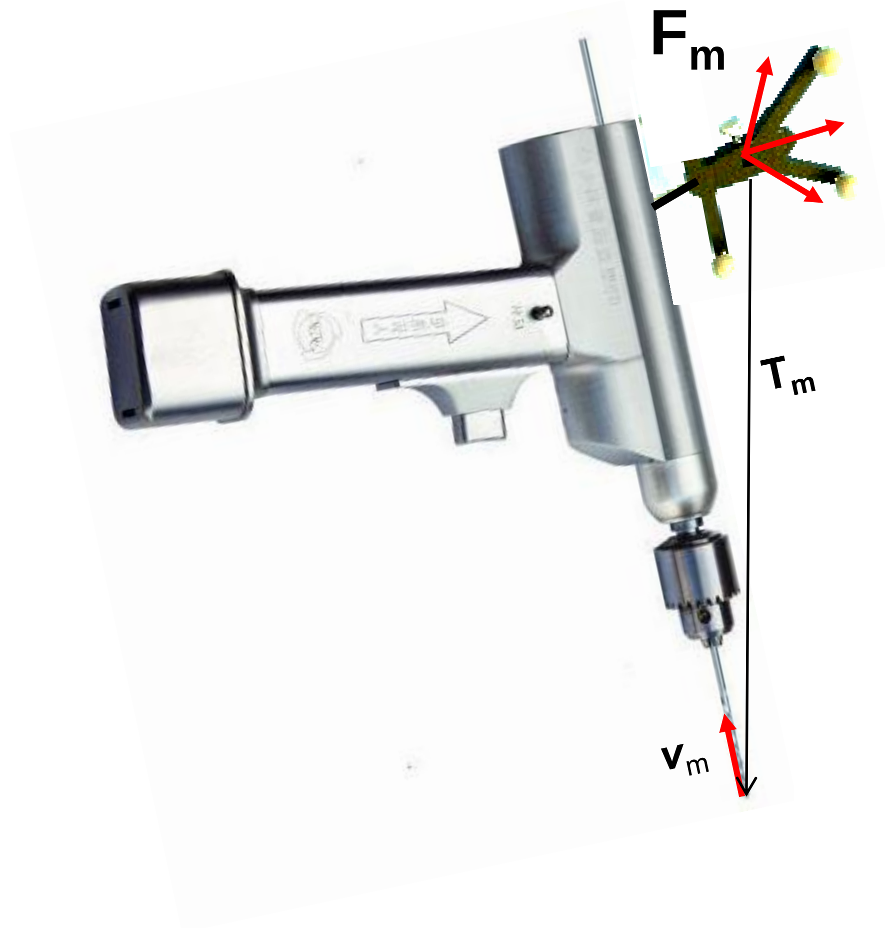
Calibration of a Tracked Surgical Drill

We designed and instrumented a strong electric bone drill with three passive reflective beads for tracking. The beads are tracked with a Polaris optical tracking camera connected to a computer. The tracker can see these reflective markers and report their locations in its own coordinate system. We want to use this drill in bone surgery to create a small pilot hole in a broken tibia for Kirschner wire placement, in conjunction with a surgical navigation system. Unfortunately, when we attach markers, we do not know exactly where the markers are relative to the drill tip and drill axis. So we must calibrate the tracked drill before using it in the operating room. K-wiring is a delicate procedure. The required calibration accuracy is 1 mm for the drill tip and 2 degrees for the drill axis.

Calibration of the tracked drill means the following: First, you assign a coordinate system to the markers (F_m). Second, you experimentally determine the position of the drill tip T_m and the direction vector of the drill axis $\boldsymbol{v}_m$ in the F_m marker coordinate system.

Note that the figure below, the depiction the F_m marker coordinate system is arbitrary. In general, you are free to fix F_m anywhere onto the 3 markers, so long it is a unique orthonormal coordinate system.

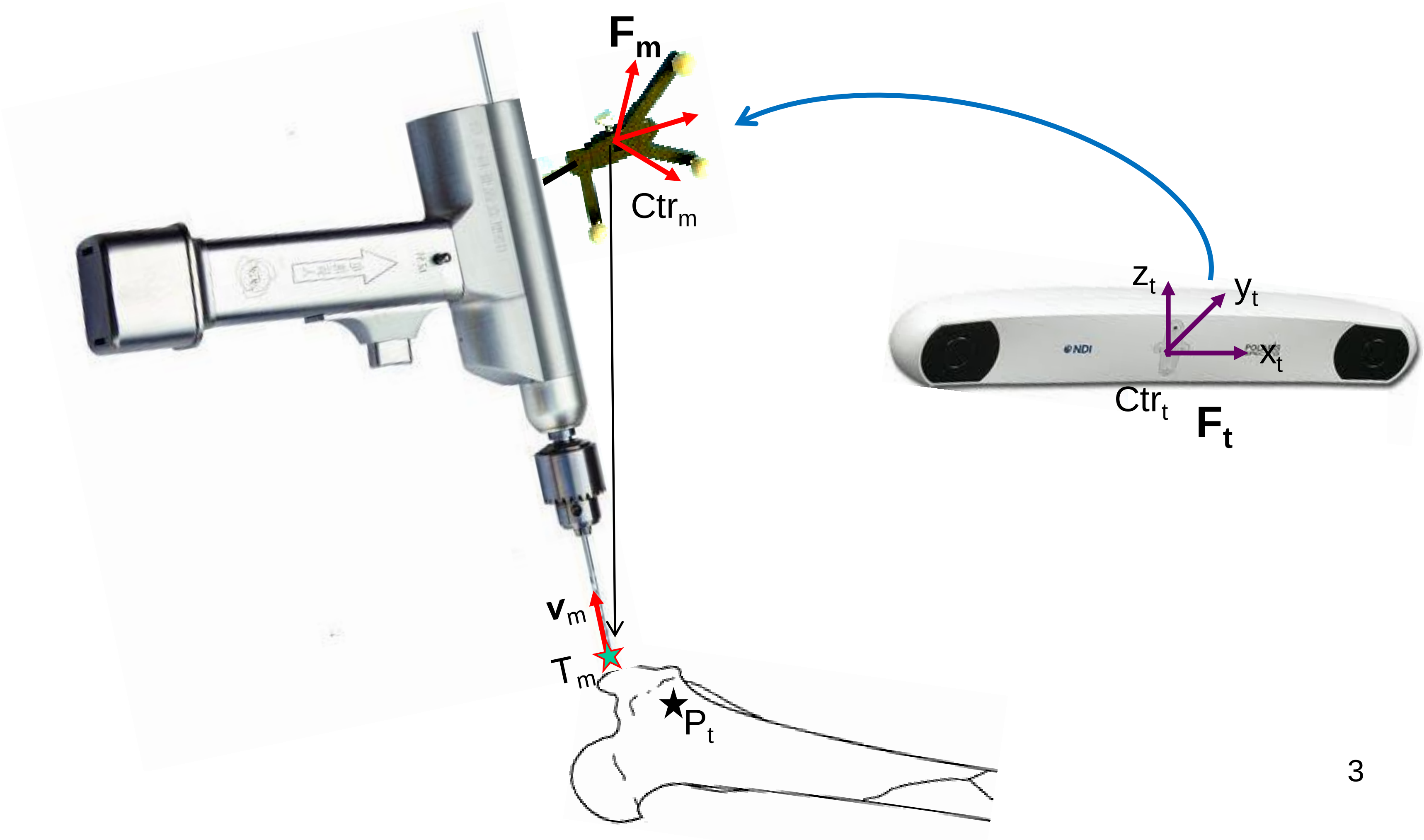


OPERATING ROOM USE

When we use the tracked drill in the operating room, we receive real-time measurements of the location of A_t,B_t,C_t in the F_t tracker coordinate system. Write up the **math** for the following and submit in PDF format:

Drill Bit Tracking: Compute the drill tip (T_t) and drill axis vector ($\boldsymbol{v}_t$) in the F_t tracker coordinate frame, assuming that the drill has been calibrated, i.e. that we know T_m and $\boldsymbol{v}_m$ in the F_m coordinate frame. **5 pts**

Targeting Error: Compute the distance by which the drill held in operating position will miss the P_t target point that we identify in tracker coordinate space. **5 pts**



CALIBRATION

We bring the drill to the lab for calibration. The drill is fully operational and a disposable sharp-point drill bit is secured in the drill head. You can use only the following tools for the calibration process:

- Tracked drill
- Optical tracker & coordinate measurements from the tracker
- Thick wooden bench top that you can scratch, cut, or drill holes into.
- Computer with MATLAB

NOTE: We discussed the tracked tool calibration process in class. We discussed the constrained motions for both tool tip and axis calibration. We discussed the resulting trajectories on which the markers travel during these constrained motions. We discussed how one can reconstruct these trajectories from measured marker positions in tracker frame (i.e. measurement space). We discussed how one can compute the calibration parameters from these, by transforming the reconstructed drill tip and drill axis vector from tracker space to marker space. We discussed redundancies present in the setup and how to take advantage of that.

Drill Tip Calibrator: Design and implement a suitable calibration process for the drill tip. (20 pts)

1. Explain the calibration process step by step with using figures, sketches, words, block diagram, etc. – anything that you find appropriate to convey your thinking. Explain the nature of the constrained motion you chose and why you chose it and how you will model these constrained motions mathematically. **(4pt)**
2. Explain how you would detect if the enforcement of the constraint failed. You do not have to implement it. **(2 pts)**
3. Explain how you would detect if there was excessive random error in tracking of the markers. You do not have to implement it. **(2 pts)**
4. Implement the calibration software in MATLAB. Input: array of (A,B,C) markers. Output: T_m **(12 pts)**

Drill Axis Calibrator: Design and implement a calibration process for the drill axis. (20 pts)

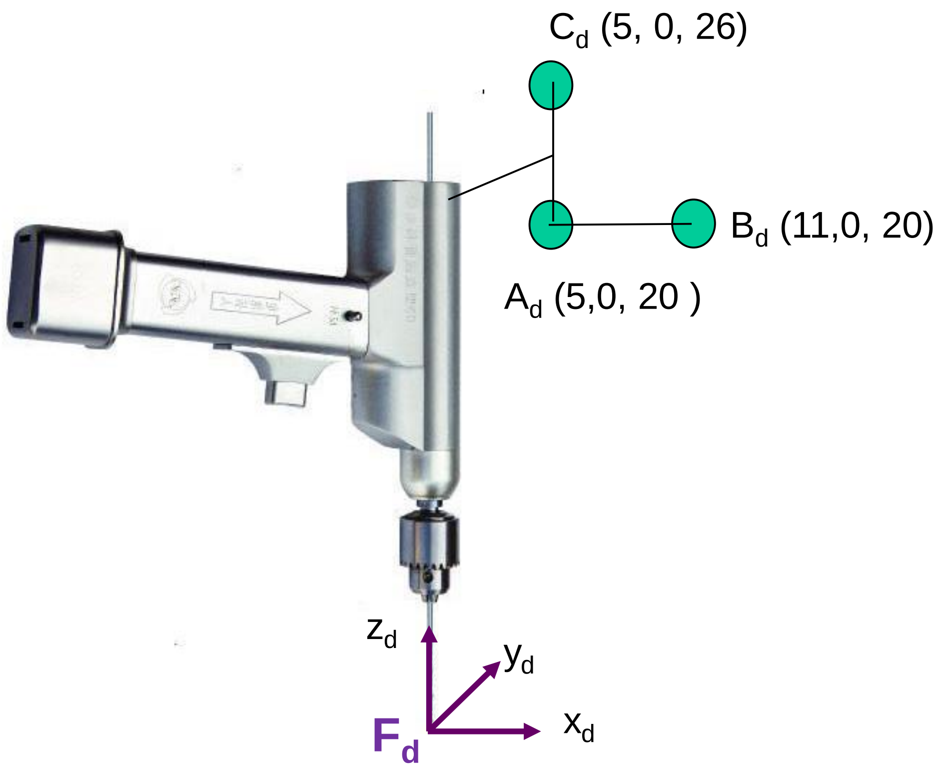
Repeat the same 1-5 steps as above. . Input: array of (A,B,C) markers. Output: v_m

SIMULATION

IDEAL DRILL MODEL: Below is an ideal model of the instrumented bone drill, as it was meant to be. (In reality, the dimensions of the drill and the placement of the markers differ from the ideal and this is why the drill must be calibrated.) The center of this $\mathbf{F_d}$ drill model coordinate system is in the drill tip. The z axis of the model coordinate system is in the drill axis, pointing from drill tip toward drill base. A, B, and C are the marker locations in cm units. (Note that real life we would not arrange for the ABC and the drill tip to be coplanar and for ABC to be an isosceles triangle - but these little simplifications do not affect our math and thus permissible in this exercise.)

Compute Marker Frame: We must define F_m marker frame. We will use this marker frame throughout the lifetime of the tracked drill. We are free to fix it anywhere relative to (A,B,C), so long it is a unique orthonormal coordinate system. Input: (A,B,C). Output: Ctr_m and base vectors (x_m y_m z_m). Compute these and draw the result in the figure below (**5 pts**).

Compute Ground Truth: Use this model to calculate the ideal (aka ground truth) calibration parameters in the F_m coordinate system for the drill axis, as $\boldsymbol{v}_\mathbf{m}$ and for the drill tip as $\mathbf{T_m}$ (**5 pts**).



Earlier, you have designed a calibration process for the drill tip and for the drill axis using constrained motions. Now you must generate motions in F_t tracker coordinate system and feed them as an input data to the calibration software as way of testing it. In the simulator, we must place the F_t tracker coordinate system somewhere with respect to the F_d drill model coordinate system. Conveniently, we can place F_t to coincide with the F_d.

Drill Tip Simulator: Design and implement software to generate calibration poses that satisfy the requirements of your drill tip calibration module developed earlier. The number of calibration poses, and the range of calibration motion should be input parameters to the simulator. The output is a series of (A_t,B_t,C_t) triplets, representing simulated marker positions. Verify the simulator visually by running it with about 20 poses and plot the resulting (A_t,B_t,C_t) marker points (**5 pts**)

Drill Axis Simulator: Design and implement software to generate calibration poses that satisfy the requirements of your drill axis calibration module developed earlier. The number of calibration poses, and the range of calibration motion should be input parameters to the simulator. The output is a series of (A_t,B_t,C_t) triplets, representing simulated marker positions. Verify the simulator visually by running it with about 20 poses and plot the resulting (A_t,B_t,C_t) marker points (**5 pts**)

CALIBRATION TEST

Drill Tip Calibration Test: Run the Drill Tip Simulator to generate poses for drill tip calibration. Design the range and number of calibration poses will use, explain your choices. Feed the simulated poses to the drill tip calibrator and ascertain that it works correctly, i.e. the received $\mathbf{T_m}$ and the ground truth are identical (or very close). (**10 pts)**

Drill Axis Calibration Test: Run the Drill Axis Simulator to generate poses for drill axis calibration. Design the range and number of calibration poses will use, explain your choices. Feed the simulated poses to the drill axis calibrator and ascertain that it works correctly, i.e. the received $\mathbf{v_m}$ and the ground truth are identical (or very close). (**10 pts)**

Drill Tip Calibration Robustness Test: Run the Drill Tip Simulator to generate poses for the drill tip calibration as above. Blend a random error of maximum E_{max} on the simulated A,B,C marker positions. Feed the simulated poses to the calibrator and compute $\mathbf{T_m}$, do this by gradually increasing E_{max} from 0 in 1mm steps. Determine the level of maximum tracking error (E_{max}) when the calibration is no longer clinically satisfactory. Make a graph between E_{max} and the calibration error (the difference between actual $\mathbf{T_m}$ and ground truth) (**10 pts)**

Bonus #1: Do the same as above for Drill Axis Calibration Robustness Test (**5 pts**)

Bonus #2: Develop alternative method for Drill Tip Calibration and test it (**5 pts**)

General Rules

- Read the online syllabus carefully for general instructions on the submission of assignments.
- Always explain how you solve a problem. Use drawings, math formulas, text, block diagram, pseudo code - anything that you find them appropriate to convey your ideas. I must know that you understand what you are doing and I must be able to follow your reasoning. Depending on the quality and depth of your reasoning and discussion or results you may pick (or lose) lots of points.
- Write proper header and richly comment your code. There is no such thing as too much comment. Good style and neatness will earn you valuable points. The lack of these will cause reduction.
- Always test the validity and deformity of the input data; lack of such testing will lead to deduction.
- Test each module fully, construct several test cases with known ground-truth answer.
- Write a testing m file(s) for each module or problem.
- Capture the output, to show that your program does what it is supposed to do. Make plots whenever it is requested or makes sense. Add explanation text as you see it useful.
- Use decimal digits sensibly and consider what is precision is practical for the given problem. (Generally, resolution finer than 10^{th} of a millimeter is not practically achievable in a surgical navigation system, so this should be your limit. Use decimal floating point format in your outputs. Do not use exponential number format.
- Use MATLAB routines (existing or written by you) for recurring tasks
- Submit the m files and the captured output file, as well as any drawing, or supplemental information you feel relevant.
- **Have fun!**