

% get ground truth Tm
Tm = Drill_Tip_Calibrator (A, B, C)

for i = 0 : 0.001 : 0.01
           (1 mm)   (Emax)

    % offset each Pose by rand Emax

    % using those points find new Tm

    % norm (new Tm - Tm)

% plot
```

Drill Tip Calibtator



Tm =

```
   -7
    0
  -21
```



Vm =

0.1459
0.0363
0.9886

Angle =

8.6474