PS Signal Processing

Cross-Correlation

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Correlation

Correlation is the degree of similarity between two time series or signal in the same time or sequence while no lag is considered.

$$c_{12} = \sum_{n=0}^{N-1} x[n]y[n]$$

Sum of products of each element

- Negative values means one variable increases while the other decreases
- The result depends on the number of samples of the signal

An alternative expression could be.

$$c_{12} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]y[n]$$

Calculate the correlation of the following vectors:

n	0	1	2	3	4	5	6	7	8
$x_1[n]$	4	2	-1	3	-2	-6	-5	4	5
$x_2[n]$	-4	1	3	7	4	-2	-8	-2	1

$$c_{12} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]y[n]$$
$$= \frac{1}{9}[4 * -4 + 2 * 1 + -1 * 3 + 3$$
$$* 7 + -2 * 4 + -6 * -2 + -5 * 8$$
$$+ 4 * -2 + 5 * 1]$$

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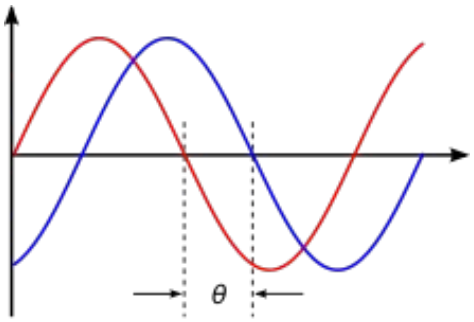
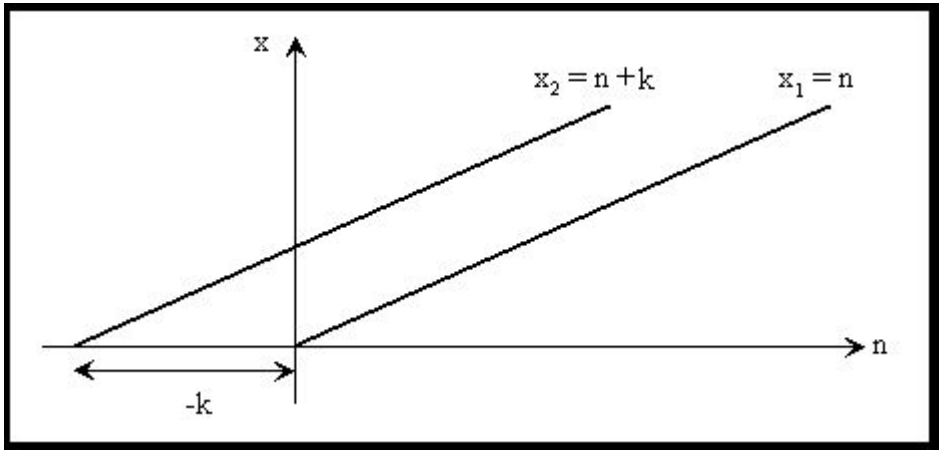


What kind of problem could be arises in this calculation?
The correlation of two signals could be zero and still being correlated each other.

Cross-Correlation

- In order to solve this problem, it is necessary rotate or introduce a lag in one of the signals

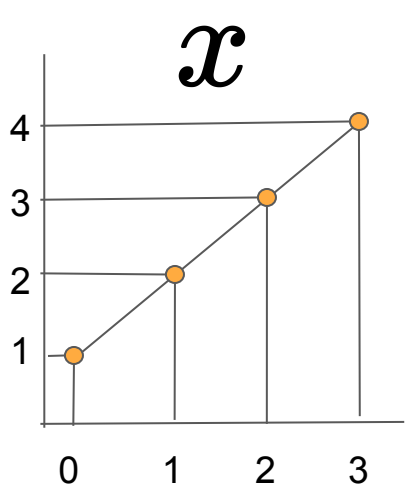


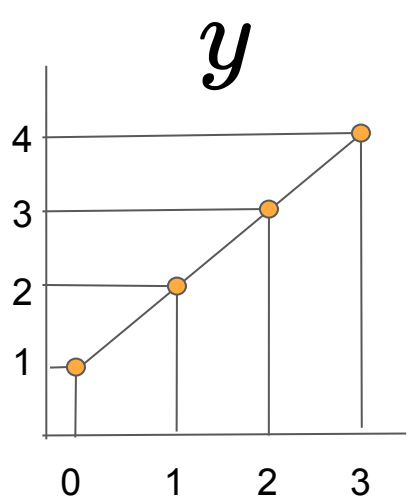
Cross-Correlation

$$z[k] = (x * y)(k - N + 1) = \sum_{l=0}^{||x||-1} x_l y^*_{l-k+N-1}$$

for $k = 0, 1, \ldots, ||x|| + ||y|| - 2$

where $||x||$ is the length of x, $N = \max(||x||, ||y||)$, and y_m is 0 when m is outside the range of y.





$||x|| = ?$

$N = ?$

$k = ?$

Cross-Correlation

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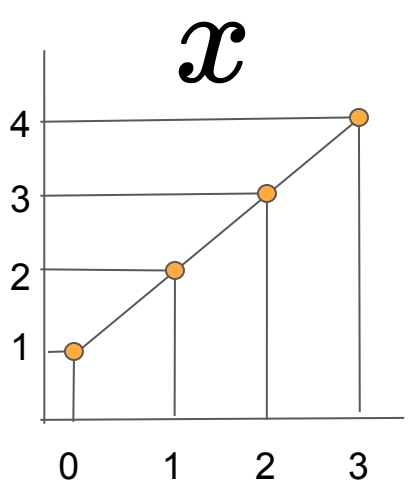
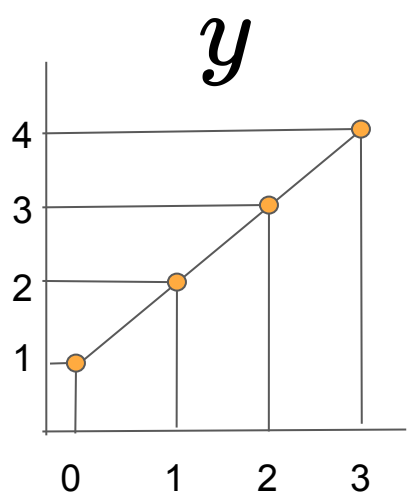




$$||x|| = 4$$

$$N = 4$$

$$k = 0, 1, .., 6$$

Calculate the Cross-Correlation

$$\|x\| = 4$$

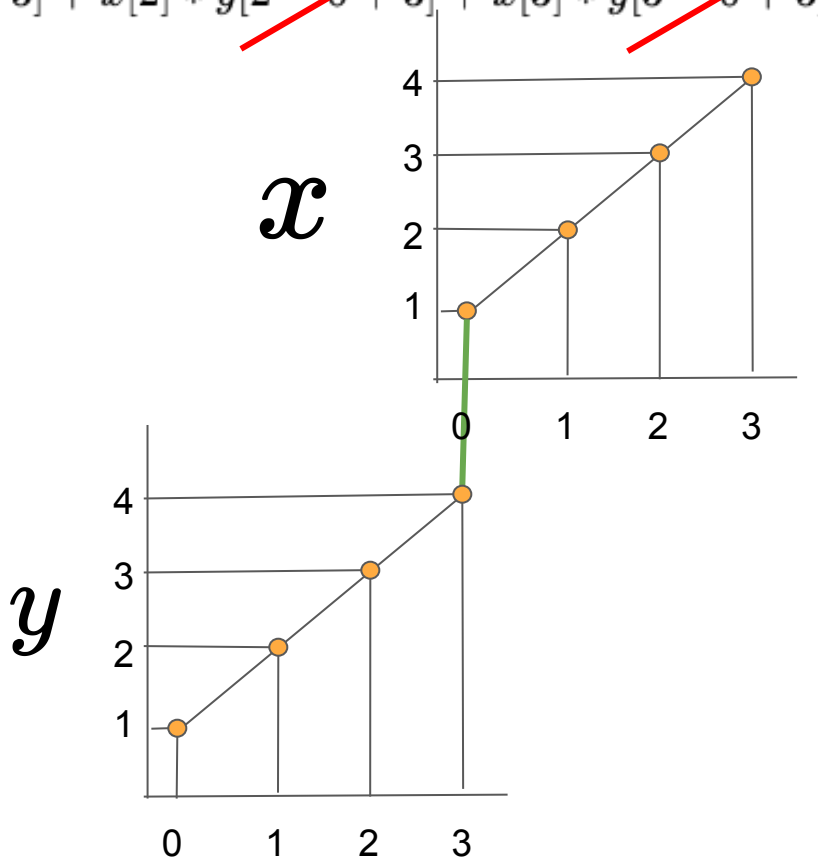
$$N = 4$$

$$z[k] = (x * y)(k - N + 1) = \sum_{l=0}^{\|x\|-1} x_l y^*_{l-k+N-1}$$

$k = 0$

$$z[k=0] = x[0] * y[0 - 0 + 3] + x[1] * y[1 - 0 + 3] + x[2] * y[2 - 0 + 3] + x[3] * y[3 - 0 + 3]$$

$$z[k=0] = 4 * 1$$



Calculate the Cross-Correlation

$$\|x\| = 4$$

$$N = 4$$

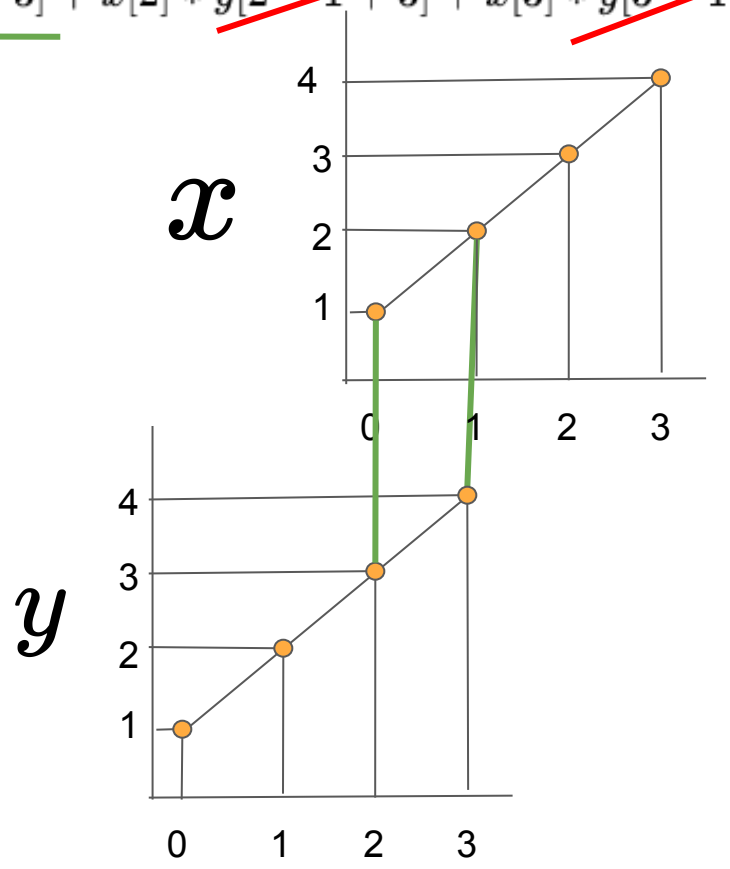
$$z[k] = (x * y)(k - N + 1) = \sum_{l=0}^{\|x\|-1} x_l y^*_{l-k+N-1}$$

$$k = 1$$

$$z[k=1] = x[0] * y[0 - 1 + 3] + x[1] * y[1 - 1 + 3] + x[2] * y[2 - 1 + 3] + x[3] * y[3 - 1 + 3]$$

$$z[k=1] = 1 * 3 + 2 * 4$$

$$z[k=1] = 11$$



Calculate the Cross-Correlation

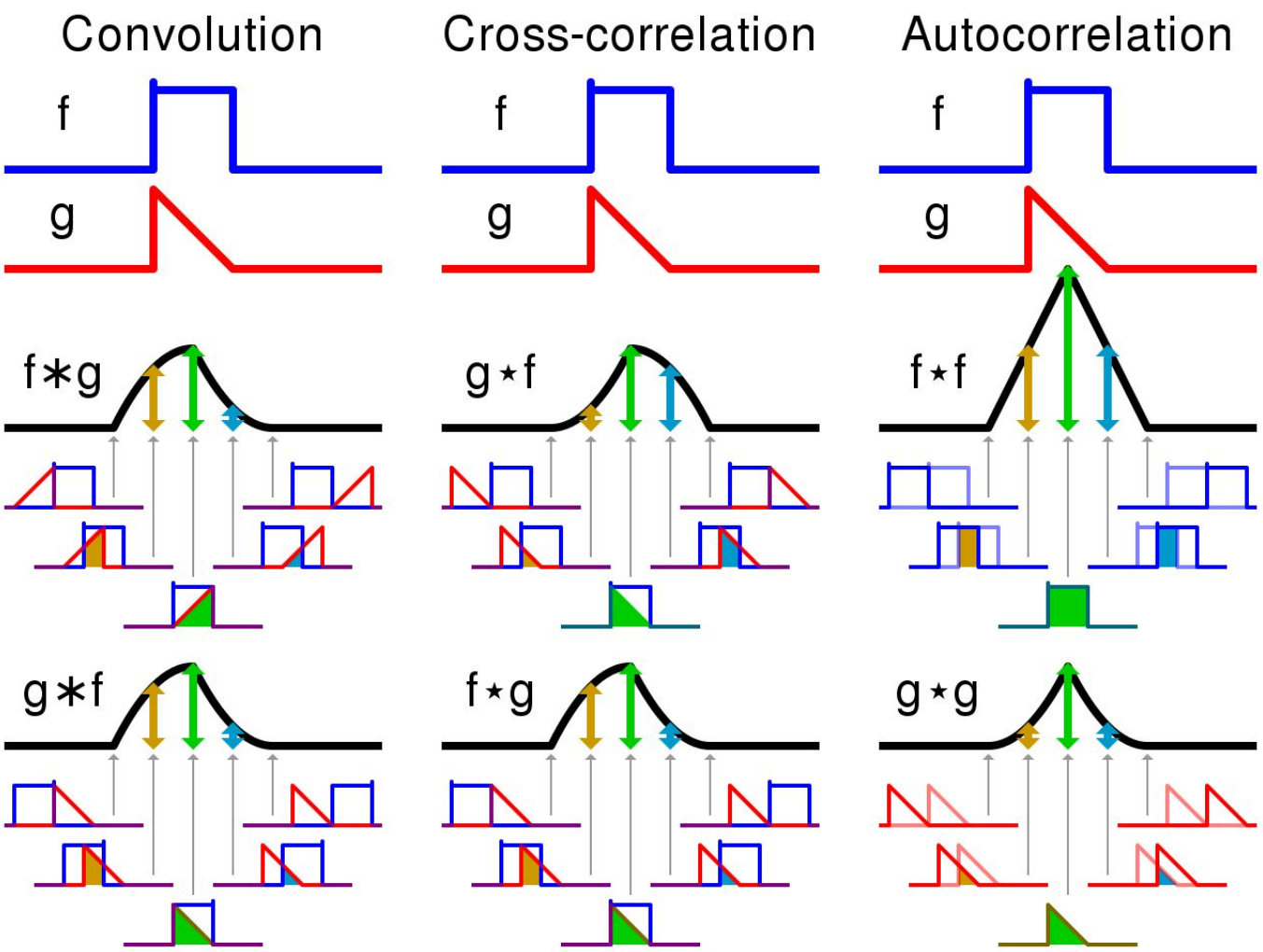
Calculate the missing values:

$$z = [4, 11, -, -, --, --, --]$$

Calculate the Cross-Correlation

Calculate the missing values:

$$z = [4, 11, 20, 30, 20, 11, 4]$$


Convolution
Cross-correlation
Autocorrelation
f
g
f∗g
g⋆f
f⋆f
g∗f
f⋆g
g⋆g