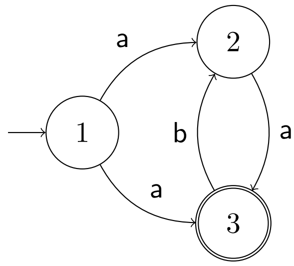
Regular Expressions and NFAs (20 pts)

Consider the following NFA $\mathcal{A}$.



1. State and give the intuition behind the proof of *Arden's Lemma* that we used in the a method to represent the language accepted by an NFA as a regular expression. This method is also called *Brzozowski method*. (10 pts)

2. Describe $\mathsf{L}(\mathcal{A})$ as a regular expression, using the Brzozowski method. (10 pts)

Weak Monadic Second-Order Logic (30 pts)

1. Define WMSO formulas $\phi_1(x, y)$ and $\phi_2(x, y)$ respectively, describing the following propositions:
   - $x = y$ (clearly without using = itself) (5 pts)
   - $y = x + k$ (for fixed $k \in \mathbb{N}$) (5 pts)

2. Define a WMSO formula ψ (over second-order variables $\{P_{\mathsf{a}} \mid \mathsf{a} \in \Sigma\}$), such that

$$\mathsf{L}(\psi) = \{w \mid \text{on each } k^{\text{th}} \text{ position of } w \text{ there is an } \mathsf{a}\} .$$

   Hint: To solve this part of the question, you may use the assertions of the first part. (10 pts)

3. A WMSO formula ϕ is called *positive*, if ϕ doesn't contain negations. Prove that satisfiability of positive WMSO formulas can be decided in polynomial time. (10 pts)

Alternating Finite Automata (20 pts)

1. Let M be a subset of states Q. M is called *minimal model* of $f \in \mathbb{B}^+(Q)$, if $M \models f$ but for all $N \subsetneq M$, $N \not\models f$.

   Prove or give a counter-example that each $f \in \mathbb{B}^+(Q)$ has a minimal model. (10 pts)

2. Let $\Sigma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$. Consider the language

   $$L = \{w \in \Sigma^* \mid \text{each } \mathsf{b} \text{ is eventually followed by a } \mathsf{a}\} .$$

   Define a memoryless AFA $\mathcal{A}$ accepting L. (10 pts)

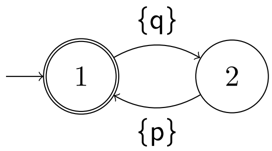
Reachability Games (10 pts)

For this question, we define reachability games between player P_0 and P_1 as *finite, acyclic directed graphs* $G = (V, V_0, V_1, E)$ with nodes V, edges E such that $V = V_0 \cup V_1$ and $V_0 \cap V_1 = \varnothing$. Let $v_0, \dots, v_n$ denote a play. Then Player P_i wins the play, if $v_n \in V_{1-i}$.

Prove that either player P_0 or P_1 has a winning strategy, that is, such reachability games are determined. (10 pts)

Büchi Automata & Linear Temporal Logic (20 pts)

Consider the following NBA $\mathcal{A}$



1. Define $\mathsf{L}(\mathcal{A})$, that is, state the language accepted by $\mathcal{A}$ as a (extended) regular expression (for infinite words). (10 pts)

2. Define an LTL-formula ϕ such that $\mathsf{L}(\phi) = \mathsf{L}(\mathcal{A})$. (10 pts)