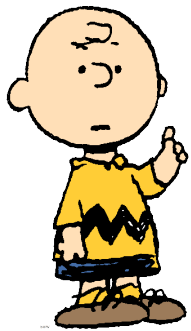


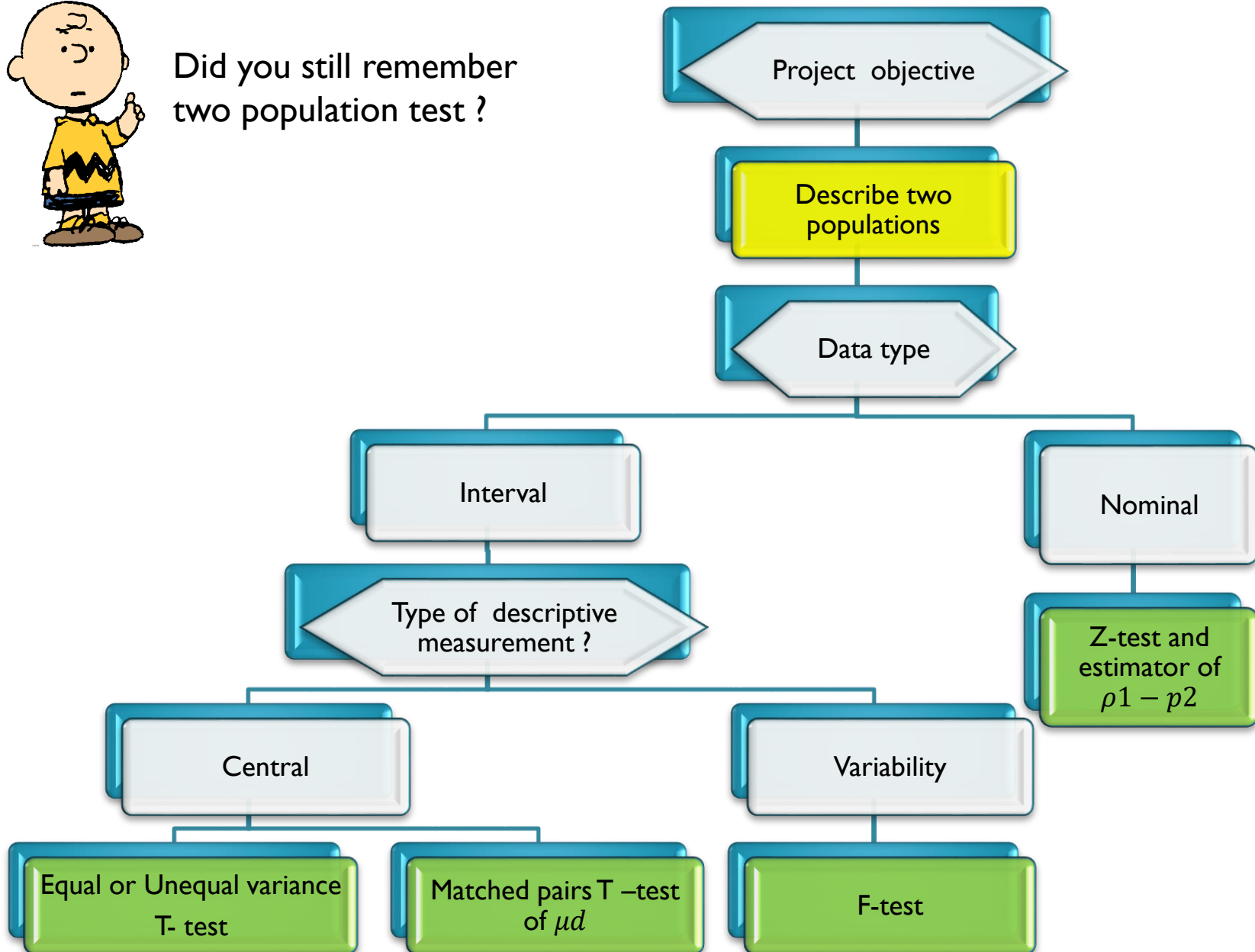


Welcome





Did you still remember
two population test ?



Topics

- One-Way ANOVA without repeated measures
 - One-way ANOVA
- One-Way ANOVA with repeated measures
 - Randomized complete block ANOVA

Batting average



A baseball player, Gary Sánchez, is shown in a batting stance, wearing a New York Yankees uniform and helmet. He is holding a bat with both hands, ready to swing. The background shows a crowd of spectators in a stadium.

FS1 ALDS
GAME 1

NEW YORK

24 C GARY SÁNCHEZ REGULAR SEASON

.278 AVG | **33** HR | **90** RBI | **.876** OPS

YANKEE RECORD BY CATCHER

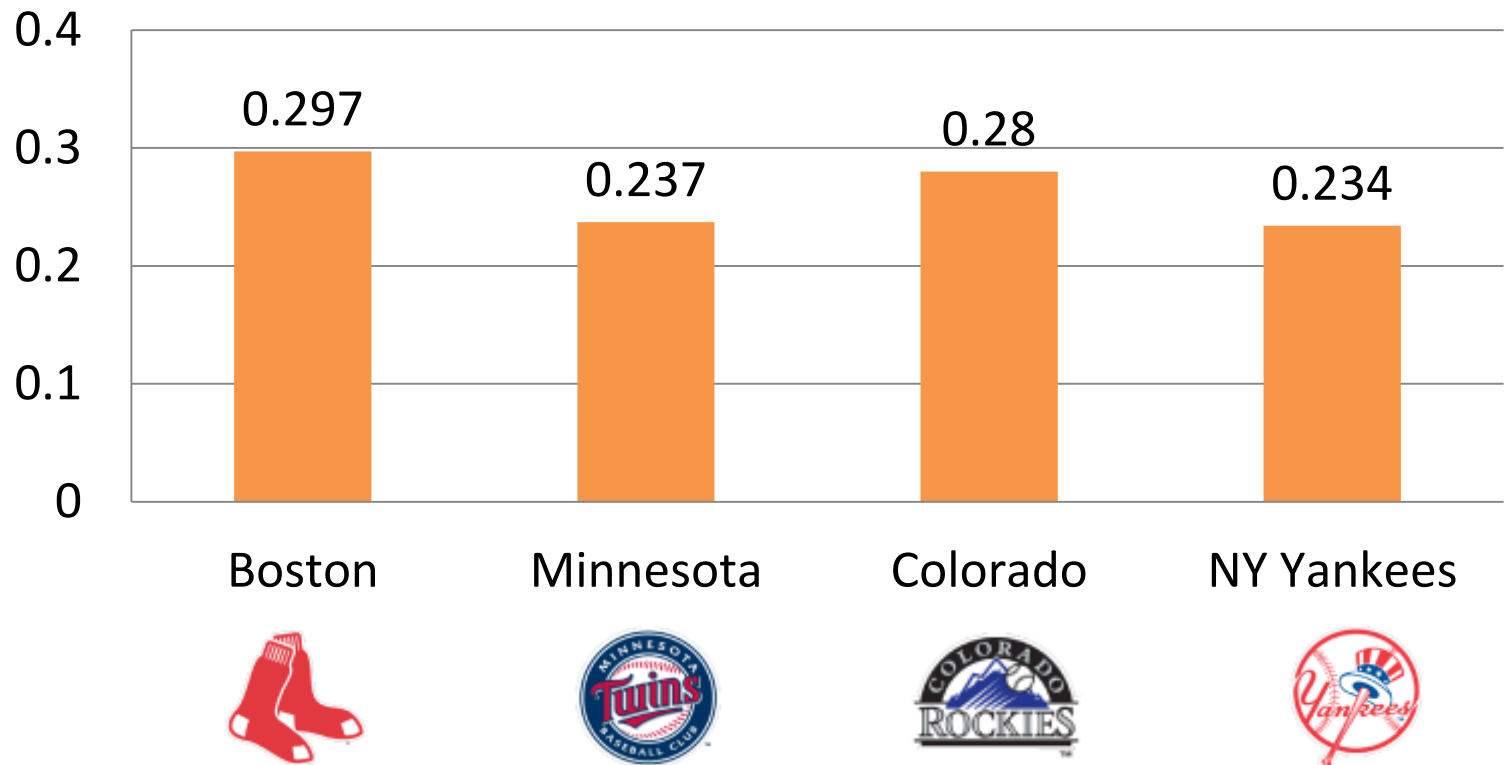
Bauer P: 12
Sánchez .278

NY 0
CLE 0

▲ 1 2 Outs **0-1**

Batting average

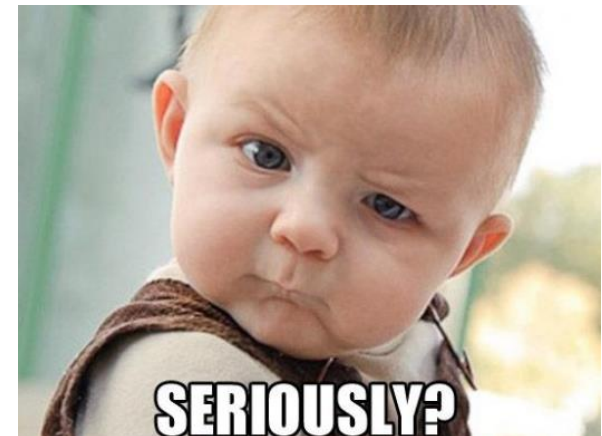
Team Batting Average- 2016



If you want to infer that the population means differ

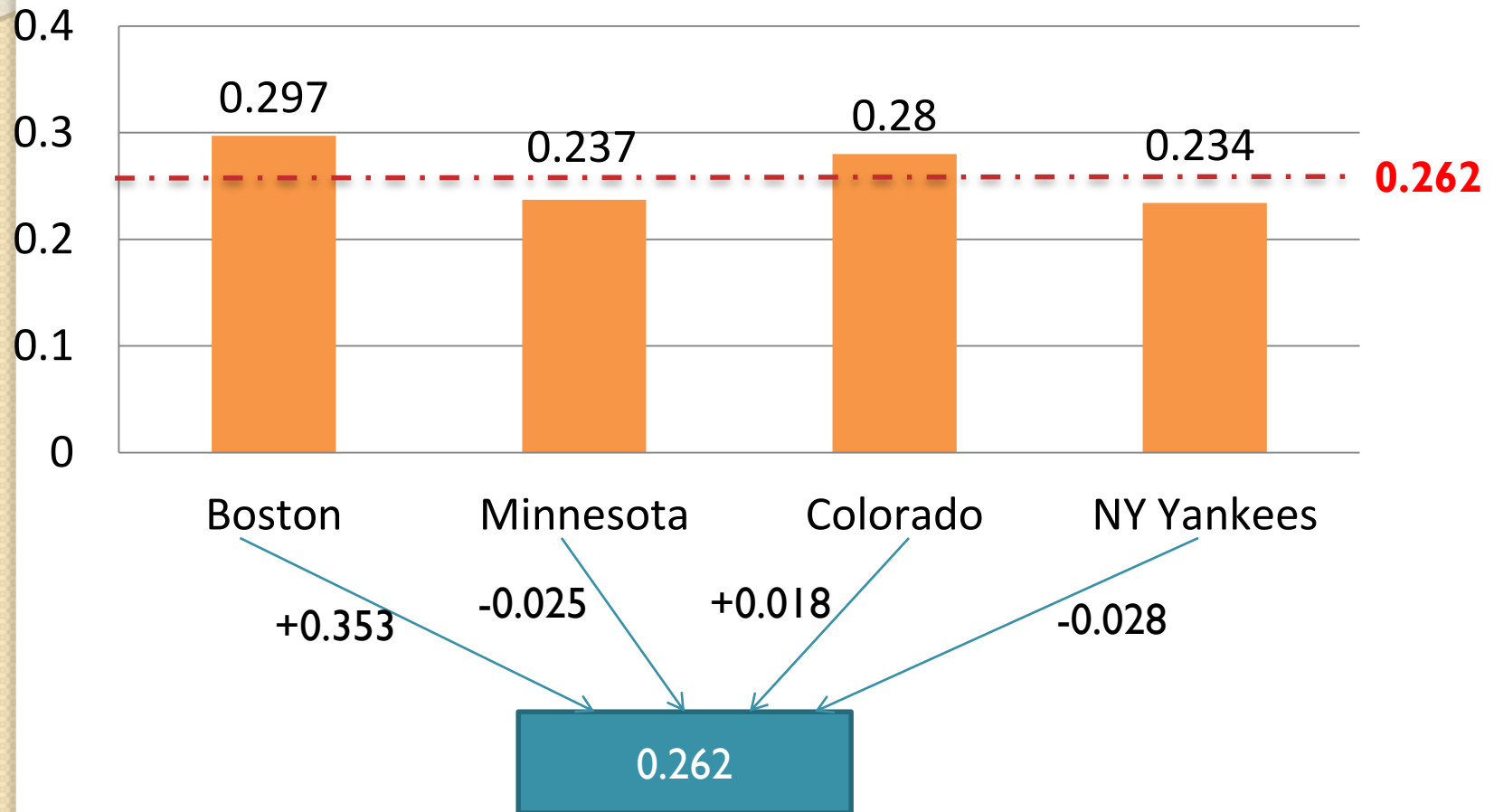
What if

- We want to compare more than two populations
- And, a mix of interval and nominal variables



Batting average problem- Eyeball test?

Team Batting Average- 2016



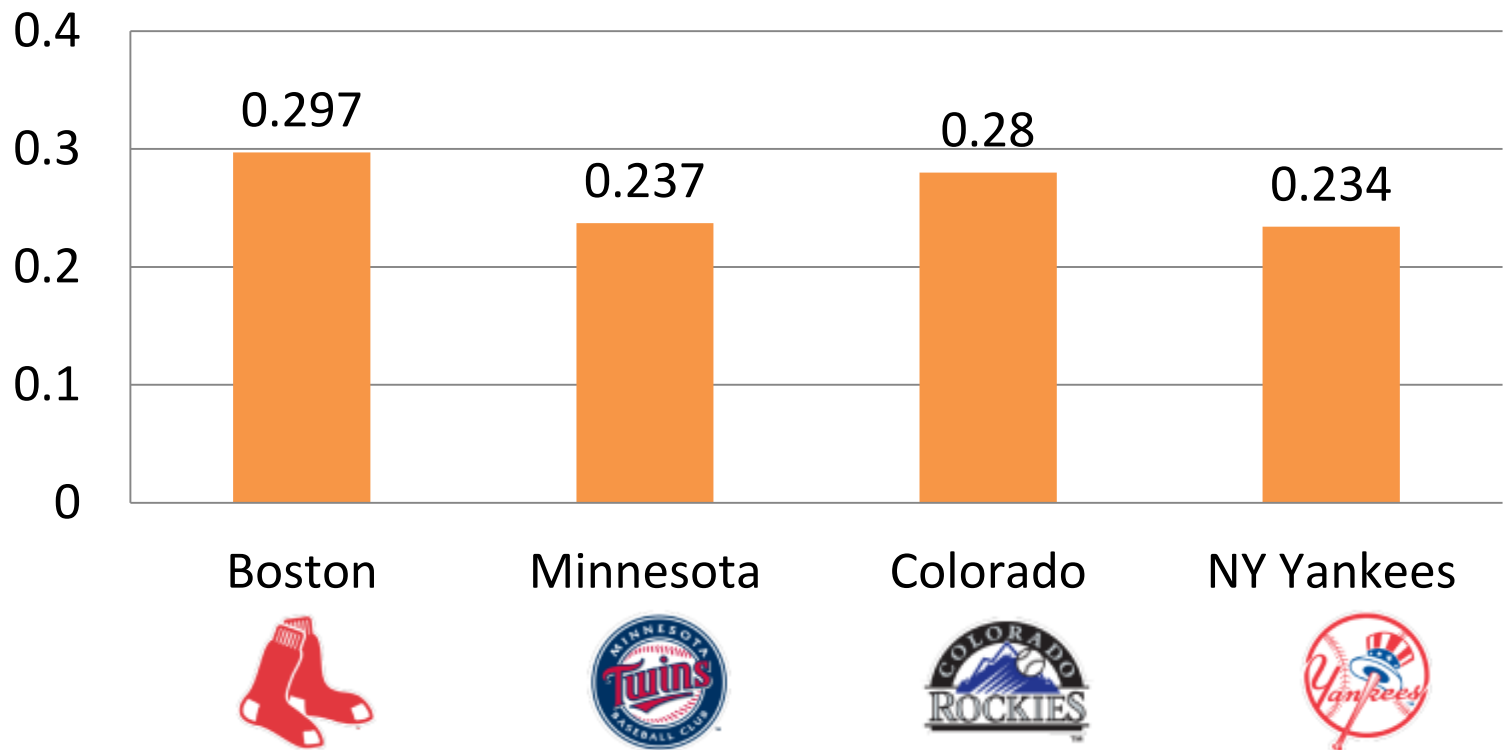
The grand mean of these four teams is 0.262

One-way ANOVA

- A mix of interval and nominal variables
- To test hypotheses about the **mean** on one variable for three or more groups
- The hypotheses:
 - Null hypothesis (H_0): all teams have the same mean
 - Research hypothesis (H_1): at least one team has a different mean

Batting average

Team Batting Average- 2016 May



If you want to infer that the population mean differ at 5 % significance level

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

H_1 : At least two means differ

The real data looks like

x_{ij} refers to the i^{th} observation in the j^{th} sample
Ex : $X_{11} = 0.284$; $X_{12} = 0.221$

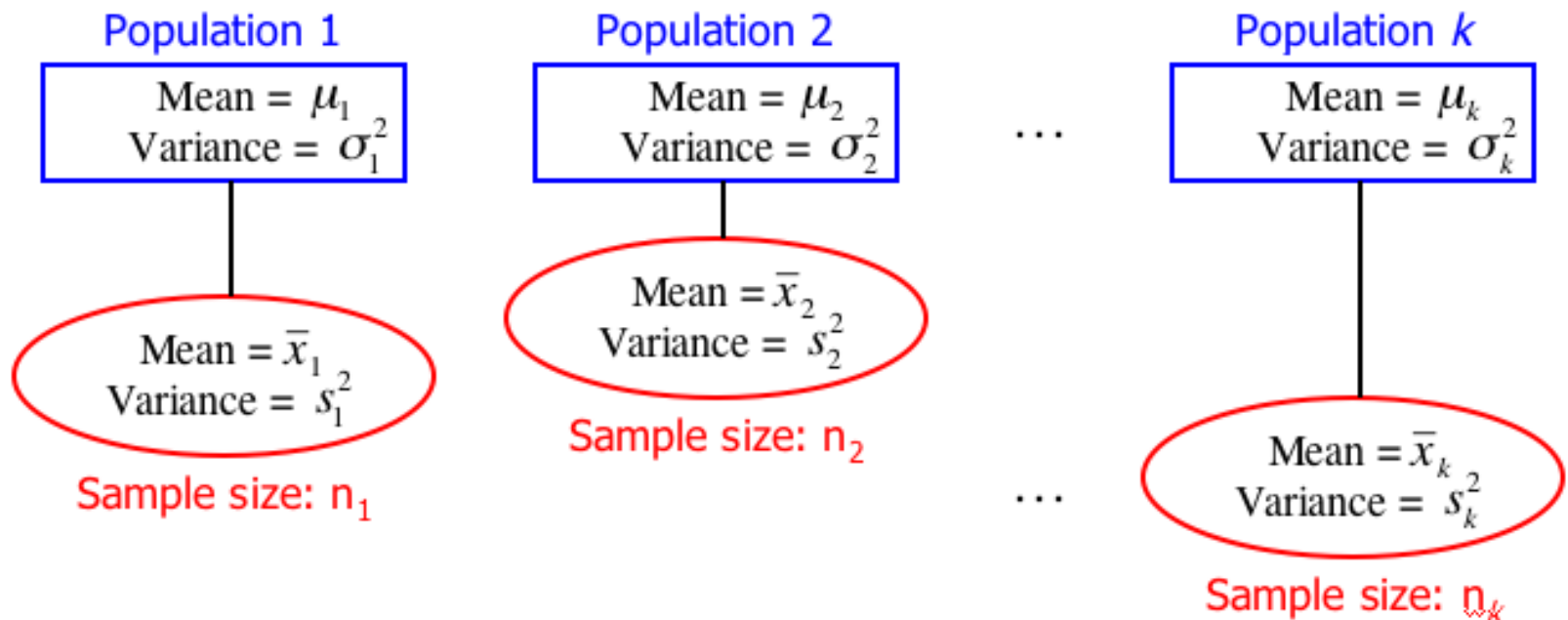
These populations are referred to as treatments

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

x is the response variable

Analysis of variance

- Analysis of variance (ANOVA) is a technique that allows us to compare more than two populations (4 baseball teams) of interval data (batting average).



ANOVA – Between treatments variation

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

The **grand mean**, $\bar{\bar{x}}$, is the mean of all the observations, i.e.:

$$\bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n}$$

$$(n = n_1 + n_2 + \dots + n_k)$$

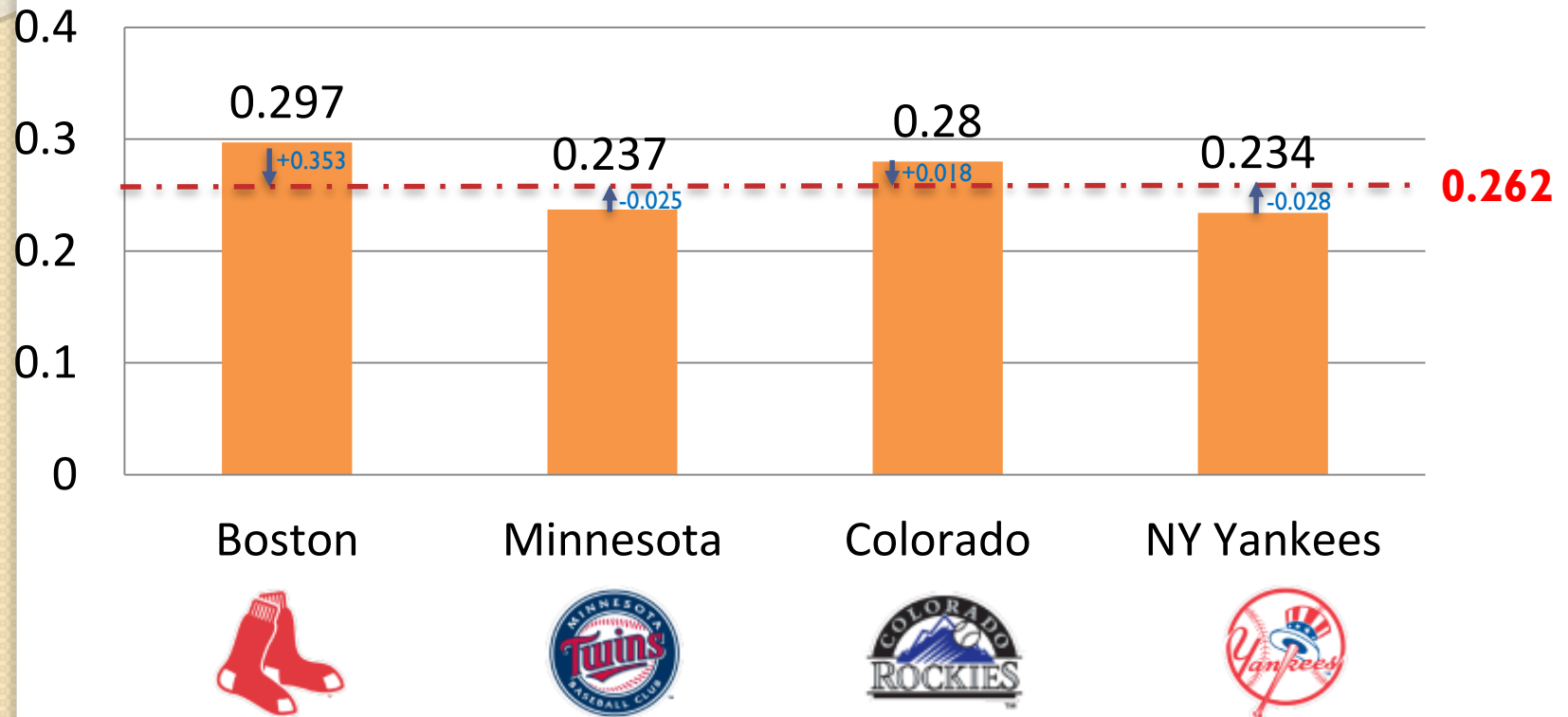
Larger or small ?

0.262

What do you think if $\mu_1 = \mu_2 = \mu_3 = \mu_4$

Batting average problem- Eyeball test?

Team Batting Average- 2016



ANOVA – Between treatments

- Between-treatments variation (Sum of squares for treatments)

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234



0.262

ANOVA – Between treatments variation

- Between-treatments variation (Sum of squares for treatments)

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$


SST

$$= 3(0.297 - 0.262)^2 + 3(0.237 - 0.262)^2 + 3(0.280 - 0.262)^2 + 3(0.234 - 0.262)^2 = 0.0089$$

- The mean square for treatments (MST)

$$MST = \frac{SST}{k-1}$$

$$MST = \frac{0.0089}{3} = 0.00297$$



What do you think if
 $\mu_1 = \mu_2 = \mu_3 = \mu_4$

ANOVA – Between treatments

What do you think if $\mu_1 = \mu_2 = \mu_3 = \mu_4$

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.281	0.280	0.287	0.289
Player 2	0.280	0.278	0.277	0.280
Player 3	0.279	0.282	0.276	0.271
BA	0.280	0.280	0.280	0.280

The **grand mean**, $\bar{\bar{x}}$ is the mean of all the observations, i.e.:

$$\bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n}$$

$$(n = n_1 + n_2 + \dots + n_k)$$

Larger or small ?

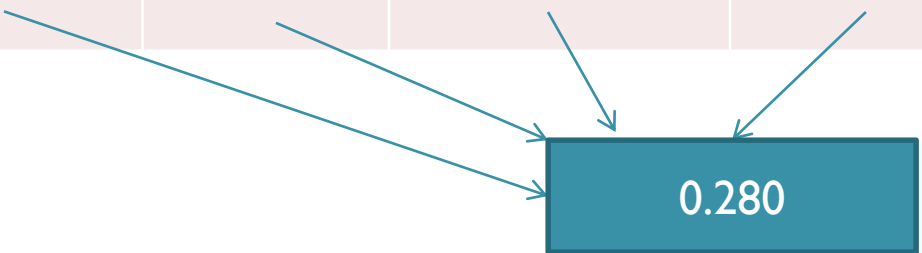
0.280

ANOVA – Between treatments

- Between-treatments variation (Sum of squares for treatments)

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.281	0.280	0.287	0.289
Player 2	0.280	0.278	0.277	0.280
Player 3	0.279	0.282	0.276	0.271
BA	0.280	0.280	0.280	0.280



0.280

ANOVA – Within treatments variation

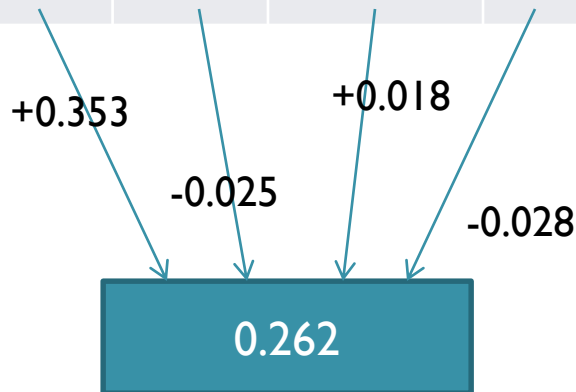
	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

SST tells us the between-treatments variation

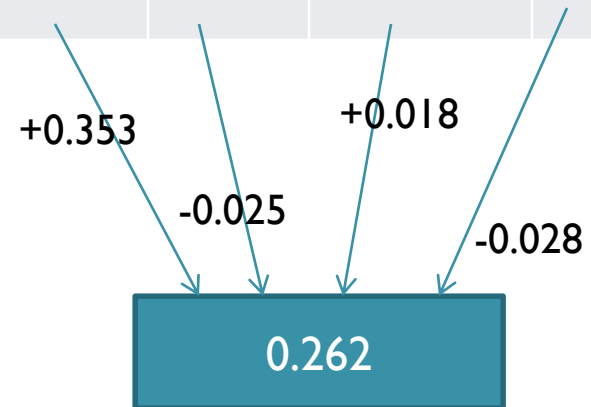
We need to know the within-treatments variation that is not caused by the treatments

ANOVA – Within treatments

	Boston	Minnesota	Colorado	NY Yankees
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234



	Boston	Minnesota	Colorado	NY Yankees
Player 1	0.335	0.221	0.287	0.112
Player 2	0.189	0.159	0.199	0.288
Player 3	0.367	0.331	0.354	0.302
BA	0.297	0.237	0.280	0.234



In which case, you will have more confidence to conclude at least two means differ?

ANOVA – Within treatments

- Within-treatments variation (Sum of Squares for Error)

$$SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$$

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

ANOVA – Within treatments variation

- Within-treatments variation (Sum of Squares for Error)

$$SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$$

SSE

$$\begin{aligned} &= (0.284 - 0.297)^2 + (0.331 - 0.297)^2 + 3(0.276 - 0.297)^2 + (0.221 - 0.237)^2 \\ &+ (0.234 - 0.237)^2 + (0.256 - 0.237)^2 + (0.287 - 0.280)^2 + (0.277 - 0.280)^2 \\ &+ (0.276 - 0.280)^2 + (0.221 - 0.234)^2 + (0.256 - 0.234)^2 + (0.225 - 0.234)^2 \\ &= 0.0032 \end{aligned}$$

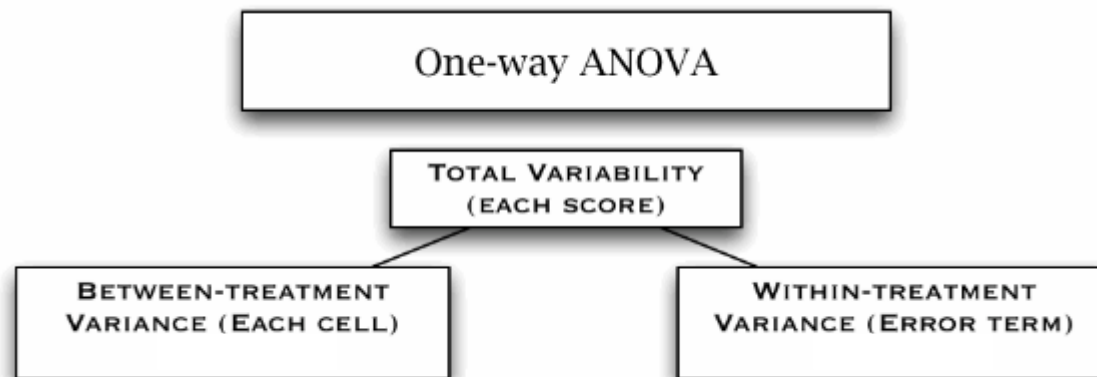
- The mean square for errors (MSE)

$$MSE = \frac{SSE}{n - k}$$

- $MSE = \frac{0.0032}{12-4} = 0.0004$

ANOVA

- We compare MST and MSE using an **F-test** and conclusions are drawn using the value of F.



$$F = \frac{MST}{MSE}$$

F-distributed with k-1 and n-k degrees of freedom.

Remember?

- When SST will equal to 0 ?
- **If SST = 0**
- Our decision : non-reject H0
 - H0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
 - H1: At least two means differ

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.281	0.280	0.287	0.289
Player 2	0.280	0.278	0.277	0.280
Player 3	0.279	0.282	0.276	0.271
BA	0.280	0.280	0.280	0.280

Hypothesis testing

- Since:

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$

- If $SST > 0$
- Our decision : **may** or **may not** reject H_0
 - $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
 - H_1 : At least two means differ
- A large value of SST rejects the H_0 .
 - But, **how large** is “large enough” to reject the H_0 ?

ANOVA table

Source of Variation	Sum of Squares	degrees of freedom	Mean Square	F
Between	SST	$k-1$	$MST = SST/(k-1)$	MST/MSE
Within	SSE	$n-k$	$MSE = SSE/(n-k)$	
Total	SS(Total)	$n-1$		

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

Anova: Single Factor

SUMMARY

<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
Boston	3	0.891	0.297	0.000883
Minnesota	3	0.711	0.237	0.000313
Colorado	3	0.84	0.28	0.00004
NY Yankees	3	0.702	0.234	0.000367

ANOVA

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	0.008874	3	0.002958	7.395	0.010774	4.066181
Within Groups	0.0032	8	0.0004			
Total	0.012074	11				

R codes

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

- `b<-c(0.284,0.331,0.276)`
- `m<-c(0.221,0.234,0.256)`
- `c<-c(0.287,0.277,0.276)`
- `ny<-c(0.221,0.256,0.225)`
- `mydata<-data.frame(b,m,c,ny)`
- `mydata2 <- stack(mydata)`
- `mydata2`
- `aov1<- aov(values ~ ind, data=mydata2)`
- `summary(aov1)`

R codes

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

```
> mydata2
  values ind
1  0.284  b
2  0.331  b
3  0.276  b
4  0.221  m
5  0.234  m
6  0.256  m
7  0.287  c
8  0.277  c
9  0.276  c
10 0.221 ny
11 0.256 ny
12 0.225 ny
```

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1 : At least two means differ

```
> aov1<- aov(values ~ ind, data=mydata2)
> summary(aov1)
            Df    Sum Sq  Mean Sq F value Pr(>F)
ind           3  0.008874  0.002958   7.395 0.0108 *
Residuals     8  0.003200  0.000400
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

	Boston	Minnesota	Colorado	NY Yankees
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

	Boston	Minnesota	Colorado	NY Yankees
Player 1	0.335	0.221	0.287	0.112
Player 2	0.189	0.159	0.199	0.288
Player 3	0.367	0.331	0.354	0.302
BA	0.297	0.237	0.280	0.234

Anova: Single Factor						
Consistent variance group						
SUMMARY						
Groups	Count	Sum	Average	Variance		
Boston	3	0.891	0.297	0.000883		
Minnesota	3	0.711	0.237	0.000313		
Colorado	3	0.84	0.28	0.00004		
NY Yankees	3	0.702	0.234	0.000367		
ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.008874	3	0.002958	7.395	0.010774	4.066181
Within Groups	0.0032	8	0.0004			
Total	0.012074	11				

Anova: Single Factor						
Inconsistent variance group						
SUMMARY						
Groups	Count	Sum	Average	Variance		
Boston	3	0.891	0.297	0.009004		
Minnesota	3	0.711	0.237	0.007588		
Colorado	3	0.84	0.28	0.006043		
NY Yankees	3	0.702	0.234	0.011212		
ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.008874	3	0.002958	0.349573	0.790752	4.066181
Within Groups	0.067694	8	0.008462			
Total	0.076568	11				

Post hoc test – Multiple comparison

- ANOVA tells you whether you have an overall difference between your treatments (four teams), but it does not tell you which specific treatments differed .
- We have multiple comparison method

Post hoc test – Multiple comparison

- `b<-c(0.284,0.331,0.276)`
- `m<-c(0.221,0.234,0.256)`
- `c<-c(0.287,0.277,0.276)`
- `ny<-c(0.221,0.256,0.225)`
- `mydata<-data.frame(b,m,c,ny)`
- `mydata2 <- stack(mydata)`
- `mydata2`
- `aov1 <- aov(values ~ ind, data=mydata2)`
- `summary(aov1)`
- `pairwise.t.test(mydata2$values,mydata2$ind)`

Post hoc test – Multiple comparison

- We are looking for a **critical number** to compare the **absolute differences** of the sample means against.

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

Post hoc test – Multiple comparison

	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player 1	0.284	0.221	0.287	0.221
Player 2	0.331	0.234	0.277	0.256
Player 3	0.276	0.256	0.276	0.225
BA	0.297	0.237	0.280	0.234

```
> pairwise.t.test(mydata2$values,mydata2$ind)
```

Pairwise comparisons using t tests with pooled SD

data: mydata2\$values and mydata2\$ind

	b	m	c
m	0.031	-	-
c	0.657	0.090	-
ny	0.029	0.859	0.090

P value adjustment method: holm

Problem

- How does an MBA major affect the number of job offers received? An MBA student randomly sampled four recent graduates, one each in finance, marketing, and management, and asked them to report the number of job offers.
- Can we conclude at the 5% significance level that there are differences in the number of job offers between the three MBA majors

Finance	Marketing	Management
3	1	8
1	5	5
4	3	4
1	4	6

R codes

- `fin<-c(3,1,4,1)`
- `market<-c(1,5,3,4)`
- `manage<-c(8,5,4,6)`
- `mydata<-data.frame(fin,market,manage)`
- `mydata2 <- stack(mydata)`
- `mydata2`
- `aov1 <- aov(values ~ ind, data=mydata2)`
- `summary(aov1)`
- `pairwise.t.test(mydata2$values,mydata2$ind)`



Happy Hour

A large, stylized clock face is positioned on the right side of the image. It has a black outline and a light blue fill. The clock face is empty, with only the hour markers visible as small black dashes. The words "Happy Hour" are written in a black, cursive script font with a yellow outline, positioned across the center of the clock face.

Exercise I

- A research study was conducted to examine the clinical efficacy of a new antidepressant. Depressed patients were randomly assigned to one of three groups: a placebo group, a group that received a low dose of the drug, and a group that received a moderate dose of the drug.
- After four weeks of treatment, the patients completed the Depression Inventory Test. The higher the score, the **more** depressed the patient. The data are presented below. Compute the appropriate test by using 5% sig. level. What is your finding?

<u>Placebo</u>	<u>Low Dose</u>	<u>Moderate Dose</u>
38	22	14
47	19	26
39	8	11
25	23	18
42	31	5



Exercise 2

- A researcher is concerned about the level of knowledge possessed by university students regarding math.
- Students completed a high school senior level standardized math exam. Major for students was also recorded. Data in terms of percent correct is recorded below for 12 students.
- Compute the appropriate test by using 5% sig. level. What is your finding?

<u>Education</u>	<u>Business</u>	<u>Social Science</u>	<u>Fine Arts</u>
62	72	42	80
81	49	52	57
75	63	31	87



Math

Exercise 3

- Neuroscience researchers examined the impact of environment on rat development. Rats were randomly assigned to be raised in one of the four following test conditions: loser (wire mesh cage - housed alone), standard (cage with other rats), enriched (cage with other rats and toys), super enriched (cage with rats and toys changes on a periodic basis).
- After two months, the rats were tested on a variety of learning a maze. Their time to complete the maze task is below. Compute the appropriate test by using 5% sig. level. What is your finding?

<u>Loser</u>	<u>Standard</u>	<u>Enriched</u>	<u>Super Enriched</u>
22	17	12	8
19	21	14	7
15	15	11	10
24	12	9	9
18	19	15	12



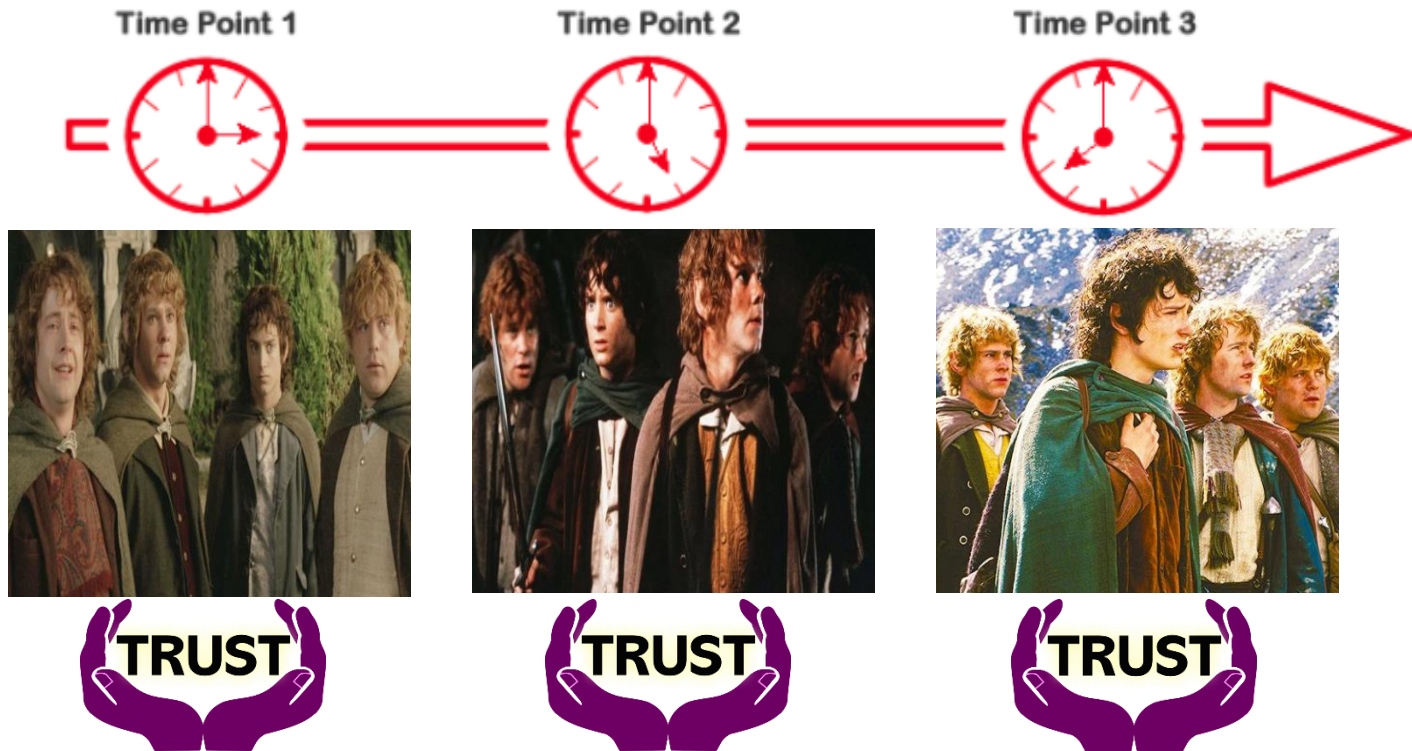
Topics

- One-Way ANOVA without repeated measures
 - One-way ANOVA
- One-Way ANOVA with repeated measures
 - Randomized complete block ANOVA

Repeated measures ANOVA

- Repeated measures ANOVA is the equivalent of the one-way ANOVA, but for related, not independent groups
- The nature of the repeated measures ANOVA, that of a test to detect any overall differences between related means
- Studies that investigate either
 - changes in mean scores over three or more time points
 - differences in mean scores under three or more different conditions

Repeated measures ANOVA



In repeated measures ANOVA, the independent variable has categories called **related groups**.

Where measurements are repeated over time in this case, such as when measuring changes in team trust due to an adventure journey, the independent variable is **time**.

Repeated measures ANOVA

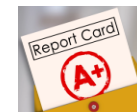
Condition1



Condition2



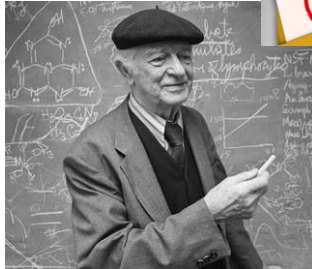
Condition3



In repeated measures ANOVA, the independent variable has categories called **related groups**.

Where measurements are repeated under different conditions, such as when measuring changes in performance due to class room, the independent variable is **conditions**.

Repeated measures ANOVA



Batting average



FS1 ALDS
GAME 1

NEW YORK

24 C GARY SÁNCHEZ REGULAR SEASON

.278 AVG | **33** HR | **90** RBI | **.876** OPS

YANKEE RECORD BY CATCHER

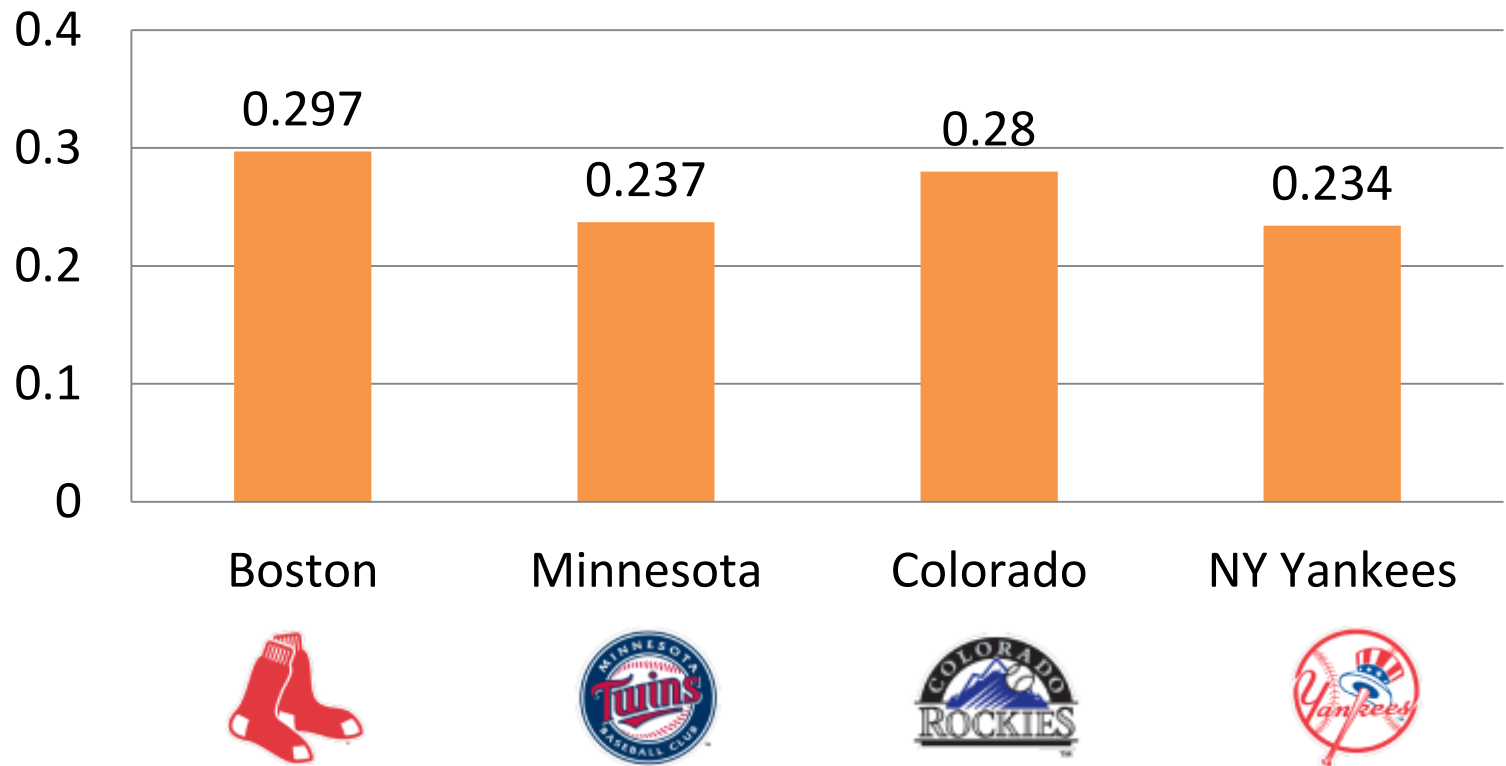
Bauer P: 12
Sánchez .278

NY 0
CLE 0

▲ 1 2 Outs 0-1

Batting average

Team Batting Average- 2016



If you want to infer that the population means differ

This time, the real data looks like



	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225

Repeated measures ANOVA



	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>	<u>Subjects</u>
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221	
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256	
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225	
BA	0.297	0.237	0.280	0.234	0.262

SST

$$\begin{aligned}
 &= 3(0.297 - 0.262)^2 + 3(0.237 - 0.262)^2 \\
 &+ 3(0.280 - 0.262)^2 + 3(0.234 - 0.262)^2 \\
 &= 0.0089
 \end{aligned}$$

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2$$

Repeated measures ANOVA







	<u>Boston</u>	<u>Minnesota</u>	<u>Colorado</u>	<u>NY Yankees</u>	<u>Subjects</u>
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221	
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256	
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225	
BA	0.297	0.237	0.280	0.234	

SSE

$$\begin{aligned}
 &= (0.284 - 0.297)^2 + (0.331 - 0.297)^2 + (0.276 - 0.297)^2 \\
 &+ (0.221 - 0.237)^2 + (0.234 - 0.237)^2 + (0.256 - 0.237)^2 \\
 &+ (0.287 - 0.280)^2 + (0.277 - 0.280)^2 + (0.276 - 0.280)^2 \\
 &+ (0.221 - 0.234)^2 + (0.256 - 0.234)^2 + (0.225 - 0.234)^2 = 0.0032
 \end{aligned}$$

$$SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$$

Repeated measures ANOVA

	 Boston	 Minnesota	 Colorado	 NY Yankees	Subjects
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221	0.253
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256	0.275
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225	0.258
BA	0.297	0.237	0.280	0.234	0.262

Player B is excellent

$$SS_{\text{subjects}} = k \cdot \sum (\bar{x}_i - \bar{\bar{x}})^2$$

$$SSB = 4(0.253 - 0.262)^2 + 4(0.275 - 0.262)^2 + 4(0.258 - 0.262)^2 = 0.001$$

	Boston	Minnesota	Colorado	NY Yankees	Subjects
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221	
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256	
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225	
BA	0.297	0.237	0.280	0.234	

$$SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2$$

Real within-treatments variation

$$= SSE - SS_{\text{subject}} = 0.0032 - 0.001 = 0.0022$$

	Boston	Minnesota	Colorado	NY Yankees	Subjects
Player A	Player A 0.284	Player A 0.221	Player A 0.287	Player A 0.221	0.253
Player B	Player B 0.331	Player B 0.234	Player B 0.277	Player B 0.256	0.275
Player C	Player C 0.276	Player C 0.256	Player C 0.276	Player C 0.225	0.258
BA	0.297	0.237	0.280	0.234	0.262

$$SS_{\text{subjects}} = k \cdot \sum (\bar{x}_i - \bar{x})^2$$

R codes

- `b<-c(0.284,0.331,0.276)`
- `m<-c(0.221,0.234,0.256)`
- `c<-c(0.287,0.277,0.276)`
- `ny<-c(0.221,0.256,0.225)`
- `mydata<-data.frame(b,m,c,ny)`
- `mydata2<- stack(mydata)`
- `id<-factor(rep(1:3, times=4))`
- `mydata2<-data.frame(mydata2,id)`
- `mydata2`
- `aov2<- aov(values ~ ind + Error(id/ind), data=mydata2)`
- `summary(aov2)`
- `pairwise.t.test(mydata2$values,mydata2$ind)`

R codes

```
> b<-c(0.284,0.331,0.276)
> m<-c(0.221,0.234,0.256)
> c<-c(0.287,0.277,0.276)
> ny<-c(0.221,0.256,0.225)
> mydata<-data.frame(b,m,c,ny)
> mydata2<- stack(mydata)
> id<-factor(rep(1:3, times=4))
> mydata2<-data.frame(mydata2,id)
> mydata2
```

	values	ind	id
1	0.284	b	1
2	0.331	b	2
3	0.276	b	3
4	0.221	m	1
5	0.234	m	2
6	0.256	m	3
7	0.287	c	1
8	0.277	c	2
9	0.276	c	3
10	0.221	ny	1
11	0.256	ny	2
12	0.225	ny	3

```
> aov2<- aov(values ~ ind + Error(id/ind), data=mydata2)
> summary(aov2)
```

Error: id

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Residuals	2	0.0009875	0.0004937		

Error: id:ind

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
ind	3	0.008874	0.0029580	8.022	0.016 *
Residuals	6	0.002212	0.0003687		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R codes

```
> pairwise.t.test(mydata2$values,mydata2$ind)
```

Pairwise comparisons using t tests with pooled SD

data: mydata2\$values and mydata2\$ind

	b	m	c
m	0.031	-	-
c	0.657	0.090	-
ny	0.029	0.859	0.090

What is the difference ?

One-way ANOVA **without** Repeated measures

```
> aov1<- aov(values ~ ind, data=mydata2)
> summary(aov1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
ind	3	0.008874	0.002958	7.395	0.0108 *
Residuals	8	0.003200	0.000400		

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

One-way ANOVA **with** Repeated measures

```
> aov2<- aov(values ~ ind + Error(id/ind), data=mydata2)
> summary(aov2)
```

```
Error: id
          Df    Sum Sq  Mean Sq F value Pr(>F)
Residuals  2 0.0009875 0.0004937
```

```
Error: id:ind
          Df    Sum Sq  Mean Sq F value Pr(>F)
ind         3 0.008874 0.0029580   8.022  0.016 *
Residuals   6 0.002212 0.0003687
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Where are we going ?



Populations
and Samples

Continuous
probability

Business
decisions
with
sampling

Decisions about
variables

- ANOVA
- Regression