Contents

1 Basic

1.1 .vimrc

```
syntax on
set nu ai bs=2 sw=2 ts=2 et ve=all cb=unnamed mouse=a
    ruler incsearch hlsearch
```

1.2 IncStack

```
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
  const rlim_t ks = 64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT_STACK, &rl);
  if(res==0){
    if(rl.rlim_cur<ks){
      rl.rlim_cur=ks;
      res=setrlimit(RLIMIT_STACK, &rl);
    }
  }
```

1.3 IncStack windows

```
//stack resize
asm( "mov %0,%%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system
```

1.4 random

```
#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }
```

1.5 time

```
cout << 1.0 * clock() / CLOCKS_PER_SEC;
```

2 Math

2.1 basic

```
PLL exd_gcd(LL a, LL b) {
  if (a % b == 0) return {0, 1};
  PLL T = exd_gcd(b, a % b);
  return {T.second, T.first - a / b * T.second};
}
LL powmod(LL x, LL p, LL mod) {
  LL s = 1, m = x % mod;
  for (; p; m = m * m % mod, p >>= 1)
    if (p&1) s = s * m % mod; // or consider int128
  return s;
}
LL LLmul(LL x, LL y, LL mod) {
  LL m = x, s = 0;
  for (; y; y >>= 1, m <<= 1, m = m >= mod? m - mod: m)
    if (y&1) s += m, s = s >= mod? s - mod: s;
  return s;
}
LL dangerous_mul(LL a, LL b, LL mod){ // 10 times
    faster than the above in average, but could be
    prone to wrong answer (extreme low prob?)
  return (a * b - (LL)((long double)a * b / mod) * mod)
      % mod;
}
vector<LL> linear_inv(LL p, int k) { // take k
```

```
  vector<LL> inv(min(p, 1ll + k));
  inv[1] = 1;
  for (int i = 2; i < inv.size(); ++i)
    inv[i] = (p - p / i) * inv[p % i] % p;
  return inv;
}
```

2.2 Chinese Remainder Theorem

```
PLL CRT(PLL eq1, PLL eq2) {
  LL m1, m2, x1, x2;
  tie(x1, m1) = eq1, tie(x2, m2) = eq2;
  LL g = __gcd(m1, m2);
  if ((x1 - x2) % g) return {-1, 0}; // NO SOLUTION
  m1 /= g, m2 /= g;
  auto p = exd_gcd(m1, m2);
  LL lcm = m1 * m2 * g, res = mul(mul(p.first, (x2 - x1
      ), lcm), m1, lcm) + x1;
  return {(res % lcm + lcm) % lcm, lcm};
}
```

2.3 Discrete Log

```
LL discrete_log(LL b, LL p, LL n) {
  map<LL, LL> att;
  LL m = sqrt((double)p) + 1, M = powmod(b, m * (p - 2)
      , p);
  for (LL cur = 1, i = 0; i < m; ++i, cur = cur * b % p
      )
    if (not att.count(cur)) att[cur] = i;
  for (LL cur = 1, i = 0; i * m < p - 1; ++i, cur = cur
       * M % p)
    if (att.count(n * cur % p))
      return (att[cur * n % p] + i * m) % (p - 1);
  return -1;
}
// find x s.t. b**x % p == n with complexity O(sqrt(N))
// return the smallest
// return -1 if ans doesn't exist
```

2.4 Discrete Kth root

```
/*
 * Solve x for x^P = A mod Q
 * https://arxiv.org/pdf/1111.4877.pdf
 * in O((lgQ)^2 + Q^0.25 (lgQ)^3)
 * Idea:
 * (P, Q-1) = 1 -> P^-1 mod (Q-1) exists
 * x has solution iff A^((Q-1) / P) = 1 mod Q
 * PP | (Q-1) -> P < sqrt(Q), solve lgQ rounds of
     discrete log
 * else -> find a s.t. s | (Pa - 1) -> ans = A^a
 */
void gcd(LL a, LL b, LL& x, LL& y, LL& g) {
  if (b == 0) {
    x = 1, y = 0, g = a;
    return;
  }
  LL tx, ty;
  gcd(b, a % b, tx, ty, g);
  x = ty;
  y = tx - ty * (a / b);
  return;
}
LL P, A, Q, g;
// x^P = A mod Q

const int X = 1e5;

LL base;
LL ae[X], aXe[X], iaXe[X];
unordered_map<LL, LL> ht;

void build(LL a) {  // ord(a) = P < sqrt(Q)
  base = a;
  ht.clear();
  ae[0] = 1;
  ae[1] = a;
  aXe[0] = 1;
  aXe[1] = pw(a, X, Q);
  iaXe[0] = 1;
  iaXe[1] = pw(aXe[1], Q - 2, Q);
  REP(i, 2, X - 1) {
    ae[i] = mul(ae[i - 1], ae[1], Q);
    aXe[i] = mul(aXe[i - 1], aXe[1], Q);
    iaXe[i] = mul(iaXe[i - 1], iaXe[1], Q);
  }
  FOR(i, X)
  ht[ae[i]] = i;
}

LL dis_log(LL x) {
  FOR(i, X) {
    LL iaXi = iaXe[i];
    LL rst = mul(x, iaXi, Q);
    if (ht.count(rst)) {
      LL res = i * X + ht[rst];
      return res;
    }
  }
}

LL main2() {
  LL t = 0, s = Q - 1;
  while (s % P == 0) {
    ++t;
    s /= P;
  }
  if (A == 0) return 0;

  if (t == 0) {
    // a^{P^-1 mod phi(Q)}
    LL x, y, _;
    gcd(P, Q - 1, x, y, _);
    if (x < 0) {
      x = (x % (Q - 1) + Q - 1) % (Q - 1);
    }
    LL ans = pw(A, x, Q);
    if (pw(ans, P, Q) != A)
      while (1)
        ;
    return ans;
  }

  // A is not P-residue
  if (pw(A, (Q - 1) / P, Q) != 1) return -1;

  for (g = 2; g < Q; ++g) {
    if (pw(g, (Q - 1) / P, Q) != 1) break;
  }
  LL alpha = 0;
  {
    LL y, _;
    gcd(P, s, alpha, y, _);
    if (alpha < 0) alpha = (alpha % (Q - 1) + Q - 1) %
        (Q - 1);
  }

  if (t == 1) {
    LL ans = pw(A, alpha, Q);
    return ans;
  }

  LL a = pw(g, (Q - 1) / P, Q);
  build(a);
  LL b = pw(A, add(mul(P % (Q - 1), alpha, Q - 1), Q -
      2, Q - 1), Q);
  LL c = pw(g, s, Q);
  LL h = 1;

  LL e = (Q - 1) / s / P;  // r^{t-1}
  REP(i, 1, t - 1) {
    e /= P;
    LL d = pw(b, e, Q);
    LL j = 0;
    if (d != 1) {
      j = -dis_log(d);
```

```
      if (j < 0) j = (j % (Q - 1) + Q - 1) % (Q - 1);
    }
    b = mul(b, pw(c, mul(P % (Q - 1), j, Q - 1), Q), Q)
      ;
    h = mul(h, pw(c, j, Q), Q);
    c = pw(c, P, Q);
  }

  LL ans = mul(pw(A, alpha, Q), h, Q);

  return ans;
}
```

2.5 FFT

```
typedef complex<double> cpx;
const double PI = acos(-1);
vector<cpx> FFT(vector<cpx> &P, bool inv = 0) {
  assert(__builtin_popcount(P.size()) == 1);
  int lg = 31 - __builtin_clz(P.size()), n = 1 << lg;
      // == P.size();
  for (int j = 1, i = 0; j < n - 1; ++j) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(P[i], P[j]);
  } //bit reverse
  auto w1 = exp((2 - 4 * inv) * PI / n * cpx(0, 1)); //
       order is 1<<lg
  for (int i = 1; i <= lg; ++i) {
    auto wn = pow(w1, 1<<(lg - i)); // order is 1<<i
    for (int k = 0; k < (1<<lg); k += 1 << i) {
      cpx base = 1;
      for (int j = 0; j < (1 << i - 1); ++j, base =
          base * wn) {
        auto t = base * P[k + j + (1 << i - 1)];
        auto u = P[k + j];
        P[k + j] = u + t;
        P[k + j + (1 << i - 1)] = u - t;
      }
    }
  }
  if(inv)
    for (int i = 0; i < n; ++i) P[i] /= n;
  return P;
} //faster performance with calling by reference
```

2.6 FWT

```
vector<LL> fast_OR_transform(vector<LL> f, bool inverse
    ) {
  for (int i = 0; (2 << i) <= f.size(); ++i)
    for (int j = 0; j < f.size(); j += 2 << i)
      for (int k = 0; k < (1 << i); ++k)
        f[j + k + (1 << i)] += f[j + k] * (inverse? -1
            : 1);
  return f;
}
vector<LL> rev(vector<LL> A) {
  for (int i = 0; i < A.size(); i += 2) swap(A[i], A[i
      ^ (A.size() - 1)]);
  return A;
}
vector<LL> fast_AND_transform(vector<LL> f, bool
    inverse) {
  return rev(fast_OR_transform(rev(f), inverse));
}
vector<LL> fast_XOR_transform(vector<LL> f, bool
    inverse) {
  for (int i = 0; (2 << i) <= f.size(); ++i)
    for (int j = 0; j < f.size(); j += 2 << i)
      for (int k = 0; k < (1 << i); ++k) {
        int u = f[j + k], v = f[j + k + (1 << i)];
        f[j + k + (1 << i)] = u - v, f[j + k] = u + v;
      }
  if (inverse) for (auto &a : f) a /= f.size();
  return f;
}
```

2.7 Gauss Lagrange Eisenstein reduced form

```
// To find min f(x, y) = a * x * x + b * x * y + c * y
    * y
// (x, y) <- Z^2 nonzero
// return (x, y)
 PLL form(LL a, LL b, LL c) {
  assert(b * b < 4 * a * c and a > 0);
  LL x, y;
  if (a > c) return tie(x, y) = form(c, b, a), {y, x};
  if (a == c and b < 0) return tie(x, y) = form(a, -b,
      c), {-x, y};
  if (b > a or b <= -a) {
    LL n = (a - b) / (2 * a);
    // -a < 2 * a * n + b <= a
    if (2 * a * n > a - b) --n;
    tie(x, y) = form(a, 2 * a * n + b, a * n * n + b *
        n + c);
    return {x - n * y, y};
  }
  // 1 <= a <= c and -a < b <= a and (a == c implies b
      >= 0)
  return {1, 0};
}
```

2.8 Lagrange Polynomial

```
struct Lagrange_poly {
  vector<LL> fac, p;
  int n;
  Lagrange_poly(vector<LL> p) : p(p) {
    n = p.size();
    fac.resize(n), fac[0] = 1;
    for (int i = 1; i < n; ++i) fac[i] = fac[i - 1] * i
        % MOD;
  }
  LL solve(LL x) {
    if (x < n) return p[x];
    LL ans = 0, to_mul = 1;
    for (int j = 0; j < n; ++j) (to_mul *= MOD - x + j)
        %= MOD;
    for (int j = 0; j < n; ++j) {
      (ans += p[j] * to_mul % MOD *
      powmod(MOD - x + j, MOD - 2, MOD) % MOD *
      powmod(fac[n - 1 - j], MOD - 2, MOD) % MOD *
      powmod(j&1? MOD - fac[j]: fac[j], MOD - 2, MOD))
          %= MOD;
    }
    return ans;
  }
};
```

2.9 Lucas

```
LL fac[100000] = {1};
LL C(LL a, LL b, LL p) {
  for (int i = 1; i <= p; ++i) fac[i] = fac[i - 1] * i
      % p;
  LL ans = 1;
  for (;a; a /= p, b /= p) {
    LL A = a % p, B = b % p;
    if (A < B) return 0;
    (ans *= fac[A] * powmod(fac[B] * fac[A - B] % p, p
        - 2, p) % p) %= p;
  }
  return ans;
}
```

2.10 Meissel-Lehmer PI

```
LL PI(LL m);
const int MAXM = 1000, MAXN = 650, UPBD = 1000000;
// 650 ~ PI(cbrt(1e11))
```

```
LL pi[UPBD] = {0}, phi[MAXM][MAXN];
vector<LL> primes;
void init() {
  fill(pi + 2, pi + UPBD, 1);
  for (LL p = 2; p < UPBD; ++p)
    if (pi[p]) {
      for (LL N = p * p; N < UPBD; N += p)
        pi[N] = 0;
      primes.push_back(p);
    }
  for (int i = 1; i < UPBD; ++i) pi[i] += pi[i - 1];
  for (int i = 0; i < MAXM; ++i)
    phi[i][0] = i;
  for (int i = 1; i < MAXM; ++i)
    for (int j = 1; j < MAXN; ++j)
      phi[i][j] = phi[i][j - 1] - phi[i / primes[j -
          1]][j - 1];
}
LL P_2(LL m, LL n) {
  LL ans = 0;
  for (LL i = n; primes[i] * primes[i] <= m and i <
      primes.size(); ++i)
    ans += PI(m / primes[i]) - i;
  return ans;
}
LL PHI(LL m, LL n) {
  if (m < MAXM and n < MAXN) return phi[m][n];
  if (n == 0) return m;
  LL p = primes[n - 1];
  if (m < UPBD) {
    if (m <= p) return 1;
    if (m <= p * p * p) return pi[m] - n + 1 + P_2(m, n
        );
  }
  return PHI(m, n - 1) - PHI(m / p, n - 1);
}
LL PI(LL m) {
  if (m < UPBD) return pi[m];
  LL y = cbrt(m) + 10, n = pi[y];
  return PHI(m, n) + n - 1 - P_2(m, n);
}
```

2.11 Miller Rabin with Pollard rho

```
bool miller_rabin(LL n, int s = 7) {
  const LL wits[7] = {2, 325, 9375, 28178, 450775,
      9780504, 1795265022};
  auto witness = [=](LL a, LL n, LL u, int t) {
    LL x = powmod(a, u, n), nx; // use LLmul, remember
    for (int i = 0; i < t; ++i, x = nx){
      nx = LLmul(x, x, n);
      if (nx == 1 and x != 1 and x != n - 1) return
          true;
    }
    return x != 1;
  };
  if (n < 2) return 0;
  if (n&1^1) return n == 2;
  LL u = n - 1, t = 0, a; // n == (u << t) + 1
  while (u&1^1) u >>= 1, ++t;
  while (s--)
    if ((a = wits[s] % n) and witness(a, n, u, t))
      return 0;
  return 1;
}
// Pollard_rho
LL pollard_rho(LL n) {
  auto f = [=](LL x, LL n) { return LLmul(x, x, n) + 1;
      };
  if (n&1^1) return 2;
  while (true) {
    LL x = rand() % (n - 1) + 1, y = 2, d = 1;
    for (int sz = 2; d == 1; y = x, sz <<= 1)
      for (int i = 0; i < sz and d <= 1; ++i)
        x = f(x, n), d = __gcd(abs(x - y), n);
    if (d and n - d) return d;
  }
}
vector<pair<LL, int>> factor(LL m) {
  vector<pair<LL, int>> ans;
  while (m != 1) {
    LL cur = m;
    while (not miller_rabin(cur)) cur = pollard_rho(cur
        );
    ans.emplace_back(cur, 0);
    while (m % cur == 0) ++ans.back().second, m /= cur;
  }
  sort(ans.begin(), ans.end());
  return ans;
}
```

2.12 Mod Mul Group Order

```
#include "Miller_Rabin_with_Pollard_rho.cpp"
LL phi(LL m) {
  auto fac = factor(m);
  return accumulate(fac.begin(), fac.end(), m, [](LL a,
      pair<LL, int> p_r) {
    return a / p_r.first * (p_r.first - 1);
  });
}
LL order(LL x, LL m) {
  // assert(__gcd(x, m) == 1);
  LL ans = phi(m);
  for (auto P: factor(ans)) {
    LL p = P.first, t = P.second;
    for (int i = 0; i < t; ++i) {
      if (powmod(x, ans / p, m) == 1) ans /= p;
      else break;
    }
  }
  return ans;
}
LL cycles(LL a, LL m) {
  if (m == 1) return 1;
  return phi(m) / order(a, m);
}
```

2.13 NTT

```
/* p == (a << n) + 1
   n    1 << n      p           a    root
   5    32          97          3    5
   6    64          193         3    5
   7    128         257         2    3
   8    256         257         1    3
   9    512         7681        15   17
   10   1024        12289       12   11
   11   2048        12289       6    11
   12   4096        12289       3    11
   13   8192        40961       5    3
   14   16384       65537       4    3
   15   32768       65537       2    3
   16   65536       65537       1    3
   17   131072      786433      6    10
   18   262144      786433      3    10 (605028353,
      2308, 3)
   19   524288      5767169     11   3
   20   1048576     7340033     7    3
   21   2097152     23068673    11   3
   22   4194304     104857601   25   3
   23   8388608     167772161   20   3
   24   16777216    167772161   10   3
   25   33554432    167772161   5    3 (1107296257, 33,
      10)
   26   67108864    469762049   7    3
   27   134217728   2013265921  15   31 */
LL root = 10, p = 786433, a = 3;
LL powM(LL x, LL b) {
  LL s = 1, m = x % p;
  for (; b; m = m * m % p, b >>= 1)
    if (b&1) s = s * m % p;
  return s;
}
vector<LL> NTT(vector<LL> P, bool inv = 0) {
  assert(__builtin_popcount(P.size()) == 1);
  int lg = 31 - __builtin_clz(P.size()), n = 1 << lg;
      // == P.size();
```

```
  for (int j = 1, i = 0; j < n - 1; ++j) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(P[i], P[j]);
  } //bit reverse
  LL w1 = powM(root, a * (inv? p - 2: 1)); // order is 1<<lg
  for (LL i = 1; i <= lg; ++i) {
    LL wn = powM(w1, 1<<(lg - i)); // order is 1<<i
    for (int k = 0; k < (1<<lg); k += 1 << i) {
      LL base = 1;
      for (int j = 0; j < (1 << i - 1); ++j, base = base * wn % p) {
        LL t = base * P[k + j + (1 << i - 1)] % p;
        LL u = P[k + j] % p;
        P[k + j] = (u + t) % p;
        P[k + j + (1 << i - 1)] = (u - t + p) % p;
      }
    }
  }
  if(inv){
    LL invN = powM(n, p - 2);
    transform(P.begin(), P.end(), P.begin(), [&](LL a) {return a * invN % p;});
  }
  return P;
} //faster performance with calling by reference
```

2.14 Number Theory Functions

```
vector<bool> Atkin_sieve(int limit) {
  assert(limit > 10 and limit <= 1e9);
  vector<bool> sieve(limit, false);
  sieve[2] = sieve[3] = true;
  for (int x = 1; x * x < limit; ++x)
    for (int y = 1; y * y < limit; ++y) {
      int n = (4 * x * x) + (y * y);
      if (n <= limit && (n % 12 == 1 || n % 12 == 5))
        sieve[n] = sieve[n] ^ true;
      n = (3 * x * x) + (y * y);
      if (n <= limit && n % 12 == 7)
        sieve[n] = sieve[n] ^ true;
      n = (3 * x * x) - (y * y);
      if (x > y && n <= limit && n % 12 == 11)
        sieve[n] = sieve[n] ^ true;
    }
  for (int r = 5; r * r < limit; ++r) if (sieve[r])
    for (int i = r * r; i < limit; i += r * r)
      sieve[i] = false;
  return sieve;
}
vector<bool> Eratosthenes_sieve(int limit) {
  assert(limit >= 10 and limit <= 1e9);
  vector<bool> sieve(limit, true);
  sieve[0] = sieve[1] = false;
  for (int p = 2; p * p < limit; ++p) if (sieve[p]) {
    for (int n = p * p; n < limit; n += p) sieve[n] = false;
  }
  return sieve;
}
template<typename T> vector<T> make_mobius(T limit) {
  auto is_prime = Eratosthenes_sieve(limit);
  vector<T> mobius(limit, 1);
  mobius[0] = 0;
  for (LL p = 2; p < limit; ++p) if (is_prime[p]) {
    for (LL n = p; n < limit; n += p)
      mobius[n] = -mobius[n];
    for (LL n = p * p; n < limit; n += p * p)
      mobius[n] = 0;
  }
  return mobius;
}
```

2.15 Polynomail root

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[10], x[10];
int n;
int sign(double x) { return (x < -eps) ? (-1) : (x > eps); }
double f(double a[], int n, double x) {
  double tmp = 1, sum = 0;
  for (int i = 0; i <= n; i++) {
    sum = sum + a[i] * tmp;
    tmp = tmp * x;
  }
  return sum;
}
double binary(double l, double r, double a[], int n) {
  int sl = sign(f(a, n, l)), sr = sign(f(a, n, r));
  if (sl == 0) return l;
  if (sr == 0) return r;
  if (sl * sr > 0) return inf;
  while (r - l > eps) {
    double mid = (l + r) / 2;
    int ss = sign(f(a, n, mid));
    if (ss == 0) return mid;
    if (ss * sl > 0)
      l = mid;
    else
      r = mid;
  }
  return l;
}
void solve(int n, double a[], double x[], int &nx) {
  if (n == 1) {
    x[1] = -a[0] / a[1];
    nx = 1;
    return;
  }
  double da[10], dx[10];
  int ndx;
  for (int i = n; i >= 1; i--) da[i - 1] = a[i] * i;
  solve(n - 1, da, dx, ndx);
  nx = 0;
  if (ndx == 0) {
    double tmp = binary(-inf, inf, a, n);
    if (tmp < inf) x[++nx] = tmp;
    return;
  }
  double tmp;
  tmp = binary(-inf, dx[1], a, n);
  if (tmp < inf) x[++nx] = tmp;
  for (int i = 1; i <= ndx - 1; i++) {
    tmp = binary(dx[i], dx[i + 1], a, n);
    if (tmp < inf) x[++nx] = tmp;
  }
  tmp = binary(dx[ndx], inf, a, n);
  if (tmp < inf) x[++nx] = tmp;
}
int main() {
  scanf("%d", &n);
  for (int i = n; i >= 0; i--) scanf("%lf", &a[i]);
  int nx;
  solve(n, a, x, nx);
  for (int i = 1; i <= nx; i++) printf("%.6f\n", x[i]);
}
```

3 Data Structure

3.1 Disjoint Set

```
struct DisjointSet{
  // save() is like recursive
  // undo() is like return
  int n, compo;
  vector<int> fa, sz;
  vector<pair<int*,int>> h;
  vector<int> sp;
  void init(int tn) {
    compo = n = tn, sz.assign(n, 1), fa.resize(n);
    for (int i = 0; i < n; ++i)
      fa[i] = i, sz[i] = 1;
    sp.clear(); h.clear();
  }
```

```
  void assign(int *k, int v) {
    h.push_back({k, *k});
    *k = v;
  }
  void save() { sp.push_back(h.size()); }
  void undo() {
    assert(!sp.empty());
    int last = sp.back(); sp.pop_back();
    while (h.size() != last) {
      auto x = h.back(); h.pop_back();
      *x.first = x.second;
    }
  }
  int f(int x) {
    while (fa[x] != x) x = fa[x];
    return x;
  }
  bool uni(int x, int y) {
    x = f(x), y = f(y);
    if (x == y) return false;
    if (sz[x] < sz[y]) swap(x, y);
    assign(&sz[x], sz[x] + sz[y]);
    assign(&fa[y], x);
    --compo;
    return true;
  }
}djs;
```

3.2 Heavy Light Decomposition

```
struct HLD {
  using Tree = vector<vector<int>>;
  vector<int> par, head, vid, len, inv;

  HLD(const Tree &g) : par(g.size()), head(g.size()),
      vid(g.size()), len(g.size()), inv(g.size()) {
    int k = 0;
    vector<int> size(g.size(), 1);
    function<void(int, int)> dfs_size = [&](int u, int
        p) {
      for (int v : g[u]) {
        if (v != p) {
          dfs_size(v, u);
          size[u] += size[v];
        }
      }
    };
    function<void(int, int, int)> dfs_dcmp = [&](int u,
         int p, int h) {
      par[u] = p;
      head[u] = h;
      vid[u] = k++;
      inv[vid[u]] = u;
      for (int v : g[u]) {
        if (v != p && size[u] < size[v] * 2) {
          dfs_dcmp(v, u, h);
        }
      }
      for (int v : g[u]) {
        if (v != p && size[u] >= size[v] * 2) {
          dfs_dcmp(v, u, v);
        }
      }
    };
    dfs_size(0, -1);
    dfs_dcmp(0, -1, 0);
    for (int i = 0; i < g.size(); ++i) {
      ++len[head[i]];
    }
  }

  template<typename T>
  void foreach(int u, int v, T f) {
    while (true) {
      if (vid[u] > vid[v]) {
        if (head[u] == head[v]) {
          f(vid[v] + 1, vid[u], 0);
          break;
        } else {
          f(vid[head[u]], vid[u], 1);
          u = par[head[u]];
        }
      } else {
        if (head[u] == head[v]) {
          f(vid[u] + 1, vid[v], 0);
          break;
        } else {
          f(vid[head[v]], vid[v], 0);
          v = par[head[v]];
        }
      }
    }
  }
};
```

3.3 KD Tree

```
#include <bits/stdc++.h>
using namespace std;

struct KDNode {
  vector<int> v;
  KDNode *lc, *rc;
  KDNode(const vector<int> &_v) : v(_v), lc(nullptr),
      rc(nullptr) {}
  static KDNode *buildKDTree(vector<vector<int>> &pnts,
       int lb, int rb, int dpt) {
    if (rb - lb < 1) return nullptr;
    int axis = dpt % pnts[0].size();
    int mb = lb + rb >> 1;
    nth_element(pnts.begin() + lb, pnts.begin() + mb,
        pnts.begin() + rb, [&](const vector<int> &a,
        const vector<int> &b) {
      return a[axis] < b[axis];
    });
    KDNode *t = new KDNode(pnts[mb]);
    t->lc = buildKDTree(pnts, lb, mb, dpt + 1);
    t->rc = buildKDTree(pnts, mb + 1, rb, dpt + 1);
    return t;
  }
  static void release(KDNode *t) {
    if (t->lc) release(t->lc);
    if (t->rc) release(t->rc);
    delete t;
  }
  static void searchNearestNode(KDNode *t, KDNode *q,
      KDNode *&c, int dpt) {
    int axis = dpt % t->v.size();
    if (t->v != q->v && (c == nullptr || dis(q, t) <
        dis(q, c))) c = t;
    if (t->lc && (!t->rc || q->v[axis] < t->v[axis])) {
      searchNearestNode(t->lc, q, c, dpt + 1);
      if (t->rc && (c == nullptr || 1LL * (t->v[axis] -
           q->v[axis]) * (t->v[axis] - q->v[axis]) <
          dis(q, c))) {
        searchNearestNode(t->rc, q, c, dpt + 1);
      }
    } else if (t->rc) {
      searchNearestNode(t->rc, q, c, dpt + 1);
      if (t->lc && (c == nullptr || 1LL * (t->v[axis] -
           q->v[axis]) * (t->v[axis] - q->v[axis]) <
          dis(q, c))) {
        searchNearestNode(t->lc, q, c, dpt + 1);
      }
    }
  }
  static int64_t dis(KDNode *a, KDNode *b) {
    int64_t r = 0;
    for (int i = 0; i < a->v.size(); ++i) {
      r += 1LL * (a->v[i] - b->v[i]) * (a->v[i] - b->v[
          i]);
    }
    return r;
  }
};

signed main() {
  ios::sync_with_stdio(false);
  int T;
  cin >> T;
```

```
  for (int ti = 0; ti < T; ++ti) {
    int N;
    cin >> N;
    vector<vector<int>> pnts(N, vector<int>(2));
    for (int i = 0; i < N; ++i) {
      for (int j = 0; j < 2; ++j) {
        cin >> pnts[i][j];
      }
    }
    vector<vector<int>> _pnts = pnts;
    KDNode *root = KDNode::buildKDTree(_pnts, 0, pnts.
        size(), 0);
    for (int i = 0; i < N; ++i) {
      KDNode *q = new KDNode(pnts[i]);
      KDNode *c = nullptr;
      KDNode::searchNearestNode(root, q, c, 0);
      cout << KDNode::dis(c, q) << endl;
      delete q;
    }
    KDNode::release(root);
  }
  return 0;
}
```

3.4 Lowest Common Ancestor

```
const int LOG = 20, N = 200000;
vector<int> g[N];
int par[N][LOG], tin[N], tout[N];
bool anc(int u, int p) {
  return tin[p] <= tin[u] and tout[u] <= tout[p];
}
void dfs(int v, int p) { // root's parent is root
  par[v][0] = p;
  for (int j = 1; j < LOG; ++j)
    par[v][j] = par[par[v][j - 1]][j - 1];
  static int timer = 0;
  tin[v] = timer++;
  for (int u: g[v]) {
    if (u == p) continue;
    dfs(u, v);
  }
  tout[v] = timer++;
}
int lca(int x, int y) {
  if (anc(x, y)) return y;
  for (int j = LOG - 1; j >= 0; --j)
    if (not anc(x, par[y][j])) y = par[y][j];
  return par[y][0];
}
```

3.5 Link Cut Tree

```
const int MXN = 100005;
const int MEM = 100005;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay (int _val=-1) : val(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d){
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push(){
    if( !rev ) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
  }
  void pull(){
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
  splayVec.clear();
  for (Splay *q=x;; q=q->f){
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir())
      rotate(x->f),rotate(x);
    else rotate(x),rotate(x);
  }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
  Splay *q = nil;
  for (;x!=nil;x=x->f){
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void chroot(Splay *x){
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y){
  access(x);
  splay(x);
  chroot(y);
  x->setCh(y, 1);
}
void cut_p(Splay *y) {
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
  chroot(x);
  cut_p(y);
}
Splay* get_root(Splay *x) {
  access(x);
  splay(x);
  for(; x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  x = get_root(x);
  y = get_root(y);
  return x == y;
}
Splay* lca(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  if (x->f == nil) return x;
```

```
  else return x->f;
}
```

3.6 PST

```
constexpr int PST_MAX_NODES = 1 << 22; // recommended:
    prepare at least 4nlgn, n to power of 2
struct Pst {
  int maxv;
  Pst *lc, *rc;
  Pst() : lc(nullptr), rc(nullptr), maxv(0) {}
  Pst(const Pst *rhs) : lc(rhs->lc), rc(rhs->rc), maxv(
      rhs->maxv) {}
  static Pst *build(int lb, int rb) {
    Pst *t = new(mem_ptr++) Pst;
    if (rb - lb == 1) return t;
    t->lc = build(lb, lb + rb >> 1);
    t->rc = build(lb + rb >> 1, rb);
    return t;
  }
  static int query(Pst *t, int lb, int rb, int ql, int
      qr) {
    if (qr <= lb || rb <= ql) return 0;
    if (ql <= lb && rb <= qr) return t->maxv;
    int mb = lb + rb >> 1;
    return max(query(t->lc, lb, mb, ql, qr), query(t->
        rc, mb, rb, ql, qr));
  }
  static Pst *modify(Pst *t, int lb, int rb, int k, int
      v) {
    Pst *n = new(mem_ptr++) Pst(t);
    if (rb - lb == 1) return n->maxv = v, n;
    int mb = lb + rb >> 1;
    if (k < mb) n->lc = modify(t->lc, lb, mb, k, v);
    else n->rc = modify(t->rc, mb, rb, k, v);
    n->maxv = max(n->lc->maxv, n->rc->maxv);
    return n;
  }
  static Pst mem_pool[PST_MAX_NODES];
  static Pst *mem_ptr;
  static void clear() {
    while (mem_ptr != mem_pool) (--mem_ptr)->~Pst();
  }
} Pst::mem_pool[PST_MAX_NODES], *Pst::mem_ptr = Pst::
    mem_pool;
/*
Usage:

vector<Pst *> version(N + 1);
version[0] = Pst::build(0, C); // [0, C)
for (int i = 0; i < N; ++i) version[i + 1] = modify(
    version[i], ...);
Pst::query(...);
Pst::clear();

*/
```

3.7 Rbst

```
constexpr int RBST_MAX_NODES = 1 << 20;
struct Rbst {
  int size, val;
  // int minv;
  // int add_tag, rev_tag;
  Rbst *lc, *rc;
  Rbst(int v = 0) : size(1), val(v), lc(nullptr), rc(
      nullptr) {
    // minv = v;
    // add_tag = 0;
    // rev_tag = 0;
  }
  void push() {
    /*
    if (add_tag) { // unprocessed subtree has tag on
        root
      val += add_tag;
      minv += add_tag;
      if (lc) lc->add_tag += add_tag;
      if (rc) rc->add_tag += add_tag;
      add_tag = 0;
    }
    if (rev_tag) {
      swap(lc, rc);
      if (lc) lc->rev_tag ^= 1;
      if (rc) rc->rev_tag ^= 1;
      rev_tag = 0;
    }
    */
  }
  void pull() {
    size = 1;
    // minv = val;
    if (lc) {
      lc->push();
      size += lc->size;
      // minv = min(minv, lc->minv);
    }
    if (rc) {
      rc->push();
      size += rc->size;
      // minv = min(minv, rc->minv);
    }
  }
  static int get_size(Rbst *t) { return t ? t->size :
      0; }
  static void split(Rbst *t, int k, Rbst *&a, Rbst *&b)
      {
    if (!t) return void(a = b = nullptr);
    t->push();
    if (get_size(t->lc) >= k) {
      b = t;
      split(t->lc, k, a, b->lc);
      b->pull();
    } else {
      a = t;
      split(t->rc, k - get_size(t->lc) - 1, a->rc, b);
      a->pull();
    }
  } // splits t, left k elements to a, others to b,
      maintaining order
  static Rbst *merge(Rbst *a, Rbst *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (a->size + b->size) < a->size) {
      a->push();
      a->rc = merge(a->rc, b);
      a->pull();
      return a;
    } else {
      b->push();
      b->lc = merge(a, b->lc);
      b->pull();
      return b;
    }
  } // merges a and b, maintaing order
  static Rbst mem_pool[RBST_MAX_NODES]; // CAUTION!!
  static Rbst *mem_ptr;
  static void clear() {
    while (mem_ptr != mem_pool) (--mem_ptr)->~Rbst();
  }
} Rbst::mem_pool[RBST_MAX_NODES], *Rbst::mem_ptr = Rbst
    ::mem_pool;

/*
Usage:

Rbst *t = new(Rbst::mem_ptr++) Rbst(val);
t = Rbst::merge(t, new(Rbst::mem_ptr++) Rbst(
    another_val));
Rbst *a, *b;
Rbst::split(t, 2, a, b); // a will have first 2
    elements, b will have the rest, in order
Rbst::clear(); // wipes out all memory; if you know the
     mechanism of clear() you can maintain many trees

*/
```

4 Flow

4.1 CostFlow

```cpp
template <class TF, class TC>
struct CostFlow {
  static const int MAXV = 205;
  static const TC INF = 0x3f3f3f3f;
  struct Edge {
    int v, r;
    TF f;
    TC c;
    Edge(int _v, int _r, TF _f, TC _c) : v(_v), r(_r),
        f(_f), c(_c) {}
  };
  int n, s, t, pre[MAXV], pre_E[MAXV], inq[MAXV];
  TF fl;
  TC dis[MAXV], cost;
  vector<Edge> E[MAXV];
  CostFlow(int _n, int _s, int _t) : n(_n), s(_s), t(_t
      ), fl(0), cost(0) {}
  void add_edge(int u, int v, TF f, TC c) {
    E[u].emplace_back(v, E[v].size(), f, c);
    E[v].emplace_back(u, E[u].size() - 1, 0, -c);
  }
  pair<TF, TC> flow() {
    while (true) {
      for (int i = 0; i < n; ++i) {
        dis[i] = INF;
        inq[i] = 0;
      }
      dis[s] = 0;
      queue<int> que;
      que.emplace(s);
      while (not que.empty()) {
        int u = que.front();
        que.pop();
        inq[u] = 0;
        for (int i = 0; i < E[u].size(); ++i) {
          int v = E[u][i].v;
          TC w = E[u][i].c;
          if (E[u][i].f > 0 and dis[v] > dis[u] + w) {
            pre[v] = u;
            pre_E[v] = i;
            dis[v] = dis[u] + w;
            if (not inq[v]) {
              inq[v] = 1;
              que.emplace(v);
            }
          }
        }
      }
      if (dis[t] == INF) break;
      TF tf = INF;
      for (int v = t, u, l; v != s; v = u) {
        u = pre[v];
        l = pre_E[v];
        tf = min(tf, E[u][l].f);
      }
      for (int v = t, u, l; v != s; v = u) {
        u = pre[v];
        l = pre_E[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      }
      cost += tf * dis[t];
      fl += tf;
    }
    return {fl, cost};
  }
};
```

4.2 MaxFlow

```cpp
template <class T>
struct Dinic {
  static const int MAXV = 10000;
  static const T INF = 0x3f3f3f3f;
  struct Edge {
    int v;
    T f;
    int re;
    Edge(int _v, T _f, int _re) : v(_v), f(_f), re(_re)
        {}
  };
  int n, s, t, level[MAXV];
  vector<Edge> E[MAXV];
  int now[MAXV];
  Dinic(int _n, int _s, int _t) : n(_n), s(_s), t(_t)
      {}
  void add_edge(int u, int v, T f, bool bidirectional =
       false) {
    E[u].emplace_back(v, f, E[v].size());
    E[v].emplace_back(u, 0, E[u].size() - 1);
    if (bidirectional) {
      E[v].emplace_back(u, f, E[u].size() - 1);
    }
  }
  bool BFS() {
    memset(level, -1, sizeof(level));
    queue<int> que;
    que.emplace(s);
    level[s] = 0;
    while (not que.empty()) {
      int u = que.front();
      que.pop();
      for (auto it : E[u]) {
        if (it.f > 0 and level[it.v] == -1) {
          level[it.v] = level[u] + 1;
          que.emplace(it.v);
        }
      }
    }
    return level[t] != -1;
  }
  T DFS(int u, T nf) {
    if (u == t) return nf;
    T res = 0;
    while (now[u] < E[u].size()) {
      Edge &it = E[u][now[u]];
      if (it.f > 0 and level[it.v] == level[u] + 1) {
        T tf = DFS(it.v, min(nf, it.f));
        res += tf;
        nf -= tf;
        it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      } else
        ++now[u];
    }
    if (not res) level[u] = -1;
    return res;
  }
  T flow(T res = 0) {
    while (BFS()) {
      T temp;
      memset(now, 0, sizeof(now));
      while (temp = DFS(s, INF)) {
        res += temp;
        res = min(res, INF);
      }
    }
    return res;
  }
};
```

4.3 KM matching

```cpp
const int MAXN = 1000;
template <class TC>
struct KM_matching { // if there's no edge, the weight
     is 0
// complexity: O(n^3), support for negetive edge
  int n, matchy[MAXN];
  bool visx[MAXN], visy[MAXN];
  TC adj[MAXN][MAXN], coverx[MAXN], covery[MAXN], slack
      [MAXN];
  KM_matching(int _n) : n(_n) {
```

```
    memset(matchy, -1, sizeof(matchy));
    memset(covery, 0, sizeof(covery));
    memset(adj, 0, sizeof(adj));
  }
  void add_edge(int x, int y, TC w) { adj[x][y] = w; }
  bool aug(int u) {
    visx[u] = true;
    for (int v = 0; v < n; ++v)
      if (not visy[v]) {
        TC t = coverx[u] + covery[v] - adj[u][v];
        if (t == 0) { // The edge is in Equality
          subgraph
          visy[v] = true;
          if (matchy[v] == -1 or aug(matchy[v]))
            return matchy[v] = u, true;
        }
        else if (slack[v] > t) slack[v] = t;
      }
    return false;
  }
  TC solve() {
    for (int u = 0; u < n; ++u)
      coverx[u] = *max_element(adj[u], adj[u] + n);
    for (int u = 0; u < n; ++u) {
      fill(slack, slack + n, INT_MAX);
      while (memset(visx, 0, sizeof(visx)),
             memset(visy, 0, sizeof(visy)),
             not aug(u)) {
        TC d = INT_MAX;
        for (int v = 0; v < n; ++v)
          if (not visy[v]) d = min(d, slack[v]);
        for (int v = 0; v < n; ++v) {
          if (visx[v]) coverx[v] -= d;
          if (visy[v]) covery[v] += d;
        }
      }
    }
    return accumulate(coverx, coverx + n, (TC)0) +
           accumulate(covery, covery + n, (TC)0);
  }
};
```

4.4 Matching

```
class matching {
  public:
  vector< vector<int> > g;
  vector<int> pa, pb, was;
  int n, m, res, iter;

  matching(int _n, int _m) : n(_n), m(_m) {
    assert(0 <= n && 0 <= m);
    pa = vector<int>(n, -1);
    pb = vector<int>(m, -1);
    was = vector<int>(n, 0);
    g.resize(n);
    res = 0, iter = 0;
  }

  void add_edge(int from, int to) {
    assert(0 <= from && from < n && 0 <= to && to < m);
    g[from].push_back(to);
  }

  bool dfs(int v) {
    was[v] = iter;
    for (int u : g[v])
      if (pb[u] == -1)
        return pa[v] = u, pb[u] = v, true;
    for (int u : g[v])
      if (was[pb[u]] != iter && dfs(pb[u]))
        return pa[v] = u, pb[u] = v, true;
    return false;
  }

  int solve() {
    while (true) {
      iter++;
      int add = 0;
      for (int i = 0; i < n; i++)
        if (pa[i] == -1 && dfs(i))
          add++;
      if (add == 0) break;
      res += add;
    }
    return res;
  }

  int run_one(int v) {
    if (pa[v] != -1) return 0;
    iter++;
    return (int) dfs(v);
  }
  pair<vector<bool>, vector<bool>> vertex_cover() {
    solve();
    vector<bool> a_cover(n, true), b_cover(m, false);
    function<void(int)> dfs_aug = [&](int v) {
      a_cover[v] = false;
      for (int u: g[v])
        if (not b_cover[u])
          b_cover[u] = true, dfs_aug(pb[u]);
    };
    for (int v = 0; v < n; ++v)
      if (a_cover[v] and pa[v] == -1)
        dfs_aug(v);
    return {a_cover, b_cover};
  }
};
```

5 Geometry

5.1 2D Geometry

```
namespace geo {
  using pt = complex<double>;
  using cir = pair<pt, double>;
  using poly = vector<pt>;
  using line = pair<pt, pt>; // point to point
  using plane = pair<pt, pt>;
  pt get_pt() { static double a, b; cin >> a >> b;
      return geo::pt(a, b);};
  const double EPS = 1e-10;
  const double PI = acos(-1);
  pt cent(cir C) { return C.first; }
  double radi(cir C) { return C.second; }
  pt st(line H) { return H.first; }
  pt ed(line H) { return H.second; }
  pt vec(line H) { return ed(H) - st(H); }
  int dcmp(double x) { return abs(x) < EPS ? 0 : x > 0
      ? 1 : -1; }
  bool less(pt a, pt b) { return real(a) < real(b) ||
      real(a) == real(b) && imag(a) < imag(b); }
  bool more(pt a, pt b) { return real(a) > real(b) ||
      real(a) == real(b) && imag(a) > imag(b); }
  double dot(pt a, pt b) { return real(conj(a) * b); }
  double cross(pt a, pt b) { return imag(conj(a) * b);
      }
  double sarea(pt a, pt b, pt c) { return cross(b - a,
      c - a); }
  double area(cir c) { return radi(c) * radi(c) * PI; }
  int ori(pt a, pt b, pt c) { return dcmp(sarea(a, b, c
      )); }
  double angle(pt a, pt b) { return acos(dot(a, b) /
      abs(a) / abs(b)); }
  pt rotate(pt a, double rad) { return a * pt(cos(rad),
       sin(rad)); }
  pt normal(pt a) { return pt(-imag(a), real(a)) / abs(
      a); }
  pt normalized(pt a) { return a / abs(a); }

  pt get_line_intersection(line A, line B) {
    pt p = st(A), v = vec(A), q = st(B), w = vec(B);
    return p + v * cross(w, p - q) / cross(v, w);
  }
  double distance_to_line(pt p, line B) {
    return abs(cross(vec(B), p - st(B)) / abs(vec(B)));
  }
  double distance_to_segment(pt p, line B) {
```

```
  pt a = st(B), b = ed(B), v1(vec(B)), v2(p - a), v3(
      p - b);
  // similar to previous function
  if (a == b) return abs(p - a);
  if (dcmp(dot(v1, v2)) < 0) return abs(v2);
  else if (dcmp(dot(v1, v3)) > 0) return abs(v3);
  return abs(cross(v1, v2)) / abs(v1);
}
pt get_line_projection(pt p, line(B)) {
  pt v = vec(B);
  return st(B) + dot(v, p - st(B)) / dot(v, v) * v;
}
bool is_segment_proper_intersection(line A, line B) {
  pt a1 = st(A), a2 = ed(A), b1 = st(B), b2 = ed(B);
  double det1 = ori(a1, a2, b1) * ori(a1, a2, b2);
  double det2 = ori(b1, b2, a1) * ori(b1, b2, a2);
  return det1 < 0 && det2 < 0;
}
double area(poly p) {
  if (p.size() < 3) return 0;
  double area = 0;
  for (int i = 1; i < p.size() - 1; ++i)
    area += sarea(p[0], p[i], p[i + 1]);
  return area / 2;
}
bool is_point_on_segment(pt p, line B) {
  pt a = st(B), b = ed(B);
  return dcmp(sarea(p, a, b)) == 0 && dcmp(dot(a - p,
       b - p)) < 0;
}
bool is_point_in_plane(pt p, line H) {
  return ori(st(H), ed(H), p) > 0;
}
bool is_point_in_polygon(pt p, poly gon) {
  int wn = 0;
  int n = gon.size();
  for (int i = 0; i < n; ++i) {
    if (is_point_on_segment(p, {gon[i], gon[(i + 1) %
         n]})) return true;
    if (not is_point_in_plane(p, {gon[i], gon[(i + 1)
         % n]})) return false;
  }
  return true;
}
poly convex_hull(vector<pt> p) {
  sort(p.begin(), p.end(), less);
  p.erase(unique(p.begin(), p.end()), p.end());
  int n = p.size(), m = 0;
  poly ch(n + 1);
  for (int i = 0; i < n; ++i) { // note that border
      is cleared
    while (m > 1 && ori(ch[m - 2], ch[m - 1], p[i])
         <= 0) --m;
    ch[m++] = p[i];
  }
  for (int i = n - 2, k = m; i >= 0; --i) {
    while (m > k && ori(ch[m - 2], ch[m - 1], p[i])
         <= 0) --m;
    ch[m++] = p[i];
  }
  ch.erase(ch.begin() + m - (n > 1), ch.end());
  return ch;
}
cir circumscribed_circle(poly tri) {
  pt B = tri[1] - tri[0];
  pt C = tri[2] - tri[0];
  double det = 2 * cross(B, C);
  pt r = pt(imag(C) * norm(B) - imag(B) * norm(C),
            real(B) * norm(C) - real(C) * norm(B)) /
                det;
  return {r + tri[0], abs(r)};
}
cir inscribed_circle(poly tri) {
  assert(tri.size() == 3);
  pt ans = 0;
  double div = 0;
  for (int i = 0; i < 3; ++i) {
    double l = abs(tri[(i + 1) % 3] - tri[(i + 2) %
         3]);
    ans += l * tri[i], div += l;
  }
  ans /= div;
  return {ans, distance_to_line(ans, {tri[0], tri
      [1]})};
}
poly tangent_line_through_point(cir c, pt p) {
  if (dcmp(abs(cent(c) - p) - radi(c)) < 0) return
      {};
  else if (dcmp(abs(cent(c) - p) - radi(c)) == 0)
      return {p};
  double theta = acos(radi(c) / abs(cent(c) - p));
  pt norm_v = normalized(p - cent(c));
  return {cent(c) + radi(c) * rotate(norm_v, +theta),
          cent(c) + radi(c) * rotate(norm_v, -theta)
              };
}
vector<pt> get_line_circle_intersection(cir d, line B
    ) {
  pt v = vec(B), p = st(B) - cent(d);
  double r = radi(d), a = norm(v), b = 2 * dot(p, v),
       c = norm(p) - r * r;
  double det = b * b - 4 * a * c;
  // t^2 * norm(v) + 2 * t * dot(p, v) + norm(p) - r
      * r = 0
  auto get_point = [=](double t) { return st(B)+ t *
      v; };
  if (dcmp(det) < 0) return {};
  if (dcmp(det) == 0) return {get_point(-b / 2 / a)};
  return {get_point((-b + sqrt(det)) / 2 / a),
          get_point((-b - sqrt(det)) / 2 / a)};
}
vector<pt> get_circle_circle_intersection(cir c, cir
    d) {
  pt a = cent(c), b = cent(d);
  double r = radi(c), s = radi(d), g = abs(a - b);
  if (dcmp(g) == 0) return {}; // may be C == D
  if (dcmp(r + s - g) < 0 or dcmp(abs(r - s) - g) >
      0) return {};
  pt C_to_D = normalized(b - a);
  double theta = acos((r * r + g * g - s * s) / (2 *
      r * g));
  if (dcmp(theta) == 0) return {a + r * C_to_D};
  else return {a + rotate(r * C_to_D, theta), a +
      rotate(r * C_to_D, -theta)};
}
cir min_circle_cover(vector<pt> A) {
  random_shuffle(A.begin(), A.end());
  cir ans = {0, 0};
  auto is_incir = [&](pt a) { return dcmp(abs(cent(
      ans) - a) - radi(ans)) < 0; };
  for (int i = 0; i < A.size(); ++i) if (not is_incir
      (A[i])) {
    ans = {A[i], 0};
    for (int j = 0; j < i; ++j) if (not is_incir(A[j
        ])) {
      ans = {(A[i] + A[j]) / 2., abs(A[i] - A[j]) /
          2};
      for (int k = 0; k < j; ++k) if (not is_incir(A[
          k]))
        ans = circumscribed_circle({A[i], A[j], A[k
            ]});
    }
  }
  return ans;
}
poly half_plane_intersection(vector<plane> A) {
  const double INF = 1e19;
  sort(A.begin(), A.end(), [=](plane a, plane b) {
    int res = dcmp(arg(vec(a)) - arg(vec(b)));
    return res == 0 ? is_point_in_plane(st(a), b) :
        res < 0;
  });
  deque<pt> ans;
  deque<plane> q;
  q.push_back(A[0]);
  for (int i = 1; i < A.size(); ++i) {
    if (dcmp(cross(vec(A[i]), vec(A[i - 1]))) == 0)
        continue;
    while (ans.size() and not is_point_in_plane(ans.
        back(), A[i]))
      q.pop_back(), ans.pop_back();
    while (ans.size() and not is_point_in_plane(ans.
        front(), A[i]))
      q.pop_front(), ans.pop_front();
```

```
      ans.push_back(get_line_intersection(A[i], q.back
          ()));
      q.push_back(A[i]);
    }
    while (ans.size() and not is_point_in_plane(ans.
        back(), q.front()))
      ans.pop_back(), q.pop_back();
    while (ans.size() and not is_point_in_plane(ans.
        front(), q.back()))
      ans.pop_front(), q.pop_front();
    if (q.size() < 3) return {};
    ans.push_back(get_line_intersection(q.back(), q.
        front()));
    return poly(ans.begin(), ans.end());
  }
  pair<pt, pt> closest_pair(vector<pt> &V, int l, int r
      ) { // l = 0, r = V.size()
    pair<pt, pt> ret = {pt(-1e18), pt(1e18)};
    const auto upd = [&](pair<pt, pt> a) {
      if (abs(a.first - a.second) < abs(ret.first - ret
          .second)) ret = a;
    };
    if (r - l < 40) { // GOD's number! It performs well
        !
      for (int i = l; i < r; ++i) for (int j = l; j < i
          ; ++j)
        upd({V[i], V[j]});
      return ret;
    }
    int m = l + r >> 1;
    const auto cmpy = [](pt a, pt b) { return imag(a) <
         imag(b); };
    const auto cmpx = [](pt a, pt b) { return real(a) <
         real(b); };
    nth_element(V.begin() + l, V.begin() + m, V.begin()
         + r, cmpx);
    pt mid = V[m];
    upd(closest_pair(V, l, m));
    upd(closest_pair(V, m, r));
    double delta = abs(ret.first - ret.second);
    vector<pt> spine;
    for (int k = l; k < r; ++k)
      if (abs(real(V[k]) - real(V[m])) < delta) spine.
          push_back(V[k]);
    sort(spine.begin(), spine.end(), cmpy);
    for (int i = 0; i < spine.size(); ++i)
      for (int j = i + 1; j - i < 8 and j < spine.size
          (); ++j) {
        upd({spine[i], spine[j]});
      }
    return ret;
  }
};
```

5.2 3D ConvexHull

```
#define SIZE(X) (int(X.size()))
#define PI 3.14159265358979323846264338327950288
struct Pt{
  Pt cross(const Pt &p) const
  { return Pt(y * p.z - z * p.y, z * p.x - x * p.z, x *
       p.y - y * p.x); }
} info[N];
int mark[N][N],n, cnt;;
double mix(const Pt &a, const Pt &b, const Pt &c)
{ return a * (b ^ c); }
double area(int a, int b, int c)
{ return norm((info[b] - info[a]) ^ (info[c] - info[a])
    ); }
double volume(int a, int b, int c, int d)
{ return mix(info[b] - info[a], info[c] - info[a], info
    [d] - info[a]); }
struct Face{
  int a, b, c; Face(){}
  Face(int a, int b, int c): a(a), b(b), c(c) {}
  int &operator [](int k)
  { if (k == 0) return a; if (k == 1) return b; return
      c; }
};
vector<Face> face;
void insert(int a, int b, int c)
{ face.push_back(Face(a, b, c)); }
void add(int v) {
  vector <Face> tmp; int a, b, c; cnt++;
  for (int i = 0; i < SIZE(face); i++) {
    a = face[i][0]; b = face[i][1]; c = face[i][2];
    if(Sign(volume(v, a, b, c)) < 0)
    mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] =
         mark[c][a] = mark[a][c] = cnt;
    else tmp.push_back(face[i]);
  } face = tmp;
  for (int i = 0; i < SIZE(tmp); i++) {
    a = face[i][0]; b = face[i][1]; c = face[i][2];
    if (mark[a][b] == cnt) insert(b, a, v);
    if (mark[b][c] == cnt) insert(c, b, v);
    if (mark[c][a] == cnt) insert(a, c, v);
}}
int Find(){
  for (int i = 2; i < n; i++) {
    Pt ndir = (info[0] - info[i]) ^ (info[1] - info[i])
        ;
    if (ndir == Pt()) continue; swap(info[i], info[2]);
    for (int j = i + 1; j < n; j++) if (Sign(volume(0,
        1, 2, j)) != 0) {
      swap(info[j], info[3]); insert(0, 1, 2); insert
          (0, 2, 1); return 1;
} } return 0; }
int main() {
  for (; scanf("%d", &n) == 1; ) {
    for (int i = 0; i < n; i++) info[i].Input();
    sort(info, info + n); n = unique(info, info + n) -
        info;
    face.clear(); random_shuffle(info, info + n);
    if (Find()) { memset(mark, 0, sizeof(mark)); cnt =
        0;
      for (int i = 3; i < n; i++) add(i); vector<Pt>
          Ndir;
      for (int i = 0; i < SIZE(face); ++i) {
        Pt p = (info[face[i][0]] - info[face[i][1]]) ^
               (info[face[i][2]] - info[face[i][1]]);
        p = p / norm( p ); Ndir.push_back(p);
      } sort(Ndir.begin(), Ndir.end());
      int ans = unique(Ndir.begin(), Ndir.end()) - Ndir
          .begin();
      printf("%d\n", ans);
    } else printf("1\n");
} }
double calcDist(const Pt &p, int a, int b, int c)
{ return fabs(mix(info[a] - p, info[b] - p, info[c] - p
    ) / area(a, b, c)); }
//compute the minimal distance of center of any faces
double findDist() { //compute center of mass
  double totalWeight = 0; Pt center(.0, .0, .0);
  Pt first = info[face[0][0]];
  for (int i = 0; i < SIZE(face); ++i) {
    Pt p = (info[face[i][0]]+info[face[i][1]]+info[face
        [i][2]]+first)*.25;
    double weight = mix(info[face[i][0]] - first, info[
        face[i][1]]
        - first, info[face[i][2]] - first);
    totalWeight += weight; center = center + p * weight
        ;
  } center = center / totalWeight;
  double res = 1e100; //compute distance
  for (int i = 0; i < SIZE(face); ++i)
    res = min(res, calcDist(center, face[i][0], face[i
        ][1], face[i][2]));
    return res; }
```

5.3 Half plane intersection

```
Pt interPnt( Line l1, Line l2, bool &res ){
  Pt p1, p2, q1, q2;
  tie(p1, p2) = l1; tie(q1, q2) = l2;
  double f1 = (p2 - p1) ^ (q1 - p1);
  double f2 = (p2 - p1) ^ (p1 - q2);
  double f = (f1 + f2);
  if( fabs(f) < eps){ res=0; return {0, 0}; }
  res = true;
  return q1 * (f2 / f) + q2 * (f1 / f);
```

```
}
bool isin( Line l0, Line l1, Line l2 ){
  // Check inter(l1, l2) in l0
  bool res; Pt p = interPnt(l1, l2, res);
  return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
 */
/* --^-- Line.FI --^-- Line.SE --^-- */
vector<Line> halfPlaneInter( vector<Line> lines ){
  int sz = lines.size();
  vector<double> ata(sz), ord(sz);
  for( int i=0; i<sz; i++) {
    ord[i] = i;
    Pt d = lines[i].SE - lines[i].FI;
    ata[i] = atan2(d.Y, d.X);
  }
  sort( ord.begin(), ord.end(), [&](int i, int j) {
    if( fabs(ata[i] - ata[j]) < eps )
      return ( (lines[i].SE - lines[i].FI) ^
               (lines[j].SE - lines[i].FI) ) < 0;
    return ata[i] < ata[j];
  });
  vector<Line> fin;
  for (int i=0; i<sz; i++)
    if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
      fin.PB(lines[ord[i]]);
  deque<Line> dq;
  for (int i=0; i<(int)(fin.size()); i++) {
    while((int)(dq.size()) >= 2 and
        not isin(fin[i], dq[(int)(dq.size())-2],
                         dq[(int)(dq.size())-1]))
      dq.pop_back();
    while((int)(dq.size()) >= 2 and
        not isin(fin[i], dq[0], dq[1]))
      dq.pop_front();
    dq.push_back(fin[i]);
  }
  while( (int)(dq.size()) >= 3 and
      not isin(dq[0], dq[(int)(dq.size())-2],
                      dq[(int)(dq.size())-1]))
    dq.pop_back();
  while( (int)(dq.size()) >= 3 and
      not isin(dq[(int)(dq.size())-1], dq[0], dq[1]))
    dq.pop_front();
  vector<Line> res(dq.begin(),dq.end());
  return res;
}
```

6 Graph

6.1 2-SAT

```
#include <bits/stdc++.h>

using namespace std;

class two_SAT {
  public:
  vector< vector<int> > g, rg;
  vector<int> visit, was;
  vector<int> id;
  vector<int> res;
  int n, iter;

  two_SAT(int _n) : n(_n) {
    g.resize(n * 2);
    rg.resize(n * 2);
    was = vector<int>(n * 2, 0);
    id = vector<int>(n * 2, -1);
    res.resize(n);
    iter = 0;
  }

  void add_edge(int from, int to) { // add (a -> b)
    assert(from >= 0 && from < 2 * n && to >= 0 && to <
        2 * n);
    g[from].emplace_back(to);
    rg[to].emplace_back(from);
  }

  void add_or(int a, int b) { // add (a V b)
    int nota = (a < n) ? a + n : a - n;
    int notb = (b < n) ? b + n : b - n;
    add_edge(nota, b);
    add_edge(notb, a);
  }

  void dfs(int v) {
    was[v] = true;
    for (int u : g[v]) {
      if (!was[u]) dfs(u);
    }
    visit.emplace_back(v);
  }

  void rdfs(int v) {
    id[v] = iter;
    for (int u : rg[v]) {
      if (id[u] == -1) rdfs(u);
    }
  }

  int scc() {
    for (int i = 0; i < 2 * n; i++) {
      if (!was[i]) dfs(i);
    }
    for (int i = 2 * n - 1; i >= 0; i--) {
      if (id[ visit[i] ] == -1) {
        rdfs(visit[i]);
        iter++;
      }
    }
    return iter;
  }

  bool solve() {
    scc();
    for (int i = 0; i < n; i++) {
      if (id[i] == id[i + n]) return false;
      res[i] = (id[i] < id[i + n]);
    }
    return true;
  }

};

/*
  usage:
    index 0 ~ n - 1 : True
    index n ~ 2n - 1 : False
    add_or(a, b) : add SAT (a or b)
    add_edge(a, b) : add SAT (a -> b)
    if you want to set x = True, you can add (not X ->
        X)
    solve() return True if it exist at least one
        solution
    res[i] store one solution
      false -> choose a
      true -> choose a + n
*/
```

6.2 BCC

```
#include <bits/stdc++.h>

using namespace std;

class biconnected_component {
  public:
  vector< vector<int> > g;
  vector< vector<int> > comp;
  vector<int> pre, depth;
  int n;

  biconnected_component(int _n) : n(_n) {
    depth = vector<int>(n, -1);
```

```cpp
    g.resize(n);
  }

  void add(int u, int v) {
    assert(0 <= u && u < n && 0 <= v && v < n);
    g[u].push_back(v);
    g[v].push_back(u);
  }

  int dfs(int v, int pa, int d) {
    depth[v] = d;
    pre.push_back(v);
    for (int u : g[v]) {
      if (u == pa) continue;
      if (depth[u] == -1) {
        int child = dfs(u, v, depth[v] + 1);
        if (child >= depth[v]) {
          comp.push_back(vector<int>(1, v));
          while (pre.back() != v) {
            comp.back().push_back(pre.back());
            pre.pop_back();
          }
        }
        d = min(d, child);
      }
      else {
        d = min(d, depth[u]);
      }
    }
    return d;
  }

  vector< vector<int> > solve() {
    for (int i = 0; i < n; i++) {
      if (depth[i] == -1) {
        dfs(i, -1, 0);
      }
    }
    return comp;
  }

  vector<int> get_ap() {
    vector<int> res, count(n, 0);
    for (auto c : comp) {
      for (int v : c ) {
        count[v]++;
      }
    }
    for (int i = 0; i < n; i++) {
      if (count[i] > 1) {
        res.push_back(i);
      }
    }
    return res;
  }
};
```

6.3 Bridge

```cpp
struct Bridge {
  vector<int> imo;
  set<pair<int, int>> bridges; // all bridges (u, v), u < v
  vector<set<int>> bcc; // bcc[i] has all vertices that belong to the i'th bcc
  vector<int> at_bcc; // node i belongs to at_bcc[i]
  int bcc_ctr;

  Bridge(const vector<vector<int>> &g) : bcc_ctr(0) {
    imo.resize(g.size());
    bcc.resize(g.size());
    at_bcc.resize(g.size());
    vector<int> vis(g.size());
    vector<int> dpt(g.size());
    function<void(int, int, int)> mark = [&](int u, int fa, int d) {
      vis[u] = 1;
      dpt[u] = d;
      for (int v : G[u]) {
        if (v == fa) continue;
        if (vis[v]) {
          if (dpt[v] > dpt[u]) {
            ++imo[v];
            --imo[u];
          }
        } else mark(v, u, d + 1);
      }
    };
    mark(0, -1, 0);
    vis.assign(g.size(), 0);
    function<int(int)> expand = [&](int u) {
      vis[u] = 1;
      int s = imo[u];
      for (int v : G[u]) {
        if (vis[v]) continue;
        int e = expand(v);
        if (e == 0) bridges.emplace(make_pair(min(u, v), max(u, v)));
        s += e;
      }
      return s;
    };
    expand(0);
    fill(at_bcc.begin(), at_bcc.end(), -1);
    for (int u = 0; u < N; ++u) {
      if (~at_bcc[u]) continue;
      queue<int> que;
      que.emplace(u);
      at_bcc[u] = bcc_ctr;
      bcc[bcc_ctr].emplace(u);
      while (que.size()) {
        int v = que.front();
        que.pop();
        for (int w : G[v]) {
          if (~at_bcc[w] || bridges.count(make_pair(min(v, w), max(v, w)))) continue;
          que.emplace(w);
          at_bcc[w] = bcc_ctr;
          bcc[bcc_ctr].emplace(w);
        }
      }
      ++bcc_ctr;
    }
  }
};
```

6.4 General Matching

```cpp
#define MAXN 505
struct Blossom {
  vector<int> g[MAXN];
  int pa[MAXN] = {0}, match[MAXN] = {0}, st[MAXN] = {0}, S[MAXN] = {0}, v[MAXN] = {0};
  int t, n;
  Blossom(int _n) : n(_n) {}
  void add_edge(int v, int u) { // 1-index
    g[u].push_back(v), g[v].push_back(u);
  }
  inline int lca(int x, int y) {
    ++t;
    while (v[x] != t) {
      v[x] = t;
      x = st[pa[match[x]]];
      swap(x, y);
      if (x == 0) swap(x, y);
    }
    return x;
  }
  inline void flower(int x, int y, int l, queue<int> &q) {
    while (st[x] != l) {
      pa[x] = y;
      if (S[y = match[x]] == 1) q.push(y), S[y] = 0;
      st[x] = st[y] = l, x = pa[y];
    }
  }
  inline bool bfs(int x) {
    for (int i = 1; i <= n; ++i) st[i] = i;
    memset(S + 1, -1, sizeof(int) * n);
    queue<int> q;
```

```
    q.push(x), S[x] = 0;
    while (q.size()) {
      x = q.front(), q.pop();
      for (size_t i = 0; i < g[x].size(); ++i) {
        int y = g[x][i];
        if (S[y] == -1) {
          pa[y] = x, S[y] = 1;
          if (not match[y]) {
            for (int lst; x; y = lst, x = pa[y])
              lst = match[x], match[x] = y, match[y] =
                  x;
            return 1;
          }
          q.push(match[y]), S[match[y]] = 0;
        } else if (not S[y] and st[y] != st[x]) {
          int l = lca(y, x);
          flower(y, x, l, q), flower(x, y, l, q);
        }
      }
    }
    return 0;
  }
  inline int blossom() {
    int ans = 0;
    for (int i = 1; i <= n; ++i)
      if (not match[i] and bfs(i)) ++ans;
    return ans;
  }
};
```

6.5 CentroidDecomposition

```
vector<int> adj[N];
int p[N], vis[N];
int sz[N], M[N]; // subtree size of u and M(u)

inline void maxify(int &x, int y) { x = max(x, y); }
int centroidDecomp(int x) {
  vector<int> q;
  { // bfs
    size_t pt = 0;
    q.push_back(x);
    p[x] = -1;
    while (pt < q.size()) {
      int now = q[pt++];
      sz[now] = 1;
      M[now] = 0;
      for (auto &nxt : adj[now])
        if (!vis[nxt] && nxt != p[now])
          q.push_back(nxt), p[nxt] = now;
    }
  }

  // calculate subtree size in reverse order
  reverse(q.begin(), q.end());
  for (int &nd : q)
    if (p[nd] != -1) {
      sz[p[nd]] += sz[nd];
      maxify(M[p[nd]], sz[nd]);
    }
  for (int &nd : q)
    maxify(M[nd], (int)q.size() - sz[nd]);

  // find centroid
  int centroid = *min_element(q.begin(), q.end(),
                              [&](int x, int y) {
                                return M[x] < M[y];
                              });

  vis[centroid] = 1;
  for (auto &nxt : adj[centroid]) if (!vis[nxt])
    centroidDecomp(nxt);
  return centroid;
}
```

6.6 Diameter

```
const int SIZE = 1e6 + 10;
struct Tree_ecc{
  vector<pair<int, LL>> g[SIZE];
  LL dp[SIZE][2] = {0}, ecc[SIZE];
  int n = -1;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      g[i].clear(), ecc[i] = dp[i][0] = dp[i][1] = 0;
  }
  void add_edge(int v, int u, LL w) { // 0-index
    g[u].emplace_back(v, w);
    g[v].emplace_back(u, w);
  }
  void dfs_length(int v, int p) {
    for (auto T: g[v]) {
      int u; LL w;
      tie(u, w) = T;
      if (u == p) continue;
      dfs_length(u, v);
      LL length_from_u = dp[u][0] + w;
      if (dp[v][0] < length_from_u)
        dp[v][1] = dp[v][0], dp[v][0] = length_from_u;
      else if (dp[v][1] < length_from_u)
        dp[v][1] = length_from_u;
    }
  }
  void dfs_ecc(int v, int p, LL pass_p) {
    ecc[v] = max(dp[v][0], pass_p);
    for (auto T: g[v]) {
      int u; LL w;
      tie(u, w) = T;
      if (u == p) continue;
      if (dp[u][0] + w == dp[v][0])
        dfs_ecc(u, v, max(pass_p, dp[v][1]) + w);
      else dfs_ecc(u, v, max(pass_p, dp[v][0]) + w);
    }
  }
  LL diameter() {
    assert(~n);
    dfs_length(0, 0);
    dfs_ecc(0, 0, 0);
    return *max_element(ecc, ecc + n);
  }
} solver;
```

6.7 DirectedGraphMinCycle

```
// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
  int to; LL w;
  edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
  LL d; int u, next;
  node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
  vector<edge> g[N], grev[N];
  LL dp[N][N], p[N], d[N], mu;
  bool inq[N];
  int n, bn, bsz, hd[N];
  void b_insert(LL d, int u){
    int i = d/mu;
    if(i >= bn) return;
    b[++bsz] = node(d, u, hd[i]);
    hd[i] = bsz;
  }
  void init( int _n ){
    n = _n;
    for( int i = 1 ; i <= n ; i ++ )
      g[ i ].clear();
  }
  void addEdge( int ai , int bi , LL ci )
  { g[ai].push_back(edge(bi,ci)); }
  LL solve(){
    fill(dp[0], dp[0]+n+1, 0);
```

```
    for(int i=1; i<=n; i++){
      fill(dp[i]+1, dp[i]+n+1, INF);
      for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
        for(int k=0; k<(int)g[j].size(); k++)
          dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                                   dp[i-1][j]+g[j][k].w);
      }
    }
    mu=INF; LL bunbo=1;
    for(int i=1; i<=n; i++) if(dp[n][i] < INF){
      LL a=-INF, b=1;
      for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
        if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
          a = dp[n][i]-dp[j][i];
          b = n-j;
        }
      }
      if(mu*b > bunbo*a)
        mu = a, bunbo = b;
    }
    if(mu < 0) return -1; // negative cycle
    if(mu == INF) return INF; // no cycle
    if(mu == 0) return 0;
    for(int i=1; i<=n; i++)
      for(int j=0; j<(int)g[i].size(); j++)
      g[i][j].w *= bunbo;
    memset(p, 0, sizeof(p));
    queue<int> q;
    for(int i=1; i<=n; i++){
      q.push(i);
      inq[i] = true;
    }
    while(!q.empty()){
      int i=q.front(); q.pop(); inq[i]=false;
      for(int j=0; j<(int)g[i].size(); j++){
        if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
          p[g[i][j].to] = p[i]+g[i][j].w-mu;
          if(!inq[g[i][j].to]){
            q.push(g[i][j].to);
            inq[g[i][j].to] = true;
          }
        }
      }
    }
    for(int i=1; i<=n; i++) grev[i].clear();
    for(int i=1; i<=n; i++)
      for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w += p[i]-p[g[i][j].to];
        grev[g[i][j].to].push_back(edge(i, g[i][j].w));
      }
    LL mldc = n*mu;
    for(int i=1; i<=n; i++){
      bn=mldc/mu, bsz=0;
      memset(hd, 0, sizeof(hd));
      fill(d+i+1, d+n+1, INF);
      b_insert(d[i]=0, i);
      for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
          b[k].next){
        int u = b[k].u;
        LL du = b[k].d;
        if(du > d[u]) continue;
        for(int l=0; l<(int)g[u].size(); l++) if(g[u][l
            ].to > i){
          if(d[g[u][l].to] > du + g[u][l].w){
            d[g[u][l].to] = du + g[u][l].w;
            b_insert(d[g[u][l].to], g[u][l].to);
          }
        }
      }
      for(int j=0; j<(int)grev[i].size(); j++) if(grev[
          i][j].to > i)
        mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
    }
    return mldc / bunbo;
  }
} graph;
```

6.8 General Weighted Matching

```
struct WeightGraph {
  static const int INF = INT_MAX;
  static const int N = 514;
  struct edge {
    int u, v, w;
    edge() {}
    edge(int ui, int vi, int wi) : u(ui), v(vi), w(wi)
        {}
  };
  int n, n_x;
  edge g[N * 2][N * 2];
  int lab[N * 2];
  int match[N * 2], slack[N * 2], st[N * 2], pa[N * 2];
  int flo_from[N * 2][N + 1], S[N * 2], vis[N * 2];
  vector<int> flo[N * 2];
  queue<int> q;
  int e_delta(const edge& e) { return lab[e.u] + lab[e.
      v] - g[e.u][e.v].w * 2; }
  void update_slack(int u, int x) {
    if (not slack[x] or e_delta(g[u][x]) < e_delta(g[
        slack[x]][x]))
      slack[x] = u;
  }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 and st[u] != x and S[st[u]] ==
          0) update_slack(u, x);
  }
  void q_push(int x) {
    if (x <= n)
      q.push(x);
    else
      for (size_t i = 0; i < flo[x].size(); i++) q_push
          (flo[x][i]);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n)
      for (size_t i = 0; i < flo[x].size(); ++i) set_st
          (flo[x][i], b);
  }
  int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) -
        flo[b].begin();
    if (pr % 2 == 1) {
      reverse(flo[b].begin() + 1, flo[b].end());
      return (int)flo[b].size() - pr;
    } else
      return pr;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i],
        flo[u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]];
      set_match(u, v);
      if (not xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u or v; swap(u, v)) {
      if (u == 0) continue;
      if (vis[u] == t) return u;
      vis[u] = t;
      u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void add_blossom(int u, int lca, int v) {
```

```
    int b = n + 1;
    while (b <= n_x and st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
      flo[b].push_back(x), flo[b].push_back(y = st[
          match[x]]), q_push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
      flo[b].push_back(x), flo[b].push_back(y = st[
          match[x]]), q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
        w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
      int xs = flo[b][i];
      for (int x = 1; x <= n_x; ++x)
        if (g[b][x].w == 0 or e_delta(g[xs][x]) <
            e_delta(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i) set_st(
        flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
      int xs = flo[b][i], xns = flo[b][i + 1];
      pa[xs] = g[xns][xs].u;
      S[xs] = 1, S[xns] = 0;
      slack[xs] = 0, set_slack(xns);
      q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
      int xs = flo[b][i];
      S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
  }
  bool on_found_edge(const edge& e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
      pa[v] = e.u, S[v] = 1;
      int nu = st[match[v]];
      slack[v] = slack[nu] = 0;
      S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
      int lca = get_lca(u, v);
      if (not lca)
        return augment(u, v), augment(v, u), true;
      else
        add_blossom(u, lca, v);
    }
    return false;
  }
  bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
      if (st[x] == x and not match[x]) pa[x] = 0, S[x]
          = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (q.size()) {
        int u = q.front();
        q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 and st[u] != st[v]) {
            if (e_delta(g[u][v]) == 0) {
              if (on_found_edge(g[u][v])) return true;
            } else
              update_slack(u, st[v]);
          }
      }
      int d = INF;
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b and S[b] == 1) d = min(d, lab[b]
            / 2);
      for (int x = 1; x <= n_x; ++x)
        if (st[x] == x and slack[x]) {
          if (S[x] == -1)
            d = min(d, e_delta(g[slack[x]][x]));
          else if (S[x] == 0)
            d = min(d, e_delta(g[slack[x]][x]) / 2);
        }
      for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
          if (lab[u] <= d) return 0;
          lab[u] -= d;
        } else if (S[st[u]] == 1)
          lab[u] += d;
      }
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b) {
          if (S[st[b]] == 0)
            lab[b] += d * 2;
          else if (S[st[b]] == 1)
            lab[b] -= d * 2;
        }
      q = queue<int>();
      for (int x = 1; x <= n_x; ++x)
        if (st[x] == x and slack[x] and st[slack[x]] !=
            x and
            e_delta(g[slack[x]][x]) == 0)
          if (on_found_edge(g[slack[x]][x])) return
              true;
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b and S[b] == 1 and lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  }
  pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
      }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
      if (match[u] and match[u] < u) tot_weight += g[u
          ][match[u]].w;
    return {tot_weight, n_matches};
  }
  void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
       g[vi][ui].w = wi; }
  void init(int _n) {  // 1-index, zero indicates
      unsaturated
    n = _n;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v) g[u][v] = edge(u, v,
           0);
  }
} graph;
```

6.9 Graph Sequence Test

```
bool is_degree_sequence(vector<LL> d) {
  if (accumulate(d.begin(), d.end(), 0ll)&1) return
      false;
  sort(d.rbegin(), d.rend());
  const int n = d.size();
  vector<LL> pre(n + 1, 0);
```

```
  for (int i = 0; i < n; ++i) pre[i + 1] += pre[i] + d[i];
  for (LL k = 0, j = 0; k < n; ++k) {
    while (j < n and (j <= k or d[j] < k)) ++j;
    if (pre[k + 1] > k * (k + 1) + pre[j] - pre[k + 1] + (k + 1) * (n - j))
      return false;
  }
  return true;
}
```

6.10 maximal cliques

```
#include <bits/stdc++.h>
using namespace std;

const int N = 60;
typedef long long LL;

struct Bron_Kerbosch {
  int n, res;
  LL edge[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i <= n; i++) edge[i] = 0;
  }
  void add_edge(int u, int v) {
    if ( u == v ) return;
    edge[u] |= 1LL << v;
    edge[v] |= 1LL << u;
  }
  void go(LL R, LL P, LL X) {
    if ( P == 0 && X == 0 ) {
      res = max( res, __builtin_popcountll(R) ); // notice LL
      return;
    }
    if ( __builtin_popcountll(R) + __builtin_popcountll(P) <= res ) return;
    for (int i = 0; i <= n; i++) {
      LL v = 1LL << i;
      if ( P & v ) {
        go( R | v, P & edge[i], X & edge[i] );
        P &= ~v;
        X |= v;
      }
    }
  }
  int solve() {
    res = 0;
    go( 0LL, ( 1LL << (n+1) ) - 1, 0LL );
    return res;
  }
/*  BronKerbosch1(R, P, X):
      if P and X are both empty:
        report R as a maximal clique
      for each vertex v in P:
        BronKerbosch1(R ▯ {v}, P ▯ N(v), X ▯ N(v))
        P := P \ {v}
        X := X ▯ {v}
*/
} MaxClique;

int main() {
  MaxClique.init(6);
  MaxClique.add_edge(1,2);
  MaxClique.add_edge(1,5);
  MaxClique.add_edge(2,5);
  MaxClique.add_edge(4,5);
  MaxClique.add_edge(3,2);
  MaxClique.add_edge(4,6);
  MaxClique.add_edge(3,4);
  cout << MaxClique.solve() << "\n";
  return 0;
}
```

6.11 MinMeanCycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
  struct Edge { int v,u; double c; };
  int n, m, prv[V][V], prve[V][V], vst[V];
  Edge e[E];
  vector<int> edgeID, cycle, rho;
  double d[V][V];
  void init( int _n )
  { n = _n; m = 0; }
  // WARNING: TYPE matters
  void addEdge( int vi , int ui , double ci )
  { e[ m ++ ] = { vi , ui , ci }; }
  void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
      fill(d[i+1], d[i+1]+n, inf);
      for(int j=0; j<m; j++) {
        int v = e[j].v, u = e[j].u;
        if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
          d[i+1][u] = d[i][v]+e[j].c;
          prv[i+1][u] = v;
          prve[i+1][u] = j;
        }
      }
    }
  }
  double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
      double avg=-inf;
      for(int k=0; k<n; k++) {
        if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
        else avg=max(avg,inf);
      }
      if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
      vst[st]++;
      edgeID.PB(prve[i][st]);
      rho.PB(st);
    }
    while (vst[st] != 2) {
      int v = rho.back(); rho.pop_back();
      cycle.PB(v);
      vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
  }
} mmc;
```

6.12 Prufer code

```
vector<int> Prufer_encode(vector<vector<int>> T) {
  int n = T.size();
  assert(n > 1);
  vector<int> deg(n), code;
  priority_queue<int, vector<int>, greater<int>> pq;
  for (int i = 0; i < n; ++i) {
    deg[i] = T[i].size();
    if (deg[i] == 1) pq.push(i);
  }
  while (code.size() < n - 2) {
    int v = pq.top(); pq.pop();
    --deg[v];
    for (int u: T[v]) {
      if (deg[u]) {
        --deg[u];
        code.push_back(u);
```

```
        if (deg[u] == 1) pq.push(u);
      }
    }
  }
  return code;
}
vector<vector<int>> Prufer_decode(vector<int> C) {
  int n = C.size() + 2;
  vector<vector<int>> T(n, vector<int>(0));
  vector<int> deg(n, 1); // outdeg
  for (int c: C) ++deg[c];
  priority_queue<int, vector<int>, greater<int>> q;
  for (int i = 0; i < n; ++i) if (deg[i] == 1) q.push(i
      );
  for (int c: C) {
    int v = q.top(); q.pop();
    T[v].push_back(c), T[c].push_back(v);
    --deg[c];
    --deg[v];
    if (deg[c] == 1) q.push(c);
  }
  int u = find(deg.begin(), deg.end(), 1) - deg.begin()
      ;
  int v = find(deg.begin() + u + 1, deg.end(), 1) - deg
      .begin();
  T[u].push_back(v), T[v].push_back(u);
  return T;
}
```

6.13 SPFA

```
struct SPFA {
  const LL INF = 1ll<<62;
  vector<vector<pair<int, LL>>> g;
  vector<int> p;
  vector<LL> d;
  int n;
  void init(int _n) {
    n = _n;
    g.assign(n, vector<pair<int, LL>>(0));
    d.assign(n, INF);
    p.assign(n, -1);
  }
  void add_edge(int u, int v, LL w) {
    g[u].push_back({v, w});
  }
  LL shortest_path(int s, int t) {
    for (int i = 0; i < n; ++i)
      sort(g[i].begin(), g[i].end(), [](pair<int, LL> A
          , pair<int, LL> B) {
        return A.second < B.second;
      });
    vector<bool> inq(n, false);
    vector<int> inq_t(n, 0);
    queue<int> q;
    q.push(s);
    d[s] = 0, inq_t[s] = 1;
    int u, v;
    LL w;
    while (q.size()) {
      inq[v = q.front()] = false; q.pop();
      for (auto P: g[v]) {
        tie(u, w) = P;
        if (d[u] > d[v] + w) {
          d[u] = d[v] + w, p[u] = v;
          if (not inq[u]) {
            q.push(u), inq[u] = true, ++inq_t[u];
            if (inq_t[u] > n) return -INF;
          }
        }
      }
    }
    return d[t];
  }
}solver;
```

7 String

7.1 AC automaton

```
// SIGMA[0] will not be considered
const string SIGMA = "
    _0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrs
    ";
vector<int> INV_SIGMA;
const int SGSZ = 63;

struct PMA {
  PMA *next[SGSZ];  // next[0] is for fail
  vector<int> ac;
  PMA *last; // state of longest accepted string that
      is pre of this
  PMA() : last(nullptr) { fill(next, next + SGSZ,
      nullptr); }
};

template<typename T>
PMA *buildPMA(const vector<T> &p) {
  PMA *root = new PMA;
  for (int i = 0; i < p.size(); ++i) {  // make trie
    PMA *t = root;
    for (int j = 0; j < p[i].size(); ++j) {
      int c = INV_SIGMA[p[i][j]];
      if (t->next[c] == nullptr) t->next[c] = new PMA;
      t = t->next[c];
    }
    t->ac.push_back(i);
  }
  queue<PMA *> que;  // make failure link using bfs
  for (int c = 1; c < SGSZ; ++c) {
    if (root->next[c]) {
      root->next[c]->next[0] = root;
      que.push(root->next[c]);
    } else root->next[c] = root;
  }
  while (!que.empty()) {
    PMA *t = que.front();
    que.pop();
    for (int c = 1; c < SGSZ; ++c) {
      if (t->next[c]) {
        que.push(t->next[c]);
        PMA *r = t->next[0];
        while (!r->next[c]) r = r->next[0];
        t->next[c]->next[0] = r->next[c];
        t->next[c]->last = r->next[c]->ac.size() ? r->
            next[c] : r->next[c]->last;
      }
    }
  }
  return root;
}

void destructPMA(PMA *root) {
  queue<PMA *> que;
  que.emplace(root);
  while (!que.empty()) {
    PMA *t = que.front();
    que.pop();
    for (int c = 1; c < SGSZ; ++c) {
      if (t->next[c] && t->next[c] != root) que.emplace
          (t->next[c]);
    }
    delete t;
  }
}

template<typename T>
map<int, int> match(const T &t, PMA *v) {
  map<int, int> res;
  for (int i = 0; i < t.size(); ++i) {
    int c = INV_SIGMA[t[i]];
    while (!v->next[c]) v = v->next[0];
    v = v->next[c];
    for (int j = 0; j < v->ac.size(); ++j) ++res[v->ac[
        j]];
    for (PMA *q = v->last; q; q = q->last) {
```

```cpp
      for (int j = 0; j < q->ac.size(); ++j) ++res[q->
          ac[j]];
    }
  }
  return res;
}

signed main() {
  INV_SIGMA.assign(256, -1);
  for (int i = 0; i < SIGMA.size(); ++i) {
    INV_SIGMA[SIGMA[i]] = i;
  }

}
```

7.2 KMP

```cpp
template<typename T>
vector<int> build_kmp(const T &s) {
  vector<int> f(s.size());
  int fp = f[0] = -1;
  for (int i = 1; i < s.size(); ++i) {
    while (~fp && s[fp + 1] != s[i]) fp = f[fp];
    if (s[fp + 1] == s[i]) ++fp;
    f[i] = fp;
  }
  return f;
}
template<typename S>
vector<int> kmp_match(vector<int> fail, const S &P,
    const S &T) {
  vector<int> res; // start from these points
  const int n = P.size();
  for (int j = 0, i = -1; j < T.size(); ++j) {
    while (~i and T[j] != P[i + 1]) i = fail[i];
    if (P[i + 1] == T[j]) ++i;
    if (i == n - 1) res.push_back(j - n + 1), i = fail[
        i];
  }
  return res;
}
```

7.3 Manacher

```cpp
template<typename T, int INF>
vector<int> manacher(const T &s) { // p = "INF" + s.
    join("INF") + "INF", returns radius on p
  vector<int> p(s.size() * 2 + 1, INF);
  for (int i = 0; i < s.size(); ++i) {
    p[i << 1 | 1] = s[i];
  }
  vector<int> w(p.size());
  for (int i = 1, j = 0, r = 0; i < p.size(); ++i) {
    int t = min(r >= i ? w[2 * j - i] : 0, r - i + 1);
    for ( ; i - t >= 0 && i + t < p.size(); ++t) {
      if (p[i - t] != p[i + t]) break;
    }
    w[i] = --t;
    if (i + t > r) r = i + t, j = i;
  }
  return w;
}
```

7.4 Suffix Array

```cpp
// -----------O(NlgNlgN)----------
pair<vector<int>, vector<int>> sa_db(const string s) {
  int n = s.size();
  vector<int> sa(n), ra(n), t(n);
  for (int i = 0; i < n; ++i) ra[sa[i] = i] = s[i];
  for (int h = 1; t[n - 1] != n - 1; h *= 2) {
    auto cmp = [&](int i, int j) {
      if (ra[i] != ra[j]) return ra[i] < ra[j];
      return i + h < n && j + h < n ? ra[i + h] < ra[j
          + h] : i > j;
    };
    sort(sa.begin(), sa.end(), cmp);
    for (int i = 0; i + 1 < n; ++i) t[i + 1] = t[i] +
        cmp(sa[i], sa[i + 1]);
    for (int i = 0; i < n; ++i) ra[sa[i]] = t[i];
  }
  return {sa, ra};
}

// O(N) -- CF: 1e6->31ms,18MB;1e7->296ms;158MB;3e7->856
    ms,471MB
bool is_lms(const string &t, int i) {
  return i > 0 && t[i - 1] == 'L' && t[i] == 'S';
}

template<typename T>
vector<int> induced_sort(const T &s, const string &t,
    const vector<int> &lmss, int sigma = 256) {
  vector<int> sa(s.size(), -1);

  vector<int> bin(sigma + 1);
  for (auto it = s.begin(); it != s.end(); ++it) {
    ++bin[*it + 1];
  }

  int sum = 0;
  for (int i = 0; i < bin.size(); ++i) {
    sum += bin[i];
    bin[i] = sum;
  }

  vector<int> cnt(sigma);
  for (auto it = lmss.rbegin(); it != lmss.rend(); ++it
      ) {
    int ch = s[*it];
    sa[bin[ch + 1] - 1 - cnt[ch]] = *it;
    ++cnt[ch];
  }

  cnt = vector<int>(sigma);
  for (auto it = sa.begin(); it != sa.end(); ++it) {
    if (*it <= 0 || t[*it - 1] == 'S') continue;
    int ch = s[*it - 1];
    sa[bin[ch] + cnt[ch]] = *it - 1;
    ++cnt[ch];
  }

  cnt = vector<int>(sigma);
  for (auto it = sa.rbegin(); it != sa.rend(); ++it) {
    if (*it <= 0 || t[*it - 1] == 'L') continue;
    int ch = s[*it - 1];
    sa[bin[ch + 1] - 1 - cnt[ch]] = *it - 1;
    ++cnt[ch];
  }

  return sa;
}

template<typename T>
vector<int> sa_is(const T &s, int sigma = 256) {
  string t(s.size(), 0);
  t[s.size() - 1] = 'S';
  for (int i = int(s.size()) - 2; i >= 0; --i) {
    if (s[i] < s[i + 1]) t[i] = 'S';
    else if (s[i] > s[i + 1]) t[i] = 'L';
    else t[i] = t[i + 1];
  }

  vector<int> lmss;
  for (int i = 0; i < s.size(); ++i) {
    if (is_lms(t, i)) {
      lmss.emplace_back(i);
    }
  }

  vector<int> sa = induced_sort(s, t, lmss, sigma);
  vector<int> sa_lms;
  for (int i = 0; i < sa.size(); ++i) {
    if (is_lms(t, sa[i])) {
      sa_lms.emplace_back(sa[i]);
    }
  }
```

```
  int lmp_ctr = 0;
  vector<int> lmp(s.size(), -1);
  lmp[sa_lms[0]] = lmp_ctr;
  for (int i = 0; i + 1 < sa_lms.size(); ++i) {
    int diff = 0;
    for (int d = 0; d < sa.size(); ++d) {
      if (s[sa_lms[i] + d] != s[sa_lms[i + 1] + d] ||
          is_lms(t, sa_lms[i] + d) != is_lms(t, sa_lms[
              i + 1] + d)) {
        diff = 1; // something different in range of
            lms
        break;
      } else if (d > 0 && is_lms(t, sa_lms[i] + d) &&
          is_lms(t, sa_lms[i + 1] + d)) {
        break; // exactly the same
      }
    }
    if (diff) ++lmp_ctr;
    lmp[sa_lms[i + 1]] = lmp_ctr;
  }

  vector<int> lmp_compact;
  for (int i = 0; i < lmp.size(); ++i) {
    if (~lmp[i]) {
      lmp_compact.emplace_back(lmp[i]);
    }
  }

  if (lmp_ctr + 1 < lmp_compact.size()) {
    sa_lms = sa_is(lmp_compact, lmp_ctr + 1);
  } else {
    for (int i = 0; i < lmp_compact.size(); ++i) {
      sa_lms[lmp_compact[i]] = i;
    }
  }

  vector<int> seed;
  for (int i = 0; i < sa_lms.size(); ++i) {
    seed.emplace_back(lmss[sa_lms[i]]);
  }

  return induced_sort(s, t, seed, sigma);
} // s must end in char(0)

// O(N) lcp, note that s must end in '\0'
vector<int> build_lcp(string &s, vector<int> &sa,
    vector<int> &ra) {
  int n = s.size();
  vector<int> lcp(n);
  for (int i = 0, h = 0; i < n; ++i) {
    if (ra[i] == 0) continue;
    if (h > 0) --h;
    for (int j = sa[ra[i] - 1]; max(j, i) + h < n; ++h)
        {
      if (s[j + h] != s[i + h]) break;
    }
    lcp[ra[i] - 1] = h;
  }
  return lcp; // lcp[i] := LCP(s[sa[i]], s[sa[i + 1]])
}

// O(N) build segment tree for lcp
vector<int> build_lcp_rmq(const vector<int> &lcp) {
  vector<int> sgt(lcp.size() << 2);
  function<void(int, int, int)> build = [&](int t, int
      lb, int rb) {
    if (rb - lb == 1) return sgt[t] = lcp[lb], void();
    int mb = lb + rb >> 1;
    build(t << 1, lb, mb);
    build(t << 1 | 1, mb, rb);
    sgt[t] = min(sgt[t << 1], sgt[t << 1 | 1]);
  };
  build(1, 0, lcp.size());
  return sgt;
}

// O(|P| + lg |T|) pattern searching, returns last
    index in sa
int match(const string &p, const string &s, const
    vector<int> &sa, const vector<int> &rmq) { // rmq
    is segtree on lcp
  int t = 1, lb = 0, rb = s.size(); // answer in [lb,
      rb)
  int lcplp = 0; // lcp(char(0), p) = 0
  while (rb - lb > 1) {
    int mb = lb + rb >> 1;
    int lcplm = rmq[t << 1];
    if (lcplp < lcplm) t = t << 1 | 1, lb = mb;
    else if (lcplp > lcplm) t = t << 1, rb = mb;
    else {
      int lcpmp = lcplp;
      while (lcpmp < p.size() && p[lcpmp] == s[sa[mb] +
          lcpmp]) ++lcpmp;
      if (lcpmp == p.size() || p[lcpmp] > s[sa[mb] +
          lcpmp]) t = t << 1 | 1, lb = mb, lcplp =
          lcpmp;
      else t = t << 1, rb = mb;
    }
  }
  if (lcplp < p.size()) return -1;
  return sa[lb];
}

int LCA(int i, int j, const vector<int> &ra, const
    vector<int> &lcp_seg) {
  // lca of ith and jth suffix
  if (ra[i] > ra[j]) swap(i, j);
  function<int(int, int, int, int, int)> query = [&](
      int L, int R, int l, int r, int v) {
    if (L <= l and r <= R) return lcp_seg[v];
    int m = l + r >> 1, ans = 1e9;
    if (L < m) ans = min(ans, query(L, R, l, m, v << 1)
        );
    if (m < R) ans = min(ans, query(L, R, m, r, v <<
        1|1));
    return ans;
  };
  return query(ra[i], ra[j], 0, ra.size(), 1);
}
vector<vector<int>> build_lcp_sparse_table(const vector
    <int> &lcp) {
  int n = lcp.size(), lg = 31 - __builtin_clz(n);
  vector<vector<int>> st(lg + 1, vector<int>(n));
  for (int i = 0; i < n; ++i) st[0][i] = lcp[i];
  for (int j = 1; (1<<j) <= n; ++j)
    for (int i = 0; i + (1<<j) <= n; ++i)
      st[j][i] = min(st[j - 1][i], st[j - 1][i + (1<<(j
          - 1))]);
  return st;
}
int sparse_rmq(int i, int j, const vector<int> &ra,
    const vector<vector<int>> &st) {
  int n = st[0].size();
  if (ra[i] > ra[j]) swap(i, j);
  int k = 31 - __builtin_clz(ra[j] - ra[i]);
  return min(st[k][ra[i]], st[k][ra[j] - (1<<k)]);
}// sparse_rmq(sa[i], sa[j], ra, st) is the lcp of sa(i
    ), sa(j)
```

7.5 Suffix Automaton

```
template<typename T>
struct SuffixAutomaton {
  vector<map<int, int>> edges;// edges[i]  : the
      labeled edges from node i
  vector<int> link;           // link[i]   : the parent
      of i
  vector<int> length;         // length[i] : the length
       of the longest string in the ith class
  int last;                   // the index of the
      equivalence class of the whole string
  vector<bool> is_terminal;   // is_terminal[i] : some
      suffix ends in node i (unnecessary)
  vector<int> occ;            // occ[i] : number of
      matches of maximum string of node i (unnecessary)
  SuffixAutomaton(const T &s) : edges({map<int, int>()
      }), link({-1}), length({0}), last(0), occ({0}) {
    for (int i = 0; i < s.size(); ++i) {
      edges.push_back(map<int, int>());
      length.push_back(i + 1);
      link.push_back(0);
```

```
      occ.push_back(1);
      int r = edges.size() - 1;
      int p = last; // add edges to r and find p with
          link to q
      while (p >= 0 && edges[p].find(s[i]) == edges[p].
          end()) {
        edges[p][s[i]] = r;
        p = link[p];
      }
      if (~p) {
        int q = edges[p][s[i]];
        if (length[p] + 1 == length[q]) { // no need to
             split q
          link[r] = q;
        } else { // split q, add qq
          edges.push_back(edges[q]);  // copy edges of
              q
          length.push_back(length[p] + 1);
          link.push_back(link[q]);  // copy parent of q
          occ.push_back(0);
          int qq = edges.size() - 1; // qq is new
              parent of q and r
          link[q] = qq;
          link[r] = qq;
          while (p >= 0 && edges[p][s[i]] == q) { //
              what points to q points to qq
            edges[p][s[i]] = qq;
            p = link[p];
          }
        }
      }
      last = r;
    } // below unnecessary
    is_terminal = vector<bool>(edges.size());
    for (int p = last; p > 0; p = link[p]) is_terminal[
        p] = 1; // is_terminal calculated
    vector<int> cnt(link.size()), states(link.size());
        // sorted states by length
    for (int i = 0; i < link.size(); ++i) ++cnt[length[
        i]];
    for (int i = 0; i < s.size(); ++i) cnt[i + 1] +=
        cnt[i];
    for (int i = link.size() - 1; i >= 0; --i) states
        [--cnt[length[i]]] = i;
    for (int i = link.size() - 1; i >= 1; --i) occ[link
        [states[i]]] += occ[states[i]]; // occ
        calculated
  }
};
```

8 Formulas

8.1 Pick's theorem

For a polygon:
A: The area of the polygon
B: Boundary Point: a lattice point on the polygon (including vertices)
I: Interior Point: a lattice point in the polygon' s interior region

$$A = I + \frac{B}{2} - 1$$

8.2 Graph Properties

1. Euler's Formula $V - E + F = 2$
2. For a planar graph, $F = E - V + n + 1$, n is the numbers of components
3. For a planar graph, $E \le 3V - 6$

For a connected graph G: $I(G)$: the size of maximum independent set $M(G)$: the size of maximum matching $Cv(G)$: be the size of minimum vertex cover $Ce(G)$: be the size of minimum edge cover

4. For any connected graph:
   (a) $I(G) + Cv(G) = |V|$
   (b) $M(G) + Ce(G) = |V|$
5. For any bipartite:
   (a) $I(G) = Cv(G)$
   (b) $M(G) = Ce(G)$

8.3 Number Theory

1. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
2. $\phi(x), \mu(x)$ are Möbius inverse
3. $\sum_{i=1}^{n} \sum_{j=1}^{m} [\gcd(i,j) = 1] = \sum \mu(d) \left\lfloor \frac{n}{d} \right\rfloor \left\lfloor \frac{m}{d} \right\rfloor$
4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

8.4 Combinatorics

1. Gray Code: $= n \oplus (n >> 1)$
2. Catalan Number:

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{n!(n+1)!} = \prod_{k=2}^{n} \frac{n+k}{k}$$

3. $\Gamma(n+1) = n!$
4. $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$
5. Stirling number of second kind: the number of ways to partition a set of n elements into k nonempty subsets.
   (a) $\left\{{0 \atop 0}\right\} = \left\{{n \atop n}\right\} = 1$
   (b) $\left\{{n \atop 0}\right\} = 0$
   (c) $\left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\}$
6. Bell numbers count the possible partitions of a set:
   (a) $B_0 = 1$
   (b) $B_n = \sum_{k=0}^{n} \left\{{n \atop k}\right\}$
   (c) $B_{n+1} = \sum_{k=0}^{n} C_k^n B_k$
   (d) $B_{p+n} \equiv B_n + B_{n+1}$ mod p, p prime
   (e) $B_{p^m+n} \equiv mB_n + B_{n+1}$ mod p, p prime
   (f) From $B_0 : 1, 1, 2, 5, 15, 52,$ $203, 877, 4140, 21147, 115975$
7. Derangement
   (a) $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n \frac{1}{n!})$
   (b) $D_n = (n-1)(D_{n-1} + D_{n-2})$
   (c) From $D_0 : 1, 0, 1, 2, 9, 44,$ $265, 1854, 14833, 133496$
8. Binomial Equality
   (a) $\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n}$
   (b) $\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$
   (c) $\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$
   (d) $\sum_{k \le l} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-1}{l-n-m}$
   (e) $\sum_{0 \le k \le l} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
   (f) $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
   (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
   (h) $\sum_{k \le n} \binom{r+k}{k} = \binom{r+n+1}{n}$
   (i) $\sum_{0 \le k \le n} \binom{k}{m} = \binom{n+1}{m+1}$
   (j) $\sum_{k \le m} \binom{m+r}{k} x^k y^k = \sum_{k \le m} \binom{-r}{k}(-x)^k(x+y)^{m-k}$

8.5 Sum of Powers

1. $a^b \% P = a^{b\%\varphi(p)+\varphi(p)}, b \ge \varphi(p)$
2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
5. $0^k + 1^k + 2^k + \ldots + n^k = P_k, P_k = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P_0 = n+1$
6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^{n} C_k^{n+1} B_k m^{n+1-k}$
7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
8. 除了 $B_1 = -1/2$，剩下的奇數項都是 0
9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

8.6 Burnside's lemma

1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
2. $X^g = t^{c(g)}$

8.7 Count on a tree

1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
2. Unrooted tree:
   (a) Odd:$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
   (b) Even:$Odd + \frac{1}{2} a_{n/2}(a_{n/2} + 1)$
3. Spanning Tree
   (a) 完全圖 $n^n - 2$
   (b) 一般圖 (Kirchhoff's theorem)$M[i][i] = \deg(V_i), M[i][j] = -1$,if have $E(i,j)$,0 if no edge. delete any one row and col in A, $ans = \det(A)$