Multivariable Feedback Control
Homework 01 - due Friday March 29th, 2024.

The goal of this homework is to get acquainted with the use of the performance weights $w_S(s)$, $w_U(s)$ and $w_T(s)$ in the definition of the closed loop specifications and solve basic $\mathcal{H}_\infty$ control problems.

We want to design a control system, based on $\mathcal{H}_\infty$ synthesis, which drives automatically the Mass-Spring-Damper system of Fig. 1 to a constant p_{des}. You already have a basic controller to compare with (see the sent file `MSD_Control_Effort_students.mlx`).

Choose your own system parameters (may be useful to consider the choice of these parameters also in view of the connection with a DC-motor).

1. First explore, using only a weight $w_S(s)$ on the sensitivity function, what reasonable performance you are able to achieve and compare the obtained results with a standard controller (PI, LQR or other of your choice). Does the resulting controller "cancel" the plant?

2. Add a control sensitivity weight $w_U(s)$ and analyze the consequences. Choose first a constant function and then move to a non-constant transfer function (think how to represent a bandwidth limitation).

3. Justify the introduction of an extra weight $w_T(s)$ on the complementary sensitivity.

4. Vary the type of reference: from a step reference to smooth functions with different slopes. Discuss the case of sinusoidal references and how the specifications/weights would change.

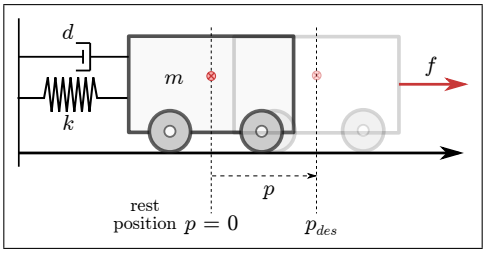

Figure 1: Mass-Spring-Damper system

Extensions:

1. Attach a second mass with spring and damper (this could have the effect of a disturbance, may be with some particular frequency profile?).

2. Let the force be provided by a DC-motor (either directly on the wheel or by a fixed mechanism). How does the control problem can be re-formulated?

Comments:

- there is a lot of flexibility in the choices you can make, be as clear as possible in explaining what you do; there is not a unique solution but rather a discussion.

- Do not fall into the trap of doing hundreds of simulations without any previous thoughts of what you should expect and stop to analyze why you have obtained a different result than the one planned.