Multivariable Feedback Control
Homework 03 - due Wednesday May 22nd, 2024.

The goal of this homework is to start exploring ***MIMO systems***.

Problem 1 The goal of this first problem is to interpret the steady state behavior of the closed loop controlled SISO system shown in Fig. 1 through the Singular Value Decomposition (interpretation of the singular input and output directions). For the simple plant $G(s)$ you will solve 2 different control problems.

$$G(s) = \frac{1}{s+10}$$

- **Spec1** Find $C_1(s)$ so that the output y tracks asymptotically any constant reference r. Let (r, d_1, d_2) be the inputs and (e_y, m) the outputs (setting all the other inputs to 0) of the 2×3 equivalent MIMO system transfer function $W_1(s)$. Interpret the SVD decomposition of the corresponding dc-gain $W_1(0)$.

- **Spec2** Find $C_2(s)$ which guarantees that the output y will track asymptotically the sinusoidal reference $r(t) = \sin 10t$. Let (r, n) be the inputs and (y, m) the outputs (setting all the other inputs to 0) of the equivalent 2×2 equivalent MIMO system transfer function $W_2(s)$. Interpret the SVD decomposition of the corresponding gain matrix $W_2(j10)$ (you may find useful the command `frd` and `frdata`).

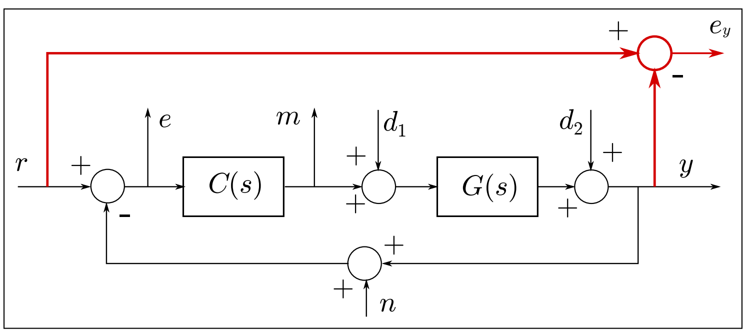

Figure 1: Control system

Problem 2 Find/create a 2 inputs – 2 outputs (possibly physical) system where it is evident the necessity (advantage) or usefulness of scaling variables in a control problem.

Problem 3 Consider the three-mass system shown in Fig. 2 and described in terms of the relative displacements $z_i = \delta_i - \delta_i(0)$, $i = 1, \ldots, 3$ by the following set of linear differential equations (see the figure for the definition of the variables δ_i, $i = 1, \ldots, 3$)

$$\begin{aligned}
m_1\ddot{z}_1 &= -k_0 z_1 - k_1(z_1 - z_2) - \mu_1(\dot{z}_1 - \dot{z}_2) - d_1\dot{z}_1 + f_1 \\
m_2\ddot{z}_2 &= k_1(z_1 - z_2) - k_2(z_2 - z_3) + \mu_1(\dot{z}_1 - \dot{z}_2) - \mu_2(\dot{z}_2 - \dot{z}_3) - d_2\dot{z}_2 \\
m_3\ddot{z}_3 &= k_2(z_2 - z_3) + \mu_2(\dot{z}_2 - \dot{z}_3) - d_3\dot{z}_3 + f_3
\end{aligned}$$

with parameters m_i (mass), k_i (elastic coefficient), μ_i (damper coefficient between masses), d_i (friction with the terrain coefficient), f_1 a control force on the mass m_1 and f_3 a second control force on the mass m_3. Note the presence of a spring from m_1 to the wall with elastic coefficient k_0

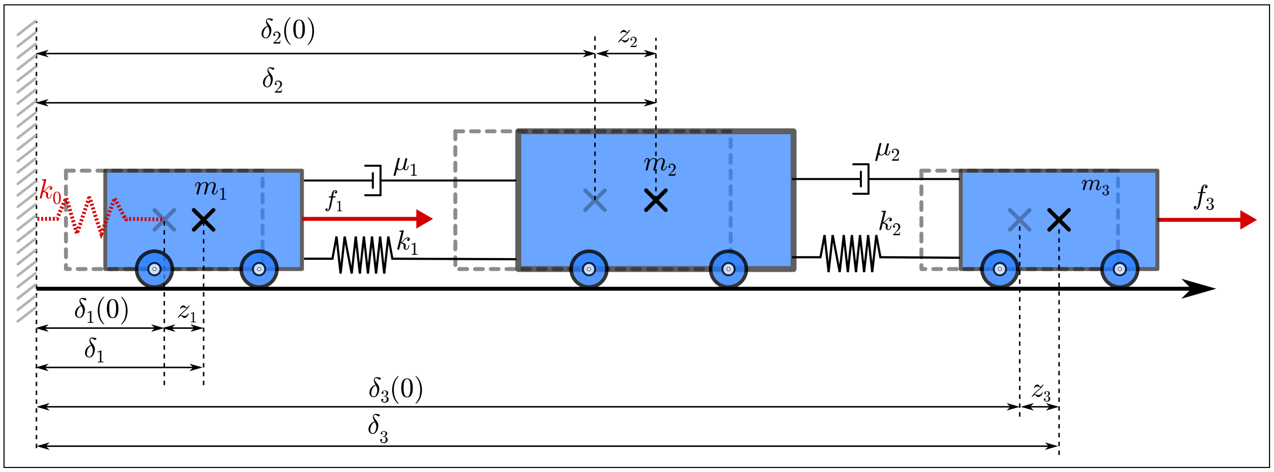

Figure 2: The three-mass system.

Main objectives:

1. Choose a nominal control problem with its performance weights, add measurement noise and a disturbance of your choice on one of the masses and finally build the augmented plant. Find a controller which guarantees this nominal performance.

2. Introduce uncertainties and show the difference between unstructured and structured uncertainty. Verify if the previous controller is also robustly stabilizing. If necessary find a controller using the SSV or see how much more uncertainty you can deal with using the SSV.

Possible values for a standard case (you can explore other situations where there is more or less friction and/or more or less rigid springs and or more differences in the masses):

$m_1 = 1$ kg, $m_2 = 1.5$ kg, $m_3 = 0.5$ kg
$k_0 = 5$ N/m, $k_1 = 10$ N/m, $k_2 = 100$ N/m
$\mu_1 = 5$ N.s/m, $\mu_2 = 15$ N.s/m
$d_1 = 0.1$ N.s/m, $d_2 = 0.2$ N.s/m, $d_3 = 0.1$ N.s/m.