DISCRETE-TIME SLIDING MODE WITH TIME DELAY ESTIMATION OF A SIX-PHASE INDUCTION MOTOR DRIVE

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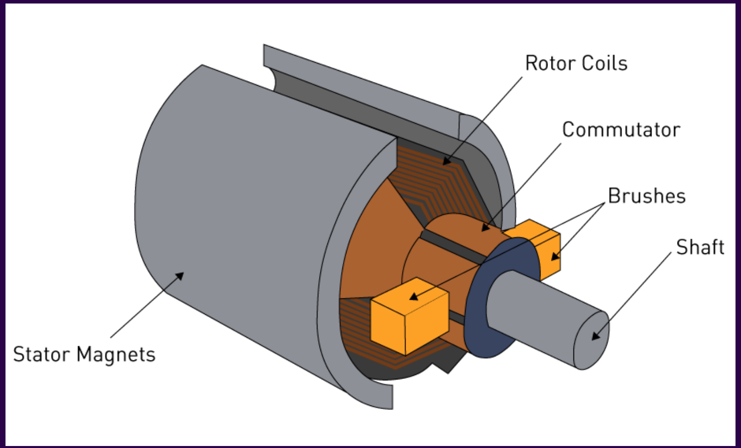
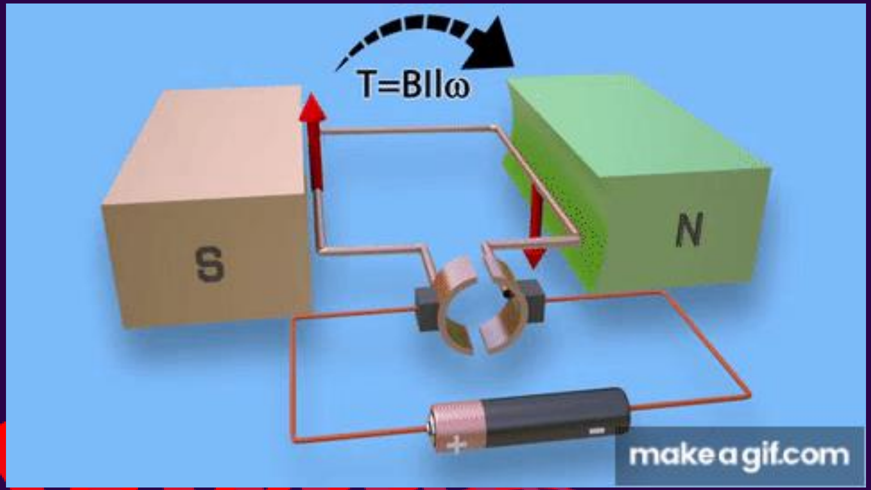
SIX-PHASE INDUCTION MOTOR

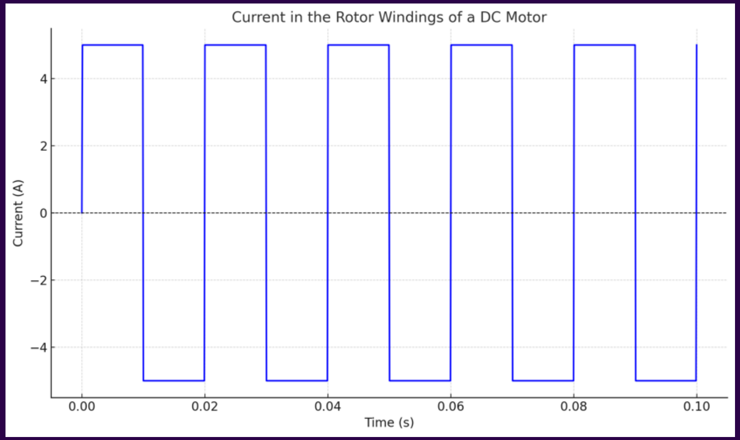
DC MOTOR



- **STATOR**: Permanent Magnets
- **ROTOR**: Coil Windings
- **COLLECTOR**: changes sign of current



CONSTANT INPUT VOLTAGE
(ex. 5V)



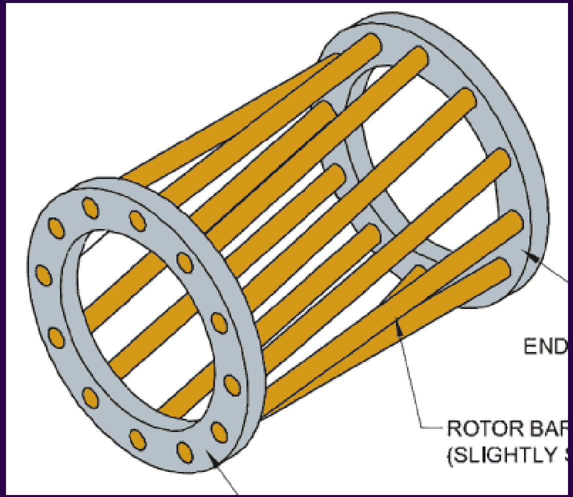
Rotor coil current

Motor Torque and Speed

SIX-PHASE INDUCTION MOTOR

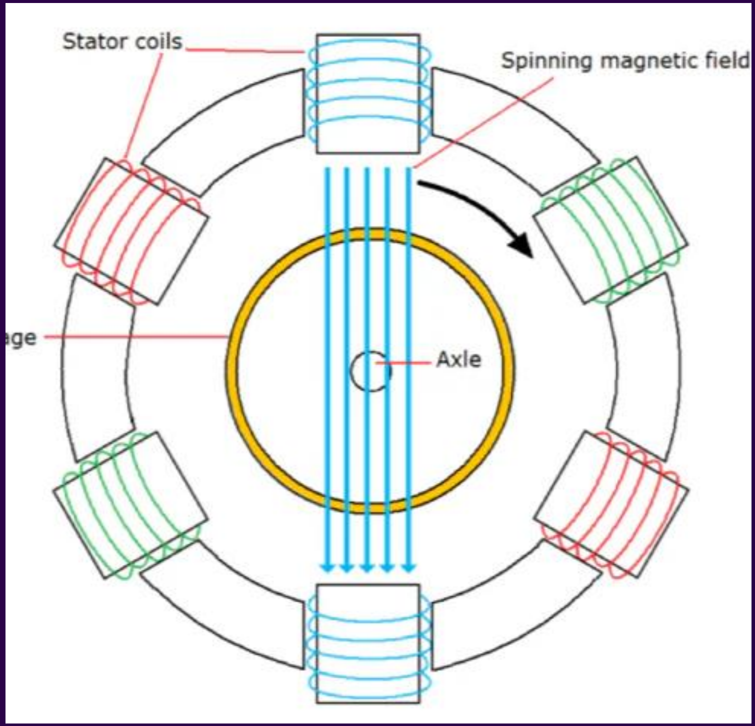
AC MOTOR

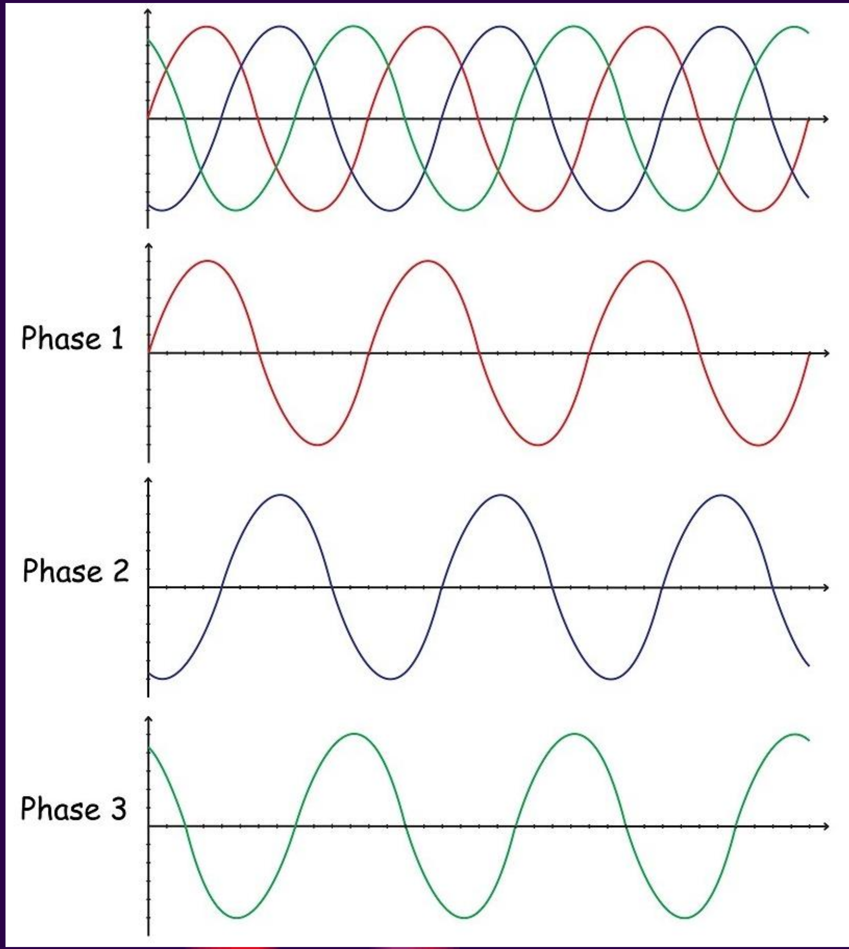
ROTOR



"Squirrel Cage"

STATOR

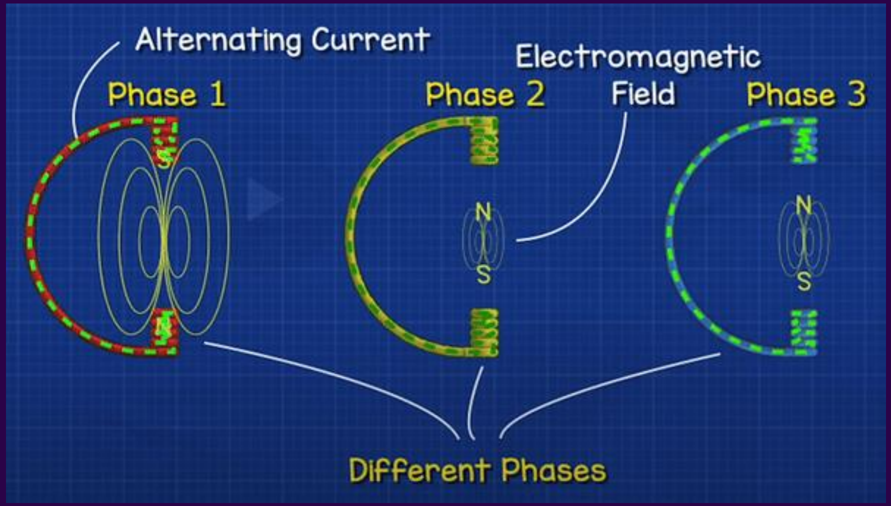


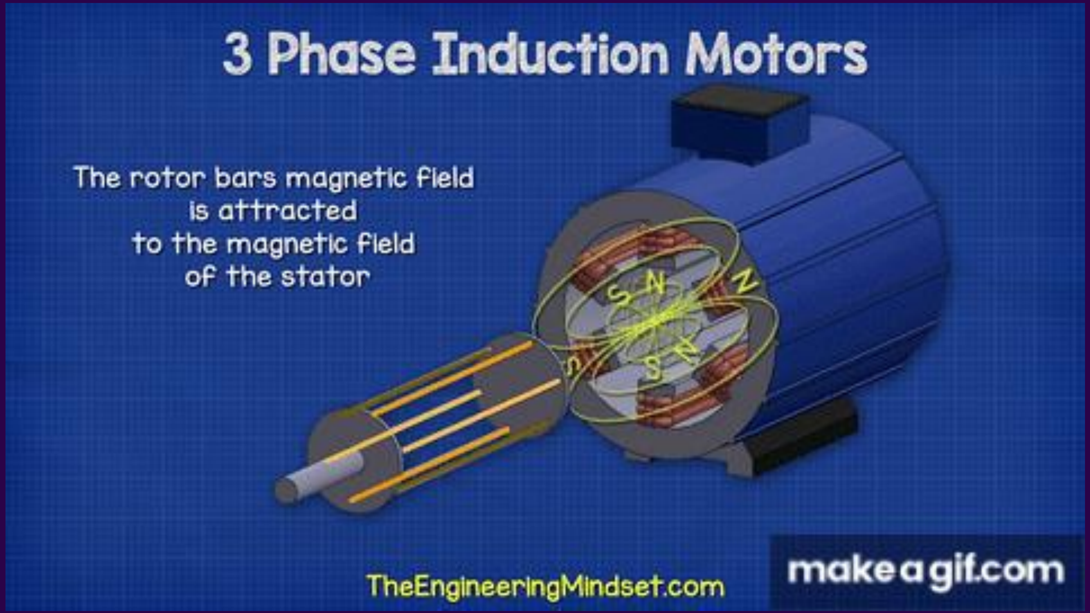


n-phase coil arrangements

SIX-PHASE INDUCTION MOTOR

AC MOTOR



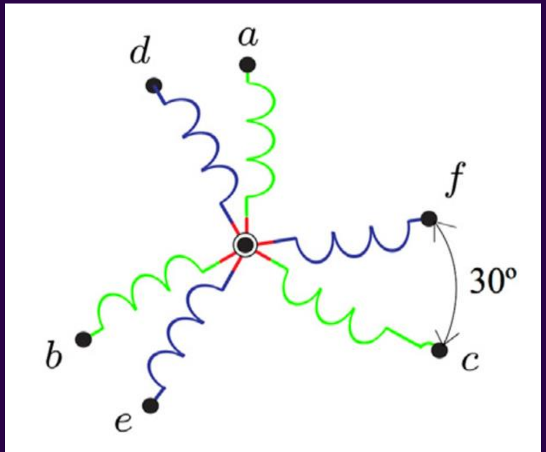


The rotating magnetic field induces a current in the rotor’s bars.
This current creates a MF itself.

The induced magnetic field, interacting with the stator’s, causes the rotor to spin.

SIX-PHASE INDUCTION MOTOR

SIX PHASE ASYMMETRICAL INDUCTION MOTOR



Asymmetrial six-phase

SIX-PHASE INDUCTION MOTOR

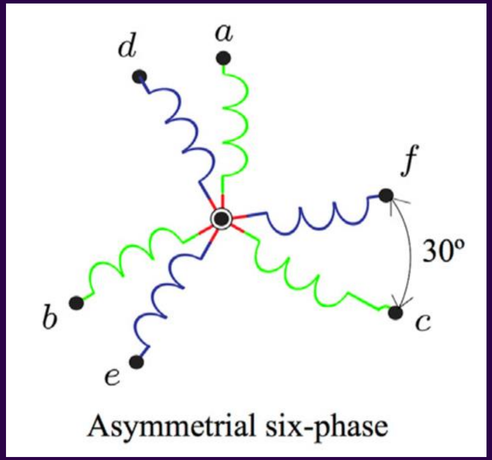
AMAZON: LION 6

- 11,800 KG
- UP TO 105 KM/H
- 335 HP

WHAT'S THE CONTROL PROBLEM?

WE WANT TO ASSIGN A CERTAIN ROTOR VELOCITY ω_r

SIX PHASES $\Longrightarrow$ SIX INPUT VOLTAGES

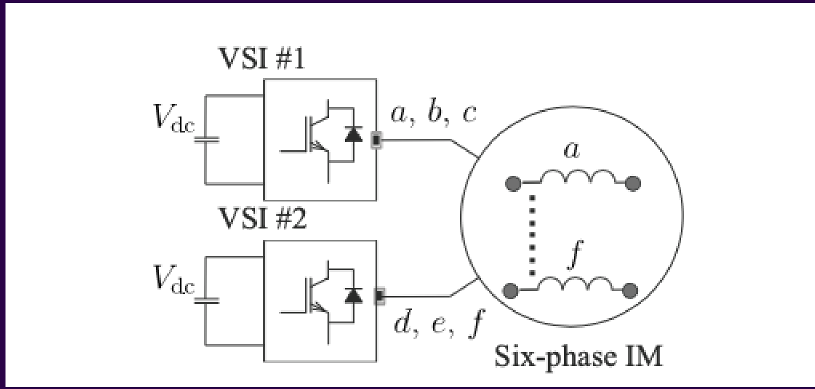


ONE REFERENCE ω_r $\Longrightarrow$ SIX INPUTS $[v_a \; v_b \; v_c \; v_d \; v_e \; v_f]$

- DOUBLE LOOP CONTROL SCHEME
- COORDINATE/INPUT TRANSFORMATIONS

MATHEMATICAL MODEL OF INDUCTION MOTOR

The system consists of an asymmetrical six-phase Induction Motor fed by two 2-level Voltage source Inverter.



Supplied by two 3-phase 2-level VSIs

To derive the state space equation the Vector space decomposition is used that brings to three sets of independent equation.

Each set of equation is related to a subspace:

- α – β subspace → k = 12n + 1 harmonics
- x – y subspace → k = 6n + 1 harmonics
- zero – sequence subspace → k = n – 3 harmonics

MATHEMATICAL MODEL OF INDUCTION MOTOR

ALPHA-BETA SUBSPACE $[v_\alpha \;\; v_\beta]$ → Flux/Torque producing components

X-Y SUBSPACE $[v_x \;\; v_y]$ → Loss producing components

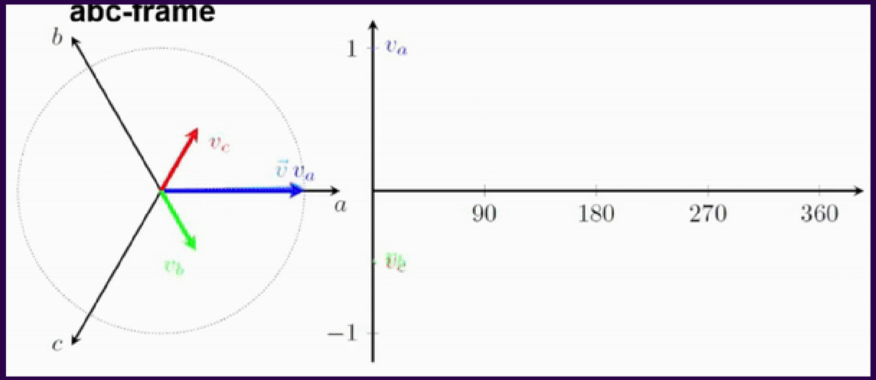
0-SEQUENCE SUBSPACE $[v_{z1} \;\; v_{z2}] = [0 \;\; 0]$ → Zero component (in balanced conditions)

CHANGE OF INPUT DIM. 6 → DIM. 4

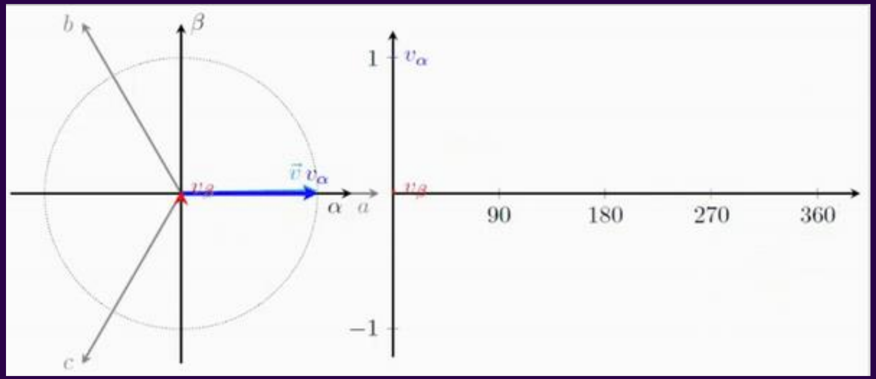
$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_x \\ v_y \\ \cancel{0} \\ \cancel{0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1 \\ 1 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\ \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \\ v_f \end{bmatrix}$$

FRAMEWORKS COMPARISON

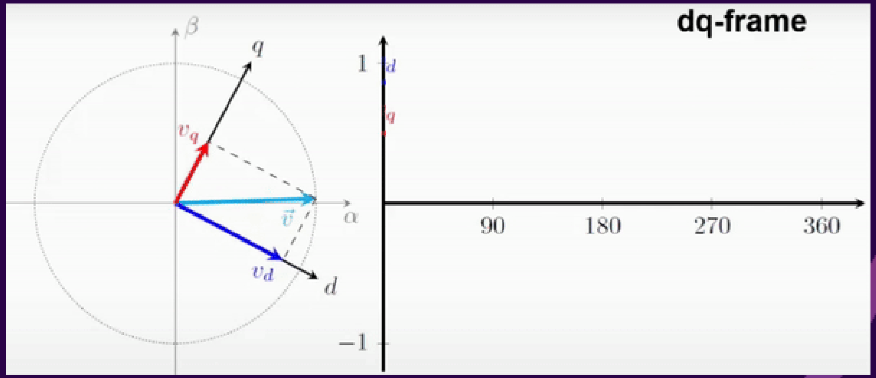
REAL INPUTS



ALPHA-BETA FRAME
(CLARK TRANSFORMATION)



DIRECT QUADRATURE FRAME
(PARK TRANSFORMATION)



VECTOR SPACE DECOMPOSITION AND DISCRETE MODEL

Stator and rotor currents state vector

$$x(k) = [x_1(k), x_2(k), x_3(k)]^T$$

with

$$x_1(k) = \left[i_{s\alpha}(k), i_{s\beta}(k)\right]^T$$

$$x_2(k) = [i_{sx}(k), i_{sy}(k)]^T$$

$$x_3(k) = [i_{r\alpha}(k), i_{r\beta}(k)]^T$$

The discrete model of the system in state-space representation

$$x_1(k+1) = A_1 x_1(k) + H_1 x_3(k) + B_1 u_1(k) + n_1(k)$$

$$x_2(k+1) = A_2 x_2(k) + B_2 u_2(k) + n_2(k)$$

$$x_3(k+1) = A_3 x_1(k) + H_2 x_3(k) + B_3 u_1(k) + n_3(k)$$

$$y(k) = Cx(k)$$

the coefficients of the matrices are obtained by combining the electrical parameters of the system

STATOR CURRENTS → OUTPUT VECTOR

$$y(k) = [x_1(k), x_2(k)]^T$$

$$= \left[i_{s\alpha}(k), i_{s\beta}(k), i_{sx}(k), i_{sy}(k)\right]^T$$

STATOR VOLTAGES → INPUT VECTORS

$$u_1(k) = \left[u_{s\alpha}(k), u_{s\beta}(k)\right]^T$$
$$u_2(k) = [u_{sx}(k), u_{sy}(k)]^T$$

VOLTAGE SOURCE INVERTER-INDUCTION MOTOR DRIVE

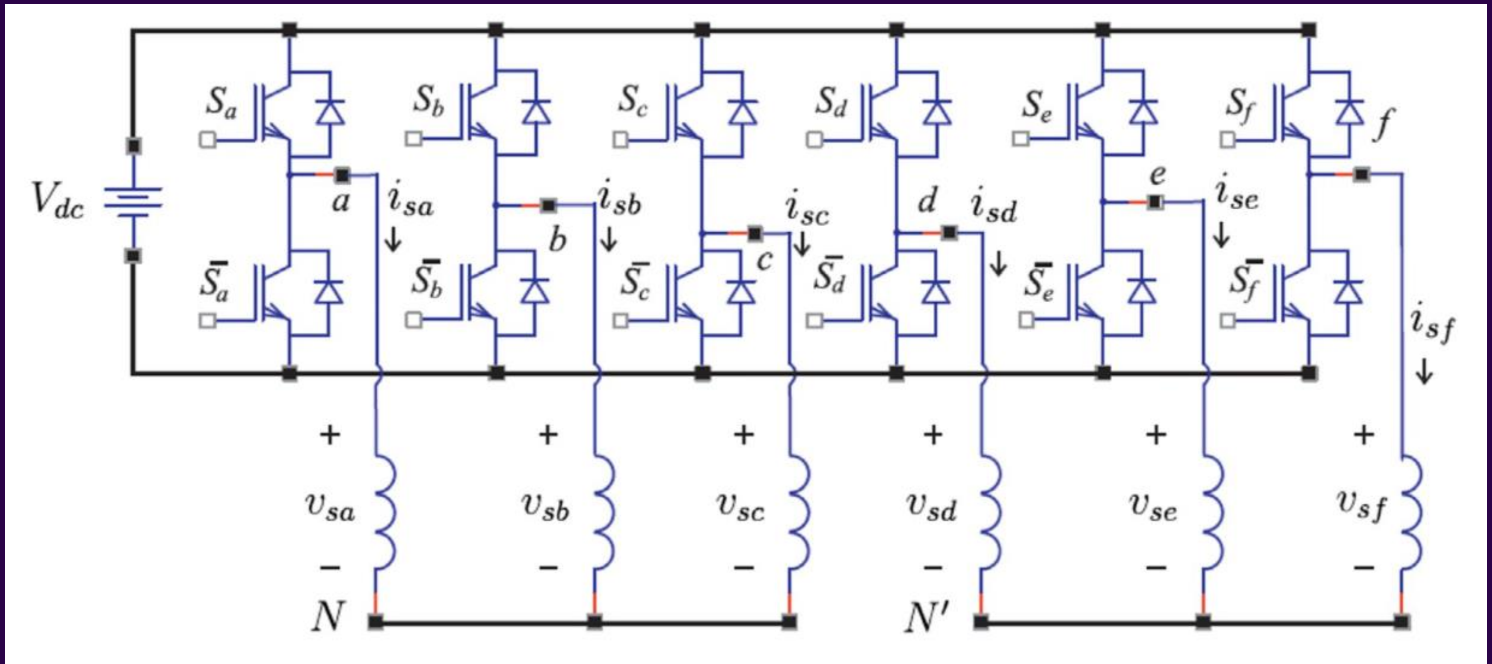
The stator voltages have a discrete nature due to VSI model and are obtained from

$$[u_{s\alpha}(k), u_{s\beta}(k), u_{sx}(k), u_{sy}(k)]^T = V_{dc}\boldsymbol{TM}$$

$$\mathbf{M} = \frac{1}{3}\begin{bmatrix} 2 & 0 & -1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 & -1 \\ -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ -1 & 0 & -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \end{bmatrix} \mathbf{S}^T$$

VSI model

DC → AC CONVERTER



PRINCIPLE OF SLIDING MODE

Robust control technique

Drives the system states to a predefined switching surface in finite time

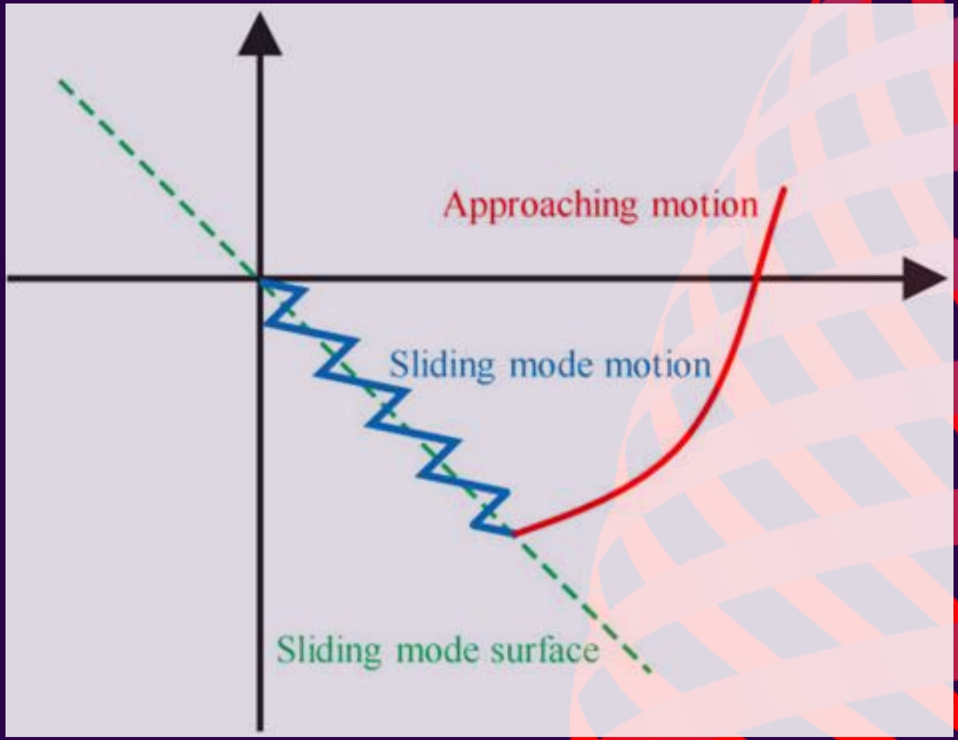
Ensures stability, despite the presence of disturbances

Sliding manifold

$$\sigma = \{x: s(x) = 0\}$$

Control law

$$u = \begin{cases} +u_0 \; if \; s(x) > 0 \\ -u_0 \; if \; s(x) < 0 \end{cases}$$



The discrete-time representation
$x(k+1) = Ax(k) + Bu(k)$

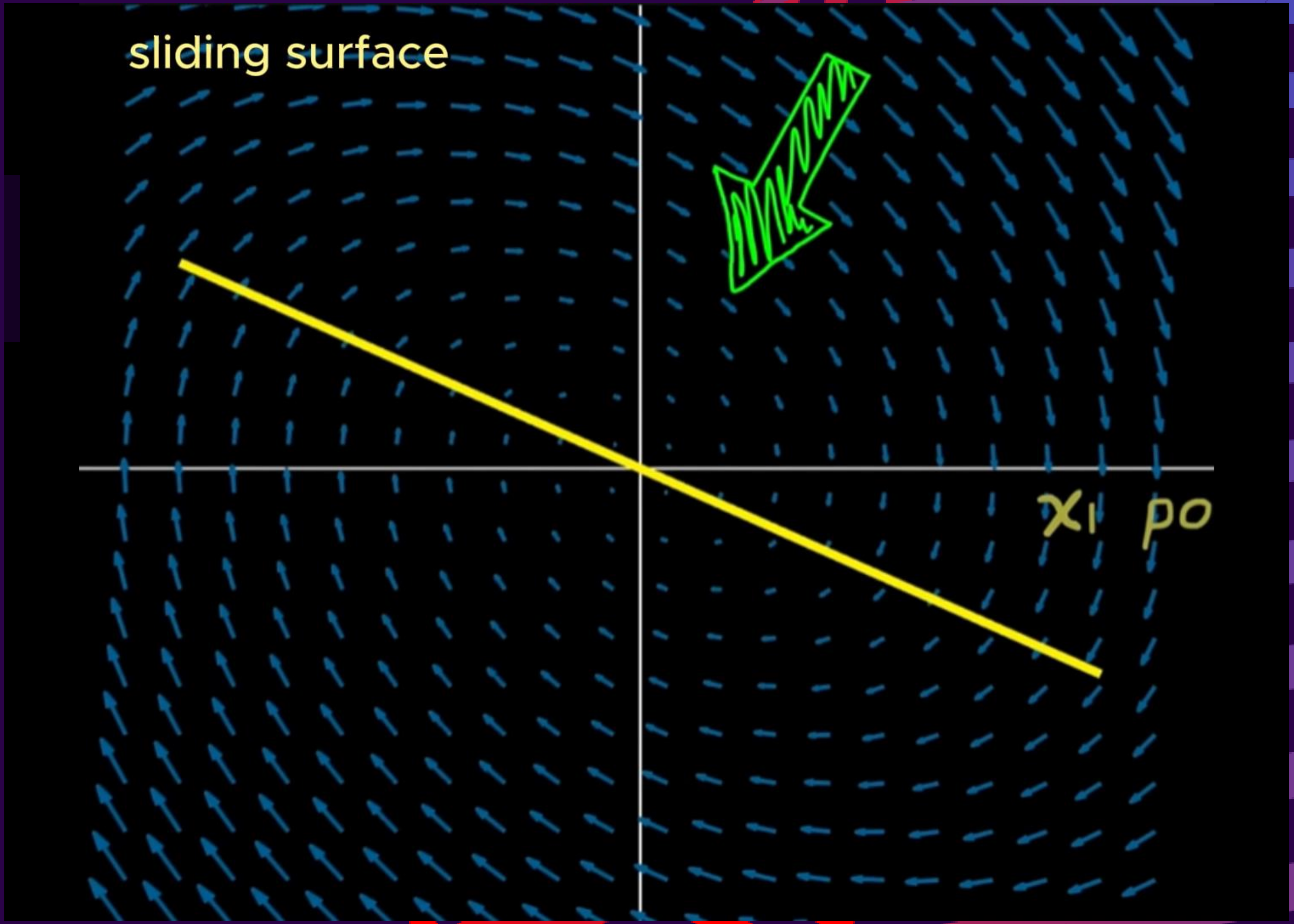
Control input $u(k)$ ensures $s(k+1) = 0$

PRINCIPLE OF SLIDING MODE

Controller Behavior
states reach the sliding surface
mantains sliding motions

Disturbances and **Chattering**
balanced by increasing control gain, leads to chattering

Chattering Mitigation
introduce a boundary layer



CHATTERING PHENOMENON

High-frequency switching can cause chattering due to

signal discontinuities,

time delays,

disturbances

Solutions:

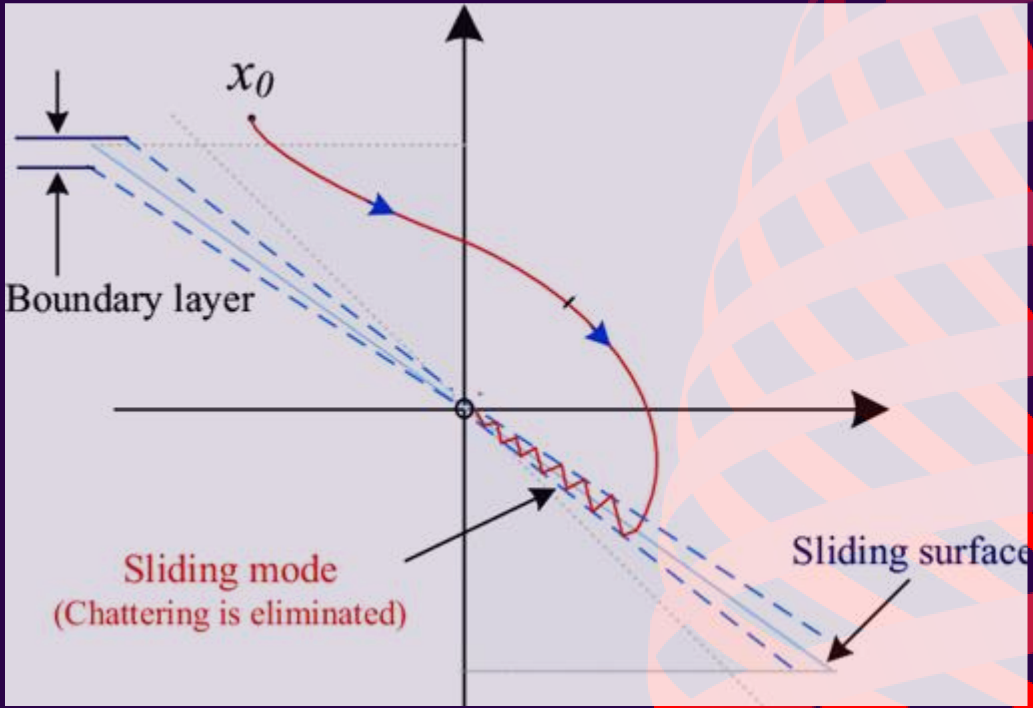
- Use **continuous functions** (e.g., saturation) instead of discontinuous signals
- Design a **robust observer** to estimate system states
- **Integrate the control input** for smoother dynamics
- Implement SMC in **discrete-time** to reduce switching effects

REAL TIME DISCRETE IMPLEMENTATION

Two options:

- analogue implementation of discontinous control law
- direct discrete implementation with digital controller

→ real time implementation
→ chattering reduction: quasi-sliding mode acts like boundary layer



→ confining trajectories
→ suppressing high-frequency oscillations

TIME DELAY CONTROL

Estimates and compansate for system uncertainties and unmodeled dynamics using time-delayed signals

Time Delay Estimation: combines SMC and TDC to reduce chattering and approximates disturbances, incorporating them into the control law for stability

Assumptions:

system controllable and observable

the uncontrollable dynamics continuously differentiable

TDE error bounded

$$E = (H_i x_3(k) + n_i(k)) - (H_i \widehat{x_3}(k) + \widehat{n}_i(k)) < \delta$$

SMC and disturbances observes handle uncertainties, with the assumption of their boundness and immutability in consecutive sampling moments

SMC OF INDUCTION MOTORS

IM are a mainstay in industrial application, due to their reliability and efficiency, despite their nonlinear dynamics and sensititvity to parameter variations

SMC provides robustness and disturbance rejection with low implementation complexity

- simplifies the decoupling and linearization of motor dynamics
- enables independent control of torque and flux
- defines a sliding surface by the error between desired anc actual stator currents

Integration of TDE into SMC

- estimate and compensate for unkwown rotor currents and disturbances

CONTROLLER DESIGN

Goal: Controlling speed and stator currents of the six-phase induction motor with robustness and precision.

Outer control loop

Inner control loop

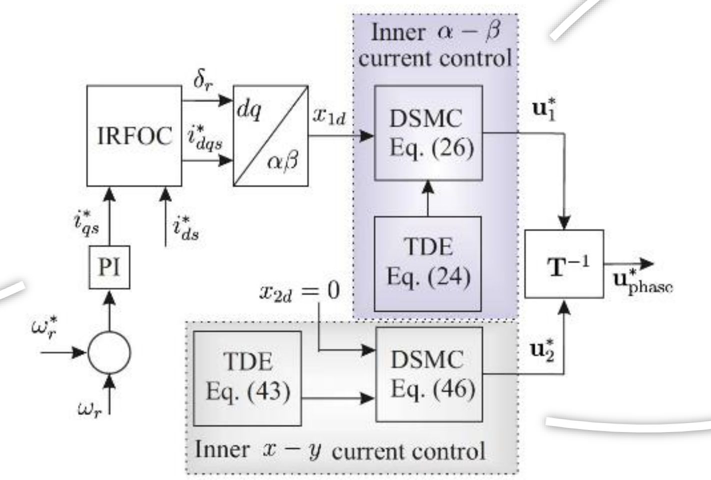


Fig. 2. Block diagram of the proposed speed control based on IRFOC technique and the DSMC with TDE method for the inner current control.

Outer loop: regulates motor speed (ω_r) using a PI controller.

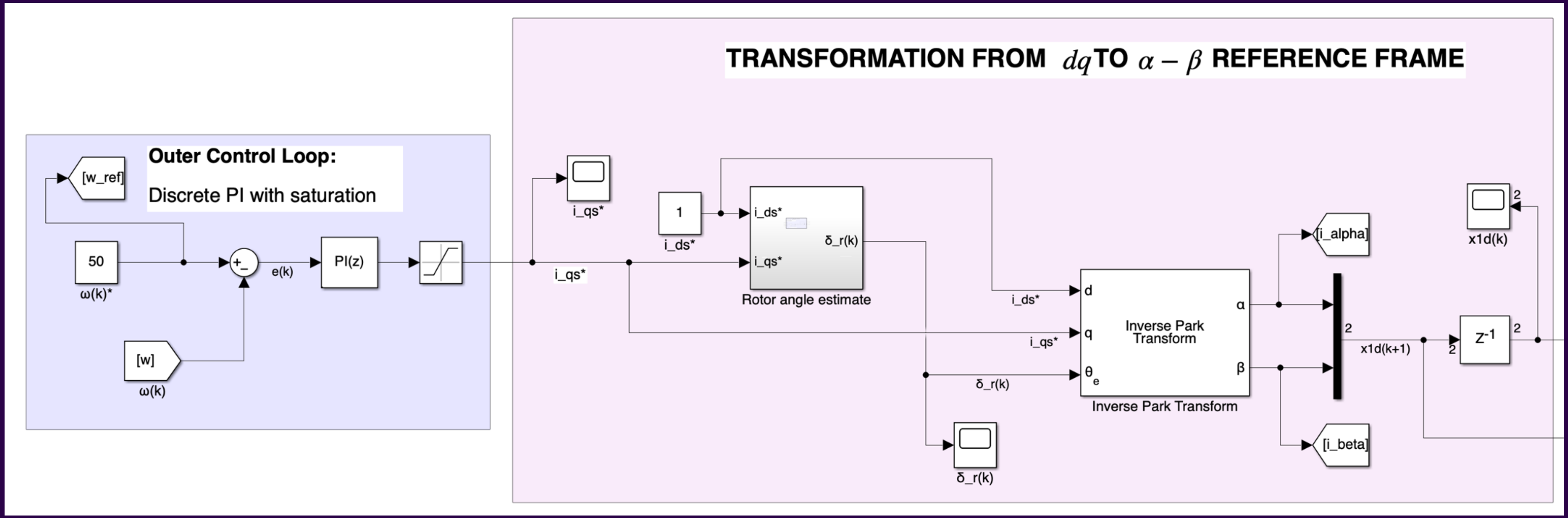
Inner loop: controls stator currents ($i_s^{\alpha}, i_s^{\beta}, i_s^{x}, i_s^{y}$,) using DSMC with TDE

OUTER CONTROL LOOP

Function: It ensures that the motor's speed ω_r matches the reference speed ω_r^*

Steps:

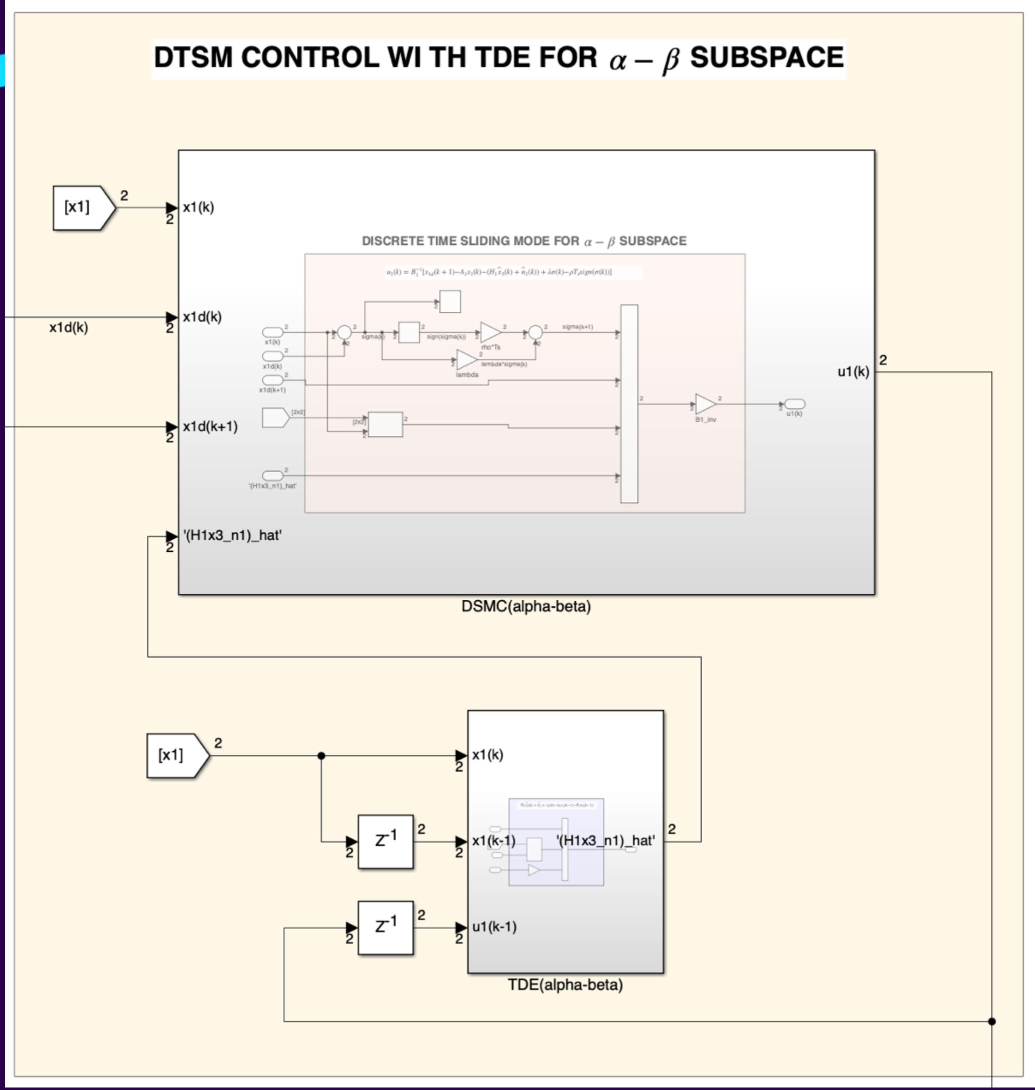
1. Calculate the speed error $(\omega_r - \omega_r^*)$
2. Generate the torque-producing current i_{qs}^* using a PI controller
3. Estimate angular rotor position δ_r from slip frequency ω_{sl}
4. Perform the Inverse Park Transformation to obtain (i_α^*, i_β^*) ← current references for the inner loop



INNER CONTROL LOOP

(α, β) ***subspace***

Goal: It forces the stator currents $(i_s^{\alpha}, i_s^{\beta})$ to track the references $(i_{\alpha}^{*}, i_{\beta}^{*})$



The selected sliding surface is $e_{\phi}(k) = x_1(k) - x_1^d(k) = i_{s\phi}(k) - i_{s\phi}^{*}(k) = \sigma(k)$ with $\phi \in \{\alpha, \beta\}$

For ideal sliding motion the following conditions must be satisfied:

$$\sigma(k) = 0\sigma(k+1) = 0$$

Hence the discrete sliding mode control (DSMC) law for the stator currents in the $\alpha - \beta$ subspace is:

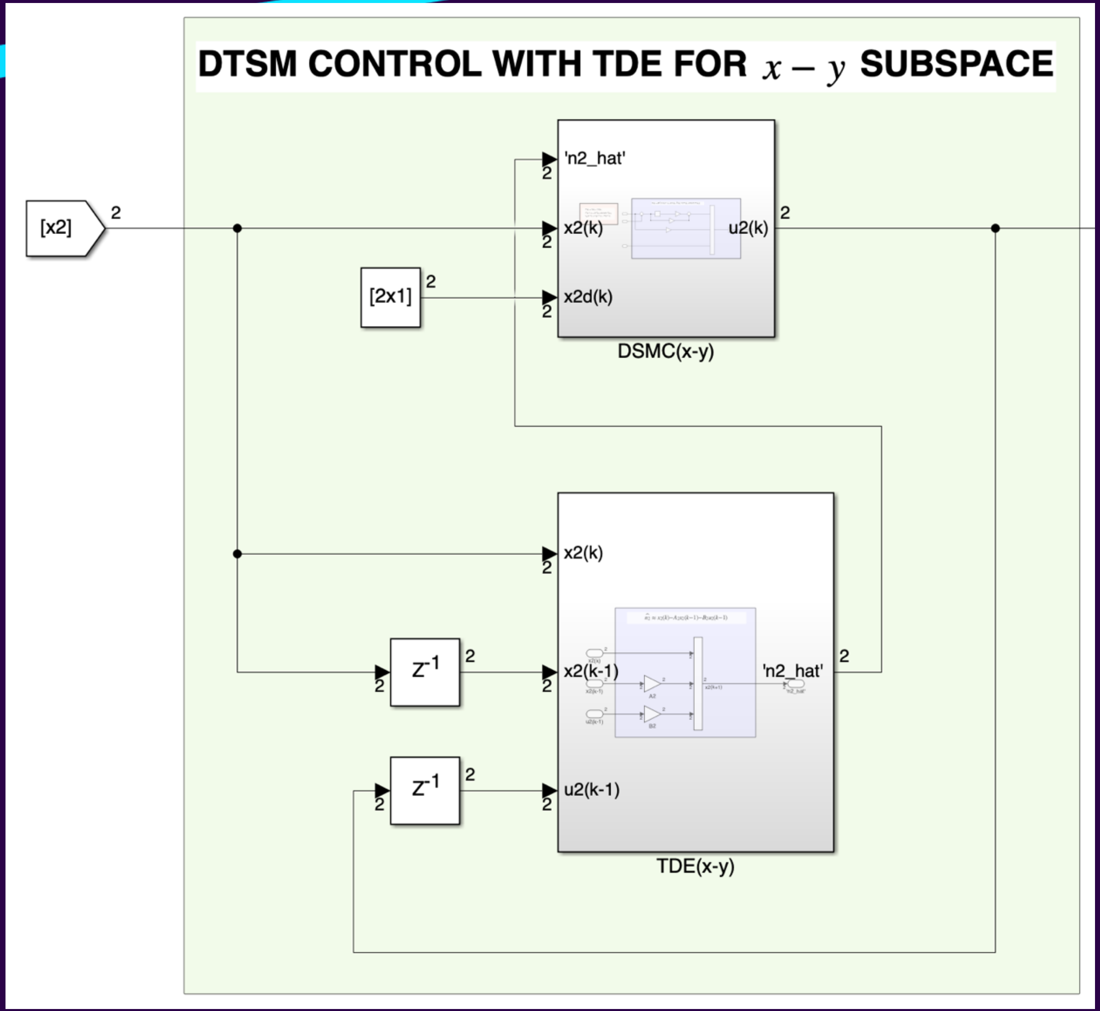
$$u_1(k) = B_1^{-1}\left[x_1^d(k+1) - A_1 x_1(k) - \underbrace{H_1 \hat{x}_3(k) - \hat{n}_1(k)}_{\textit{Estimated from TDE}} + \lambda\sigma(k) - T_s \rho\, sign(\sigma(k))\right]$$

With $\sigma(k+1) = \lambda\sigma(k) - T_s \rho\, sign(\sigma(k))$

INNER CONTROL LOOP

$(x-y)$ subspace

Goal: it minimizes the currents in the $(x-y)$ subspace to reduce losses.



The sliding surface is selected as

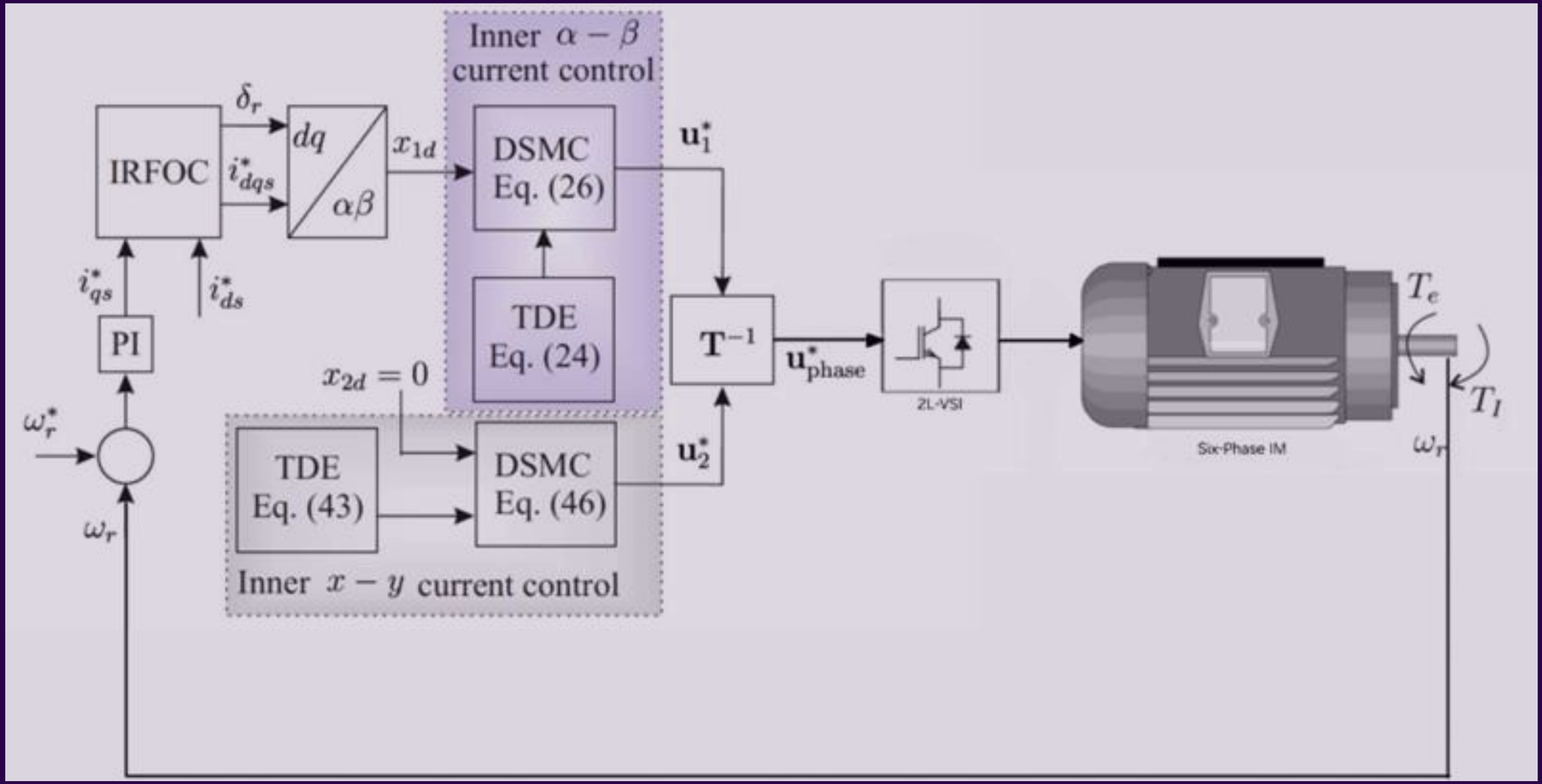
$$e_{s_{xy}}(k) = x_2(k) - x_2^d(k) = \sigma^*(k) \text{ with } x_2^d(k) = [i_{sx}^*(k), i_{sy}^*(k)]^T$$

The final control law is

$$u_2(k) = B_2^{-1}\left[x_2^d(k+1) - A_2x_2(k) - \hat{n}_2(k) + \Gamma\sigma^*(k) - T_s\varrho\, sign(\sigma^*(k))\right]$$

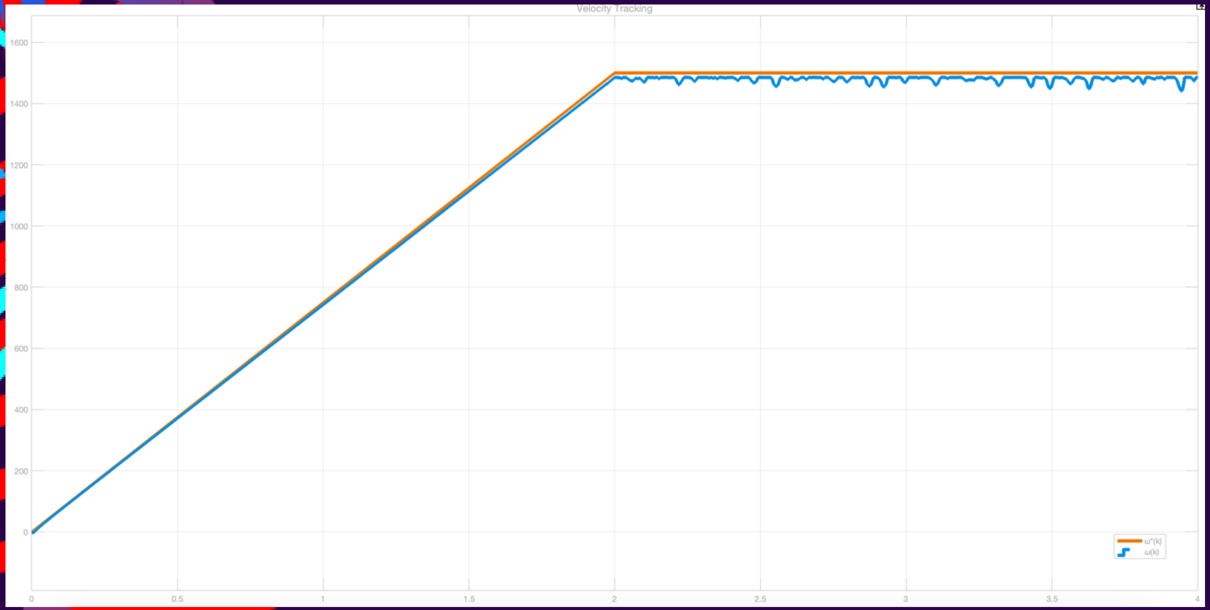
Where $\widehat{n_2(k)}$ is estimated using TDE

SIMULATION



Inner $\alpha - \beta$ current control
IRFOC
δ_r
i^*_{dqs}
dq
$\alpha\beta$
x_{1d}
DSMC Eq. (26)
TDE Eq. (24)
$\mathbf{u}^*_1$
$\mathbf{T}^{-1}$
$\mathbf{u}^*_{\text{phase}}$
2L-VSI
Six-Phase IM
T_e
T_l
ω_r
i^*_{qs}
i^*_{ds}
PI
ω^*_r
ω_r
$x_{2d} = 0$
TDE Eq. (43)
DSMC Eq. (46)
$\mathbf{u}^*_2$
Inner $x - y$ current control

VELOCITY TRACKING



Friction coefficient

Load torque

The **rotor electrical speed** is derived from $J_m \dot{\omega}_r + B_m \omega_r = P(T_e - T_l)$

Inertia coefficient

Generated torque

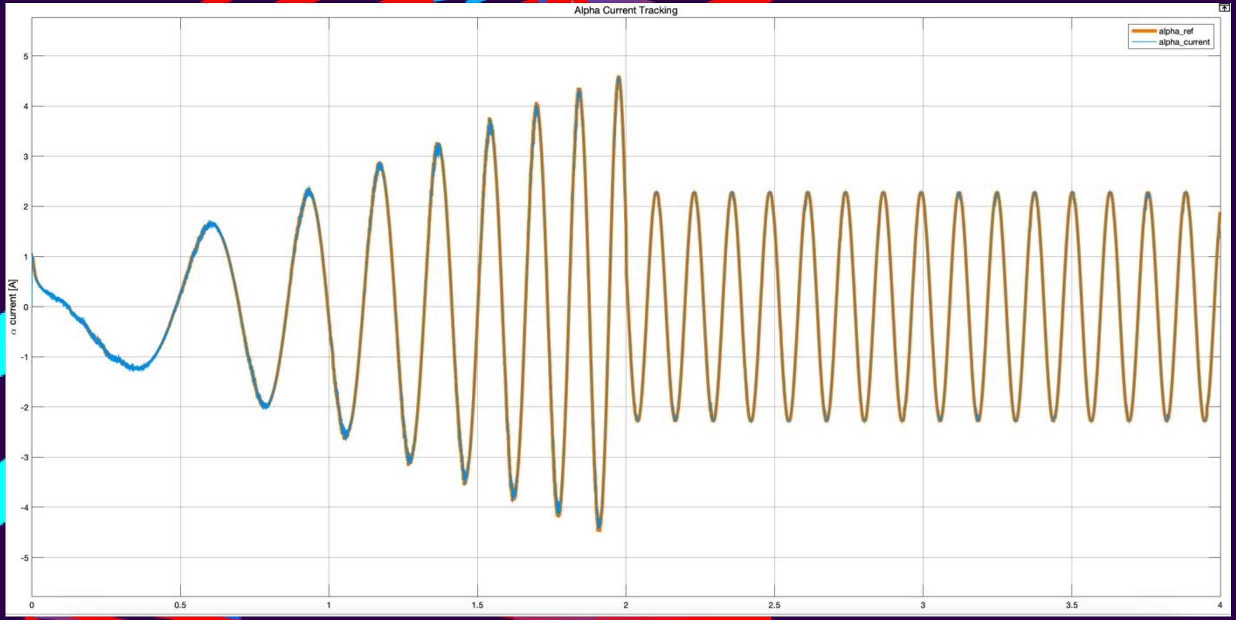
$T_e = 3PM(i_{r\beta} i_{s\alpha} - i_{r\alpha} i_{s\beta})$

Magnetized inductance

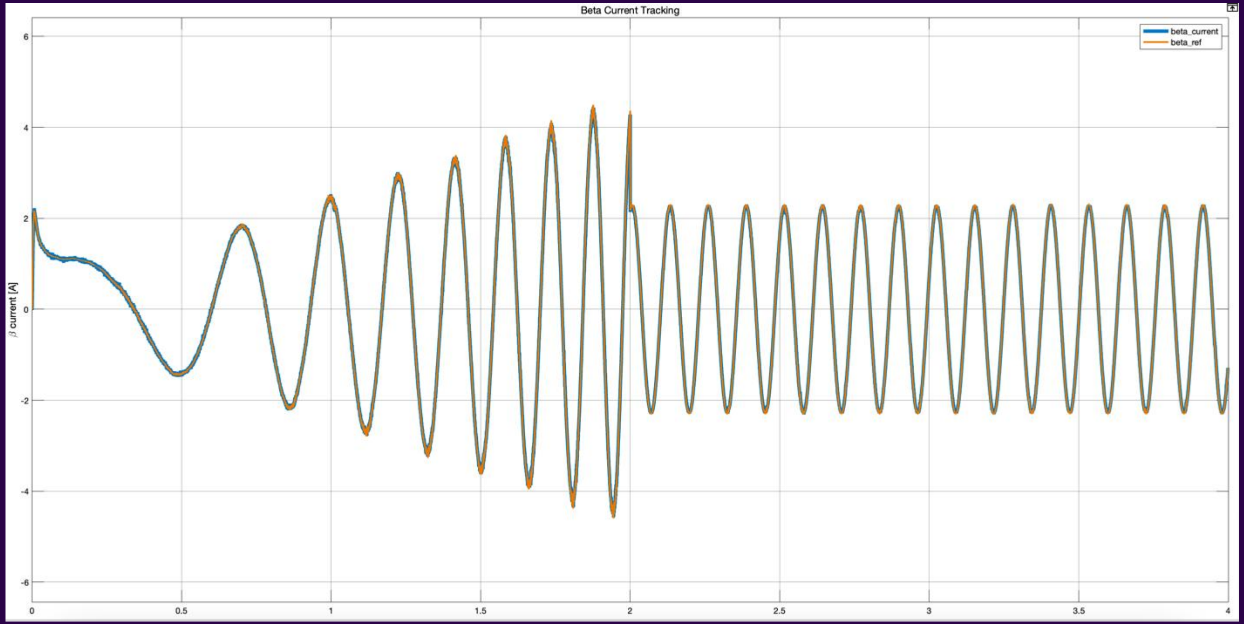
Number of poles

CURRENT TRACKING

ALPHA SUBSPACE

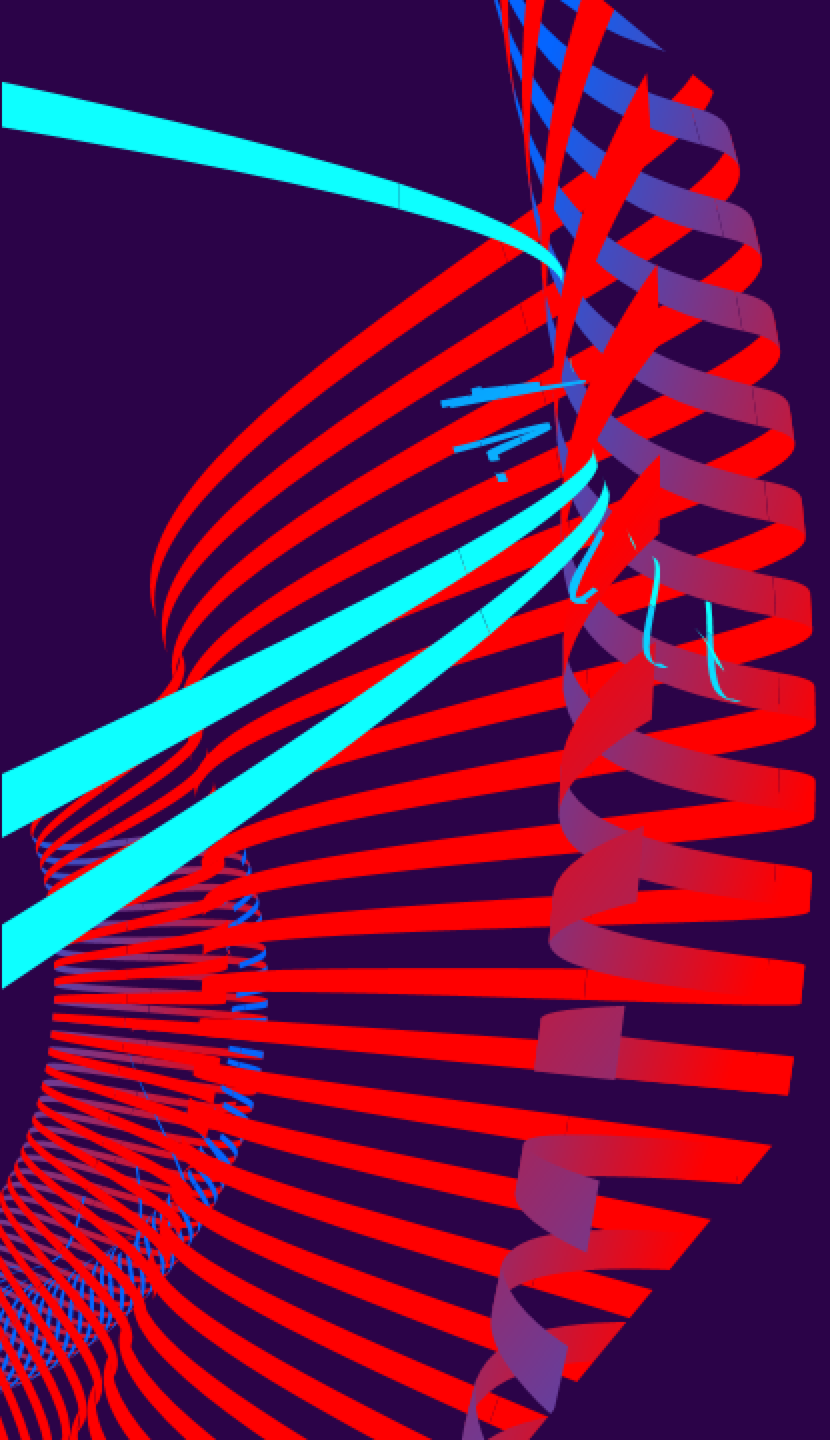


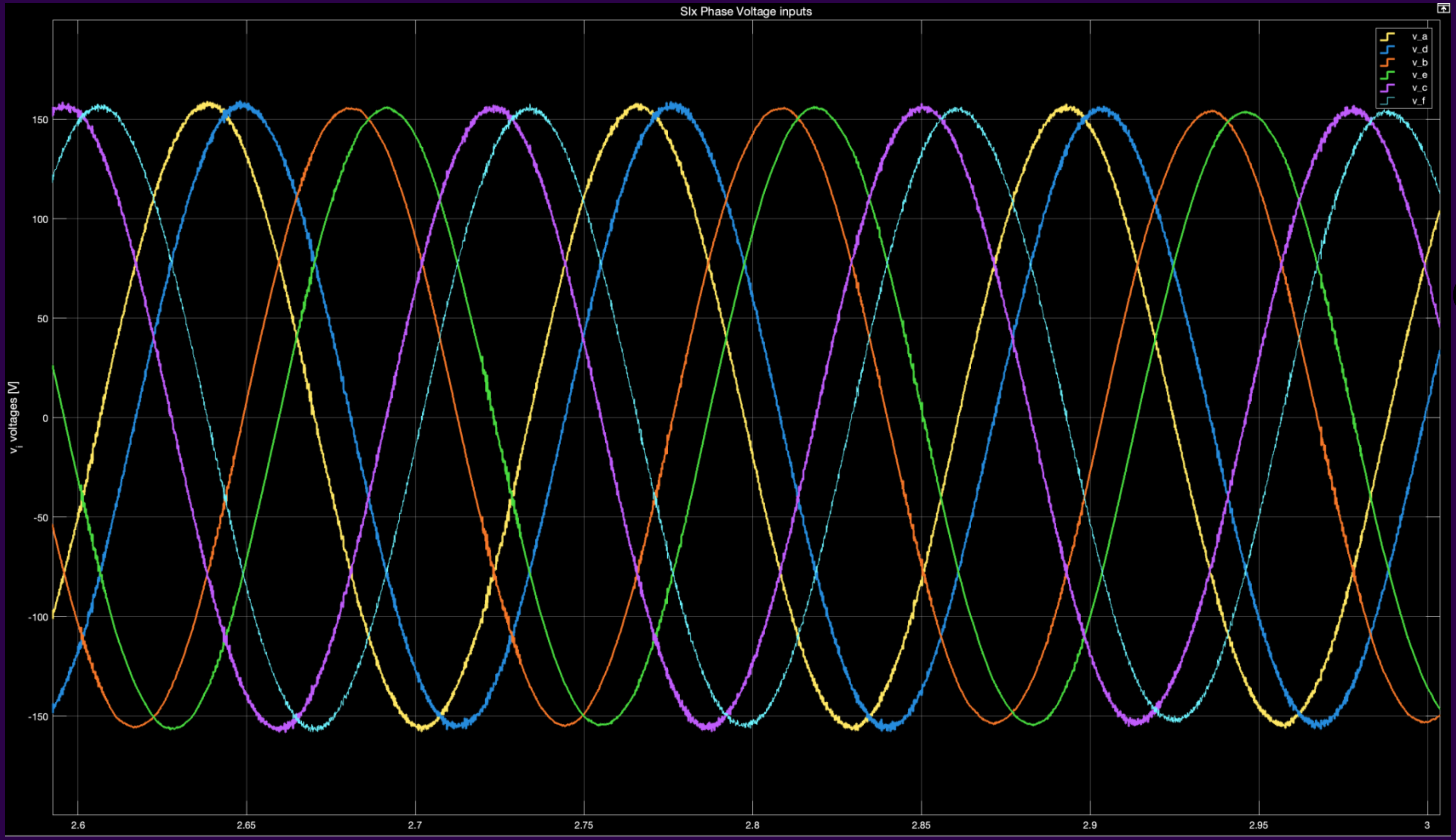
BETA SUBSPACE

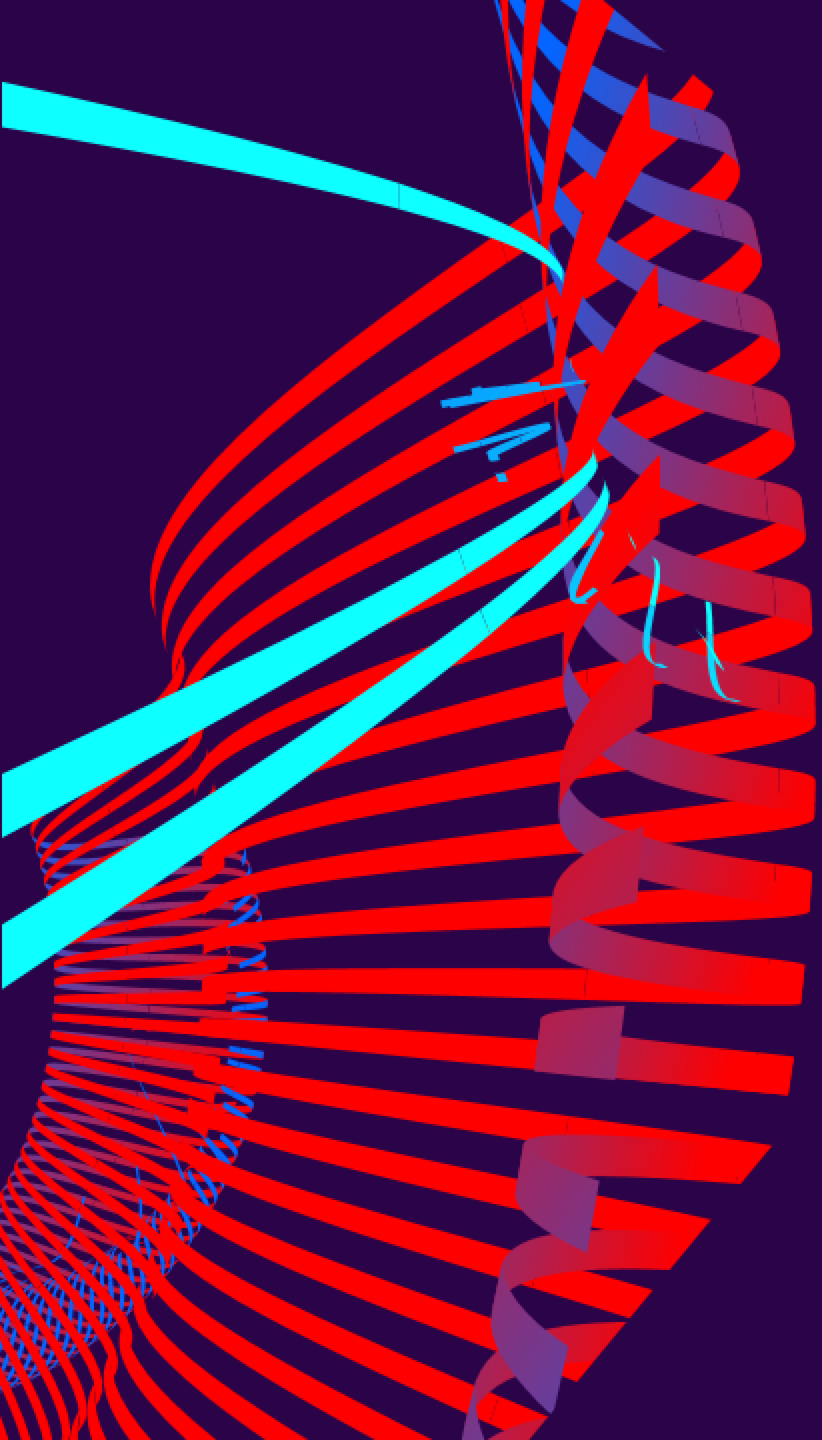


VOLTAGE INPUTS

$$[v_a \;\; v_b \;\; v_c \;\; v_d \;\; v_e \;\; v_f]$$







CONCLUSIONS

There are many advantages in using this kind of control.
In fact, it is based on TDE method, that estimates uncertainties and disturbances, and on DSM that provides robustness against TDE error, finite-time convergence and chattering reduction.
The average switching frequency of the proposed method is lower than the conventional SMC and other controllers.

THANK YOU