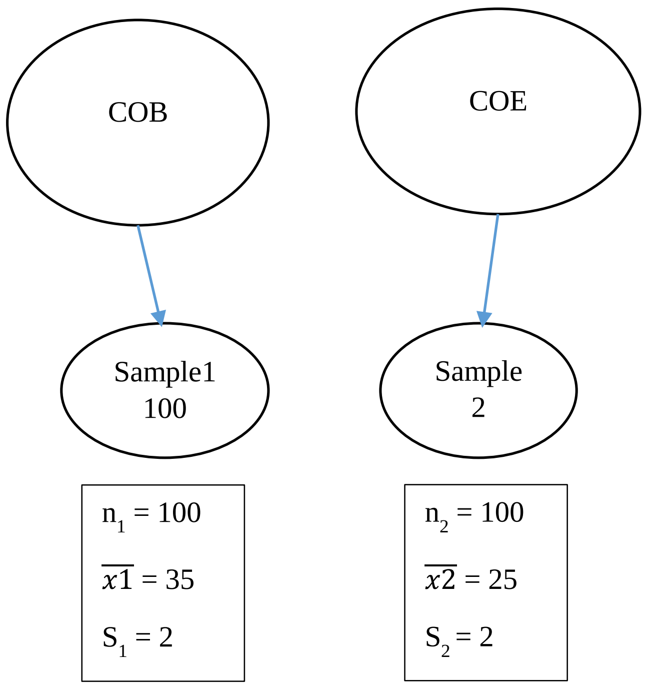
In order to test and estimate the difference between two population means, we will draw random samples from each of two populations. The best estimator of the difference between two population means $\mu 1 - \mu 2$ is the different between two sample means, $\overline{x1} - \overline{x2}$.

If I try to compare the number of job offers (X) between college of business (COB) students and college of education (COE) students, I will randomly sample 200 recently graduated college students, half of whom are COB students and half are COE students.

In average, COB students have 35 job offers while COE students have 25 job offers. I want to test whether the number of job offers is different between COB students and COE students at 5% significance level.



1. We **assume** equal variances between COB and COE students

2. Set up the hypotheses: Because I want to test whether the number of job offers is different between COB students and COE students, the alternative hypothesis will be $\mu_1 - \mu_2 \neq 0$

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

3. $\alpha = 0.05$

4. $s_p^2 = \left(\dfrac{(100-1)2^2 + (100-1)2^2}{100+100-2} \right)$

5. $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \dfrac{(35-25)-0}{\sqrt{\left(\frac{(100-1)2^2 + (100-1)2^2}{100+100-2} \right) \left(\frac{1}{100} + \frac{1}{100} \right)}} = 35.71$

6. It is a two tailed test

Rejection region: $t < -t_{\alpha/2,\nu} = -t_{.025,198} = -1.972$ or $t > t_{\alpha/2,\nu} = t_{.025,198} = 1.972$

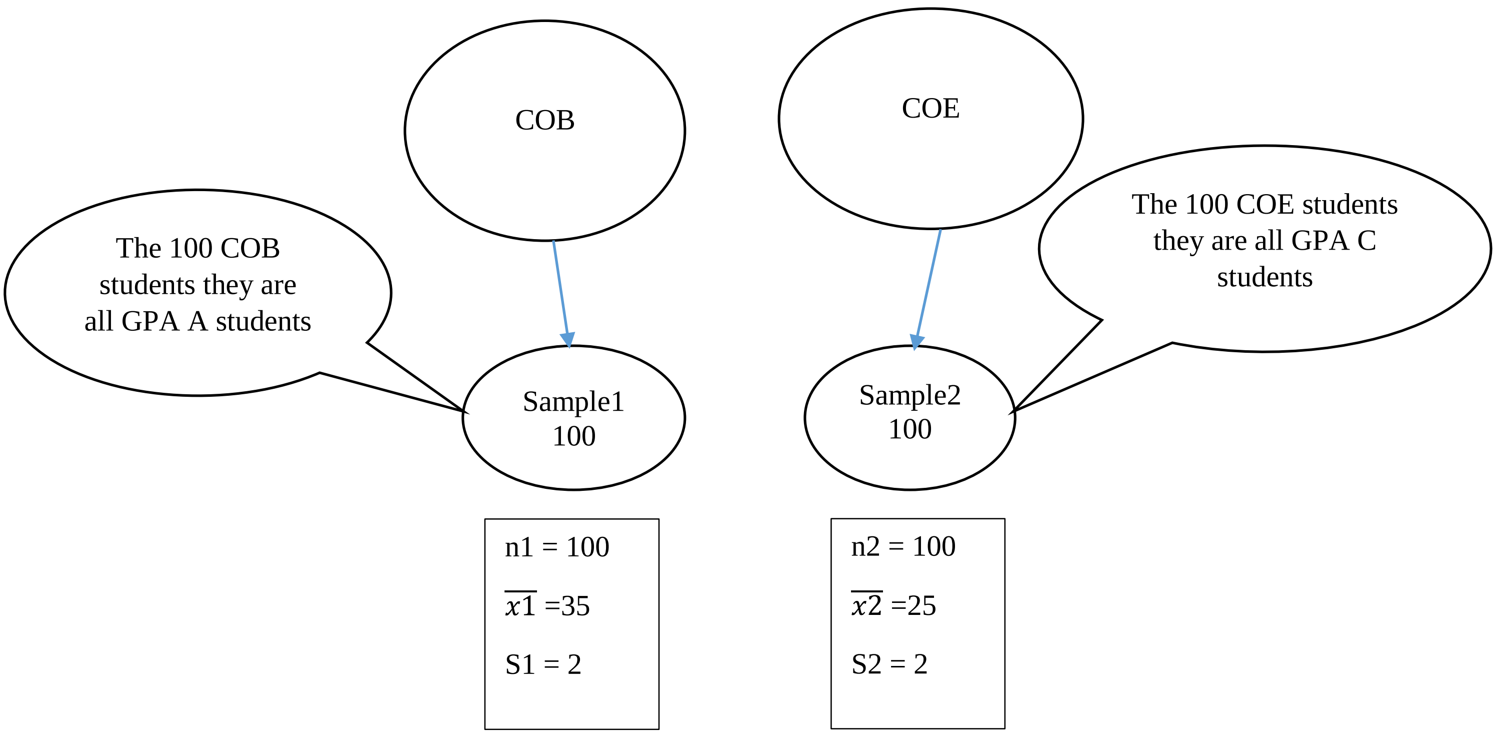
7. We reject H_0

8. We have enough evidence to infer that the population means differ.

Wait a minute! Suppose I raise one question!

How important is GPA in your job hunt out of college?

As we all know, GPA does matter when students are applying to jobs. So, in the above example, if the 100 COB students they are all GPA A students and the 100 COE students they are all GPA C students. I would say this comparison is totally UNFAIR because you should compare students whose GPA are similar.



	COB students	COE students
GPA A	35 jobs	
GPA B		
GPA C		25 jobs

If you take the effect of "GPA" into account, you should make a comparison by selecting COB and COE students with similar GPAs.

Right now, the raw data looks like this:

	COB students	COE students
GPA A	35	33
GPA B	30	29
GPA C	26	25

In the first example, there is no relationship between the observations in one sample (COB students) and the observations in the second sample (COE students). However, in the second example the study is designed in such a way that each observation in one sample is matched with an observation in the other sample. The matching is conducted by selecting COB students and COE students with similar GPAs.

So, I want to test whether the number of job offers is different between COB students and COE students at 5% significance level.

1. Set up the hypotheses:

We firstly define $\mu_1 - \mu_2 = \mu_D$

Because I want to test whether the number of job offers is different between COB students and COE students, the alternative hypothesis will be $H_1 : \mu_D \neq 0$

$H_0 : \mu_D = 0$

$H_1 : \mu_D \neq 0$

2. $\alpha = 0.05$

3.

	COB	COE	Difference scores
GPA A	35	33	2
GPA B	30	29	1
GPA C	26	25	1

$\bar{x}_D$ = (2+1+1)/3 = 1.33

s_D = 0.58

4.

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}} = \frac{1.33 - 0}{0.58 / \sqrt{3}} = 3.97$$

5. It is a two tailed test. Rejection region: $t < -t_{\alpha/2,2} = -t_{.025,2} \approx -4.303$ or

$t > t_{\alpha/2,2} = t_{.025,2} \approx 4.303$

6. Non reject H_0

7. There is no enough evidence to infer that the population means differ.

Did you find any differences in the Example 1 and Example 2?

Obviously, in the first one example, we have evidence to conclude that the population means differ (of course, you compare GPA A COB students and GPA C COE students). In the second one example, by redoing the study as matched pairs we do not have enough evidence to infer that the population means differ (you compare COB students and COE students with similar GPAs).

In the real world, ask yourself a question, does some natural relationship exist between each pair of observations that provides a logical reason to compare the first observation of sample 1 and the first observation of sample 2, the second observation of sample 1 and the second observation of sample 2, and so on. If yes, the study should conduct by matched pairs. If not, the study should conduct by independent samples.