Contents

1 Basic

1.1 .vimrc

```
syntax on
set nu ai bs=2 sw=2 et ve=all cb=unnamed mouse=a ruler
    incsearch hlsearch
```

2 Combinatorics

2.1 FFT

```
typedef complex<double> cpx;
const double PI = acos(-1);
vector<cpx> FFT(vector<cpx> &P, bool inv = 0) {
  assert(__builtin_popcount(P.size()) == 1);
  int lg = 31 - __builtin_clz(P.size()), n = 1 << lg;
      // == P.size();
  for (int j = 1, i = 0; j < n - 1; ++j) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(P[i], P[j]);
  } //bit reverse
  auto w1 = exp((2 - 4 * inv) * PI / n * cpx(0, 1)); //
       order is 1<<lg
  for (int i = 1; i <= lg; ++i) {
    auto wn = pow(w1, 1<<(lg - i)); // order is 1<<i
    for (int k = 0; k < (1<<lg); k += 1 << i) {
      cpx base = 1;
      for (int j = 0; j < (1 << i - 1); ++j, base =
          base * wn) {
        auto t = base * P[k + j + (1 << i - 1)];
        auto u = P[k + j];
        P[k + j] = u + t;
        P[k + j + (1 << i - 1)] = u - t;
      }
    }
  }
  if(inv)
    for (int i = 0; i < n; ++i) P[i] /= n;
  return P;
} //faster performance with calling by reference
```

2.2 FWT

```
vector<int> fast_OR_transform(vector<int> f, bool
    inverse) {
  for (int i = 0; (2 << i) <= f.size(); ++i)
    for (int j = 0; j < f.size(); j += 2 << i)
      for (int k = 0; k < (1 << i); ++k)
        f[j + k + (1 << i)] += f[j + k] * (inverse? -1
            : 1);
  return f;
}
vector<int> rev(vector<int> A) {
  for (int i = 0; i < A.size(); i += 2) swap(A[i], A[i
      ^ (A.size() - 1)]);
  return A;
}
vector<int> fast_AND_transform(vector<int> f, bool
    inverse) {
  return rev(fast_OR_transform(rev(f), inverse));
}
vector<int> fast_XOR_transform(vector<int> f, bool
    inverse) {
  for (int i = 0; (2 << i) <= f.size(); ++i)
    for (int j = 0; j < f.size(); j += 2 << i)
      for (int k = 0; k < (1 << i); ++k) {
        int u = f[j + k], v = f[j + k + (1 << i)];
        f[j + k + (1 << i)] = u - v, f[j + k] = u + v;
      }
  if (inverse) for (auto &a : f) a /= f.size();
  return f;
}
```

2.3 NTT

```
/* p == (a << n) + 1
   n    1 << n     p          a    root
   5    32         97         3    5
   6    64         193        3    5
   7    128        257        2    3
   8    256        257        1    3
   9    512        7681       15   17
   10   1024       12289      12   11
   11   2048       12289      6    11
   12   4096       12289      3    11
   13   8192       40961      5    3
   14   16384      65537      4    3
   15   32768      65537      2    3
   16   65536      65537      1    3
   17   131072     786433     6    10
   18   262144     786433     3    10 (605028353,
       2308, 3)
   19   524288     5767169    11   3
   20   1048576    7340033    7    3
   21   2097152    23068673   11   3
   22   4194304    104857601  25   3
   23   8388608    167772161  20   3
   24   16777216   167772161  10   3
   25   33554432   167772161  5    3 (1107296257, 33,
       10)
   26   67108864   469762049  7    3
   27   134217728  2013265921 15   31 */
LL root = 10, p = 786433, a = 3;
LL powM(LL x, LL b) {
  LL s = 1, m = x % p;
  for (; b; m = m * m % p, b >>= 1)
    if (b&1) s = s * m % p;
  return s;
}
vector<LL> NTT(vector<LL> P, bool inv = 0) {
  assert(__builtin_popcount(P.size()) == 1);
  int lg = 31 - __builtin_clz(P.size()), n = 1 << lg;
      // == P.size();
  for (int j = 1, i = 0; j < n - 1; ++j) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(P[i], P[j]);
  } //bit reverse
  LL w1 = powM(root, a * (inv? p - 2: 1)); // order is
      1<<lg
  for (LL i = 1; i <= lg; ++i) {
    LL wn = powM(w1, 1<<(lg - i)); // order is 1<<i
    for (int k = 0; k < (1<<lg); k += 1 << i) {
      LL base = 1;
      for (int j = 0; j < (1 << i - 1); ++j, base =
          base * wn % p) {
        LL t = base * P[k + j + (1 << i - 1)] % p;
        LL u = P[k + j] % p;
        P[k + j] = (u + t) % p;
        P[k + j + (1 << i - 1)] = (u - t + p) % p;
      }
    }
  }
  if(inv){
    LL invN = powM(n, p - 2);
    transform(P.begin(), P.end(), P.begin(), [&](LL a)
        {return a * invN % p;});
  }
  return P;
} //faster performance with calling by reference
```

2.4 permanent

```
typedef vector<vector<LL> > mat;
LL permanent(mat A) {
  LL n = A.size(), ans = 0, *tmp = new LL[n], add;
  for (int pgray = 0, s = 1, gray, i; s < 1 << n; ++s)
      {
    gray = s ^ s >> 1, add = 1;
    i = __builtin_ctz(pgray ^ gray);
    for (int j = 0; j < n; ++j)
      add *= tmp[j] += A[i][j] * (gray>>i&1 ? 1 : -1);
    ans += add * (s&1^n&1? -1 : 1), pgray = gray;
  }
  return ans;
}
// how many ways to put rooks on a matrix with 0,1 as
    constrain
// 1 - ok to put
// 0 - not ok to put
```

3 Data Structure

3.1 Heavy Light Decomposition

```
struct HLD {
  using Tree = vector<vector<int>>;
  vector<int> par, head, vid, len, inv;

  HLD(const Tree &g) : par(g.size()), head(g.size()),
      vid(g.size()), len(g.size()), inv(g.size()) {
    int k = 0;
    vector<int> size(g.size(), 1);
    function<void(int, int)> dfs_size = [&](int u, int
        p) {
      for (int v : g[u]) {
        if (v != p) {
          dfs_size(v, u);
          size[u] += size[v];
        }
      }
    };
    function<void(int, int, int)> dfs_dcmp = [&](int u,
         int p, int h) {
      par[u] = p;
      head[u] = h;
      vid[u] = k++;
      inv[vid[u]] = u;
      for (int v : g[u]) {
        if (v != p && size[u] < size[v] * 2) {
          dfs_dcmp(v, u, h);
        }
      }
      for (int v : g[u]) {
        if (v != p && size[u] >= size[v] * 2) {
          dfs_dcmp(v, u, v);
        }
      }
    };
    dfs_size(0, -1);
    dfs_dcmp(0, -1, 0);
    for (int i = 0; i < g.size(); ++i) {
      ++len[head[i]];
    }
  }

  template<typename T>
  void foreach(int u, int v, T f) {
    while (true) {
      if (vid[u] > vid[v]) {
        if (head[u] == head[v]) {
          f(vid[v] + 1, vid[u], 0);
          break;
        } else {
          f(vid[head[u]], vid[u], 1);
          u = par[head[u]];
        }
      } else {
        if (head[u] == head[v]) {
          f(vid[u] + 1, vid[v], 0);
          break;
        } else {
          f(vid[head[v]], vid[v], 0);
          v = par[head[v]];
        }
      }
    }
  }
};
```

4 Flow

4.1 CostFlow

```
template <class TF, class TC>
struct CostFlow {
  static const int MAXV = 205;
  static const TC INF = 0x3f3f3f3f;
  struct Edge {
    int v, r;
    TF f;
    TC c;
    Edge(int _v, int _r, TF _f, TC _c) : v(_v), r(_r),
        f(_f), c(_c) {}
  };
  int n, s, t, pre[MAXV], pre_E[MAXV], inq[MAXV];
  TF fl;
  TC dis[MAXV], cost;
  vector<Edge> E[MAXV];
  CostFlow(int _n, int _s, int _t) : n(_n), s(_s), t(_t
      ), fl(0), cost(0) {}
  void add_edge(int u, int v, TF f, TC c) {
    E[u].emplace_back(v, E[v].size(), f, c);
    E[v].emplace_back(u, E[u].size() - 1, 0, -c);
  }
  pair<TF, TC> flow() {
    while (true) {
      for (int i = 0; i < n; ++i) {
        dis[i] = INF;
        inq[i] = 0;
      }
      dis[s] = 0;
      queue<int> que;
      que.emplace(s);
      while (not que.empty()) {
        int u = que.front();
        que.pop();
        inq[u] = 0;
        for (int i = 0; i < E[u].size(); ++i) {
          int v = E[u][i].v;
          TC w = E[u][i].c;
          if (E[u][i].f > 0 and dis[v] > dis[u] + w) {
            pre[v] = u;
            pre_E[v] = i;
            dis[v] = dis[u] + w;
            if (not inq[v]) {
              inq[v] = 1;
              que.emplace(v);
            }
          }
        }
      }
      if (dis[t] == INF) break;
      TF tf = INF;
      for (int v = t, u, l; v != s; v = u) {
        u = pre[v];
        l = pre_E[v];
        tf = min(tf, E[u][l].f);
      }
      for (int v = t, u, l; v != s; v = u) {
        u = pre[v];
        l = pre_E[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      }
      cost += tf * dis[t];
      fl += tf;
    }
    return {fl, cost};
  }
};
```

4.2 Dinic

```
template <class T>
struct Dinic {
  static const int MAXV = 10000;
  static const T INF = 0x3f3f3f3f;
  struct Edge {
    int v;
    T f;
    int re;
    Edge(int _v, T _f, int _re) : v(_v), f(_f), re(_re)
        {}
  };
  int n, s, t, level[MAXV];
  vector<Edge> E[MAXV];
  int now[MAXV];
  Dinic(int _n, int _s, int _t) : n(_n), s(_s), t(_t)
      {}
  void add_edge(int u, int v, T f, bool bidirectional =
      false) {
    E[u].emplace_back(v, f, E[v].size());
    E[v].emplace_back(u, 0, E[u].size() - 1);
    if (bidirectional) {
      E[v].emplace_back(u, f, E[u].size() - 1);
    }
  }
  bool BFS() {
    memset(level, -1, sizeof(level));
    queue<int> que;
    que.emplace(s);
    level[s] = 0;
    while (not que.empty()) {
      int u = que.front();
      que.pop();
      for (auto it : E[u]) {
        if (it.f > 0 and level[it.v] == -1) {
          level[it.v] = level[u] + 1;
          que.emplace(it.v);
        }
      }
    }
    return level[t] != -1;
  }
  T DFS(int u, T nf) {
    if (u == t) return nf;
    T res = 0;
    while (now[u] < E[u].size()) {
      Edge &it = E[u][now[u]];
      if (it.f > 0 and level[it.v] == level[u] + 1) {
        T tf = DFS(it.v, min(nf, it.f));
        res += tf;
        nf -= tf;
        it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      } else
        ++now[u];
    }
    if (not res) level[u] = -1;
    return res;
  }
  T flow(T res = 0) {
    while (BFS()) {
      T temp;
      memset(now, 0, sizeof(now));
      while (temp = DFS(s, INF)) {
        res += temp;
        res = min(res, INF);
      }
    }
    return res;
  }
};
```

5 Geometry

5.1 2D Geometry

```
namespace geo {
  using pt = complex<double>;
  using cir = pair<pt, double>;
  const double EPS = 1e-4;
  const double PI = acos(-1);
  pt cent(cir C) { return C.first; }
```

```
double radi(cir C) { return C.second; }
int dcmp(double x) {
  if (abs(x) < EPS) return 0;
  return x > 0 ? 1 : -1;
}
bool less(pt a, pt b) {
  return real(a) < real(b) || real(a) == real(b) &&
      imag(a) < imag(b);
}
bool more(pt a, pt b) {
  return real(a) > real(b) || real(a) == real(b) &&
      imag(a) > imag(b);
}
double dot(pt a, pt b) { return real(conj(a) * b); }
double cross(pt a, pt b) { return imag(conj(a) * b);
    }
double sarea(pt a, pt b, pt c) { return cross(b - a,
    c - a); }
int ori(pt a, pt b, pt c) { return dcmp(sarea(a, b, c
    )); }
double angle(pt a, pt b) { return acos(dot(a, b) /
    abs(a) / abs(b)); }
pt rotate(pt a, double rad) { return a * pt(cos(rad),
     sin(rad)); }
pt normal(pt a) { return pt(-imag(a) / abs(a), real(a
    ) / abs(a)); }
pt normalized(pt a) { return a / abs(a); }
pt get_line_intersection(pt p, pt v, pt q, pt w) {
  // L = p + t * v, J = q + s * w
  assert(dcmp(cross(v, w)));
  return p + v * cross(w, p - q) / cross(v, w);
}
double distance_to_line(pt p, pt a, pt b) {
  // the line contains two distinct points a, b
  return abs(cross(b - a, p - a) / abs(b - a));
}
double distance_to_segment(pt p, pt a, pt b) {
  // similar to previous function
  if (a == b) return abs(p - a);
  pt v1(b - a), v2(p - a), v3(p - b);
  if (dcmp(dot(v1, v2)) < 0) return abs(v2);
  else if (dcmp(dot(v1, v3)) > 0) return abs(v3);
  return abs(cross(v1, v2)) / abs(v1);
}
pt get_line_projection(pt p, pt a, pt b) {
  pt v = b - a;
  return a + dot(v, p - a) / dot(v, v) * v;
}
bool segment_proper_intersection(pt a1, pt a2, pt b1,
     pt b2) {
  double det1 = ori(a1, a2, b1) * ori(a1, a2, b2);
  double det2 = ori(b1, b2, a1) * ori(b1, b2, a2);
  return det1 < 0 && det2 < 0;
}
double polygon_area(vector<pt> p) {
  double area = 0;
  for (int i = 1; i < p.size() - 1; ++i) {
    area += sarea(p[0], p[i], p[i + 1]);
  }
  return area / 2;
}
bool is_point_on_segment(pt p, pt a1, pt a2) {
  return dcmp(sarea(p, a1, a2)) == 0 && dcmp(dot(a1 -
       p, a2 - p)) < 0;
}
int is_point_in_polygon(pt p, vector<pt> gon) {
  int wn = 0;
  int n = gon.size();
  for (int i = 0; i < n; ++i) {
    if (is_point_on_segment(p, gon[i], gon[(i + 1) %
        n])) return -1;
    int k = dcmp(cross(gon[(i + 1) % n] - gon[i], p -
         gon[i]));
    int d1 = dcmp(imag(gon[i]) - imag(p));
    int d2 = dcmp(imag(gon[(i + 1) % n] - imag(p)));
    wn += k > 0 && d1 <= 0 && d2 > 0;
    wn -= k < 0 && d2 <= 0 && d1 > 0;
  }
  return wn != 0;
}
vector<pt> convex_hull(vector<pt> p) {
  sort(p.begin(), p.end(), less);
  p.erase(unique(p.begin(), p.end()), p.end());
  int n = p.size(), m = 0;
  vector<pt> ch(n + 1);
  for (int i = 0; i < n; ++i) { // note that border
      is cleared
    while (m > 1 && ori(ch[m - 2], ch[m - 1], p[i])
        <= 0) --m;
    ch[m++] = p[i];
  }
  for (int i = n - 2, k = m; i >= 0; --i) {
    while (m > k && ori(ch[m - 2], ch[m - 1], p[i])
        <= 0) --m;
    ch[m++] = p[i];
  }
  ch.erase(ch.begin() + m - (n > 1), ch.end());
  return ch;
}
cir circumscribed_circle(vector<pt> tri) {
  pt B = tri[1] - tri[0];
  pt C = tri[2] - tri[0];
  double det = 2 * cross(B, C);
  pt r = pt(imag(C) * norm(B) - imag(B) * norm(C),
            real(B) * norm(C) - real(C) * norm(B)) /
                det;
  return {r + tri[0], abs(r)};
}
cir inscribed_circle(vector<pt> tri) {
  assert(tri.size() == 3);
  pt ans = 0;
  double div = 0;
  for (int i = 0; i < 3; ++i) {
    double l = abs(tri[(i + 1) % 3] - tri[(i + 2) %
        3]);
    ans += l * tri[i], div += l;
  }
  ans /= div;
  return {ans, distance_to_line(ans, tri[0], tri[1])
      };
}
vector<pt> tangent_line_through_point(cir c, pt p) {
  if (dcmp(abs(cent(c) - p) - radi(c)) < 0) return
      {};
  else if (dcmp(abs(cent(c) - p) - radi(c)) == 0)
      return {p};
  auto theta = acos(radi(c) / abs(cent(c) - p));
  auto norm_v = normalized(p - cent(c));
  return {cent(c) + radi(c) * rotate(norm_v, +theta),
          cent(c) + radi(c) * rotate(norm_v, -theta)
              };
}
vector<pt> get_line_circle_intersection(cir d, pt st,
     pt ed) {
  pt v = ed - st, p = st - cent(d);
  double r = radi(d), a = norm(v), b = 2 * dot(p, v),
       c = norm(p) - r * r;
  double det = b * b - 4 * a * c;
  // t^2 * norm(v) + 2 * t * dot(p, v) + norm(p) - r
      * r = 0
  auto get_point = [=](double t) { return st + t * v;
       };
  if (dcmp(det) < 0) return {};
  if (dcmp(det) == 0) return {get_point(-b / 2 / a)};
  return {get_point((-b + sqrt(det)) / 2 / a),
          get_point((-b - sqrt(det)) / 2 / a)};
}
vector<pt> get_circle_circle_intersection(cir c, cir
     d) {
  auto a = cent(c), b = cent(d);
  auto r = radi(c), s = radi(d), g = abs(a - b);
  if (dcmp(g) == 0) return {}; // may be C == D
  if (dcmp(r + s - g) < 0 or dcmp(abs(r - s) - g) >
      0) return {};
  auto C_to_D = normalized(b - a);
  auto theta = acos((r * r + g * g - s * s) / (2 * r
      * g));
  if (dcmp(theta) == 0) return {a + r * C_to_D};
  else return {a + rotate(r * C_to_D, theta), a +
      rotate(r * C_to_D, -theta)};
}
};
```

6 Graph

6.1 2-SAT

```
#include <cstdio>
#include <vector>
#include <stack>
#include <cstring>
using namespace std;

const int N = 2010;
struct two_SAT {
  int n;
  vector<int> G[N], revG[N];
  stack<int> finish;
  bool sol[N], visit[N];
  int cmp[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < N; i++) {
      G[i].clear();
      revG[i].clear();
    }
  }
  void add_edge(int u, int v) {
    // 2 * i -> i is True, 2 * i + 1 -> i is False
    G[u].push_back(v);
    G[v^1].push_back(u^1);
    revG[v].push_back(u);
    revG[u^1].push_back(v^1);
  }
  void dfs(int v) {
    visit[v] = true;
    for ( auto i:G[v] ) {
      if ( !visit[i] ) dfs(i);
    }
    finish.push(v);
  }
  void revdfs(int v, int id) {
    visit[v] = true;
    for ( auto i:revG[v] ) {
      if ( !visit[i] ) revdfs(i,id);
    }
    cmp[v] = id;
  }
  int scc() {
    memset( visit, 0, sizeof(visit) );
    for (int i = 0; i < 2 * n; i++) {
      if ( !visit[i] ) dfs(i);
    }
    int id = 0;
    memset( visit, 0, sizeof(visit) );
    while ( !finish.empty() ) {
      int v = finish.top(); finish.pop();
      if ( visit[v] ) continue;
      revdfs(v,++id);
    }
    return id;
  }
  bool solve() {
    scc();
    for (int i = 0; i < n; i++) {
      if ( cmp[2*i] == cmp[2*i+1] ) return 0;
      sol[i] = ( cmp[2*i] > cmp[2*i+1] );
    }
    return 1;
  }
} sat;

int main() {
  // ( a or not b ) and ( b or c ) and ( not c or not a
    )
  sat.init(3);
  sat.add_edge( 2*0+1, 2*1+1 );
  sat.add_edge( 2*1+1, 2*2+0 );
  sat.add_edge( 2*2+0, 2*0+1 );
  printf("%d\n", sat.solve() );
  return 0;
}
```

6.2 maximal cliques

```
#include <bits/stdc++.h>
using namespace std;

const int N = 60;
typedef long long LL;

struct Bron_Kerbosch {
  int n, res;
  LL edge[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i <= n; i++) edge[i] = 0;
  }
  void add_edge(int u, int v) {
    if ( u == v ) return;
    edge[u] |= 1LL << v;
    edge[v] |= 1LL << u;
  }
  void go(LL R, LL P, LL X) {
    if ( P == 0 && X == 0 ) {
      res = max( res, __builtin_popcountll(R) ); //
        notice LL
      return;
    }
    if ( __builtin_popcountll(R) + __builtin_popcountll
      (P) <= res ) return;
    for (int i = 0; i <= n; i++) {
      LL v = 1LL << i;
      if ( P & v ) {
        go( R | v, P & edge[i], X & edge[i] );
        P &= ~v;
        X |= v;
      }
    }
  }
  int solve() {
    res = 0;
    go( 0LL, ( 1LL << (n+1) ) - 1, 0LL );
    return res;
  }
/*  BronKerbosch1(R, P, X):
      if P and X are both empty:
        report R as a maximal clique
      for each vertex v in P:
        BronKerbosch1(R □ {v}, P □ N(v), X □ N(v))
        P := P \ {v}
        X := X □ {v}
*/
} MaxClique;

int main() {
  MaxClique.init(6);
  MaxClique.add_edge(1,2);
  MaxClique.add_edge(1,5);
  MaxClique.add_edge(2,5);
  MaxClique.add_edge(4,5);
  MaxClique.add_edge(3,2);
  MaxClique.add_edge(4,6);
  MaxClique.add_edge(3,4);
  cout << MaxClique.solve() << "\n";
  return 0;
}
```

6.3 Tarjan SCC

```
#include <cstdio>
#include <vector>
#include <stack>
#include <cstring>
using namespace std;

const int N = 10010;
struct Tarjan {
  int n;
  vector<int> G[N], revG[N];
  stack<int> finish;
  bool visit[N];
```

```
  int cmp[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i <= n; i++) {
      G[i].clear();
      revG[i].clear();
    }
  }
  void add_edge(int u, int v) {
    G[u].push_back(v);
    revG[v].push_back(u);
  }
  void dfs(int v) {
    visit[v] = true;
    for ( auto i:G[v] ) {
      if ( !visit[i] ) dfs(i);
    }
    finish.push(v);
  }
  void revdfs(int v, int id) {
    visit[v] = true;
    for ( auto i:revG[v] ) {
      if ( !visit[i] ) revdfs(i,id);
    }
    cmp[v] = id;
  }
  int solve() {
    memset( visit, 0, sizeof(visit) );
    for (int i = 0; i < n; i++) {
      if ( !visit[i] ) dfs(i);
    }
    int id = 0;
    memset( visit, 0, sizeof(visit) );
    while ( !finish.empty() ) {
      int v = finish.top(); finish.pop();
      if ( visit[v] ) continue;
      revdfs(v,++id);
    }
    return id;
  }
} scc;

int main() {
  int V, E;
  scanf("%d %d", &V, &E);
  scc.init(V);
  for (int i = 0; i < E; i++) {
    int u, v;
    scanf("%d %d", &u, &v);
    scc.add_edge(u-1,v-1);
  }
  printf("%d\n", scc.solve() );
  return 0;
}
```

7 Number Theory

7.1 basic

```
PLL exd_gcd(LL a, LL b) {
  if (a % b == 0) return {0, 1};
  PLL T = exd_gcd(b, a % b);
  return {T.second, T.first - a / b * T.second};
}
LL mul(LL x, LL y, LL mod) {
  LL ans = 0, m = x, s = 0, sgn = (x > 0) xor (y > 0)?
      -1: 1;
  for (x = abs(x), y = abs(y); y; y >>= 1, m <<= 1, m =
       m >= mod? m - mod: m)
    if (y&1) s += m, s = s >= mod? s - mod: s;
  return s * sgn;
}
LL dangerous_mul(LL a, LL b, LL mod){ // 10 times
    faster than the above in average, but could be
    prone to wrong answer (extreme low prob?)
  return (a * b - (LL)((long double)a * b / mod) * mod)
       % mod;
}
LL powmod(LL x, LL p, LL mod) {
  LL s = 1, m = x % mod;
  for (; p; m = mul(m, m, mod), p >>= 1)
    if (p&1) s = mul(s, m, mod);
  return s;
}
```

7.2 Chinese Remainder Theorem

```
typedef long long LL;
typedef pair<LL, LL> PLL;
PLL exd_gcd(LL a, LL b);
LL CRT(vector<PLL> &eqs) {
  LL prod = 1, ans = 0, ni, ns;
  for (auto P: eqs) prod *= P.second;
  for (auto P: eqs) {
    ni = P.second, ns = prod / ni;
    (ans += ns * P.first % prod * exd_gcd(ni, ns).
        second) %= prod;
  }
  return (ans + prod) % prod;
}
```

7.3 Discrete Log

```
LL discrete_log(LL b, LL p, LL n) {
  map<LL, LL> att;
  LL m = sqrt((double)p) + 1, M = powmod(b, m * (p - 2)
      , p);
  for (LL cur = 1, i = 0; i < m; ++i, cur = cur * b % p
      )
    if (not att.count(cur)) att[cur] = i;
  for (LL cur = 1, i = 0; i * m < p - 1; ++i, cur = cur
       * M % p)
    if (att.count(n * cur % p))
      return (att[cur * n % p] + i * m) % (p - 1);
  return -1;
}
// find x s.t. b**x % p == n with complexity O(sqrt(N))
// return the smallest
// return -1 if ans doesn't exist
```

7.4 Lucas

```
LL fac[100000] = {1};
LL C(LL a, LL b, LL p) {
  for (int i = 1; i <= p; ++i) fac[i] = fac[i - 1] * i
      % p;
  LL ans = 1;
  for (;a; a /= p, b /= p) {
    LL A = a % p, B = b % p;
    if (A < B) return 0;
    (ans += fac[A] * powmod(fac[B] * fac[A - B] % p, p
        - 2, p) % p) %= p;
  }
  return ans;
}
```

7.5 Meissel-Lehmer PI

```
LL PI(LL m);
const int MAXM = 1000, MAXN = 650, UPBD = 1000000;
// 650 ~ PI(cbrt(1e11))
LL pi[UPBD] = {0}, phi[MAXM][MAXN];
vector<LL> primes;
void init() {
  fill(pi + 2, pi + UPBD, 1);
  for (LL p = 2; p < UPBD; ++p)
    if (pi[p]) {
      for (LL N = p * p; N < UPBD; N += p)
        pi[N] = 0;
      primes.push_back(p);
    }
```

```
  for (int i = 1; i < UPBD; ++i) pi[i] += pi[i - 1];
  for (int i = 0; i < MAXM; ++i)
    phi[i][0] = i;
  for (int i = 1; i < MAXM; ++i)
    for (int j = 1; j < MAXN; ++j)
      phi[i][j] = phi[i][j - 1] - phi[i / primes[j -
          1]][j - 1];
}
LL P_2(LL m, LL n) {
  LL ans = 0;
  for (LL i = n; primes[i] * primes[i] <= m and i <
      primes.size(); ++i)
    ans += PI(m / primes[i]) - i;
  return ans;
}
LL PHI(LL m, LL n) {
  if (m < MAXM and n < MAXN) return phi[m][n];
  if (n == 0) return m;
  LL p = primes[n - 1];
  if (m < UPBD) {
    if (m <= p) return 1;
    if (m <= p * p * p) return pi[m] - n + 1 + P_2(m, n
        );
  }
  return PHI(m, n - 1) - PHI(m / p, n - 1);
}
LL PI(LL m) {
  if (m < UPBD) return pi[m];
  LL y = cbrt(m) + 10, n = pi[y];
  return PHI(m, n) + n - 1 - P_2(m, n);
}
```

7.6 Miller Rabin with Pollard rho

```
// Miller_Rabin
LL abs(LL a) {return a > 0? a: -a;}
bool witness(LL a, LL n, LL u, int t) {
  LL x = modpow(a, u, n), nx;
  for (int i = 0; i < t; ++i, x = nx){
    nx = mul(x, x, n);
    if (nx == 1 and x != 1 and x != n - 1) return 1;
  }
  return x != 1;
}
const LL wits[7] = {2, 325, 9375, 28178, 450775,
    9780504, 1795265022};
bool miller_rabin(LL n, int s = 7) {
  if (n < 2) return 0;
  if (n&1^1) return n == 2;
  LL u = n - 1, t = 0, a; // n == (u << t) + 1
  while (u&1^1) u >>= 1, ++t;
  while (s--)
    if (a = wits[s] % n and witness(a, n, u, t)) return
        0;
  return 1;
}
// Pollard_rho
LL f(LL x, LL n) {
  return mul(x, x, n) + 1;
}
LL pollard_rho(LL n) {
  if (n&1^1) return 2;
  while (true) {
    LL x = rand() % (n - 1) + 1, y = 2, d = 1;
    for (int sz = 2; d == 1; y = x, sz <<= 1)
      for (int i = 0; i < sz and d <= 1; ++i)
        x = f(x, n), d = __gcd(abs(x - y), n);
    if (d and n - d) return d;
  }
}
```

7.7 Primitive Root

```
vector<LL> factor(LL N) {
  vector<LL> ans;
  for (LL p = 2, n = N; p * p <= n; ++p)
    if (N % p == 0) {
      ans.push_back(p);
      while (N % p == 0) N /= p;
    }
  if (N != 1) ans.push_back(N);
  return ans;
}
LL find_root(LL p) {
  LL ans = 1;
  for (auto q: factor(p - 1)) {
    LL a = rand() % (p - 1) + 1, b = (p - 1) / q;
    while (powmod(a, b, p) == 1) a = rand() % (p - 1) +
        1;
    while (b % q == 0) b /= q;
    ans = mul(ans, powmod(a, b, p), p);
  }
  return ans;
}
bool is_root(LL a, LL p) {
  for (auto q: factor(p - 1))
    if (powmod(a, (p - 1) / q, p) == 1)
      return false;
  return true;
}
```

8 String

8.1 AC automaton

```
// SIGMA[0] will not be considered
const string SIGMA = "
    _0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrs
    ";
vector<int> INV_SIGMA;
const int SGSZ = 63;

struct PMA {
  PMA *next[SGSZ];  // next[0] is for fail
  vector<int> ac;
  PMA *last; // state of longest accepted string that
      is pre of this
  PMA() : last(nullptr) { fill(next, next + SGSZ,
      nullptr); }
};

template<typename T>
PMA *buildPMA(const vector<T> &p) {
  PMA *root = new PMA;
  for (int i = 0; i < p.size(); ++i) {  // make trie
    PMA *t = root;
    for (int j = 0; j < p[i].size(); ++j) {
      int c = INV_SIGMA[p[i][j]];
      if (t->next[c] == nullptr) t->next[c] = new PMA;
      t = t->next[c];
    }
    t->ac.push_back(i);
  }
  queue<PMA *> que;  // make failure link using bfs
  for (int c = 1; c < SGSZ; ++c) {
    if (root->next[c]) {
      root->next[c]->next[0] = root;
      que.push(root->next[c]);
    } else root->next[c] = root;
  }
  while (!que.empty()) {
    PMA *t = que.front();
    que.pop();
    for (int c = 1; c < SGSZ; ++c) {
      if (t->next[c]) {
        que.push(t->next[c]);
        PMA *r = t->next[0];
        while (!r->next[c]) r = r->next[0];
        t->next[c]->next[0] = r->next[c];
        t->next[c]->last = r->next[c]->ac.size() ? r->
            next[c] : r->next[c]->last;
      }
    }
  }
  return root;
}
```

```
void destructPMA(PMA *root) {
  queue<PMA *> que;
  que.emplace(root);
  while (!que.empty()) {
    PMA *t = que.front();
    que.pop();
    for (int c = 1; c < SGSZ; ++c) {
      if (t->next[c] && t->next[c] != root) que.emplace
          (t->next[c]);
    }
    delete t;
  }
}

template<typename T>
map<int, int> match(const T &t, PMA *v) {
  map<int, int> res;
  for (int i = 0; i < t.size(); ++i) {
    int c = INV_SIGMA[t[i]];
    while (!v->next[c]) v = v->next[0];
    v = v->next[c];
    for (int j = 0; j < v->ac.size(); ++j) ++res[v->ac[
        j]];
    for (PMA *q = v->last; q; q = q->last) {
      for (int j = 0; j < q->ac.size(); ++j) ++res[q->
          ac[j]];
    }
  }
  return res;
}
```

8.2 Gusfield

```
template<typename T>
vector<int> gusfield(const T &s) {
  vector<int> z(s.size(), s.size()); // z[i] := max k
      for z[0, k) = z[i, i + k)
  for (int i = 1, L = 0, R = 0; i < s.size(); ++i) {
    if (R < i) {
      L = R = i;
      while (R < s.size() && s[R] == s[R - L]) ++R;
      z[i] = R - L;
      --R;
    } else {
      int k = i - L;
      if (z[k] < R - i + 1) {
        z[i] = z[k];
      } else {
        L = i;
        while (R < s.size() && s[R] == s[R - L]) ++R;
        z[i] = R - L;
        --R;
      }
    }
  }
  return z;
}
```

8.3 KMP

```
template<typename T>
vector<int> build_kmp(const T &s) {
  vector<int> f(s.size());
  int fp = f[0] = -1;
  for (int i = 1; i < s.size(); ++i) {
    while (~fp && s[fp + 1] != s[i]) fp = f[fp];
    if (s[fp + 1] == s[i]) ++fp;
    f[i] = fp;
  }
  return f;
}
```

8.4 Manacher

```
template<typename T, int INF>
vector<int> manacher(const T &s) { // p = "INF" + s.
    join("INF") + "INF", returns radius on p
  vector<int> p(s.size() * 2 + 1, INF);
  for (int i = 0; i < s.size(); ++i) {
    p[i << 1 | 1] = s[i];
  }
  vector<int> w(p.size());
  for (int i = 1, j = 0, r = 0; i < p.size(); ++i) {
    int t = min(r >= i ? w[2 * j - i] : 0, r - i + 1);
    for ( ; i - t >= 0 && i + t < p.size(); ++t) {
      if (p[i - t] != p[i + t]) break;
    }
    w[i] = --t;
    if (i + t > r) r = i + t, j = i;
  }
  return w;
}
```

8.5 Suffix Array

```
// -----------O(NlgNlgN)----------
vector<int> sa_db(const string &s) {
  int n = s.size();
  vector<int> sa(n), r(n), t(n);
  for (int i = 0; i < n; ++i) r[sa[i] = i] = s[i];
  for (int h = 1; t[n - 1] != n - 1; h *= 2) {
    auto cmp = [&](int i, int j) {
      if (r[i] != r[j]) return r[i] < r[j];
      return i + h < n && j + h < n ? r[i + h] < r[j +
          h] : i > j;
    };
    sort(sa.begin(), sa.end(), cmp);
    for (int i = 0; i + 1 < n; ++i) t[i + 1] = t[i] +
        cmp(sa[i], sa[i + 1]);
    for (int i = 0; i < n; ++i) r[sa[i]] = t[i];
  }
  return sa;
}

// O(N) -- CF: 1e6->31ms,18MB;1e7->296ms;158MB;3e7->856
    ms,471MB
bool is_lms(const string &t, int i) {
  return i > 0 && t[i - 1] == 'L' && t[i] == 'S';
}

template<typename T>
vector<int> induced_sort(const T &s, const string &t,
    const vector<int> &lmss, int sigma = 256) {
  vector<int> sa(s.size(), -1);

  vector<int> bin(sigma + 1);
  for (auto it = s.begin(); it != s.end(); ++it) {
    ++bin[*it + 1];
  }

  int sum = 0;
  for (int i = 0; i < bin.size(); ++i) {
    sum += bin[i];
    bin[i] = sum;
  }

  vector<int> cnt(sigma);
  for (auto it = lmss.rbegin(); it != lmss.rend(); ++it
      ) {
    int ch = s[*it];
    sa[bin[ch + 1] - 1 - cnt[ch]] = *it;
    ++cnt[ch];
  }

  cnt = vector<int>(sigma);
  for (auto it = sa.begin(); it != sa.end(); ++it) {
    if (*it <= 0 || t[*it - 1] == 'S') continue;
    int ch = s[*it - 1];
    sa[bin[ch] + cnt[ch]] = *it - 1;
    ++cnt[ch];
  }

  cnt = vector<int>(sigma);
  for (auto it = sa.rbegin(); it != sa.rend(); ++it) {
```

```cpp
    if (*it <= 0 || t[*it - 1] == 'L') continue;
    int ch = s[*it - 1];
    sa[bin[ch + 1] - 1 - cnt[ch]] = *it - 1;
    ++cnt[ch];
  }

  return sa;
}

template<typename T>
vector<int> sa_is(const T &s, int sigma = 256) {
  string t(s.size(), 0);
  t[s.size() - 1] = 'S';
  for (int i = int(s.size()) - 2; i >= 0; --i) {
    if (s[i] < s[i + 1]) t[i] = 'S';
    else if (s[i] > s[i + 1]) t[i] = 'L';
    else t[i] = t[i + 1];
  }

  vector<int> lmss;
  for (int i = 0; i < s.size(); ++i) {
    if (is_lms(t, i)) {
      lmss.emplace_back(i);
    }
  }

  vector<int> sa = induced_sort(s, t, lmss, sigma);
  vector<int> sa_lms;
  for (int i = 0; i < sa.size(); ++i) {
    if (is_lms(t, sa[i])) {
      sa_lms.emplace_back(sa[i]);
    }
  }

  int lmp_ctr = 0;
  vector<int> lmp(s.size(), -1);
  lmp[sa_lms[0]] = lmp_ctr;
  for (int i = 0; i + 1 < sa_lms.size(); ++i) {
    int diff = 0;
    for (int d = 0; d < sa.size(); ++d) {
      if (s[sa_lms[i] + d] != s[sa_lms[i + 1] + d] ||
          is_lms(t, sa_lms[i] + d) != is_lms(t, sa_lms[
            i + 1] + d)) {
        diff = 1; // something different in range of
          lms
        break;
      } else if (d > 0 && is_lms(t, sa_lms[i] + d) &&
          is_lms(t, sa_lms[i + 1] + d)) {
        break; // exactly the same
      }
    }
    if (diff) ++lmp_ctr;
    lmp[sa_lms[i + 1]] = lmp_ctr;
  }

  vector<int> lmp_compact;
  for (int i = 0; i < lmp.size(); ++i) {
    if (~lmp[i]) {
      lmp_compact.emplace_back(lmp[i]);
    }
  }

  if (lmp_ctr + 1 < lmp_compact.size()) {
    sa_lms = sa_is(lmp_compact, lmp_ctr + 1);
  } else {
    for (int i = 0; i < lmp_compact.size(); ++i) {
      sa_lms[lmp_compact[i]] = i;
    }
  }

  vector<int> seed;
  for (int i = 0; i < sa_lms.size(); ++i) {
    seed.emplace_back(lmss[sa_lms[i]]);
  }

  return induced_sort(s, t, seed, sigma);
} // s must end in char(0)

// O(N) lcp, note that s must end in '\0'
vector<int> build_lcp(const string &s, const vector<int
    > &sa, const vector<int> &rank) {
  int n = s.size();
  vector<int> lcp(n);
  for (int i = 0, h = 0; i < n; ++i) {
    if (rank[i] == 0) continue;
    int j = sa[rank[i] - 1];
    if (h > 0) --h;
    for ( ; j + h < n && i + h < n; ++h) {
      if (s[j + h] != s[i + h]) break;
    }
    lcp[rank[i] - 1] = h;
  }
  return lcp; // lcp[i] := lcp(s[sa[i]..-1], s[sa[i +
      1]..-1])
}

// O(N) build segment tree for lcp
vector<int> build_lcp_rmq(const vector<int> &lcp) {
  vector<int> sgt(lcp.size() << 2);
  function<void(int, int, int)> build = [&](int t, int
      lb, int rb) {
    if (rb - lb == 1) return sgt[t] = lcp[lb], void();
    int mb = lb + rb >> 1;
    build(t << 1, lb, mb);
    build(t << 1 | 1, mb, rb);
    sgt[t] = min(sgt[t << 1], sgt[t << 1 | 1]);
  };
  build(1, 0, lcp.size());
  return sgt;
}

// O(|P| + lg |T|) pattern searching, returns last
    index in sa
int match(const string &p, const string &s, const
    vector<int> &sa, const vector<int> &rmq) { // rmq
    is segtree on lcp
  int t = 1, lb = 0, rb = s.size(); // answer in [lb,
      rb)
  int lcplp = 0; // lcp(char(0), p) = 0
  while (rb - lb > 1) {
    int mb = lb + rb >> 1;
    int lcplm = rmq[t << 1];
    if (lcplp < lcplm) t = t << 1 | 1, lb = mb;
    else if (lcplp > lcplm) t = t << 1, rb = mb;
    else {
      int lcpmp = lcplp;
      while (lcpmp < p.size() && p[lcpmp] == s[sa[mb] +
          lcpmp]) ++lcpmp;
      if (lcpmp == p.size() || p[lcpmp] > s[sa[mb] +
          lcpmp]) t = t << 1 | 1, lb = mb, lcplp =
          lcpmp;
      else t = t << 1, rb = mb;
    }
  }
  if (lcplp < p.size()) return -1;
  return sa[lb];
}
```

8.6 Suffix Automaton

```cpp
template<typename T>
struct SuffixAutomaton {
  vector<map<int, int>> edges;// edges[i]  : the
      labeled edges from node i
  vector<int> link;           // link[i]   : the parent
       of i
  vector<int> length;         // length[i] : the length
       of the longest string in the ith class
  int last;                   // the index of the
      equivalence class of the whole string
  vector<bool> is_terminal;   // is_terminal[i] : some
      suffix ends in node i (unnecessary)
  vector<int> occ;            // occ[i] : number of
      matches of maximum string of node i (unnecessary)
  SuffixAutomaton(const T &s) : edges({map<int, int>()
      }), link({-1}), length({0}), last(0), occ({0}) {
    for (int i = 0; i < s.size(); ++i) {
      edges.push_back(map<int, int>());
      length.push_back(i + 1);
      link.push_back(0);
      occ.push_back(1);
      int r = edges.size() - 1;
```

```
      int p = last; // add edges to r and find p with link to q
      while (p >= 0 && edges[p].find(s[i]) == edges[p].end()) {
        edges[p][s[i]] = r;
        p = link[p];
      }
      if (~p) {
        int q = edges[p][s[i]];
        if (length[p] + 1 == length[q]) { // no need to split q
          link[r] = q;
        } else { // split q, add qq
          edges.push_back(edges[q]);  // copy edges of q
          length.push_back(length[p] + 1);
          link.push_back(link[q]);  // copy parent of q
          occ.push_back(0);
          int qq = edges.size() - 1; // qq is new parent of q and r
          link[q] = qq;
          link[r] = qq;
          while (p >= 0 && edges[p][s[i]] == q) { // what points to q points to qq
            edges[p][s[i]] = qq;
            p = link[p];
          }
        }
      }
      last = r;
    } // below unnecessary
    is_terminal = vector<bool>(edges.size());
    for (int p = last; p > 0; p = link[p]) is_terminal[p] = 1; // is_terminal calculated
    vector<int> cnt(link.size()), states(link.size()); // sorted states by length
    for (int i = 0; i < link.size(); ++i) ++cnt[length[i]];
    for (int i = 0; i < s.size(); ++i) cnt[i + 1] += cnt[i];
    for (int i = link.size() - 1; i >= 0; --i) states[--cnt[length[i]]] = i;
    for (int i = link.size() - 1; i >= 1; --i) occ[link[states[i]]] += occ[states[i]]; // occ calculated
  }
};
```

9 Formulas

9.0.1 Pick's theorem

Pick's theorem provides a simple formula for calculating the area A of this polygon in terms of the number i of lattice points in the interior located in the polygon and the number b of lattice points on the boundary placed on the polygon's perimeter:

$$A = i + \frac{b}{2} - 1$$

9.0.2 Graph Properties

1. Euler's Formula $V - E + F = 2$
2. For a planar graph, $F = E - V + n + 1$, n is the numbers of components
3. For a planar graph, $E \le 3V - 6$ For a connected graph G, let $I(G)$ be the size of maximum independent set, $M(G)$ be the size of maximum matching, $Cv(G)$ be the size of minimum vertex cover, $Ce(G)$ be the size of minimum edge cover。
4. For any connected graph:
   (a) $I(G) + Cv(G) = |V|$
   (b) $M(G) + Ce(G) = |V|$
5. For any bipartite:
   (a) $I(G) = Cv(G)$
   (b) $M(G) = Ce(G)$

```
double l=0,=m,stop=1.0/n/n;
while(r-l>=stop){
  double(mid);
  if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
  else r=mid;
}
build(l);
sol.maxFlow(s,t);
vector<int> ans;
for(int i=1;i<=n;++i)
  if(sol.vis[i])ans.push_back(i);
```

9.0.3 Number Theory

1. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
2. $\phi(x), \mu(x)$ are Möbius inverse
3. $\sum_{i=1}^{n} \sum_{j=1}^{m} [\text{gcd}(i,j) = 1] = \sum \mu(d) \left\lfloor \frac{n}{d} \right\rfloor \left\lfloor \frac{m}{d} \right\rfloor$
4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

9.0.4 Combinatorics

1. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
2. $\gamma = 0.57721566490153286060651209008240243104215$
3. Gray Code: $= n \oplus (n >> 1)$
4. Catalan Number: $\frac{C_n^{kn}}{n(k-1)+1}$, $C_m^n = \frac{n!}{m!(n-m)!}$
5. $\gamma(n+1) = n!$
6. $H(n,m) \cong x_1 + x_2 \ldots + x_n = k, num = C_k^{n+k-1}$
7. Stirling number of 2^{nd} kind: n 人分 k 組方法數目
   (a) $S(0,0) = S(n,n) = 1$
   (b) $S(n,0) = 0$
   (c) $S(n,k) = kS(n-1,k) + S(n-1,k-1)$
8. Bell number,n 人分任意多組方法數目
   (a) $B_0 = 1$
   (b) $B_n = \sum_{i=0}^{n} S(n,i)$
   (c) $B_{n+1} = \sum_{k=0}^{n} C_k^n B_k$
   (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime
   (e) $B_{p^m+n} \equiv mB_n + B_{n+1} mod p$, p is prime
   (f) From $B_0 : 1, 1, 2, 5, 15, 52,$ $203, 877, 4140, 21147, 115975$
9. Derangement, 錯排, □有人在自己位置上
   (a) $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n \frac{1}{n!})$
   (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
   (c) From $D_0 : 1, 0, 1, 2, 9, 44,$ $265, 1854, 14833, 133496$
10. Binomial Equality
    (a) $\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n}$
    (b) $\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$
    (c) $\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$
    (d) $\sum_{k \le l} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-1}{l-n-m}$
    (e) $\sum_{0 \le k \le l} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
    (f) $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
    (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
    (h) $\sum_{k \le n} \binom{r+k}{k} = \binom{r+n+1}{n}$
    (i) $\sum_{0 \le k \le n} \binom{k}{m} = \binom{n+1}{m+1}$
    (j) $\sum_{k \le m} \binom{m+r}{k} x^k y^k = \sum_{k \le m} \binom{-r}{k}(-x)^k (x+y)^{m-k}$

9.0.5 □次, □次和

1. $a^b \% P = a^{b \% \varphi(p) + \varphi(p)}, b \ge \varphi(p)$
2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
5. $0^k + 1^k + 2^k + \ldots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^{n} C_k^{n+1} B_k m^{n+1-k}$
7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
8. 除了 $B_1 = -1/2$，剩下的奇數項都是 0
9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

9.0.6 Burnside's lemma

1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
2. $X^g = t^{c(g)}$
3. G 表示有幾種轉法，X^g 表示在那種轉法下，有幾種是會保持對稱的，t 是□色數，$c(g)$ 是循環節不動的面數。
4. 正立方體塗三□色，轉 0 有 3^6 個元素不變，轉 90 有 6 種，每種有 3^3 不變，180 有 3×3^4，120(角) 有 8×3^2，180(邊) 有 6×3^3，全部 $\frac{1}{24}\left(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3\right) = 57$

9.0.7 Count on a tree

1. Rooted tree: $s_{n+1} = \frac{1}{n}\sum_{i=1}^{n}(i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i\times j})$
2. Unrooted tree:
   (a) Odd:$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
   (b) Even:$Odd + \frac{1}{2}a_{n/2}(a_{n/2}+1)$
3. Spanning Tree
   (a) 完全圖 $n^n - 2$
   (b) 一般圖 (Kirchhoff's theorem)$M[i][i] = degree(V_i)$,$M[i][j] = -1$,if have $E(i,j)$,0 if no edge. delete any one row and col in A, $ans = det(A)$