INTERCONNECTION AND DAMPING ASSIGNMENT PASSIVITY-BASED CONTROL OF MECHANICAL SYSTEMS WITH UNDERACTUATION DEGREE ONE

PASSIVITY BASED POSITION CONTROL FOR VTOL DYNAMICS (PORT-HAMILTONIAN SYSTEM)

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What is a VTOL?

Vertical Take-off and Landing Aircraft

Doesn’t need a landing strip.
Can land and take-off like an helicopter

Jet Structure (F-35B)

Drone-Like Structure

Tri-rotor Structure

Used for what? EXPORT DEMOCRACY

How does it work?

Phase 1:
Takeoff

Phase 2:
Flight

Phase 3:
Landing

Our Goal:

Reach a desired position x_d with zero velocity

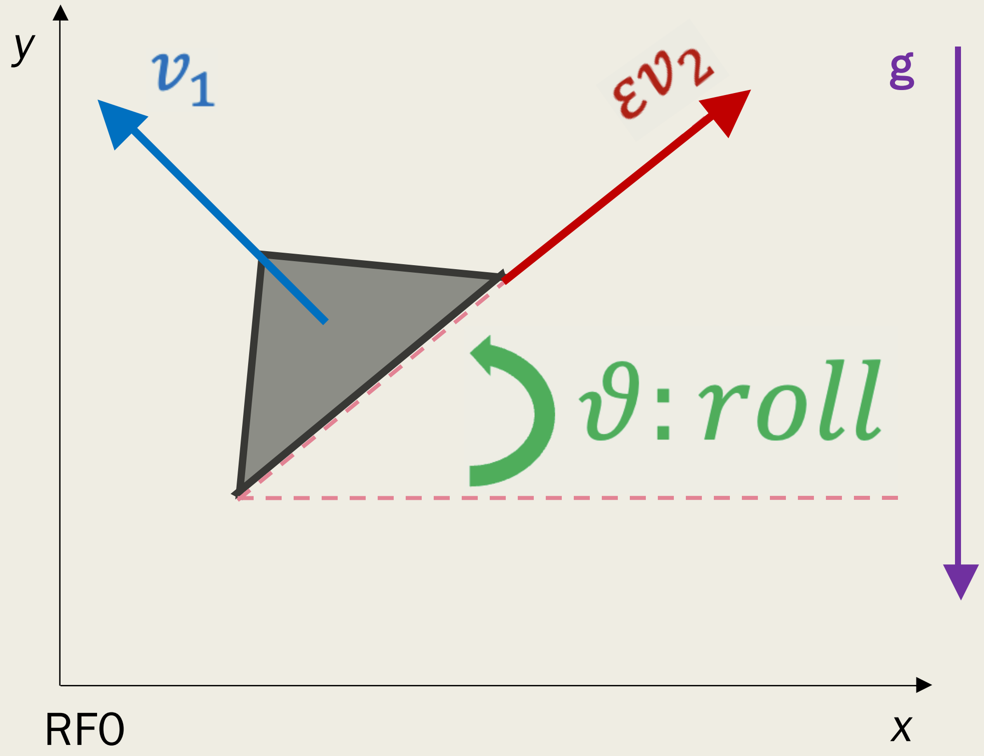
Start From an initial position x_0

Model

$$\begin{cases} \ddot{x} = -\sin\vartheta\, v_1 + \varepsilon\cos\vartheta\, v_2 \\ \ddot{y} = \cos\vartheta\, v_1 + \varepsilon\sin\vartheta\, v_2 - g \\ \ddot{\vartheta} = v_2 \end{cases}$$

v_1: Vertical Acceleration

v_2: (Angular) Roll Acceleration



Change of input

$$\begin{cases} \ddot{x} = -\sin\vartheta\, v_1 + \varepsilon\cos\vartheta\, v_2 \\ \ddot{y} = \cos\vartheta\, v_1 + \varepsilon\sin\vartheta\, v_2 - g \\ \ddot{\vartheta} = v_2 \end{cases}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = g \begin{bmatrix} \cos\vartheta \\ \sin\vartheta/\varepsilon \end{bmatrix} + \begin{bmatrix} -\sin\vartheta & \cos\vartheta \\ \cos\vartheta/\varepsilon & \sin\vartheta/\varepsilon \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Globally defined!

$$\begin{cases} \ddot{x} = u_1 \\ \ddot{y} = u_2 \\ \ddot{\vartheta} = \frac{1}{\varepsilon}\left(g\sin\vartheta + u_1\cos\vartheta + u_2\sin\vartheta\right) \end{cases}$$

State Space Representation

$$\begin{cases} q_1 := x \\ q_2 := y \\ q_3 := \vartheta \\ \hline p_1 := \dot{x} \\ p_2 := \dot{y} \\ p_3 := \dot{\vartheta} \end{cases} \Rightarrow \begin{cases} \dot{q_1} = p_1 \\ \dot{q_2} = p_2 \\ \dot{q_3} = p_3 \\ \dot{p_1} = u_1 \\ \dot{p_2} = u_2 \\ \dot{p_3} = \frac{g}{\varepsilon}\, sinq_3 + \frac{1}{\varepsilon} cosq_3\, u_1 + \frac{1}{\varepsilon} sinq_3\, u_2 \end{cases}$$

Compact Form

$$\begin{cases} \dot{q} = p \\ \dot{p} = \begin{bmatrix} 0 \\ 0 \\ \frac{g}{\varepsilon}\, sinq_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{\varepsilon} cosq_3 & \frac{1}{\varepsilon} sinq_3 \end{bmatrix}}_{G} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{cases}$$

Port-Hamiltonian form

$$\begin{cases} \dot{q} = \mathrm{p} \\ \dot{p} = \dfrac{g}{\varepsilon}\sin(q_3)\, e_3\, G\, u \end{cases}$$

We need 3 basic assumptions:

- $m = n - 1 = 2$
- $\exists\, G^{\perp} s.t{:}\;\; G^{\perp}\nabla_q(p^T M^{-1} p) = 0 \Rightarrow M\ doesn't\ depend\ on\ q$
- $G\ depends\ solely\ on\ q_3$

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{3\times 3} & I_{3\times 3} \\ -I_{3\times 3} & 0_{3\times 3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{g}{\varepsilon}\cos(q_3) \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} + \begin{bmatrix} 0_{3\times 2} \\ G_{3\times 2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Passivity

- **Storage Function:** $H(q,p) = \frac{1}{2}p^T M^{-1} p + \frac{g}{\varepsilon} cos(q_3)$

- **Passivating output:** $y = g^T \nabla H(q,p) = \begin{bmatrix} p_1 - \frac{p_3}{\varepsilon}\cos(q_3) \\ p_2 + \frac{p_3}{\varepsilon}\sin(q_3) \end{bmatrix}$

$$\dot{H}(q,p) = \nabla^T H(q,p) \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \mathrm{y}^{\mathrm{T}} u$$

The system is **passive** w.r.t this storage function and this passivating output.
In addiction the system is lossless as we expected, since there isn't any damping.

Necessity of using IDA-PBC

- $$\nabla H(q_d, p_d) = \begin{bmatrix} 0 \\ 0 \\ -\frac{g}{\varepsilon}\sin(q_3) \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}_{(q_d, p_d)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ ✔

- $$\nabla^2 H(q_d, p_d) = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g}{\varepsilon}\cos(q_3) & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]_{(q_d, p_d)} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g}{\varepsilon} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \not> 0$$ ✘

⇒ We need $\boldsymbol{u} = \boldsymbol{u_{ES}} + \boldsymbol{u_{DI}}$ to makes $\left(q_{1_d}, q_{2_d}, 0, 0, 0, 0\right)^T$ **asymptotically stable.**

What now?

REMEMBER:

Passive System! (Port-Hamiltonian Form)

Energy Shaping

+

Damping Injection

under

Zero State Detectability

=

Asymptotic Stability

Energy Shaping… (wishful thinking)

WHAT WE HAVE (OPEN LOOP)

Port Hamiltonian Form

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = [\mathcal{J} - \cancel{\mathcal{R}}]\begin{bmatrix}\nabla_q H\\ \nabla_p H\end{bmatrix} + \begin{bmatrix}0\\ G\end{bmatrix}u$$

Storage Function

H(q,p) such that (q^*,p^*) is <u>NOT</u> a minimum of H

H(q,p) only positive semidefinite (not a LF)

Physical Constraint

$\dot{q} = p$

WHAT WE WANT (CLOSED LOOP)

Desired Port Hamiltonian Form

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = [\mathcal{J}_d - \mathcal{R}_d]\begin{bmatrix}\nabla_q H_d\\ \nabla_p H_d\end{bmatrix} + \begin{bmatrix}0\\ G\end{bmatrix}v$$

Desired Storage Function

$H_d(q,p)$ such that (q^*, p^*) is a minimum of H_d

Choose $H_d(q,p)$ that qualifies as a Lyapunov Function

Physical Constraint (can't change)

$\dot{q} = p$

Interconnection and Damping Assignment

BEFORE ENERGY SHAPING:

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}0 & I\\ -I & 0\end{bmatrix}\begin{bmatrix}\nabla_q H\\ \nabla_p H\end{bmatrix} + \begin{bmatrix}0\\ G\end{bmatrix}u_{ES}$$

EQUATE

MATCHING EQUATION

$$\begin{bmatrix}0\\ Gu_{ES}\end{bmatrix} = \begin{bmatrix}-\nabla_p H\\ \nabla_q H\end{bmatrix} + \begin{bmatrix}M_d\nabla_p H_d\\ -M_d\nabla_q H_d\end{bmatrix}$$

AFTER ENERGY SHAPING:

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}0 & M_d\\ -M_d & 0\end{bmatrix}\begin{bmatrix}\nabla_q H_d\\ \nabla_p H_d\end{bmatrix}$$

There exist a solution IF AND ONLY IF

THE MATCHING CONDITION HOLDS

ENERGY SHAPING FEEDBACK

$$\boxed{u_{ES} = G^{\#}(\nabla_q H - M_d\nabla_q H_d)}$$

SOLVE PDE

MATCHING CONDITION

$$G^{\perp}(\nabla_q H - M_d\nabla_q H_d) = 0$$

What about Hd?

$$H_d = \frac{1}{2} p^T {M_d}^{-1}(q) p + V_d(q - q^*)$$

$$V_d = -\frac{g}{k_1 - k_2 \varepsilon} cos q_3 + \frac{1}{2} \varphi(q - q^*)^T P \varphi(q - q^*)$$

RECENTER Potential Energy in q^*

$$M_d = \begin{bmatrix} k_1 \varepsilon\, cos^2 q_3 + k_3 & k_1 \varepsilon\, cos q_3 sin q_3 & k_1\, cos q_3 \\ k_1 \varepsilon\, cos q_3 sin q_3 & -k_1 \varepsilon\, cos^2 q_3 + k_3 & k_1\, sin q_3 \\ k_1\, cos q_3 & k_1\, sin q_3 & k_2 \end{bmatrix}$$

Needed due to underactuation

Doesn't change the physical contstraint:

$$\dot{q} = M_d \nabla_p H_d = M_d {M_d}^{-1} p = p$$

Zero State Detectability

To use the damping injection, we need that the system is Zero State Detectable:

- Computing the new passivating output $y_d = g^T \nabla H_d(q, p)$
- And putting $y_d = 0$

$$\boldsymbol{M} = \left\{ \left(\boldsymbol{q_{1_d}}, \boldsymbol{q_{2_d}}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0} \right)^{\boldsymbol{T}} \right\}$$

$\Rightarrow$ the system is **Z.S.D** and moreover it is Z.S.O

Damping Injection

Since we have proved that the system is Z.S.D, we can apply the damping injection:

$$\boldsymbol{u_{DI}} = -\boldsymbol{K_v y_d} = -\begin{bmatrix} \boldsymbol{k_{v_{11}}} & \boldsymbol{k_{v_{12}}} \\ \boldsymbol{k_{v_{21}}} & \boldsymbol{k_{v_{22}}} \end{bmatrix} \begin{bmatrix} \boldsymbol{y_{d_1}} \\ \boldsymbol{y_{d_2}} \end{bmatrix}$$

and now the point $\left(q_{1_d}, q_{2_d}, 0, 0, 0, 0\right)^T$ is **G.A.S** for the system.