功與能 (Work and Energy)

- 一般古典力學問題皆可用牛頓定律解決，但如果對質點的作用力所知有限，則可採用功與能的觀念來解決。

- 能量表示作功的能力，但能量不一定可完全轉換為功。

功的定義

定力作功(work done by a constant force)

- 物體在定力 $\vec{F}$ 作用下產生位移 $\vec{S}$，

  定力作功的定義如下：

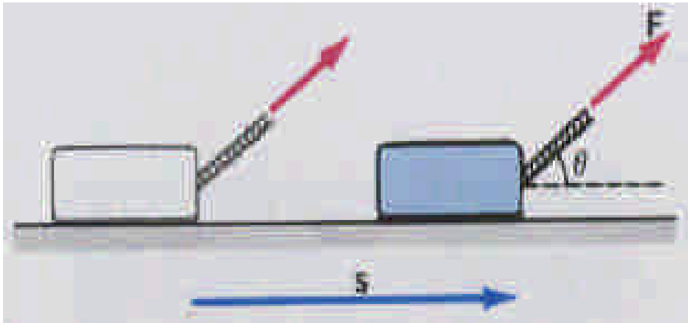


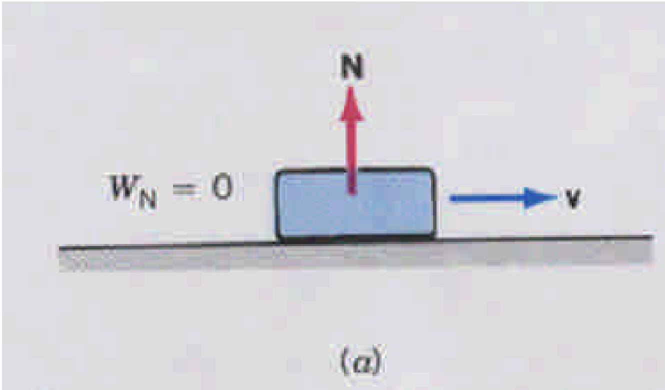
Fig.7.1

$$W = Fs\cos\theta = \vec{F}\cdot\vec{s}$$

$$= (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}) = F_x\Delta x + F_y\Delta y + F_z\Delta z$$

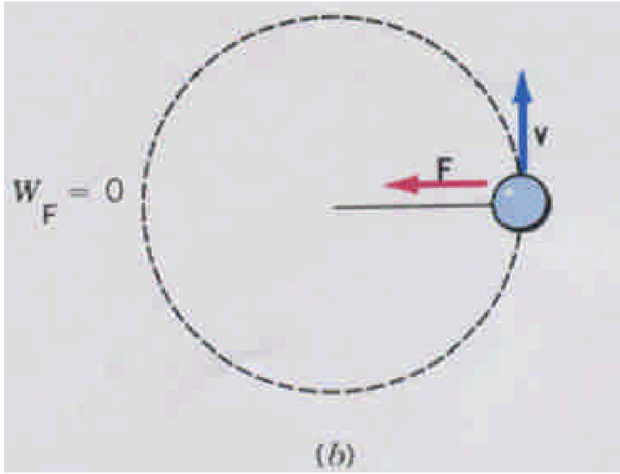
- 僅沿位移 $\vec{S}$ 方向的力或分力($F\cos\theta$)才會作功。

➢垂直於位移方向的力或分力($F\sin\theta$)不會作功，如：正向力、向心力。



正向力N

Fig.7.2



向心力F

•淨功(net work)—考慮多力作功。

$$W_{net} = \vec{F}_1 \cdot \vec{s}_1 + \vec{F}_2 \cdot \vec{s}_2 + \cdots + \vec{F}_N \cdot \vec{s}_N$$

$$= W_1 + W_2 + \cdots + W_N$$ (因考慮多力作用在同一質點上，位移應相同。)

$$= \vec{F}_1 \cdot \vec{s} + \vec{F}_2 \cdot \vec{s} + \cdots + \vec{F}_N \cdot \vec{s}$$

$$= \vec{F}_{net} \cdot \vec{s}$$ (其中 $\vec{F}_{net} = \sum \vec{F}_i$)

- **正功**(position work)**與負功**(negative work)

—作用力與位移方向同向為作正功，而反向則為作負功。

➢作用力與反作用力

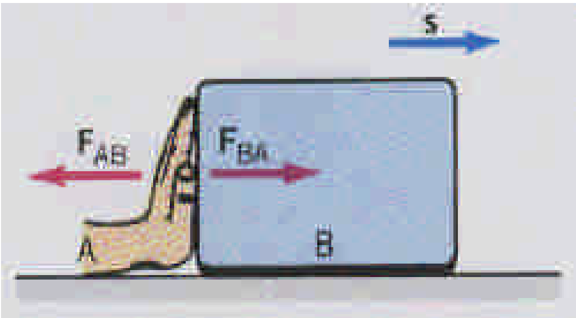


Fig.7.4

➢摩擦力

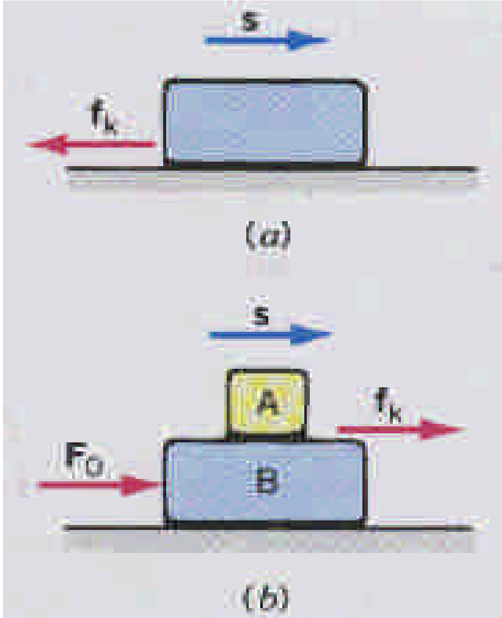


Fig.7.5

➢**重力作功**(work done by gravity)

$$W_g = -m\vec{g}\cdot\vec{s} = \left(-mg\hat{j}\right)\cdot\left(\Delta x\hat{i} + \Delta y\hat{j}\right)$$

$$= -mg\Delta y$$

僅與垂直座標(初始、最終位置)有關，但與路徑無關。

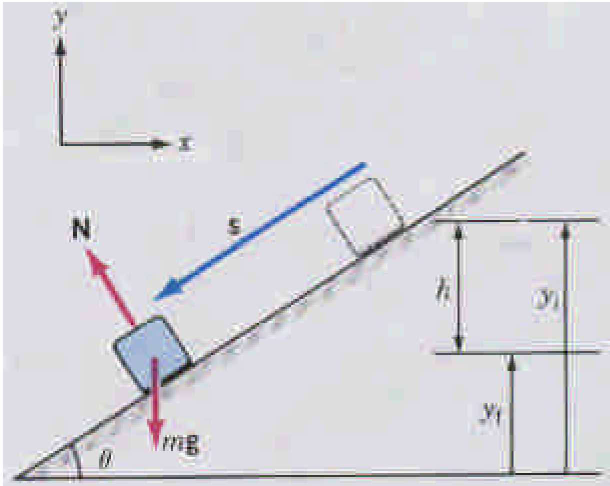


Fig.7.6

變力作功(work done by a variable force)

- 一維空間：
  - 定力作功 $\Rightarrow W = \vec{F}\cdot\vec{s}$

$$= (F_x\hat{i})\cdot(\Delta x\hat{i}) = F_x\Delta x$$

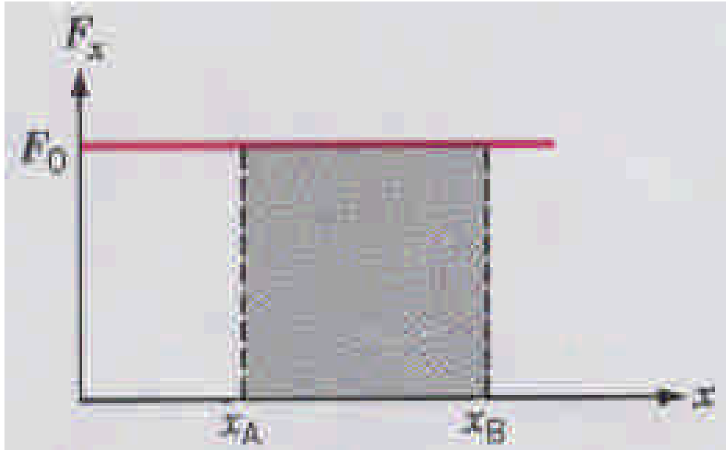


Fig.7.8

  - 變力作功：

$$\Delta W_n = F_n\Delta x_n \Rightarrow W \approx \sum F_n\Delta x_n \Rightarrow \lim_{\Delta x_n\to 0}\left(\sum F_n\Delta x_n\right) = \int F_x dx$$

$$\Rightarrow W_{A\to B} = \int_{x_A}^{x_B} F_x dx \Rightarrow \; W_{A\to B} = -W_{B\to A}$$

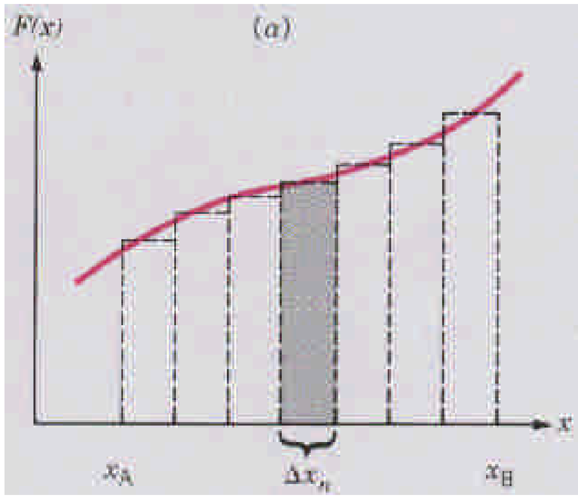


Fig.7.11a

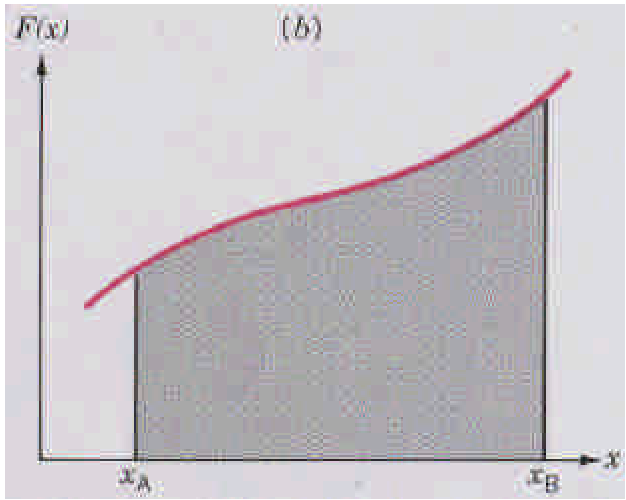


Fig.7.11b

➢彈力作功(work done by a spring)

$$F_{sp} = -kx$$ （負號表彈力一直抗拒彈簧的伸長或壓縮。）

$$W_{sp} = \int_{x_i}^{x_f} F_{sp} dx = \int_{x_i}^{x_f} -kx dx = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

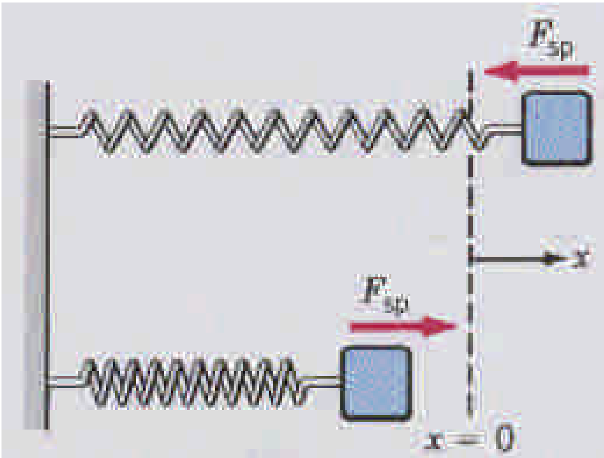


Fig.7.9

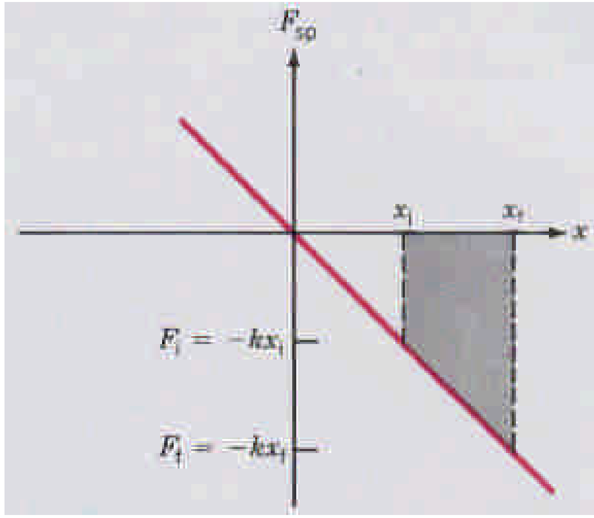


Fig.7.10

- 一維⇒多維

$$W_{A\to B} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

功-能定理 (Work-Energy Theorem)

$$W_{net} = \Delta k \qquad \text{，其中 } \Delta k = \frac{1}{2}m\mathrm{v}_f^2 - \frac{1}{2}m\mathrm{v}_i^2$$

(即動能變化量)

證明：

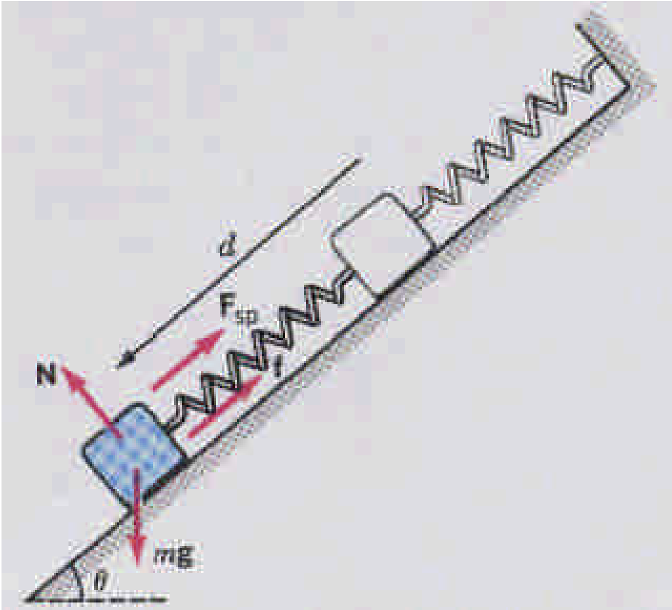
定力作功 $\Rightarrow W = F\Delta x = ma\Delta x$ $\quad (\because\ 2a\Delta x = \mathrm{v}_f^2 - \mathrm{v}_i^2)$

$$= m\left[\frac{1}{2}\left(\mathrm{v}_f^2 - \mathrm{v}_i^2\right)\right] = \frac{1}{2}m\mathrm{v}_f^2 - \frac{1}{2}m\mathrm{v}_i^2$$

變力作功 $\Rightarrow W = \int F dx = \int madx = m\int (\frac{d\mathrm{v}}{dt})dx = m\int d\mathrm{v}(\frac{dx}{dt}) = m\int_{\mathrm{v}_i}^{\mathrm{v}_f} \mathrm{v}d\mathrm{v}$

$$= \frac{1}{2}m\mathrm{v}^2\Big|_{\mathrm{v}_i}^{\mathrm{v}_f} = \frac{1}{2}m\mathrm{v}_f^2 - \frac{1}{2}m\mathrm{v}_i^2$$

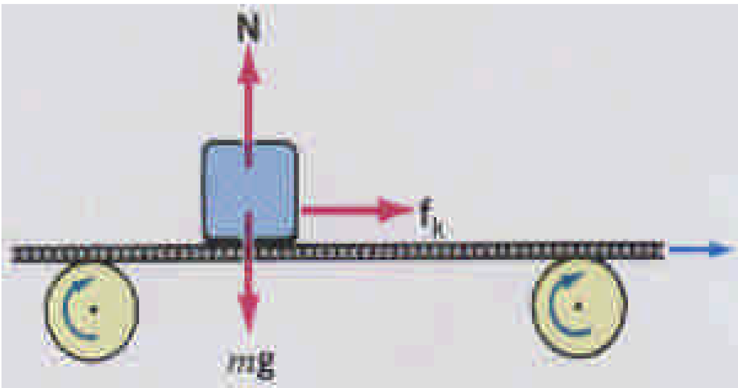
Example 7.4 求 block 滑下速率v？



$$W_g = m\vec{g}\cdot\vec{s} = mgd\sin\theta$$
$$W_f = \vec{f}\cdot\vec{s} = -\mu_k(mg\cos\theta)d$$
$$W_{sp} = -\frac{1}{2}kd^2$$
$$W_N = 0$$
$$\Rightarrow W_{net} = W_g + W_f + W_{sp} + W_N = \Delta K$$

$$\Rightarrow mgd\sin\theta - \mu_k(mg\cos\theta)d - \frac{1}{2}kd^2 = \frac{1}{2}m\mathrm{v}^2$$

Example 7.5 $W_f = ?,\ d(crate) = ?,\ d(belt) = ?$



$$W_{net} = W_f = \Delta K = \frac{1}{2}m\mathrm{v}^2 - 0 = \frac{1}{2}m\mathrm{v}^2$$
$$W_f = \mu_k mgd = \frac{1}{2}m\mathrm{v}^2 \Rightarrow d(crate) = \frac{\mathrm{v}^2}{2\mu_k g}$$
$$d(crate) = \frac{1}{2}at^2 = \frac{1}{2}\mathrm{v}t \Rightarrow d(belt) = \mathrm{v}t = 2d(crate)$$

✦功率 (Power)

- 平均功率(average power) ⇒ $P_{av} = \dfrac{\Delta W}{\Delta t}$

- 瞬間功率(instantaneous power) ⇒ $P = \lim\limits_{\Delta t \to 0} \dfrac{\Delta W}{\Delta t} = \dfrac{dW}{dt}$

  (若定力作功 ⇒ $dW = \vec{F} \cdot d\vec{s}$) $= \vec{F} \cdot \dfrac{d\vec{s}}{dt} = \vec{F} \cdot \vec{\mathrm{v}}$

- 功率單位(unit) ⇒ 1 W(瓦特)=1 J/s ；1 hp(馬力)=746 W

  (Note: W⇒Watt；hp ⇒horsepower)

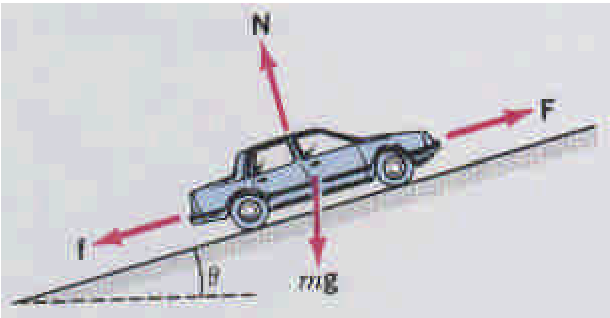
- 功率廣義的定義 ⇒ $P = \dfrac{dE}{dt}$ (其中E為任何形式能量)

Example 7.6： A pump raises water from a well of depth 20m at a rate 10 kg/s and discharges it at 6 m/s. What is the power of the motor?

$$W_{net} = \Delta K \Rightarrow W_p + W_g = \frac{1}{2}m\mathrm{v}^2 - 0 \Rightarrow W_p - mgh = \frac{1}{2}m\mathrm{v}^2 \Rightarrow W_p = mgh + \frac{1}{2}m\mathrm{v}^2$$

$$\Rightarrow P = \frac{dW_P}{dt} = \frac{dm}{dt}\left(gh + \frac{\mathrm{v}^2}{2}\right) = (10kg/s)(200\mathrm{m}^2/s^2 + 18\mathrm{m}^2/s^2) = 2180\ \mathrm{W}$$

Example 7.7： A 10^3-kg requires 12 hp to cruise at a steady 80 km/h on a level road. What would be the power required to move up to a 10^0 incline at the same speed?



$$\mathrm{v} = 80\ \mathrm{km/h} = 22.2\ \mathrm{m/s},\ \ \mathrm{P{=}12hp{=}8.95 \times 10^3\ W}$$

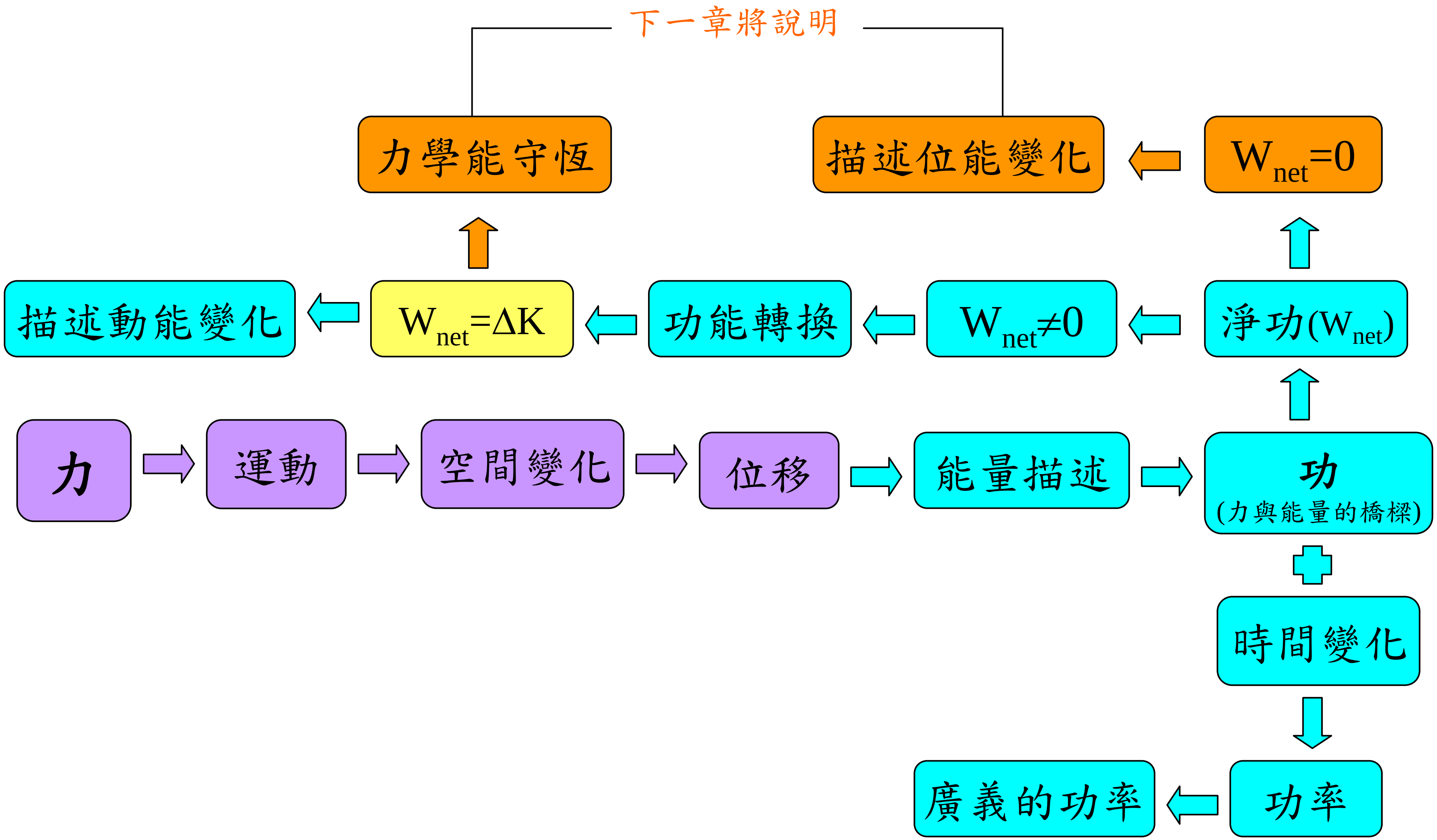
等速表合力為零，
即 F ＝f (水平分量)
P(功率)=Fv=fv

$$\Rightarrow f = \frac{P}{\mathrm{v}} = \frac{8.95 \times 10^3 \mathrm{W}}{22.2\ \mathrm{m/s}} = 403N$$

同理，沿斜面分量

$$\Rightarrow P(incline) = (f + mg\sin 10^0)\mathrm{v} = 46.6 \times 10^3 (W) = 62.7(hp)$$

本章重要觀念發展脈絡彙整



習　題

- 教科書習題 (p.138~p.143)

  Exercise: 11,13,25,29,31,39,41,45,59,65,71

  Problem: 5,7,9,11

- 基本觀念問題：

  1.何謂正功與負功？有何物理意義？

- 延伸思考問題：

  1.請問動能的定義式可否加以推導？

  2.請舉例說明哪些能量不能完全轉換為功？

位能(Potential Energy)

- 擺錘爬升高度不受釘子阻斷影響(即不受運動路徑影響)且$v^2 \propto h$。

- 位能與位置有關，它是由兩個或更多個交互作用的粒子因彼此相對位置而具有的能量，換言之，若位置發生變化，則必須有外力作功(外功)。

- 位能與外功(external work)

$$W_{EXT} = +\Delta U = U_f - U_i$$

若設定零位面(即$U=0$)，則可定義位能。

(重力位能 $\xrightarrow{U=0}$ 任意；彈力位能 $\xrightarrow{U=0}$ 原長)

伽利略單擺實驗

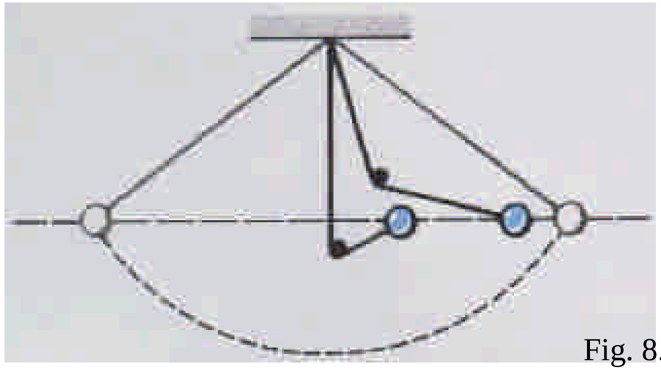


Fig. 8.1

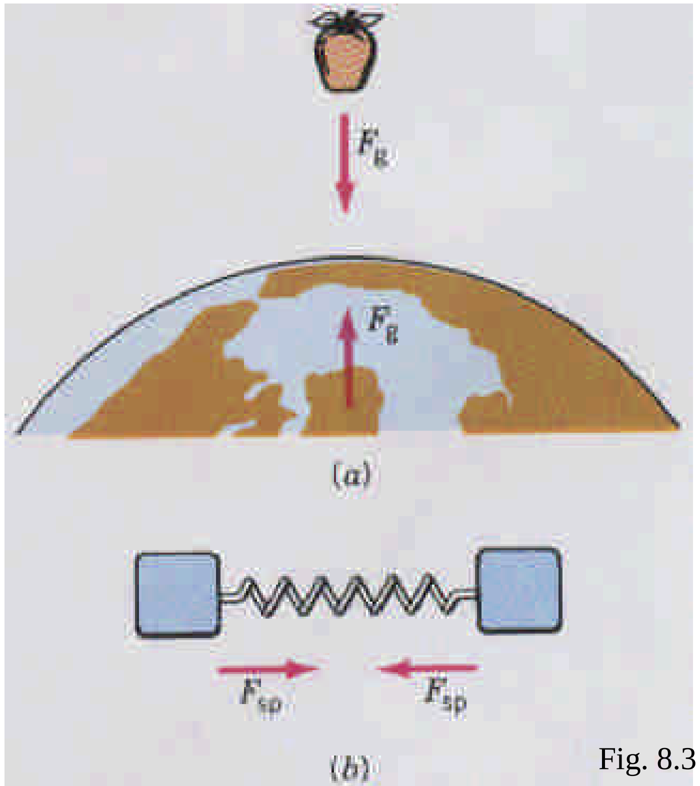


Fig. 8.3

➢某一位置的**位能**乃是將質點從零位面 (即U=0) 的位置以等速移至該位置時所需的外功(即外力所作的功)。

上述位能觀念有兩項缺點：

1.無法確定作外功的外力。

2.等速率移動質點，則質點上的作用力必須保持平衡，即可能有一內力與外力作用，但什麼是內力?

保守力(Conservative Force)

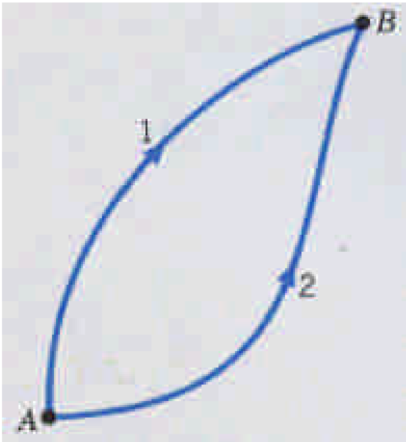


Fig.8.5

- 保守力作功與路徑無關，即：$W^{(1)}_{A\to B}=W^{(2)}_{A\to B}$ 。

  如：保守力⇒重力、彈力; 非保守力⇒摩擦力

- 保守力為位置的函數，而與速率或時間無關。

  如：非保守力⇒磁力、流體阻尼力

- 保守力沿任意封閉路徑所作的功為零，即：$W^{(1)}_{A\to B}+W^{(2)}_{B\to A}=0$

Example：

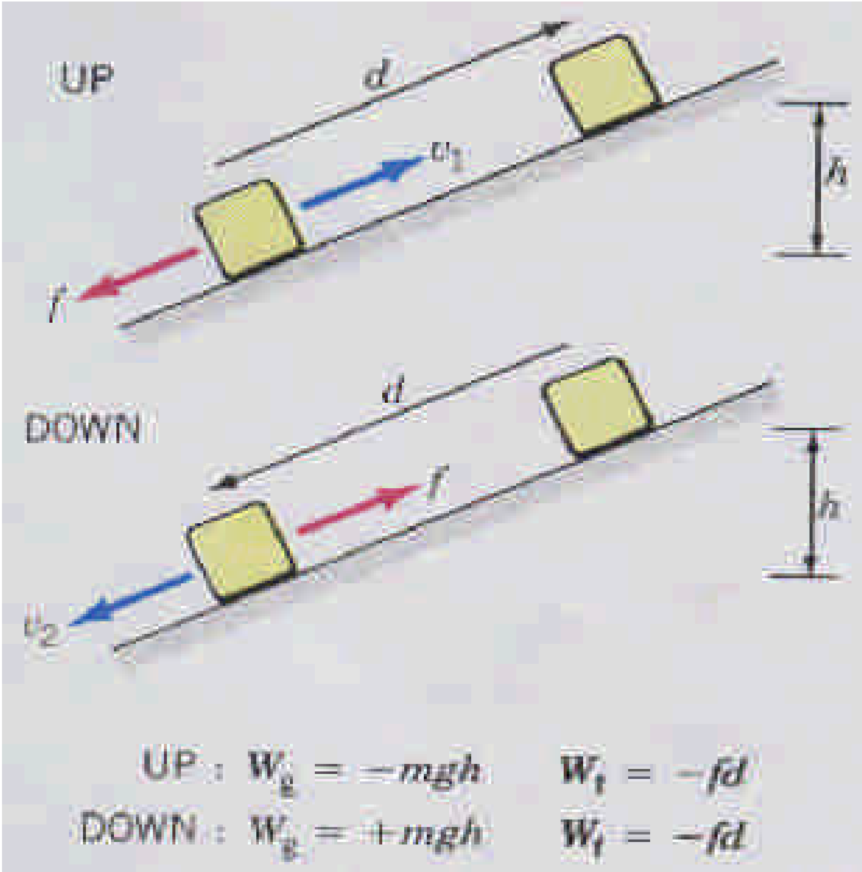


Fig. 8.4

For the force of gravity $\Rightarrow W_{up} + W_{down} = 0$

For the force of friction $\Rightarrow W_{up} + W_{down} = -2fd$

位能與保守力(potential energy and conservative force)

- $W_{net} = \Delta k = 0$ （考慮質點等速率改變位置）

$$\because W_{net} = W_{ext} + W_c = 0$$

$$\therefore W_c = -W_{ext} = -\Delta U = -(U_f - U_i)$$

- 保守力作正功將導致位能減少，意味可促使任何系統內的位能降至最低值(即趨於自然界的穩定態)。

- 考慮可變的保守力大小與方向，可利用無限小的微積分概念表示：

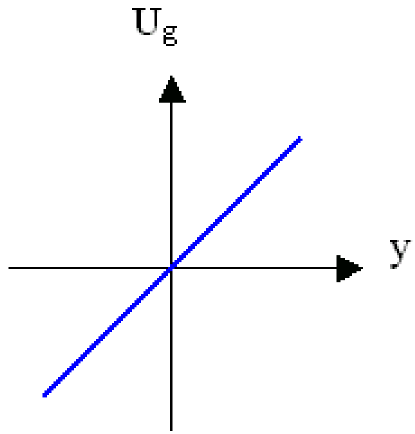
$$dU = -dW_c = -\vec{F}_c \cdot d\vec{s} \;\Rightarrow\; \Delta U = U_B - U_A = -\int_A^B \vec{F}_c \cdot d\vec{s}$$

- 保守力可具有相關的純量位能函數(scalar potential energy)，而非保守力則否。

✦ 位能函數(potential energy functions)

- 位能是位置的函數。

➢重力位能 (gravitational potential energy)



$$W_g = -\Delta U_g = -\left(U_f - U_i\right) = -mg\Delta y = -mg(y_f - y_i)$$

$$\Rightarrow U_f - U_i = mg(y_f - y_i) \xrightarrow{\text{Let } U_i = 0 \text{ at } y_i = 0} U_f = mgy_f$$

$$\Rightarrow U_g = mgy$$ (Note：僅指地表附近)

➢彈力位能 (spring potential energy)

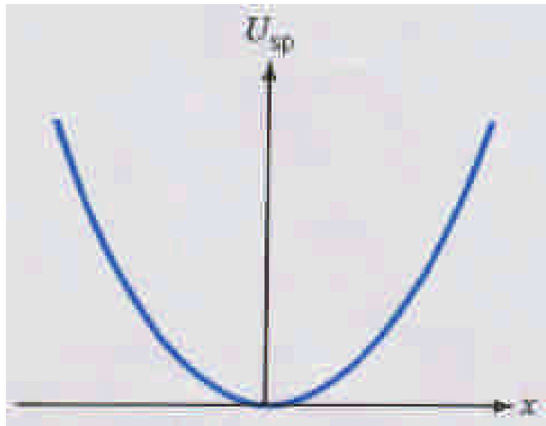


Fig. 8.6

$$W_{sp} = -\Delta U_{sp} = -\left(U_f - U_i\right) = -\frac{1}{2}k(x_f^2 - x_i^2)$$

$$\Rightarrow U_f - U_i = \frac{1}{2}k(x_f^2 - x_i^2)$$

$$\xrightarrow{\text{Let } U_i = 0 \text{ at } x_i = 0 \text{(即彈簧原長處)}} U_f = \frac{1}{2}kx_f^2 \Rightarrow U_{sp} = \frac{1}{2}kx^2 \geq 0$$

保守力與位能函數(Conservative force and potential energy function)

- 保守力可由純量位能函數導出。

$$dU = -\vec{F}_c \cdot d\vec{s} \xrightarrow{\text{考慮}F_c\text{沿位移}ds\text{方向的分量爲}F_s} dU = -F_s ds \Rightarrow F_s = -\frac{dU}{ds}$$

$$dU = -(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k}) \Rightarrow \vec{F}_c = -(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k})$$

$$\xrightarrow[\text{直角座標}]{\text{考慮一維分量}} F_x = -\frac{dU}{dx}$$

$$\xrightarrow[\text{極座標}]{\text{考慮徑向分量}} F_r = -\frac{dU}{dr}$$

Example：

$$U_g = mgy \Rightarrow F_y = -\frac{dU_g}{dy} = -mg$$

$$U_{sp} = \frac{1}{2}kx^2 \Rightarrow F_x = -\frac{dU_{sp}}{dx} = -kx$$

Example：

$(r < r_0)$: $F_r > 0 \Rightarrow$ 受斥力作用，r愈小，斥力愈大

$(r = r_0)$: $F_r = 0 \Rightarrow$ 穩定平衡點 (stable equilibrium)

$(r_0 < r < r_2)$: $F_r < 0 \Rightarrow$ 受引力作用，r_1處引力最大

$(r = r_2)$: $F_r = 0 \Rightarrow$ 不穩定平衡點 (unstable equilibrium)

$(r_3 > r > r_2)$: $F_r > 0 \Rightarrow$ 弱斥力作用

$(r \geq r_3)$: $F_r = 0 \Rightarrow$ 隨遇平衡區(neutral equilibrium) （右圖未標示）

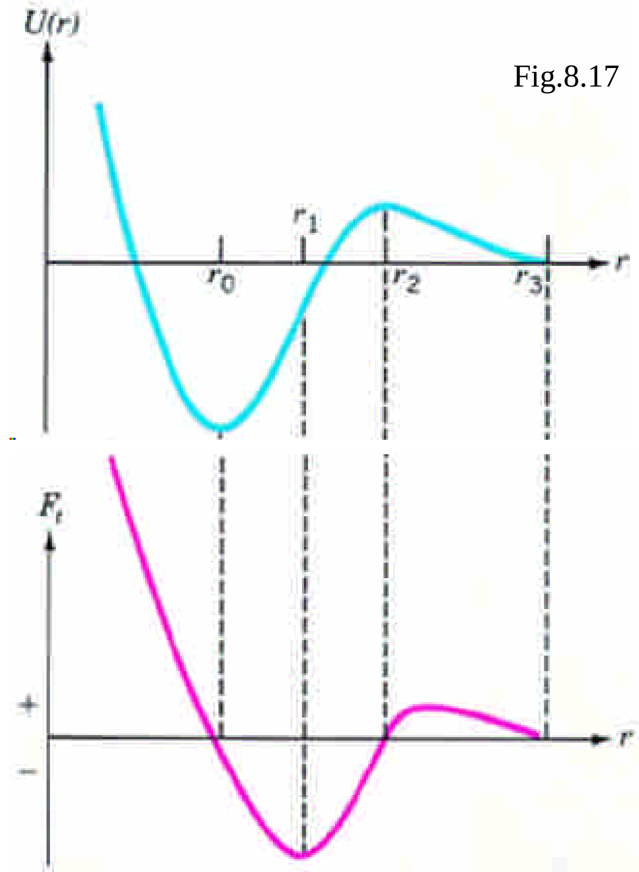


Fig.8.17

- 穩定平衡：質點稍偏離穩定點，仍會回復，如：不倒翁。
  (stable equilibrium)

- 不穩定平衡：質點稍偏離穩定點，無法回復且會一直偏離，如：雞蛋倒立。
  (unstable equilibrium)

- 隨遇平衡：質點稍偏離穩定點，即會停留在新的穩定點。
  (neutral equilibrium)

力學能守恆(Conservation of Mechanical Energy)
（或機械能守恆）

- 力學能$(E) \Rightarrow E = K + U$ ，其中K表動能， U表位能。
- $E_f = E_i \Rightarrow K_f + U_f = K_i + U_i$ 或 $\Delta E = \Delta K + \Delta U = 0$
  - 證明⇒若僅考慮保守力作功，則：$W_{net} = W_c \quad (\because W_{ext} = 0)$

    又 $W_{net} = \Delta K$ ， 故 $W_c = \Delta K = -\Delta U$

    $\Rightarrow \Delta K + \Delta U = 0$ 或 $\Delta E = 0$
- 適用條件：

  1. 必須在保守力的條件下。

  2. 能量的量測必須在同一個慣性座標系中。

Example：

$$\text{重力} \Rightarrow E = \frac{1}{2}m\mathrm{v}^2 + mgy = \frac{1}{2}m\mathrm{v}_{\max}^2 = mgH$$

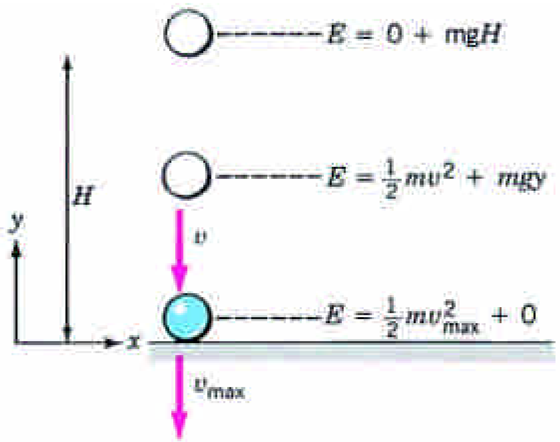
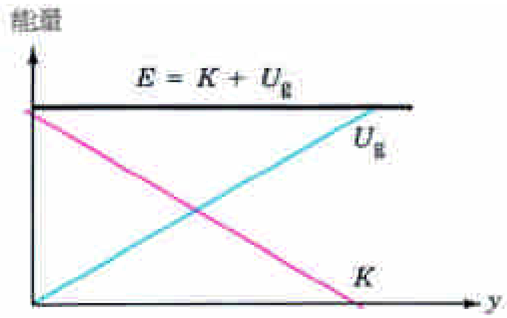


Fig.8.7



$$\text{彈力} \Rightarrow E = \frac{1}{2}m\mathrm{v}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\mathrm{v}_{\max}^2 = \frac{1}{2}kA^2$$

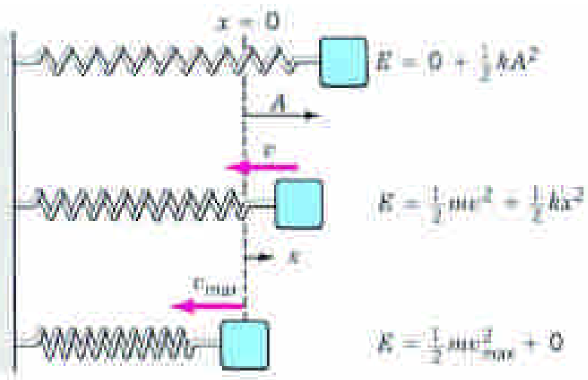
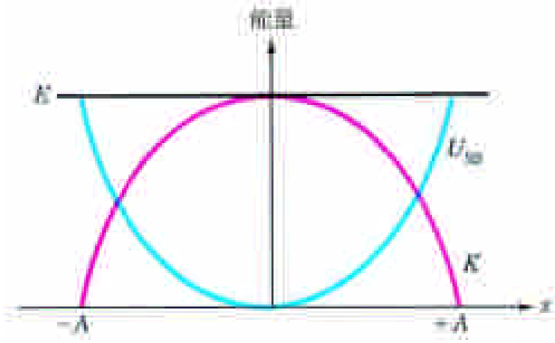


Fig.8.9



- 力學能守恆求解力學問題指引：

  1.在力學系統中，可能不只一個質點具有動能，而位能類型也可能不只一種。 $E = K + U_g + U_{sp}$

  2.力學守恆定律式有兩種類型：

  $(a)\ K_f + U_f = K_i + U_i$ ⇔ 必須設定零位面(U=0)。

  (b) $\Delta K + \Delta U = 0$ ⇔ 不必設零位面，但須小心正負號。

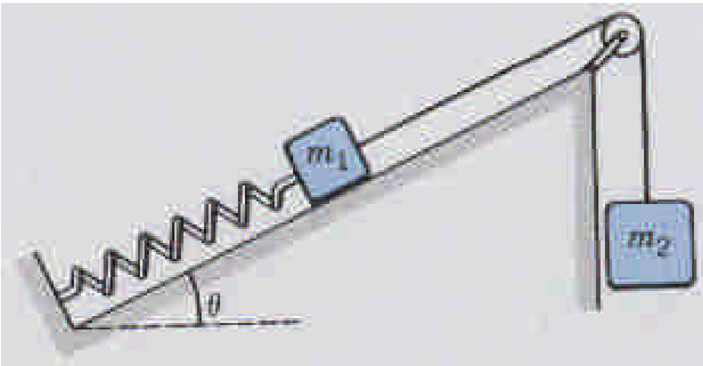
- 處理力學問題的優點(advantages)：

  1.不需考慮力的向量，因功與能為純量。

  2.只需考慮系統最初與最終的狀態。

  3.當無法量測作用力時(即牛頓第二定律不適用)，能量的觀念仍可適用。

Example 8.6： (a) the maximum extension of the spring ? (b) the speed of m_2 when the extension is 0.5 m?



$$\Delta K + \Delta U_g + \Delta U_{sp} = 0$$

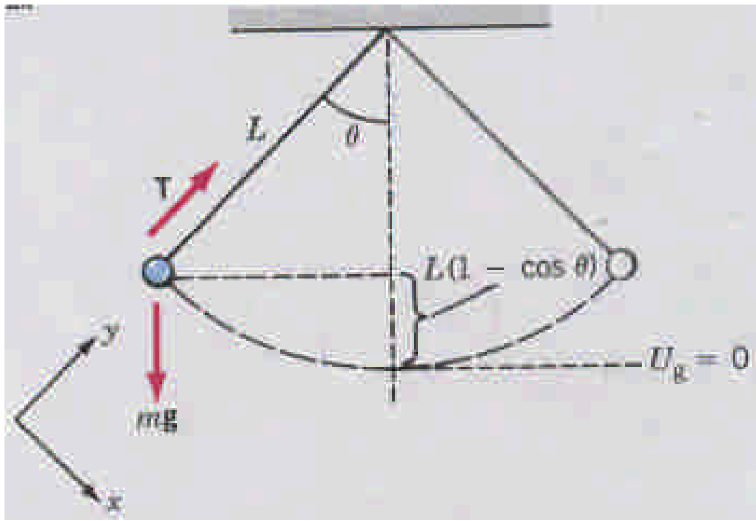
$$0 + (-m_2 g D + m_1 g D \sin\theta) + \frac{1}{2} k D^2 = 0$$

$$D = \frac{2g}{k}(m_2 - m_1 \sin\theta) = 0.98 \text{ m} \qquad \text{Ans(a)}$$

$$\frac{1}{2}(m_1 + m_2)\text{v}^2 + (-m_2 g d + m_1 g d \sin\theta) + \frac{1}{2} k d^2 = 0 \;\Rightarrow\; \text{v} = 1.39(\text{m}/s)$$

Ans(b)

Example 8.7： The bob of a simple pendulum of length L=2 m has a mass m = 2 kg and a speed v=1.2 m/s when the string is at 35° to the vertical. Find the tension in the sting at: (a) the lowest point in its swing; (b) the highest point.



$$\vec{T} + m\vec{g} = m\vec{a} \Rightarrow \begin{cases} \sum F_x = mg\sin\theta = ma_t \\ \sum F_y = T - mg\cos\theta = \dfrac{m\mathrm{v}^2}{\mathrm{L}} \end{cases}$$

$$E = \frac{1}{2}m\mathrm{v}^2 + mgL(1-\cos\theta) = 8.5 \quad (\text{where } \theta = 35^0)$$

At the lowest point $\theta = 0 \Rightarrow E = \frac{1}{2}m\mathrm{v}_{\mathrm{max}}^2 + 0 = 8.5$

$$\Rightarrow T - mg = \frac{m\mathrm{v}_{\mathrm{max}}^2}{L} \Rightarrow T = mg + \frac{m\mathrm{v}_{\mathrm{max}}^2}{L} = 28.1\ \mathrm{N}$$

Ans(a)

At the highest point $\mathrm{v} = 0 \Rightarrow E = 0 + mgL(1-\cos\theta_{\mathrm{max}}) = 8.5\ \mathrm{J} \Rightarrow \cos\theta_{\mathrm{max}} = 0.783$

$$\Rightarrow \mathrm{T} = \mathrm{mg}\cos\theta_{\mathrm{max}} = 15.3\ \mathrm{N} \quad (\because \mathrm{v} = 0)$$ Ans(b)

能量圖(energy diagram)

—考慮位能阱(potential well)

- E < 0 ，處於束縛態(bound state)，如：E_1, E_2。
- E > 0 ，處於非束縛(unbound)，如：E_3。 $(r \to \infty，U = 0)$

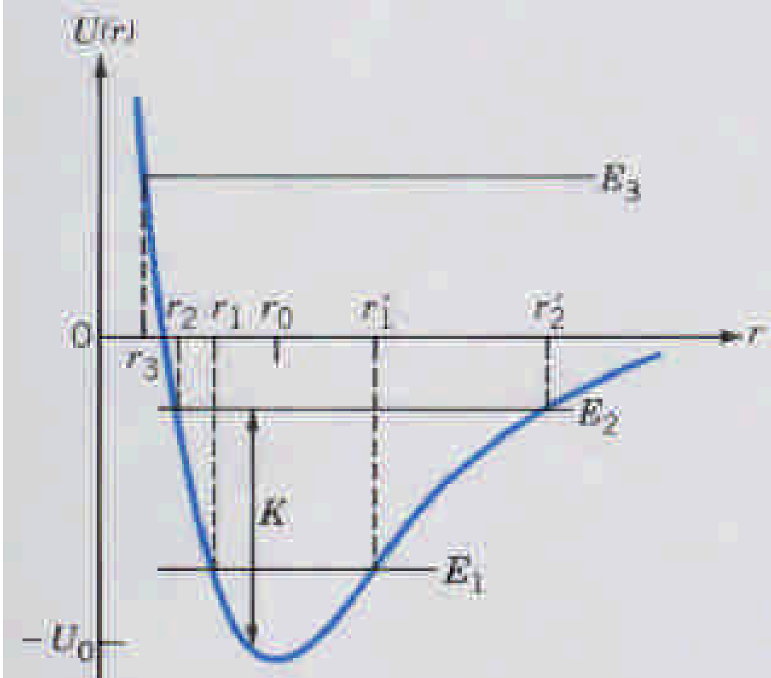


Fig. 8.18

- **束縛能**(binding energy)—使束縛態質點成為不受束縛的質點所需的最小能量。

例如：原子中的電子最低能態束縛能，就是游離能(ionization energy)；對分子而言，就是解離能(dissociation energy)。

✦力學能與非保守力(nonconservative force)

- 力學能的變化量(△E≠0)即為非保守力作的功。

➢證明：

$$W_{net} = W_c + W_{nc} = \Delta K \quad \text{，其中 } W_c = -\Delta U$$

$$\Rightarrow \quad -\Delta U + W_{nc} = \Delta K$$

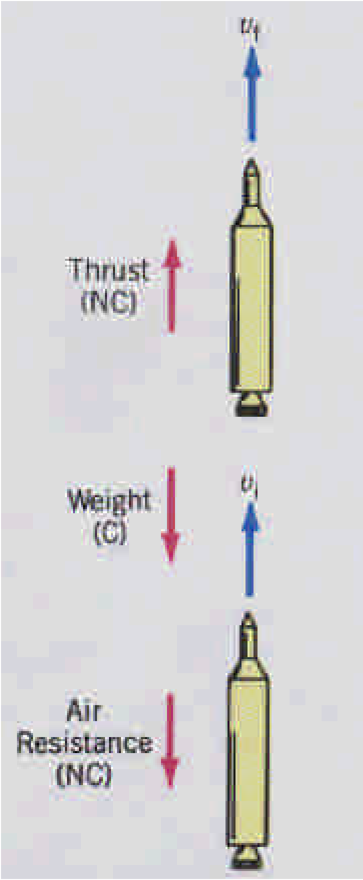
$$\Rightarrow \quad \underline{W_{nc} = \Delta K + \Delta U = \Delta E}$$

➢例如：火箭垂直向上發射的各種作用力

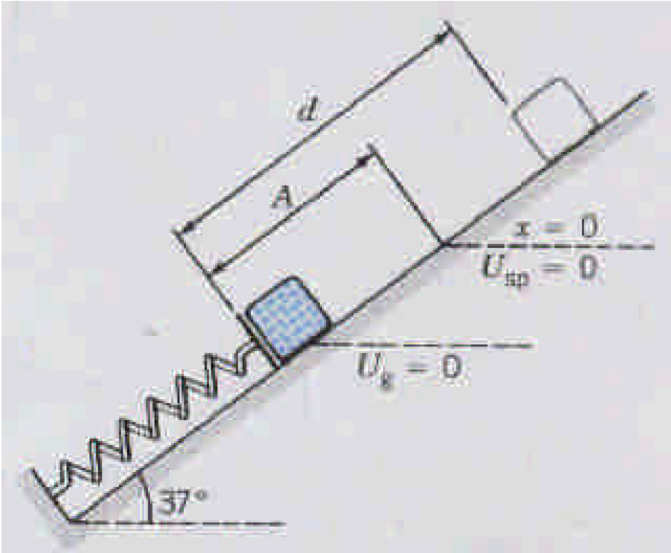
包括：保守力⇒重力(weight)

非保守力⇒空氣阻力(air resistance)及

引擎推進力(thrust)。



Example 8.9： Find (a) the force of friction; (b) the speed of the block just as it leaves the spring.



$$E = K + U_g + U_{sp} \Rightarrow \begin{cases} E_i = \frac{1}{2}KA^2 \\ E_f = mgd\sin\theta \end{cases}$$

$$\Rightarrow E_f - E_i = -fd$$

$$\Rightarrow mgd\sin\theta - \frac{1}{2}KA^2 = -fd$$

$$\Rightarrow f = 0.82N \qquad \text{Ans (a)}$$

$$\begin{cases} E_i = \frac{1}{2}KA^2 \\ E_f = \frac{1}{2}m\mathrm{v}^2 + mgA\sin\theta \end{cases} \Rightarrow E_f - E_i = -fA \Rightarrow \mathrm{v} = 2.45\ (\mathrm{m/s}) \quad \text{Ans(b)}$$

✦重力位能(gravitational potential energy)

- $U_g = mgh$ 適用於地表附近(因$h << R_e$，重力場 $g \simeq \frac{GM_e}{R_e^2}$，重力(mg)可視為定值)，實際上重力場(g) 隨距離改變，重力應為一變力。

- 萬有引力定律： $\vec{F}_r = -\frac{GmM}{r^2}\hat{r}$

$$W_c = W_g = \int_{r_A}^{r_B} \vec{F}_r \cdot d\vec{r} = -\int_{r_A}^{r_B} F_r dr$$ ，其中 $r_B > r_A$ 。

$$\Delta U = U_B - U_A = -W_g = \int_{r_A}^{r_B} F_r dr = \int_{r_A}^{r_B} \left(\frac{GmM}{r^2}\right) dr = \frac{GmM}{r_A} - \frac{GmM}{r_B}$$

$$Assume\ U_B = 0 \text{ at } r_B = \infty \Rightarrow U_A = -\frac{GmM}{r_A} \Rightarrow \underline{U(r) = -\frac{GmM}{r}}$$

Example 8.10： What is the connection $U(r) = -\frac{GmM}{r}$ between and the equation $U_g = mgh$?

$$\begin{cases} U(R_E) = -\dfrac{GmM_E}{R_E} \\ U(R_E + h) = -\dfrac{GmM_E}{R_E + h} \end{cases} \Rightarrow \Delta U = U(R_E + h) - U(R_E) = \frac{GmM_E h}{R_E(R_E + h)}$$

$$\Rightarrow \Delta U \approx \frac{GmM_E h}{R_E^2} = mgh \qquad (\because h \ll R_E \Rightarrow (R_E + h) \approx R_E)$$

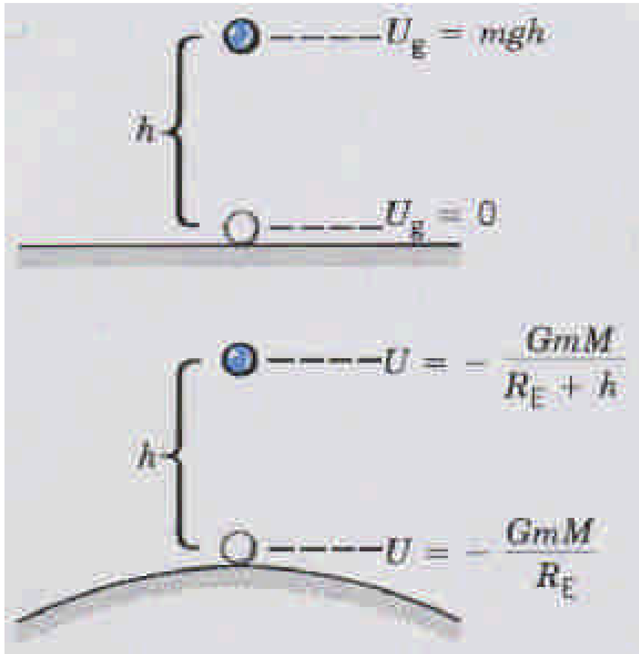


Fig. 8.19

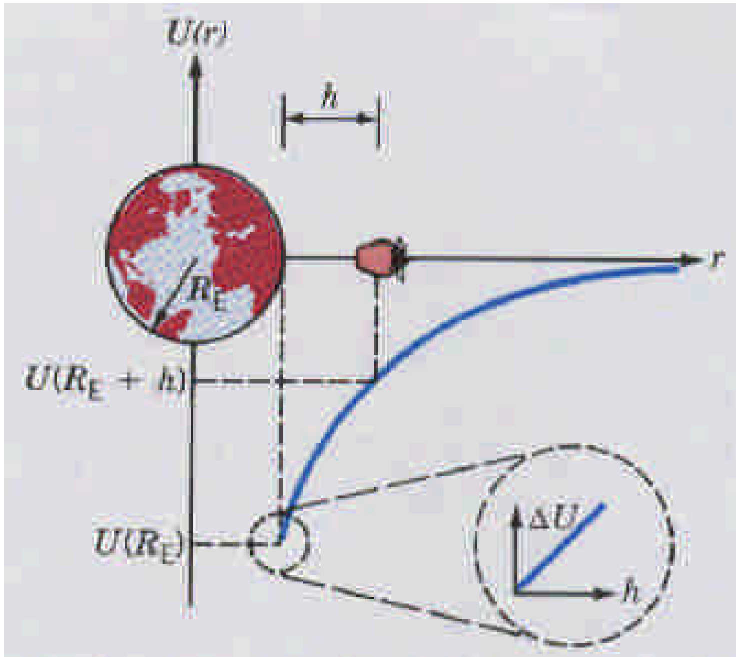


Fig. 8.20

- 衛星軌道運動的力學能 E < 0，衛星處於束縛態。

證明⇒ 假設衛星軌道運動為圓周運動，向心力＝重力

$$\Rightarrow \frac{m\text{v}_{\text{orb}}^2}{r} = \frac{GmM_E}{r^2} \quad \Rightarrow \frac{1}{2}m\text{v}_{\text{orb}}^2 = \frac{GmM_E}{2r}$$

$$\therefore E = K + U = \frac{1}{2}m\text{v}_{\text{orb}}^2 - \frac{GmM_E}{r} = -\frac{GmM_E}{2r} < 0$$

- 地表脫離速率(escape speed) — $\text{v}_{esc} = \sqrt{2GM_E / R_E}$

  - **質點脫離束縛的力學能 E ≥ 0。**
  - **火箭脫離地球至無窮遠時**，$\text{v} \to 0$ and $U \to 0$ at $r \to \infty$

$$\left.\begin{aligned} E_f &= K_f + U_f = 0 \quad at\ r \to \infty \\ E_i &= \frac{1}{2}m\text{v}_{esc}^2 - \frac{GmM_E}{R_E} \ \ at\ r = R_E \end{aligned}\right\} \Rightarrow E_f = E_i \ \Rightarrow \ \text{v}_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Example 8.12： A rocket is fired vertically with half the escape speed. What is its maximum altitude in terms of the radius of the earth R_E? Ignore the earth's rotation.

$$\left.\begin{array}{l} E_i = \frac{1}{2} m (\frac{v_{esc}}{2})^2 - \frac{GmM_E}{R_E} = \frac{GmM_E}{4R_E} - \frac{GmM_E}{R_E} = -\frac{3GmM_E}{4R_E} \\ E_f = 0 - \frac{GmM}{R_E + h} \end{array}\right\} \Rightarrow E_i = E_f \Rightarrow h = \frac{R_E}{3}$$

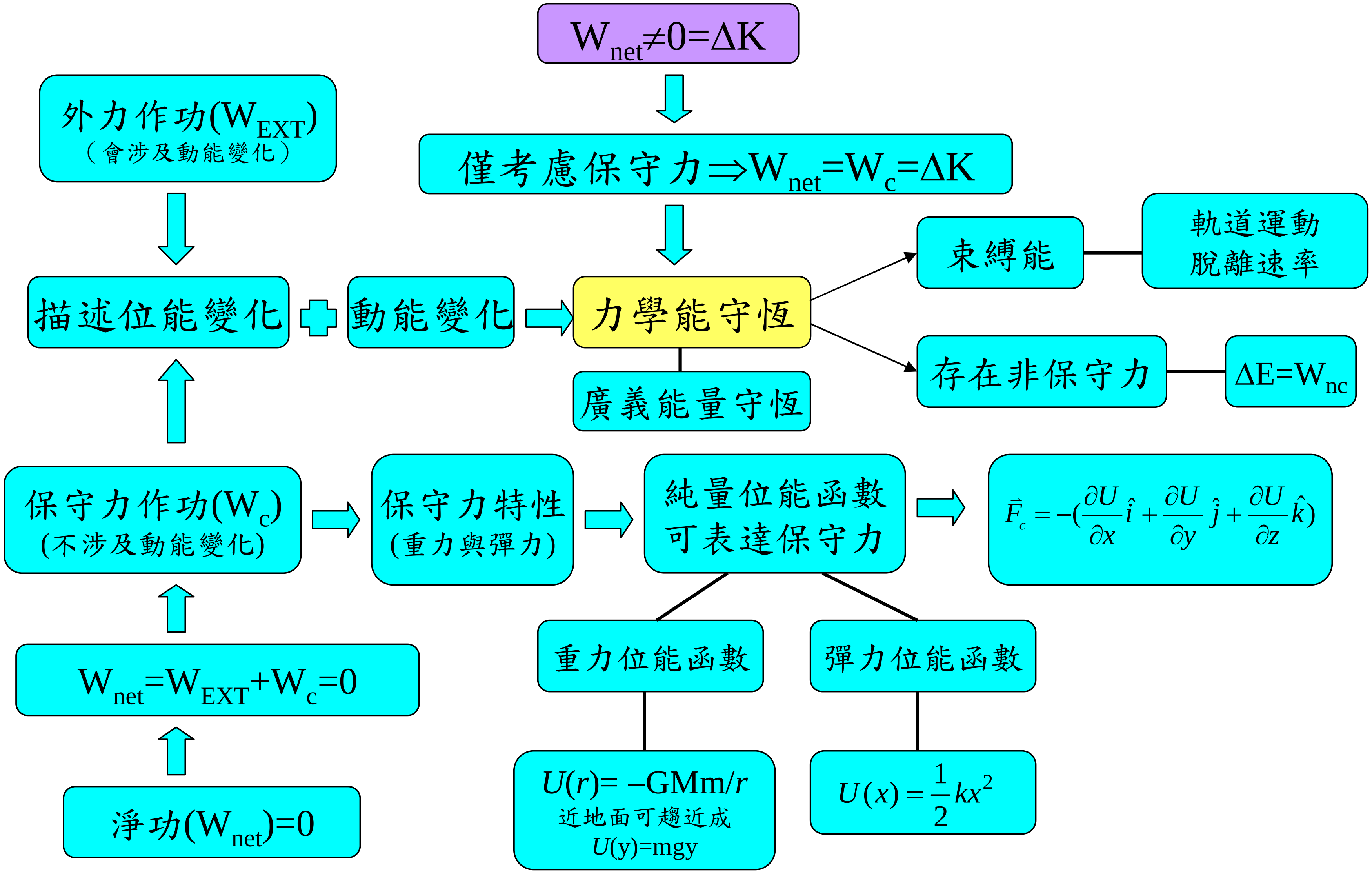
廣義的能量守恆(Generalized conservation of energy)

—能量可轉換成各種形式，但無法創造(create)或毀滅(destroy)。

範例：

- 系統的內能可因作功或熱傳遞而改變，此為熱力學第一定律。
- 能量轉移未作巨觀功的情形，如:太陽能電池－光能→電能 或 燈泡－熱能→光能。

本章重要觀念發展脈絡彙整



習　題

- 教科書習題(p.164~p.170)

  Exercise: 1,3,11,13,15,19,31,33,39,47,57,61,63,73,75

  Problem: 1,5,11

- 基本觀念問題：

  1.利用外力作功定義位能變化會有何缺失?

  2.保守力的特性有哪些?

  3.何謂穩定平衡、不穩定平衡與隨遇平衡?

  4.力學能守恆成立的條件為何?

- 延伸思考問題：

  1.請說明廣義能量守恆與力學能守恆有何區別？