線動量 (Linear momentum)

➢描述物質本體之運動量

運動量(quantity of motion) $mv \Rightarrow$ 線動量 $\vec{p} = m\vec{v}$

(其中 m 為質量， $\vec{v}$ 為速度。)

- **若淨外力為零，則線動量維持不變。**

➢ $\sum \vec{F}_{ie} = \vec{F}_{ext} = 0 \Rightarrow \sum \vec{p}_i =$ 定值

➢ 單一質點(single particle) $\Rightarrow \vec{p} =$ 定值 or $\Delta\vec{p} = 0$

(相當於牛頓第一定律)

➢ 兩質點碰撞(collisions) $\Rightarrow \vec{p}_1 + \vec{p}_2 =$ 定值 or $\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$

- **作用於質點的淨力相當於質點線動量隨時間的變化量。**

➢ $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$ ($\vec{F}$為定力) $\Rightarrow \vec{F} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{d\vec{p}}{dt}$

➢ 若質量 m 不變，則： $\vec{F}=\dfrac{d\vec{p}}{dt}=\dfrac{dm\vec{v}}{dt}=m\dfrac{d\vec{v}}{dt}=m\vec{a}$

（ 相當於牛頓第二定律 ）

- **<u>兩質點碰撞</u>**

$\because\ \vec{F}=\dfrac{\Delta\vec{p}}{\Delta t}\Rightarrow\Delta\vec{p}=\vec{F}\Delta t$

考慮
$\vec{F}_{12}$ 為質點 2 作用於質點 1 的作用力
$\vec{F}_{21}$ 為質點 1 作用於質點 2 的作用力
$\vec{F}_{12}$, $\vec{F}_{21}$ 為碰撞系統的內力。

$\therefore\Delta\vec{p}_1=\vec{F}_{12}\Delta t$ ； $\Delta\vec{p}_2=\vec{F}_{21}\Delta t$

If $\sum\vec{F}_{ie}=0$ during collision ，then

$\Delta\vec{p}_1+\Delta\vec{p}_2=0 \qquad \Rightarrow \qquad \vec{F}_{12}=-\vec{F}_{21}$

（線動量守恆） （作用力與反作用）

（相當於牛頓第三定律）

✦ 線動量守恆(Conservation of Linear Momentum)

- 線動量守恆條件 $\Rightarrow$ 淨外力為零(即 $\sum \vec{F}_{ie} = 0$)

$$\sum \vec{F}_{ie} = \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = \sum \vec{p}_i = \text{定值}$$

- 若淨外力不為零(即 $\sum \vec{F}_{ie} \neq 0$),則:$\sum \vec{F}_{ie} = \vec{F}_{ext} = \dfrac{d\vec{P}}{dt}$

$$\vec{F}_{1e} + \vec{F}_{12} = \frac{d\vec{p}_1}{dt} \quad (1) \qquad ; \qquad \vec{F}_{2e} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} \quad (2)$$

$$(1)+(2) \Rightarrow (\vec{F}_{1e} + \vec{F}_{2e}) = \frac{d(\vec{p}_1 + \vec{p}_2)}{dt} \qquad (\because \vec{F}_{12} + \vec{F}_{21} = 0)$$

$$\Rightarrow \quad \vec{F}_{ext} = \sum \vec{F}_{ie} = \frac{d\vec{P}}{dt} \qquad (\text{其中 } \vec{P} = \sum \vec{p}_i)$$

(Note: 若淨外力作用時間甚短,則仍可趨近線動量守恆。)

碰撞型式：

- **彈性碰撞** ⇒ 總線動量守恆，總動能守恆，碰撞後分開運動。
  (elastic collision)

- **非彈性碰撞**⇒總線動量守恆，總動能不守恆(會減小)，碰撞後分開運動。
  (inelastic collision)

- **完全非彈性碰撞** ⇒ 總線動量守恆，總動能不守恆(會減小)，碰撞後黏在一起運動。
  (completely inelastic collision)

- **超彈性碰撞** ⇒ 總線動量守恆，總動能不守恆(會增加)，碰撞後分開運動。
  (superelastic collision)

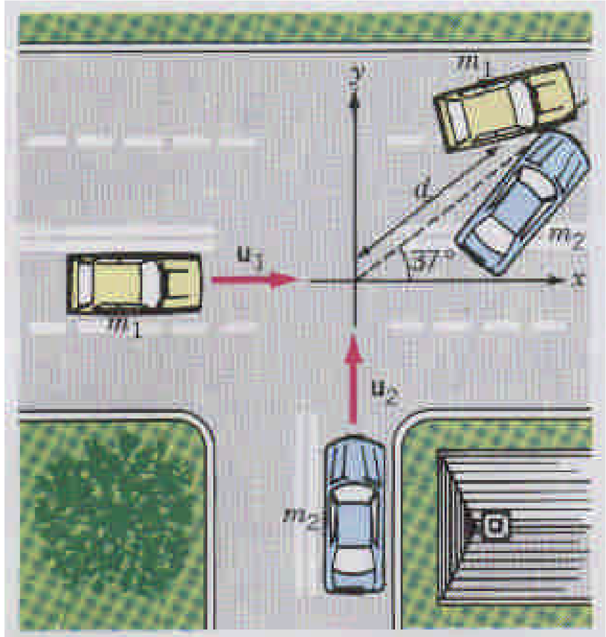
➢例如：

考慮兩質點碰撞

總線動量守恆⇒ $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

總動能守恆 ⇒ $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Example 9.4 Was either car exceeding the 15 m/s speed limit?



From Newton's second law :

$$f_k = ma \;\Rightarrow \mu_k(m_1+m_2)g = (m_1+m_2)a$$

$$\Rightarrow a = \mu_k g$$

$$\mathrm{v}^2 = \mathrm{v}_0^2 + 2a\Delta x \;\Rightarrow\; 0 = \mathrm{v}_0^2 + 2(-\mu_k g)d$$

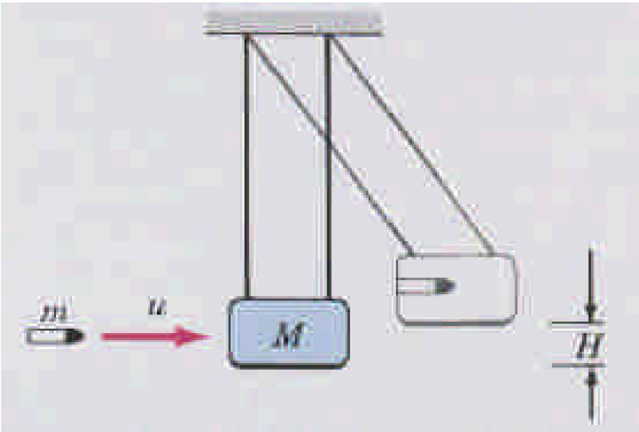
$$\Rightarrow\; \mathrm{v}_0 = (2\mu_k g d)^{1/2} = 8.5 m/s$$

From the conservation of linear momentum :

$$\sum \vec{p} \;\Rightarrow\; m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1+m_2)\vec{\mathrm{v}}_0 \;\Rightarrow \begin{cases} \sum p_x \Rightarrow m_1u_1 + 0 = (m_1+m_2)\mathrm{v}_0\cos\theta \\ \sum p_y \Rightarrow 0 + m_2u_2 = (m_1+m_2)\mathrm{v}_0\sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} u_1 = \dfrac{(m_1+m_2)\mathrm{v}_0\cos\theta}{m_1} = 16.4 \text{ m/s} > 15 \text{ m/s} \\ u_2 = \dfrac{(m_1+m_2)\mathrm{v}_0\sin\theta}{m_2} = 8.7 \text{ m/s} < 15 \text{ m/s} \end{cases}$$

Example 9.5 (a) How can one determine u from H? (b) What is the thermal energy generated?



From the conservation of linear momentum :

$$\Rightarrow mu = (m+M)V \Rightarrow V = \frac{mu}{m+M}$$

From the conservation of mechanical energy :

$$\Rightarrow \frac{1}{2}(m+M)\mathrm{V}^2 = (m+M)gH$$

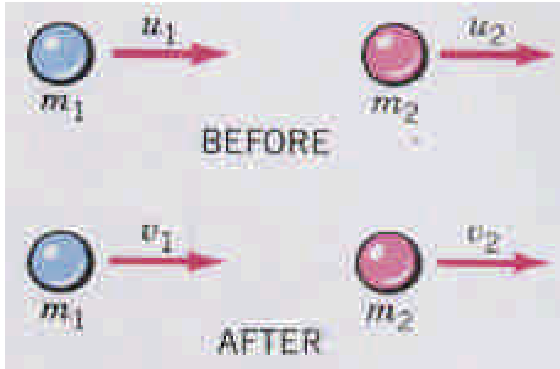
$$\Rightarrow u = \frac{(m+M)\sqrt{2gH}}{m} \quad \text{Ans(a)}$$

$$\begin{cases} K_i = \frac{1}{2}mu^2 = 200J \\ K_f = \frac{1}{2}(m+M)\mathrm{V}^2 = 1J \end{cases} \Rightarrow \text{thermal energy} = K_i - K_f = 199 \text{ J} \quad \text{Ans(b)}$$

✦一維彈性碰撞(Elastic collision in one dimension)

- 質點碰撞前後的相對速度大小維持不變，但方向相反。

證明：　　假設 $\vec{u} = u\hat{i}$；$\vec{v} = v\hat{i}$



由總線動量守恆式可得

$$\Rightarrow\ m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad (1)$$

由總動能守恆式可得

$$\Rightarrow\ m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$\Rightarrow\ m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \qquad (2)$$

$$(2)/(1) \Rightarrow (u_1 + v_1) = (u_2 + v_2)\ \Rightarrow \underline{(v_2 - v_1) = -(u_2 - u_1)} \qquad (3)$$

- 質量相等 ($m_1 = m_2$)，線動量可完全交換，即：

$$\underline{v_1 = u_2\ ,\quad v_2 = u_1} \qquad \text{(由(1), (3)式可推得)}$$

- **質量不等 ($m_1 \neq m_2$) ，可推得末速與初速關係如下：**

$$\mathrm{v}_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \quad ; \quad \mathrm{v}_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{(m_2 - m_1)}{m_1 + m_2} u_2$$

證明：

From (3) $\Rightarrow$ $\mathrm{v}_2 = u_1 - u_2 + \mathrm{v}_1$ 代入(1)式

$$\mathrm{v}_1 = u_1 - \frac{m_2}{m_1}(\mathrm{v}_2 - u_2) = u_1 - \frac{m_2}{m_1}(u_1 - u_2 + \mathrm{v}_1 - u_2)$$

$$= (\frac{m_1 - m_2}{m_1})u_1 + (\frac{2m_2}{m_1})u_2 - \frac{m_2}{m_1}\mathrm{v}_1$$

$$\Rightarrow (\frac{m_1 + m_2}{m_1})\mathrm{v}_1 = (\frac{m_1 - m_2}{m_1})u_1 + (\frac{2m_2}{m_1})u_2 \Rightarrow \mathrm{v}_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

同理將 $\mathrm{v}_1 = u_2 - u_1 + \mathrm{v}_2$ 代入(1)式 $\Rightarrow \mathrm{v}_2 = \dfrac{2m_1}{m_1 + m_2} u_1 + \dfrac{(m_2 - m_1)}{m_1 + m_2} u_2$

- 令 $u_2 = 0 \quad \Rightarrow \mathrm{v}_1 = \dfrac{(m_1 - m_2)}{m_1 + m_2} u_1 \;\; ; \;\; \mathrm{v}_2 = \dfrac{2m_1}{m_1 + m_2} u_1$

(a) If $m_1 > m_2$, then $\mathrm{v}_1 > 0$, $\mathrm{v}_2 > 0$.

(b) If $m_1 < m_2$, then $\mathrm{v}_1 < 0$, $\mathrm{v}_2 > 0$.

(c) If $m_1 >> m_2$, then $\mathrm{v}_1 \approx u_1$, $\mathrm{v}_2 \approx 2u_1$.

(d) If $m_1 << m_2$, then $\mathrm{v}_1 \approx -u_1$, $\mathrm{v}_2 \approx 0$.

✦ 衝量(Impulse)

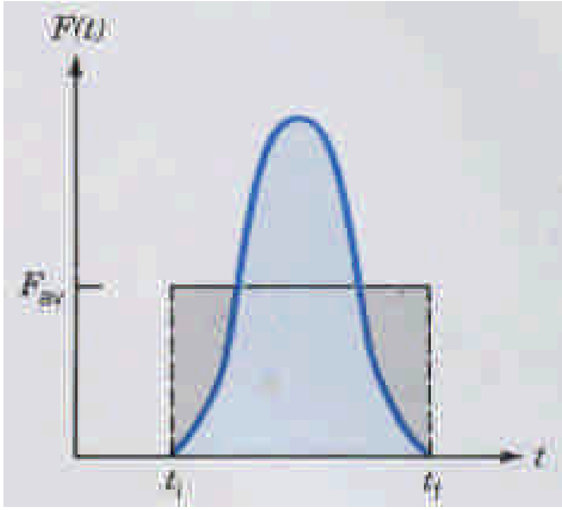
- 定義為線動量的改變，即：

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$= \int \vec{F} dt = \vec{F}_{av}\Delta t$$

$$(\because \ \vec{F} = d\vec{p}/dt \ \text{ and } \ \Delta\vec{p} = \int d\vec{p} = \int \vec{F} dt)$$

Fig. 9.11



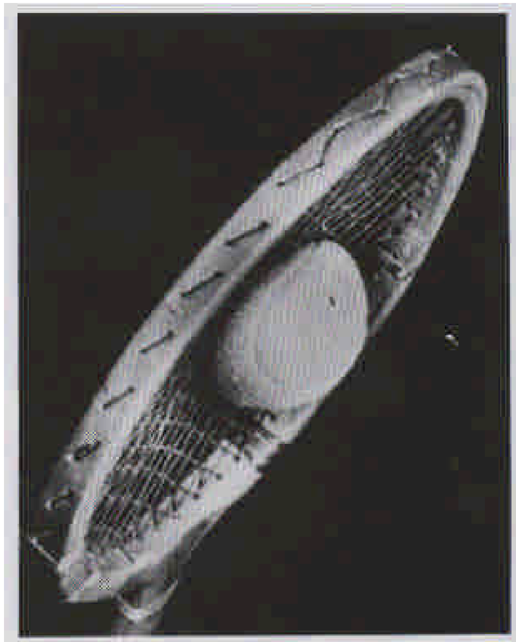


Fig. 9.12

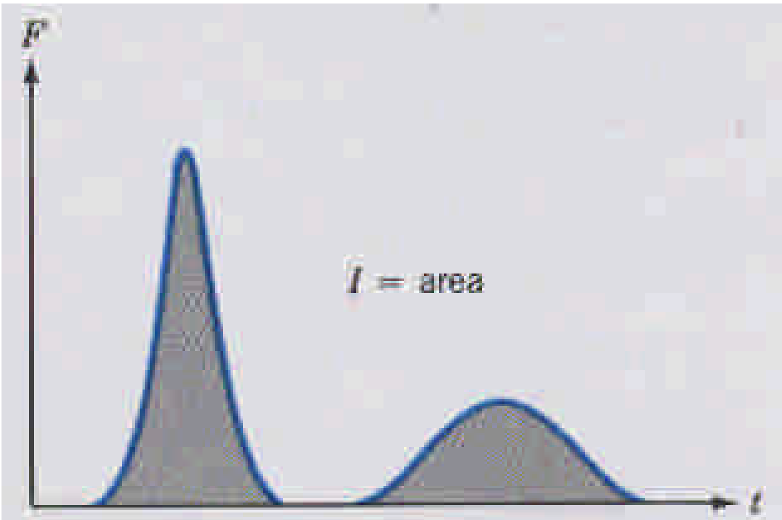


Fig. 9.13

線動量與動能的比較

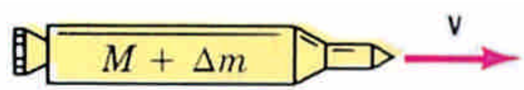
(1)線動量守恆在任何形式的碰撞過程中皆成立，而動能守恆僅在彈性碰撞才成立。

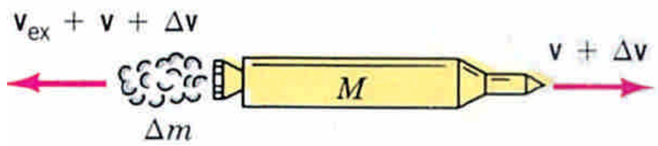
(2)線動量是一個向量，而動能是純量。

(3) $F=\dfrac{\Delta p}{\Delta t}$ (力相當於線動量對時間的改變率，若力非定值，則為對時間的平均力。)

$F=\dfrac{\Delta K}{\Delta x}$ (力相當於動能對位置的改變率，若力非定值，則為對空間的平均力。)

✦ 火箭推進力(Rocket Propulsion)－選擇性





線動量守恆$\Rightarrow \left(M+\Delta m\right)v = M\left(v+\Delta v\right)+\Delta m\left(-v_{ex}+v+\Delta v\right)$

(氣體未排放)　　　　(氣體排放時)

$$\Rightarrow 0 = M\Delta v+\Delta m\left(-v_{ex}+\Delta v\right) \xrightarrow{\text{忽略}\Delta m\Delta v} M\Delta v = \Delta m v_{ex}$$

$$\Rightarrow \Delta v = v_{ex}\frac{\Delta m}{M} \xrightarrow{\text{考慮微量變化}} \mathrm{d}v = -v_{ex}\frac{dM}{M}$$

$$\int_{v_i}^{v_f} dv = -\int_{M_i}^{M_f} v_{ex}\frac{dM}{M} \quad \Rightarrow v_f - v_i = v_{ex}\ln\frac{M_i}{M_f} \quad \text{(火箭方程式)}$$

M_i(火箭+燃料質量)，M_f(火箭質量)

➢當火箭快速推進時，可從燃料獲得較多動能。

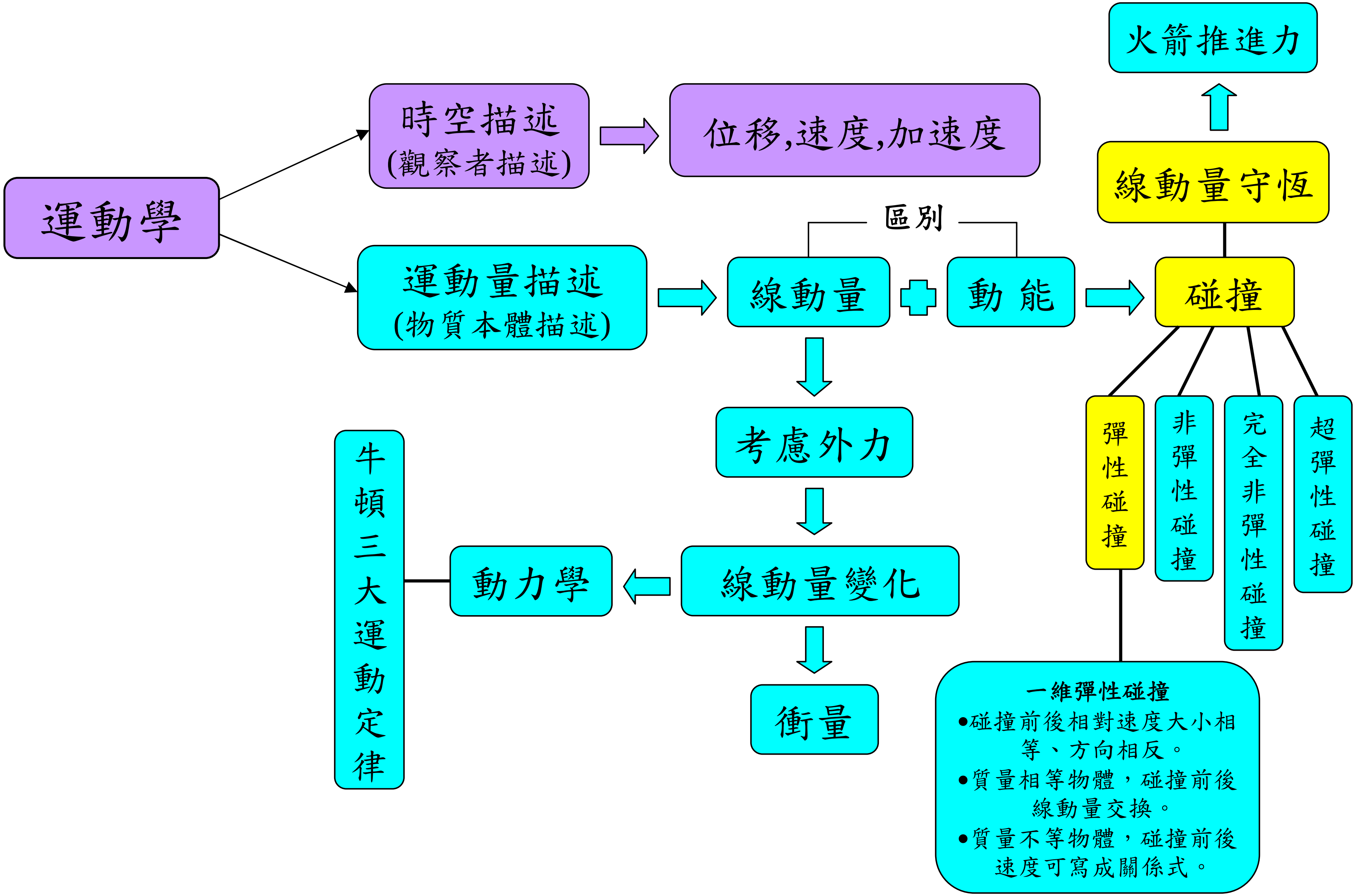
●考慮變質量的線動量分析

$$\frac{dP}{dt} = M\frac{dv}{dt} + v\frac{dM}{dt} \xrightarrow{\text{考慮線動量守恆}} M\frac{dv}{dt} + v\frac{dM}{dt} = 0$$

$$\Rightarrow M\frac{dv}{dt} = -v\frac{dM}{dt}$$ ，其中 $M\dfrac{dv}{dt} = Ma = F$

若視火箭與廢氣為不同物體，則F 相當於火箭推進力。

本章重要觀念發展脈絡彙整



習　題

- 教科書習題 (p.187~p.193)

  Exercise: 13,15,17,25,29,33,41,51,75

  Problem: 3,7,9,13,15

- 基本觀念問題：

  1.請以線動量說明牛頓三大運動定律。

  2.請問碰撞的類型有哪幾種？並說明其中的不同處。

  3.請簡述線動量(Linear Momentum)與動能(Kinetic energy)的區別。

- 延伸思考問題：

  1.若物體運動速度接近光速，則線動量是否會增加？請申述其中原理。

多質點系統 (systems of particles)

- 平移運動(translation motion)雖可適用於單質點模型，但若考慮轉動(rotation)與振動(vibration)，則必須應用多質點系統處理。

質心 (center of mass)

- 作用於質心的力僅會產生平移，而質心的平移可代表整個系統平移。

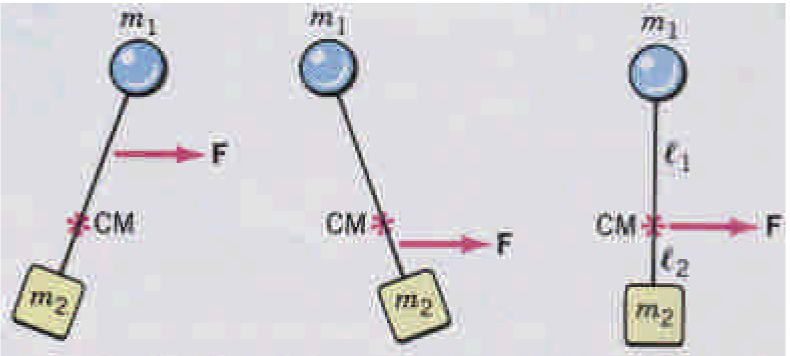


Fig.10.1

- 質心位置的估算方程式：

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M} \quad (\ \vec{r}\ \text{為位置向量})$$

$$\Rightarrow x_{CM} = \frac{\sum m_i x_i}{M}; y_{CM} = \frac{\sum m_i y_i}{M}; z_{CM} = \frac{\sum m_i z_i}{M}$$

- 推導：

➢質點質量與質心距離的關係

$$\Rightarrow m_1 l_1 = m_2 l_2$$

➢考慮座標系統 $\Rightarrow \begin{cases} l_1 = x_{CM} - x_1 \\ l_2 = x_2 - x_{CM} \end{cases}$

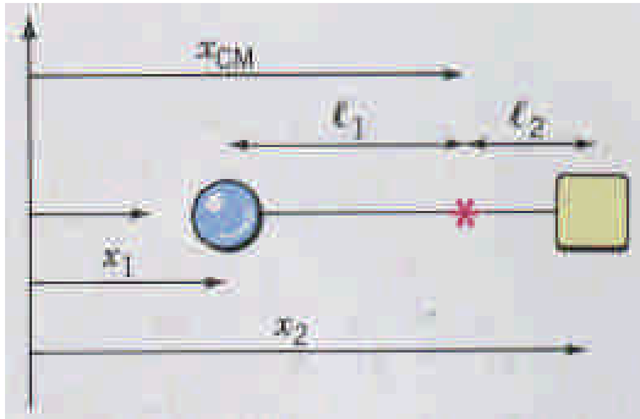


Fig.10.2

$$\therefore m_1(x_{CM} - x_1) = m_2(x_2 - x_{CM})$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \xrightarrow{\text{考慮}N\text{個質點}}$$

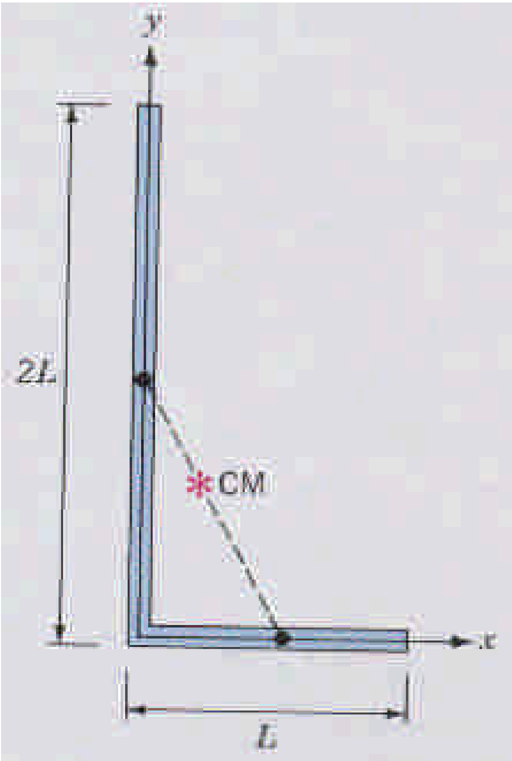
$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdot\cdot\cdot + m_N x_N}{m_1 + m_2 + \cdot\cdot\cdot + m_N} \Rightarrow x_{CM} = \frac{\sum m_i x_i}{M}$$

考慮三維分量，以此類推 $\Rightarrow y_{CM} = \dfrac{\sum m_i y_i}{M}, \; z_{CM} = \dfrac{\sum m_i z_i}{M}$

Example 10.2： A thin rod of length 3L is bent at right angles at a distance L from one end. Locate the CM with respect to the corner.

Sol：

Assume that the linear mass density of a thin rod is λ (kg/m) and the corner position is (0,0)



$$m_1 = \lambda L \xrightarrow{CM} (\frac{L}{2}, 0)$$

$$m_2 = \lambda 2L \xrightarrow{CM} (0, L)$$

$$M = \lambda 3L \xrightarrow{CM} (x_{CM}, y_{CM})$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{M} = \frac{\lambda L \times \frac{L}{2} + \lambda 2L \times 0}{\lambda 3L} = \frac{L}{6}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{M} = \frac{\lambda L \times 0 + \lambda 2L \times L}{\lambda 3L} = \frac{2L}{3}$$

- 質心位於對稱軸或對稱面上。
  - 可應用於對稱物體。

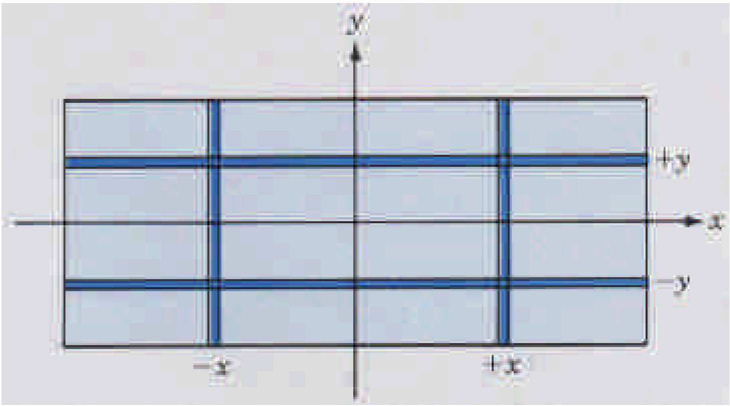


Fig.10.3

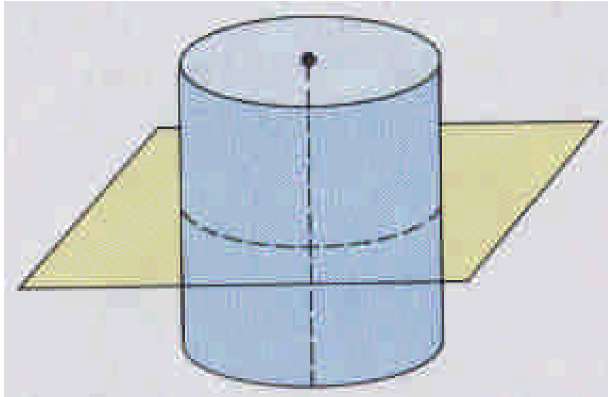


Fig.10.4

- 質心位於任一懸點的垂線上。
  - 可應用於非對稱物體。
  - 利用兩不共線的懸點可決定平面物的質心。
  - 利用三個不共面懸點可決定非平面物的質心。

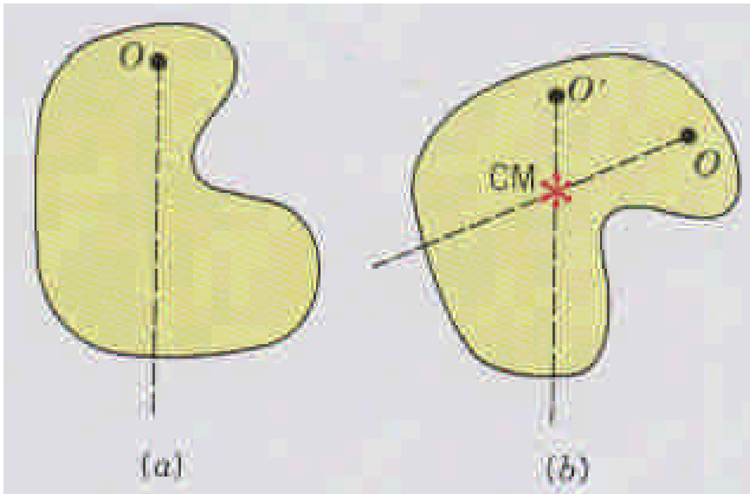
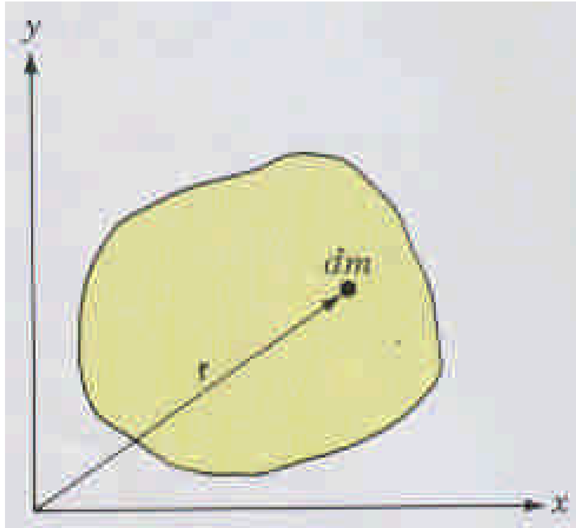


Fig.10.5

• 連續體的質心 (center of mass of continuous bodies)

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M} = \frac{1}{M}\int \vec{r}dm$$

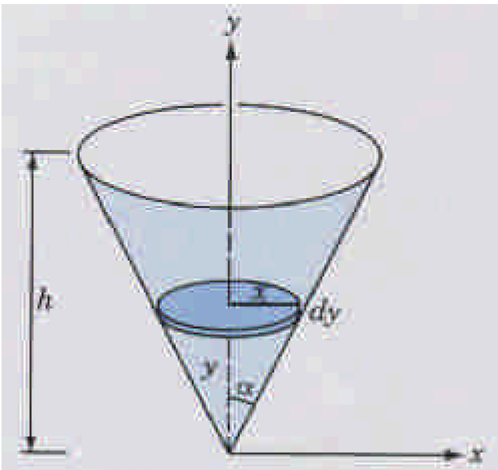


$$\Rightarrow x_{CM} = \frac{1}{M}\int xdm \; ; \; y_{CM} = \frac{1}{M}\int ydm \; ;$$

$$z_{CM} = \frac{1}{M}\int zdm$$

Fig.10.8

Example 10.4：Find the CM of a uniform solid cone of height h and semiangle α .



$$dV = \pi x^2 dy = \pi (y\tan\alpha)^2 dy$$

$$M = \int dm = \int \rho dV = \pi\rho\tan^2\alpha \int_0^h y^2 dy = \pi\rho(\tan^2\alpha)(\frac{h^3}{3})$$

$$y_{CM} = \frac{\int ydm}{M} = \frac{1}{M}\pi\rho\tan^2\alpha\int_0^h y^3 dy = \frac{\pi\rho\tan^2\alpha}{M}(\frac{h^4}{4}) = \frac{3h}{4}$$

✦ 質心運動 (motion of the center of mass)

- 質心速度 $\Rightarrow \quad \vec{v}_{CM} = \dfrac{\sum m_i \vec{v}_i}{M}$

$$\because \vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{d}{dt}\left(\frac{\sum m_i \vec{r}_i}{M}\right) = \frac{\sum m_i}{M}\left(\frac{d\vec{r}_i}{dt}\right) = \underline{\frac{\sum m_i \vec{v}_i}{M}}$$

- 質心動量等於多質點系統的總動量，即：

$$\vec{P} = M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdot \cdot \cdot + m_N\vec{v}_N$$

$$\left(\because \vec{v}_{CM} = \frac{\sum m_i \vec{v}_i}{M} \Rightarrow M\vec{v}_{CM} = \sum m_i \vec{v}_i \right)$$

- 多質點系統的總動量變化率等於淨外力，即：

$$\vec{F}_{EXT} = \frac{d\vec{P}}{dt} = \sum m_i \vec{a}_i = M\vec{a}_{CM} \qquad \left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- 質點間的內力(internal forces) $\vec{F}_{INT}$ 會成對消去 $\Rightarrow \sum m_i \vec{a}_i = \sum \vec{F}_i = \vec{F}_{EXT} + \vec{F}_{INT} = \vec{F}_{EXT}$

If $\vec{F}_{EXT} = 0$, then $\vec{\mathrm{v}}_{CM} = \text{constant}$.

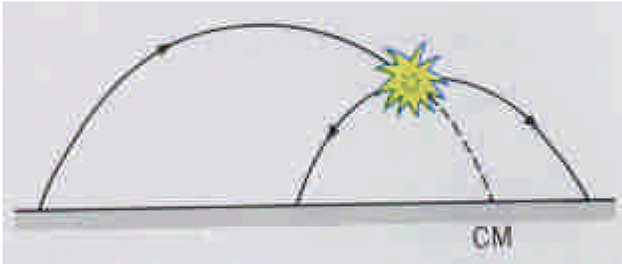


Fig.10.13

Fig.10.14

Example 10.5：

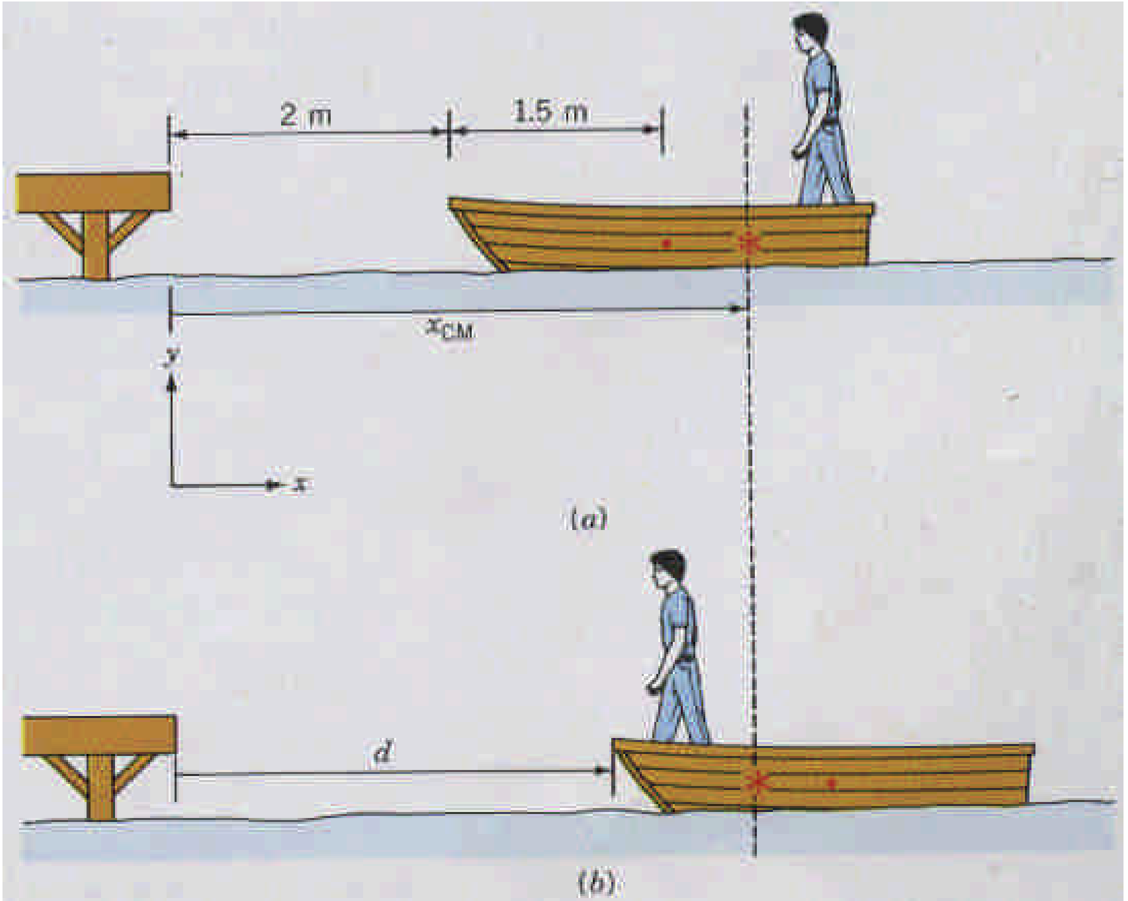


Fig.10.15

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{M} = \frac{60kg \times 5m + 40kg \times 3.5m}{100kg} = 4.4\text{m}$$

$$x_{CM} = \frac{m_1 d + m_2 (d + 1.5)}{100} \quad \Rightarrow d = 3.8\text{m} \ \ (\text{船後退}1.8\text{m})$$

Example 10.6： Find the displacement of (a) the platform; (b) the man; (c) the center of mass.

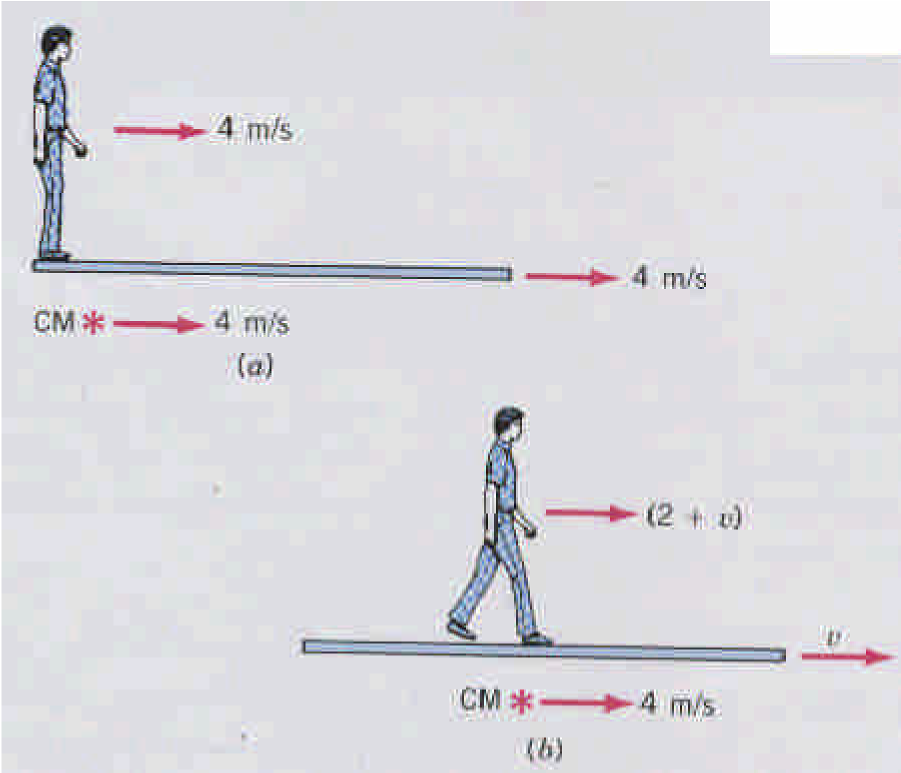


Fig.10.17

已知：

1.the length of platform=4 m

2.man walks at 2m/s relative to the platform

3.the platform moves initally at 4 m/s relative to ground

$$\underset{\text{(CM)}}{100\times 4} = \underset{\text{(man)}}{75(2+v_P)} + \underset{\text{(platform)}}{25v_P}$$

$$\Rightarrow v_P = 2.5\text{ m/s},\ v_m = 4.5\text{ m/s}$$

$$\Delta t = 4\text{m}/2(\text{m/s}) = 2\text{ (s)} \Rightarrow \begin{cases} \Delta x_p = 2.5\times 2 = 5\text{ m} \\ \Delta x_m = 4.5\times 2 = 9\text{ m} \\ \Delta x_{CM} = 4\times 2 = 8\text{ m} \end{cases}$$

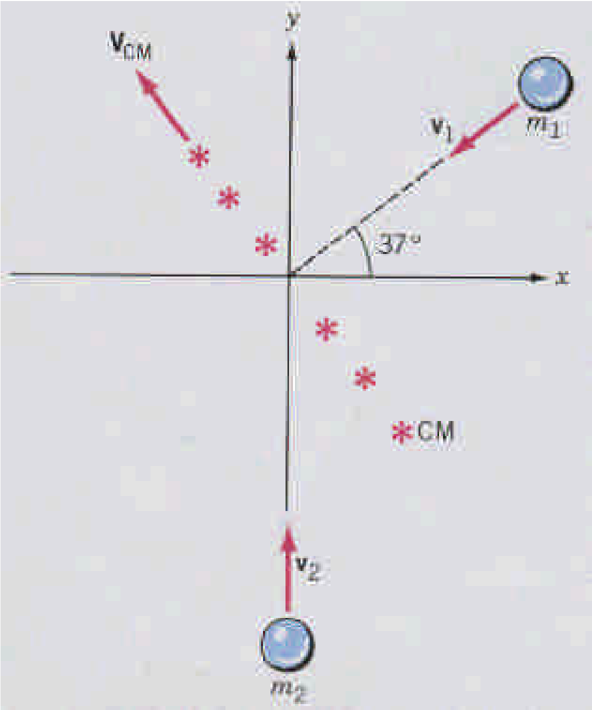


Fig.10.16

Example 10.6： Find the velocity of CM.

$$\mathrm{v}_{CMx} = \frac{m_1 \mathrm{v}_{1x} + m_2 \mathrm{v}_{2x}}{\mathrm{M}} \quad ; \quad \mathrm{v}_{CM\,y} = \frac{m_1 \mathrm{v}_{1\,y} + m_2 \mathrm{v}_{2\,y}}{\mathrm{M}}$$

$$\vec{\mathrm{v}}_{CM} = \mathrm{v}_{CMx}\hat{i} + \mathrm{v}_{CM\,y}\hat{j}$$

$$= \frac{1}{M}(m_1 \mathrm{v}_{1x}\hat{i} + m_1 \mathrm{v}_{1\,y}\hat{j}) + \frac{1}{M}(m_2 \mathrm{v}_{2x}\hat{i} + m_2 \mathrm{v}_{2\,y}\hat{j})$$

$$= \frac{1}{M}(m_1 \vec{\mathrm{v}}_1 + m_2 \vec{\mathrm{v}}_2) \quad \Leftarrow \text{碰撞前}$$

$$= \frac{1}{M}(m_1 \vec{\mathrm{v}}_1' + m_2 \vec{\mathrm{v}}_2') \quad \Leftarrow \text{碰撞後}$$

➢碰撞前後若線動量維持不變，則質心速度不變。

✦多質點系統的動能(kinetic energy of a system of particles)

$$\Rightarrow \quad K = K_{CM} + K_{rel} \quad \begin{cases} K_{CM} = \dfrac{1}{2}Mv_{CM}^2 & \text{(相對於原點 } O \text{ 的質心動能)} \\ K_{rel} = \sum \dfrac{1}{2}m_i v_i'^2 & \text{(相對於質心的質點總動能)} \end{cases}$$

推導：

$$\text{如圖} \Rightarrow \vec{r}_i = \vec{r}_{CM} + \vec{r}_i' \quad \Rightarrow \vec{v}_i = \vec{v}_{CM} + \vec{v}_i'$$

$$K_i = \frac{1}{2}m_i(\vec{v}_i \cdot \vec{v}_i) = \frac{1}{2}m_i(v_{CM}^2 + v_i'^2 + 2\vec{v}_{CM} \cdot \vec{v}_i')$$

$$K = \sum K_i = \frac{1}{2}(\sum m_i)v_{CM}^2 + \sum \frac{1}{2}m_i v_i'^2 + \vec{v}_{CM} \cdot \sum m_i \vec{v}_i')$$

$$= \frac{1}{2}Mv_{CM}^2 + \sum \frac{1}{2}m_i v_i'^2 \qquad (\because \sum m_i \vec{v}_i' = 0)$$

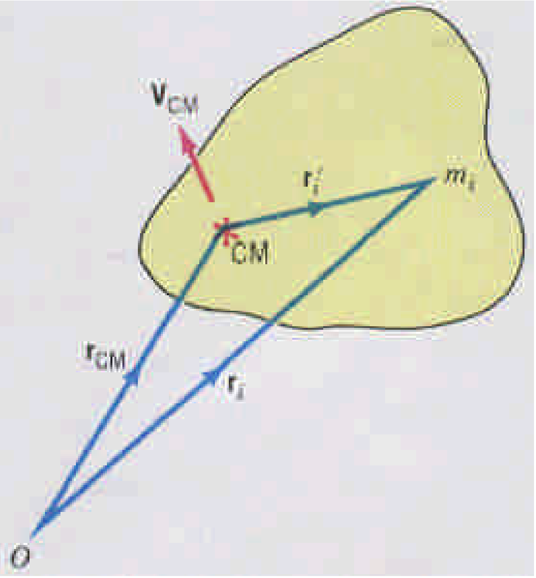
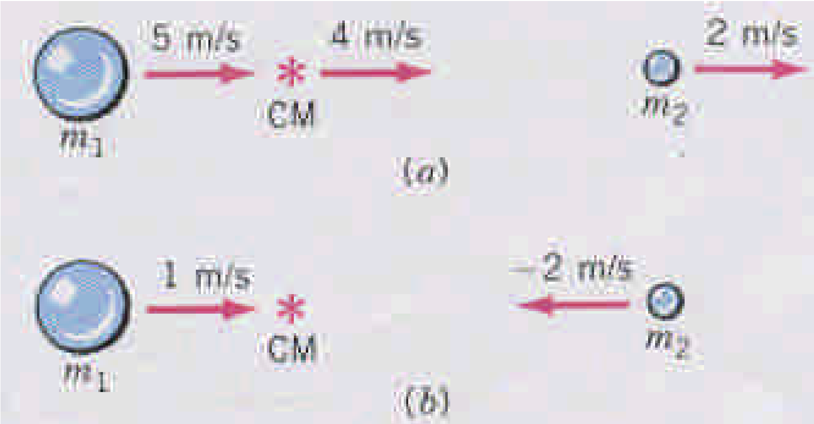


Fig.10.18

$$\text{p.s.} \sum m_i \vec{v}_i' = \sum m_i(\vec{v}_i - \vec{v}_{CM}) = \sum m_i \vec{v}_i - \vec{v}_{CM} \sum m_i$$

$$= M\vec{v}_{CM} - \vec{v}_{CM} M = 0$$

Example 10.8 A particle of mass $m_1 = 4$ kg moves at $5\hat{i}$ m/s, while $m_2 = 2$ kg moves at $2\hat{i}$ m/s. Find K_{CM} and K_{rel} .



$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 4 \text{ m/s}$$

the velocities relative to the CM :

$$v_1' = v_1 - v_{CM} = +1 \text{ m/s}$$

$$v_2' = v_2 - v_{CM} = -2 \text{ m/s}$$

$$\left.\begin{aligned} K_{CM} &= \frac{1}{2}(m_1 + m_2)v_{CM}^2 = 48 \text{ J} \\ K_{rel} &= \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 = 6 \text{ J} \end{aligned}\right\} \Rightarrow \text{prove} : K_{CM} + K_{rel} = K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

多質點系統的功-能轉換定理
(work-energy theorem for a system of particles)

$$W_{net} = \Delta K \xrightarrow{\text{考慮多質點系統}} W_{net} = \Delta K_{CM} + \Delta K_{rel} \Rightarrow W_{EXT} = \Delta K_{CM} + \Delta E_{INT}$$

- 說明：多質點系統的淨功W_{net}包含外力與內力所作的功，其中內力所作淨功不一定為零。

$$\Rightarrow W_{net} = W_{EXT} + W_{INT} = \Delta K_{CM} + \Delta K_{rel} \quad \text{(多質點系統功能轉換定理)}$$

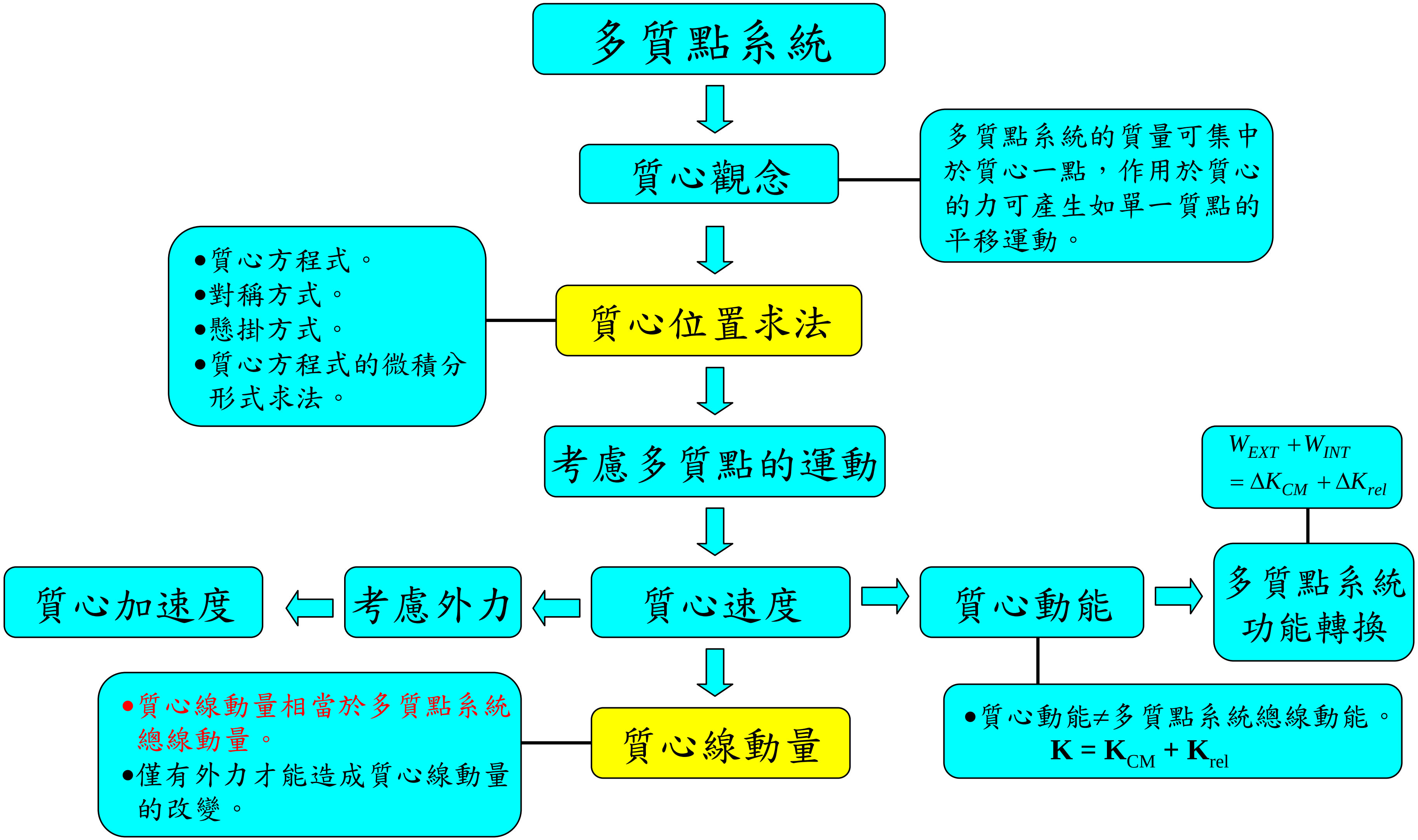
- 考慮 $W_{INT} = -\Delta U_{INT}$且令 $E_{INT} = K_{rel} + U_{INT}$

可改寫為： $W_{EXT} = \Delta K_{CM} + \Delta E_{INT}$

- 質心功能轉換定理：

$$W_{EXT} = W_{CM} + W_{rel} \xrightarrow{\text{僅考慮質心運動}} W_{CM} = \Delta K_{CM}$$

本章重要觀念發展脈絡彙整



習　題

- 教科書習題 (p.206~p.210)

  Exercise: 5,11,19,21,23,25,27,31

  Problem: 7

- 延伸思考問題：

  1. 爲何兩不共線的懸點可決定不規則平面體的質心？請申述其原理。