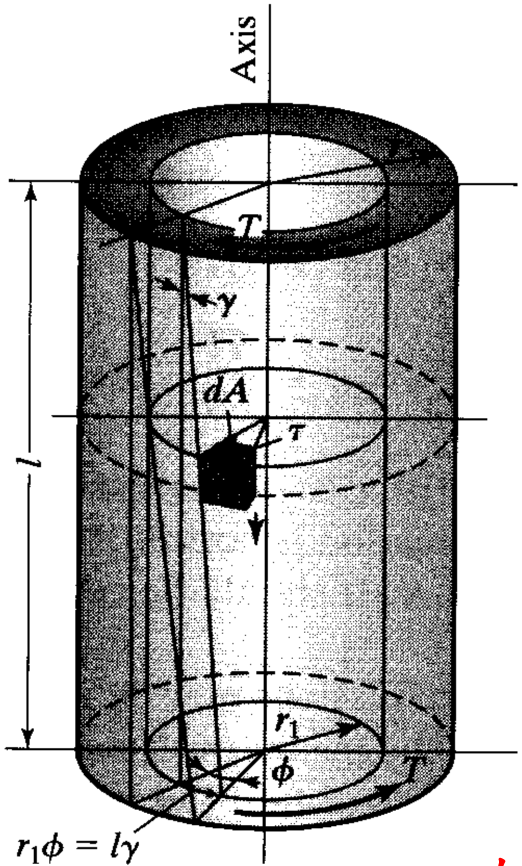
# 3. DESIGN OF SHAFTS

***Most shafts are subjected to fluctuating loads of combined bending and torsion with various degrees of stress concentration.***

- In addition to shaft itself, the design usually must include keys and couplings (連軸器).
- Determine the stress and deformation of noncircular cross sections shafts

# 3-1 TORSION OF CIRCULAR SHAFT




***The element is stressed in pure shear***

$$r_1\varphi = l\gamma \qquad \gamma = \frac{\tau}{G}$$

$$\tau = \frac{\varphi G r_1}{l}$$

在圓截面上：$\dfrac{\tau}{r_1} = \dfrac{\varphi G}{l} = \text{constant}$

**Balance of torque**

$$T = \int_0^r \tau r_1 dA = \int_0^r \frac{\tau}{r_1} r_1^2 dA = \frac{\tau}{r_1}\int_0^r r_1^2 dA = \frac{\tau}{r_1} J$$

***J*** **is called polar moment of inertia**

**At the outer surface $r_1=r$**

$$\boxed{\tau = \frac{Tr}{J}}$$

**Solid circular cross section**

$$J = \frac{\pi d^4}{32} = \frac{\pi r^4}{2}$$

**Hollow shaft**

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)$$

**Torsion angle**

$$\boxed{T = \frac{\tau}{r_1} J = \left( \frac{\varphi G}{l} \right) J \quad \Rightarrow \quad \varphi = \frac{Tl}{GJ}}$$

# EXAMPLE 3-1

**Problem Statement:**

The shaft in Fig. 3-2 does not rotate. Loads are steady. Assume simple supports. Diameter is 50 mm. Length is 180 mm. Elements are located similarly to those of the figure. Load at center is 9000 N. Torques at ends are equal to 1,000,000 Nmm each. Answer the same questions as for Example 1.

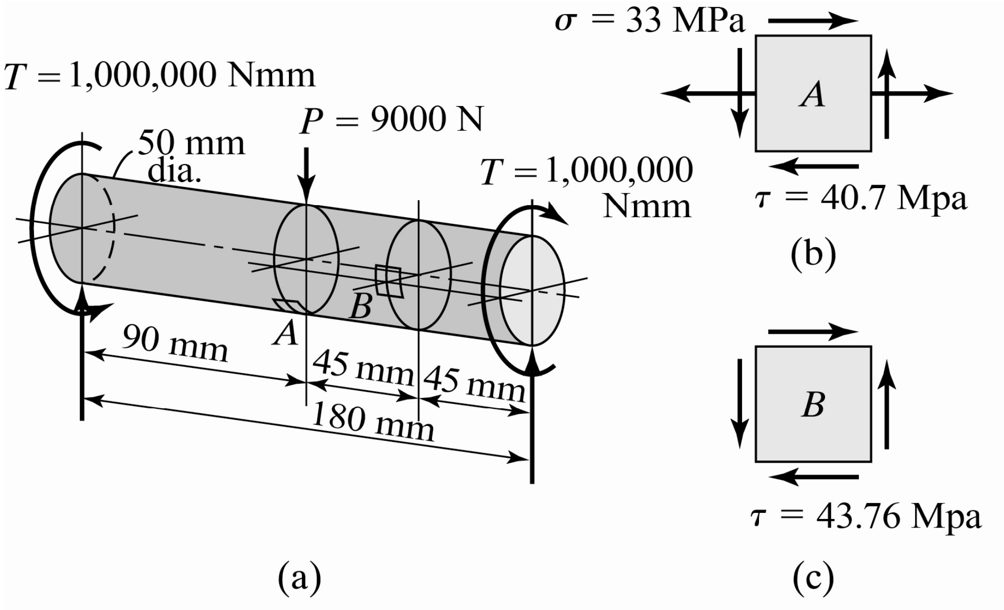



**Figure 3-2** Example 3-1.

$$R = 4500 \quad \text{N}$$

$$M_A = 4500 \times 90 = 405000 \quad \text{Nmm}$$
$$M_B = 4500 \times 45 = 202500 \quad \text{Nmm}$$

$$V_B = 4500 \quad \text{N}$$

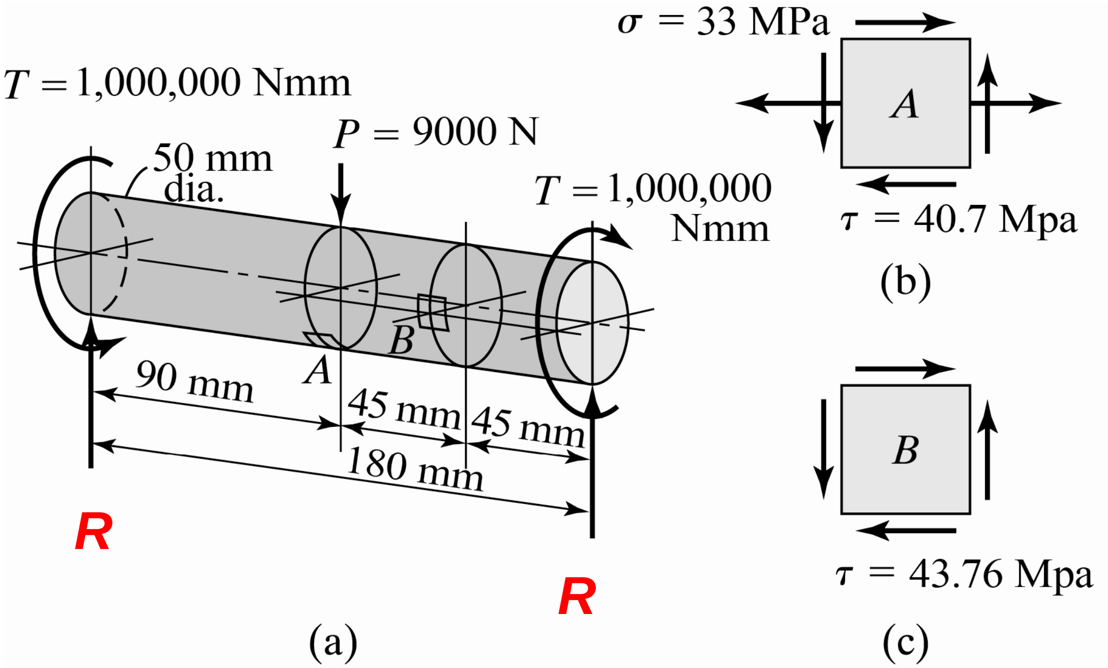



**Figure 3-2** Example 3-1.

**Element A**

$$\sigma = \frac{32M}{\pi d^3} = \frac{32 \times 405000}{\pi \left(50\right)^3} = 33 \quad \text{MPa}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 1,000,000}{\pi \left(50\right)^3} = 40.7 \quad \text{MPa}$$

**Element B**

$$\sigma = 0 \quad \text{psi}$$

$$\tau = \frac{4V}{3A} + \frac{16T}{\pi d^3} = \frac{4 \times 4500}{3 \dfrac{\pi \times 50^2}{4}} + 40.7 = 43.76 \quad \text{MPa}$$

# EXAMPLE 3-2

**Problem Statement:** A hollow shaft must carry a torque of 30,000 in. lb at a shearing stress of 8000 psi. The inside diameter is to be 0.65 of the outside diameter. Find the value of the outside diameter.

$$d_i = 0.65 d_o$$

$$J = \frac{\pi}{32}\left(d_o^4 - \left(0.65 d_o\right)^4\right) = 0.0865 d_o^4$$

$$J = \frac{Tr}{\tau} = \frac{30000 \times 0.5 d_o}{8000} = 0.0865 d_o^4 \quad \Rightarrow \quad d_o = 2.854 \quad \text{in.}$$

# EXAMPLE 3-3

**Problem Statement:**

Suppose it is specified that the angular deformation in a shaft should not exceed l$^\circ$ in a length of 1800 mm. The permissible shearing stress is 83 MPa. Find the diameter of the shaft. The material has a shear modulus of 77,000 MPa.

$$G = 77,000 \quad \text{MPa}$$

$$\varphi = 1^\circ = \frac{\pi}{180}\ \text{rad} = 0.01745\ \text{rad}$$

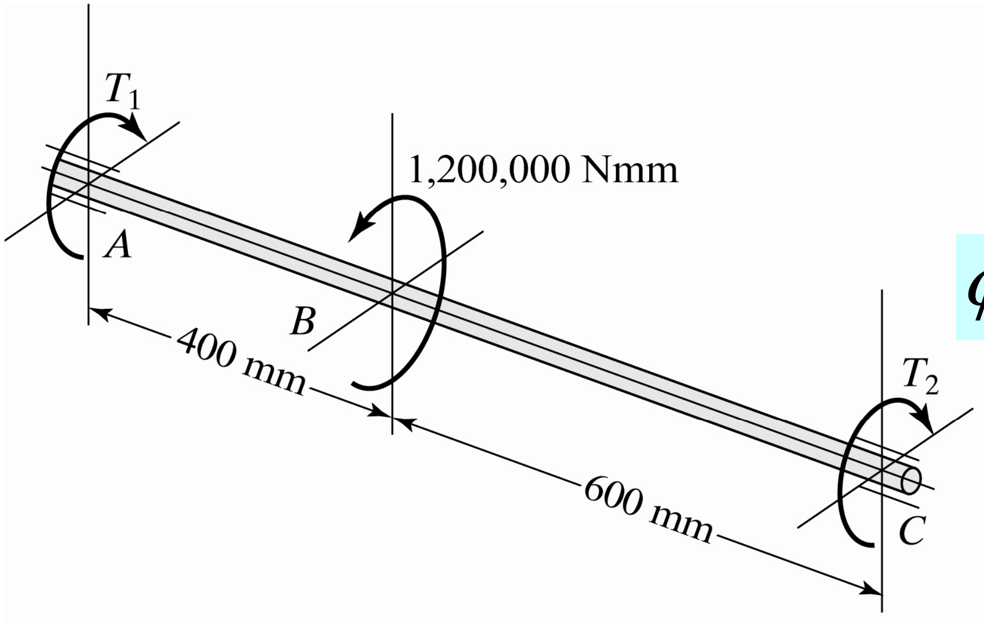
$$T = \frac{\tau J}{r} = \frac{\varphi G}{l} J \quad \Rightarrow \quad r = \frac{\tau l}{\varphi G} = \frac{83 \times 1800}{0.01745 \times 77,000} = 111.2\ \text{mm}$$

$$d = 222.4\ \text{mm}$$

# EXAMPLE 3-4

**Problem Statement:**

The shaft in Fig. 3-3 carries the torque of 1,200,000 Nmm at the location shown. If the ends of the shaft are fixed against rotation, find the values of the torque reactions $T_1$, and $T_2$.




Figur

$$\varphi_1 = \frac{400T_1}{GJ}, \quad \varphi_2 = \frac{600T_2}{GJ}$$

$$\varphi_1 = \varphi_2 \quad \Rightarrow \quad 400T_1 - 600T_2 = 0$$

$$T_1 + T_2 = 1,200,000$$

$$T_1 = 720,000 \text{ Nmm}$$
$$T_2 = 480,000 \text{ Nmm}$$

# 3-2 POWER TRANSMITTED

**For rotating systems**

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$1\text{hp} = 33000\,\frac{\text{ft.lb}}{\text{min}}, \quad \text{hp} = \frac{FV}{33000}$$

# EXAMPLE 3-5

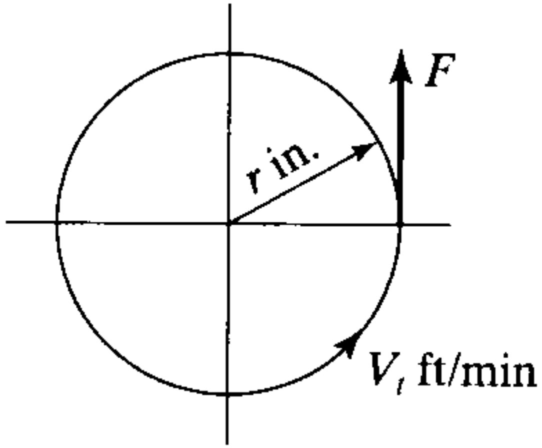
**Problem Statement:** If a draft horse walks at the rate of 3 miles per hour, what uniform force must he exert if the output power is exactly 1 horsepower?

$$V = \frac{3 \times 5280}{60} = 264 \ \text{ft/min}$$

$$F = \frac{33000\text{hp}}{V} = \frac{33000 \times 1}{264} = 125 \ \text{lb}$$

$$F = \frac{T}{r} \quad \text{lb}, \quad V = \frac{2\pi r n}{12} \quad \text{ft/min}$$

$$\boxed{\text{hp} = \frac{\frac{T}{r} \times \frac{2\pi r n}{12}}{33000} = \frac{Tn}{63025} \approx \frac{Tn}{63000}}$$




$$\omega = \frac{2n\pi}{60} \quad \Rightarrow \quad n = \frac{60\omega}{2\pi}$$

$$\boxed{\text{hp} = \frac{T\omega}{6600}}$$

# EXAMPLE 3-6

**Problem Statement:** A shaft carries a torque of 10,000 in. lb and turns 900 rpm. Find the horsepower transmitted.

$$\text{hp} = \frac{10000 \times 900}{63025} = 142.8$$

**SI unit**

Force in Newton

$$W = FV \quad (\text{watt})$$

$$\text{kW} = \frac{NV}{1000}$$

N.mm

rpm

$$F = \frac{T}{r}, \quad V = \frac{2\pi rn}{60 \times 1000}$$

mm

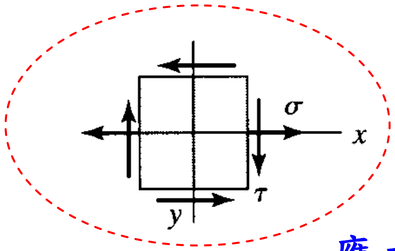
m/sec

$$\text{kW} = \frac{1}{1000}\frac{T}{r} \times \frac{2\pi rn}{60 \times 1000} = \frac{Tn}{9550000}$$

$$1 \text{ hp} = 745.7 \text{ watt} = 0.7457 \text{ kW}$$

# 3-3 MAXIMUM STATIC SRHEARING STRESS

***When shafts carry combined loads of bending and torque, the bending moment M causes a normal stress in the axial direction, and the torque T produces the shear stress***

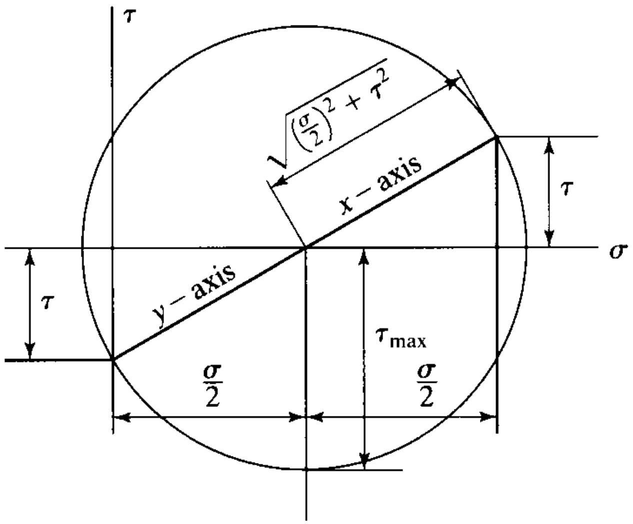


(a)

應力狀態

$$\tau_{\max} = \frac{0.5\sigma_{yp}}{N_{fs}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma = \frac{32M}{\pi d^3}, \quad \tau = \frac{16T}{\pi d^3}$$



(b)

$$\tau_{\max} = \frac{0.5\sigma_{yp}}{N_{fs}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{32M}{2\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\frac{0.5\sigma_{yp}}{N_{fs}} \geq \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

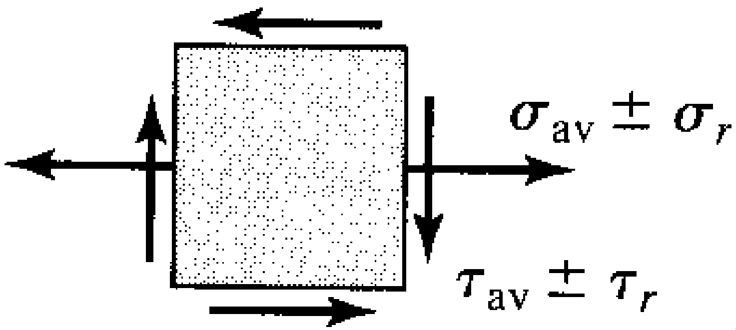
以最大剪應力破壞理論評估

**Max. shear stress theory, why?**

# 3-4 DESIGN OF SHAFTS FOR FLUCTUATING LOADS

***Consider the most general state of stress associated with combined reversed bending and reversed torsion:***

利用 **Soderberg** 理論定義等效應力:

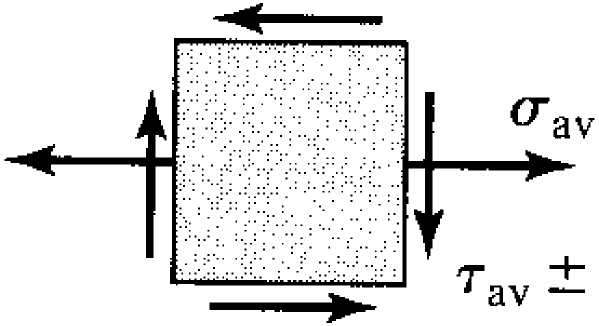



$$\sigma_x = \sigma_{avg} + \sigma_r K_{fb}\left(\frac{S_{yp}}{S_e}\right)$$

$$\tau_{xy} = \tau_{avg} + \tau_r K_{ft}\left(\frac{S_{yp}}{S_e}\right)$$

用來取代動態應力的等效應力

分析原理：疲勞理論 ＋ 破壞理論




$$S_1 = \overbrace{\frac{\sigma_x}{2}}^{a} + \overbrace{\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}}^{b}$$

$$S_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

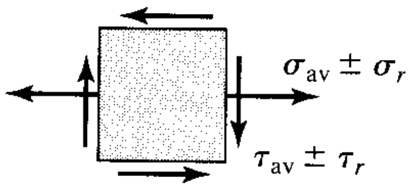
**See page 134** 二維應力狀態時的畸變能破壞準則

$$\begin{aligned}\left(S_1^2 + S_2^2 - S_1 S_2\right) &= (a+b)^2 + (a-b)^2 - (a+b)(a-b) \\ &= a^2 + 3b^2 \\ &= \left(\frac{\sigma_x}{2}\right)^2 + 3\left(\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}\right)^2 \\ &= \sigma_x^2 + 3\tau_{xy}^2\end{aligned}$$

## *von Mises-Hencky Theory (Max. distortion energy theory)*

$$\sigma_x^2 + 3\tau_{xy}^2 \le \left( \frac{S_{yp}}{N_{fs}} \right)^2$$

***Design equation***




***bending*** **+** ***torsion***

$$\boxed{\left( \sigma_{av} + \sigma_r K_{fb} \left( \frac{S_{yp}}{S_e} \right) \right)^2 + 3\left( \tau_{av} + \tau_r K_{ft} \left( \frac{S_{yp}}{S_e} \right) \right)^2 \le \left( \frac{S_{yp}}{N_{fs}} \right)^2}$$

## Case I : Reversed bending with static torque

$$\begin{cases} \sigma_{av} = 0 \\ \tau_r = 0 \end{cases}$$

$$\left( \sigma_r K_{fb} \left( \frac{S_{yp}}{S_e} \right) \right)^2 + 3(\tau_{av})^2 \le \left( \frac{S_{yp}}{N_{fs}} \right)^2$$

Let $\sigma_r K_{fb} = S_b$ and $N_{fs} = 1$

$$\left( S_b \left( \frac{S_{yp}}{S_e} \right) \right)^2 + 3(\tau_{av})^2 \le (S_{yp})^2 \quad \xrightarrow{\div S_{yp}^2} \quad \left( \frac{S_b}{S_e} \right)^2 + 3\left( \frac{\tau_{av}}{S_{yp}} \right)^2 \le 1$$

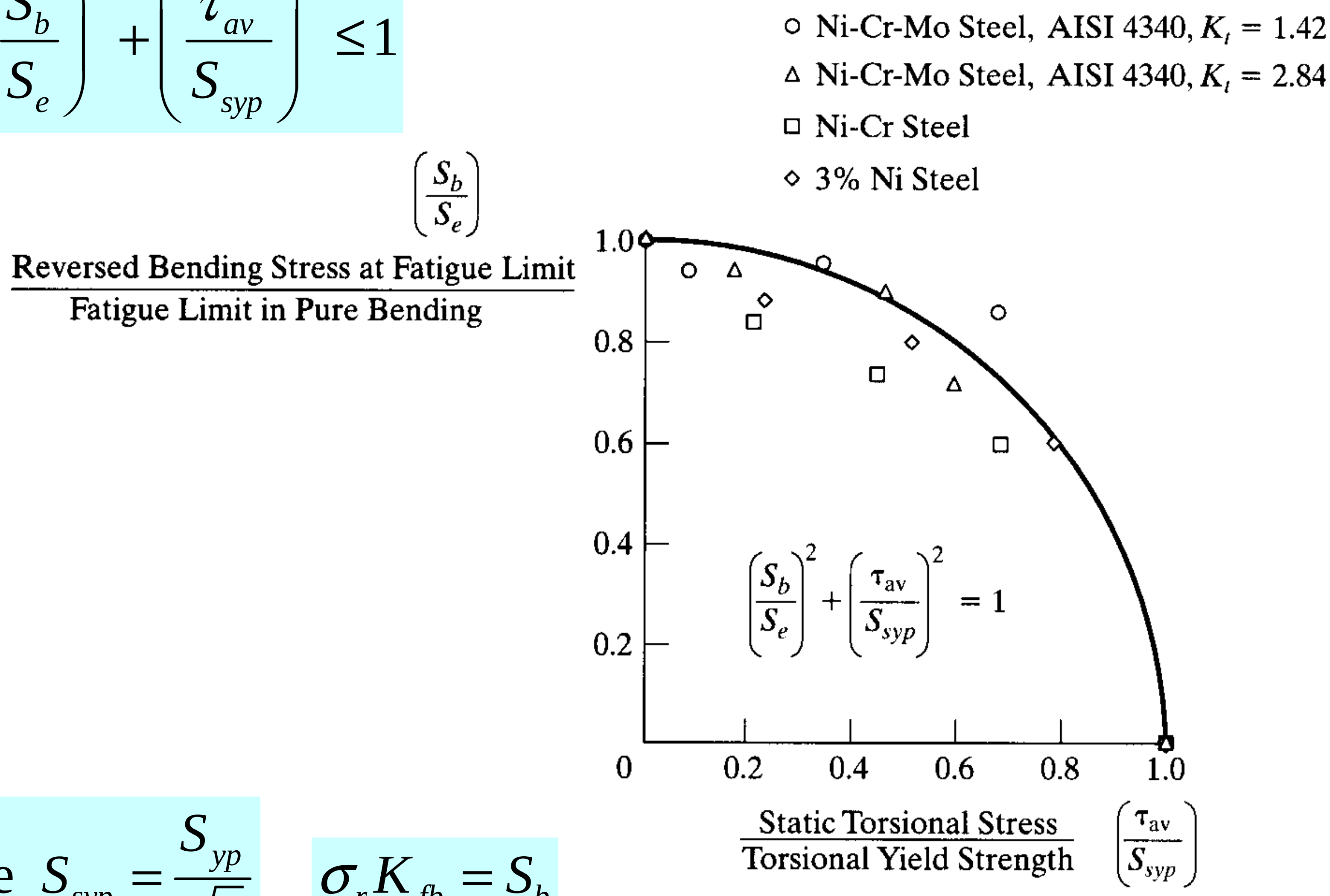
Let $S_{syp} = \dfrac{S_{yp}}{\sqrt{3}}$

$$\left( \frac{S_b}{S_e} \right)^2 + \left( \frac{\tau_{av}}{S_{syp}} \right)^2 \le 1$$

圓方程式

$$\left(\frac{S_b}{S_e}\right)^2+\left(\frac{\tau_{av}}{S_{syp}}\right)^2\leq 1$$

## 實驗測試結果：




where $S_{syp}=\frac{S_{yp}}{\sqrt{3}}$

$$\sigma_r K_{fb}=S_b$$

## *Case II : Reversed bending with reversed torque*

$$\begin{cases} \sigma_{av} = 0 \\ \tau_{av} = 0 \end{cases}$$

$$\left( \sigma_r K_{fb} \left( \frac{S_{yp}}{S_e} \right) \right)^2 + 3 \left( \tau_r K_{ft} \left( \frac{S_{yp}}{S_e} \right) \right)^2 \le \left( \frac{S_{yp}}{N_{fs}} \right)^2$$

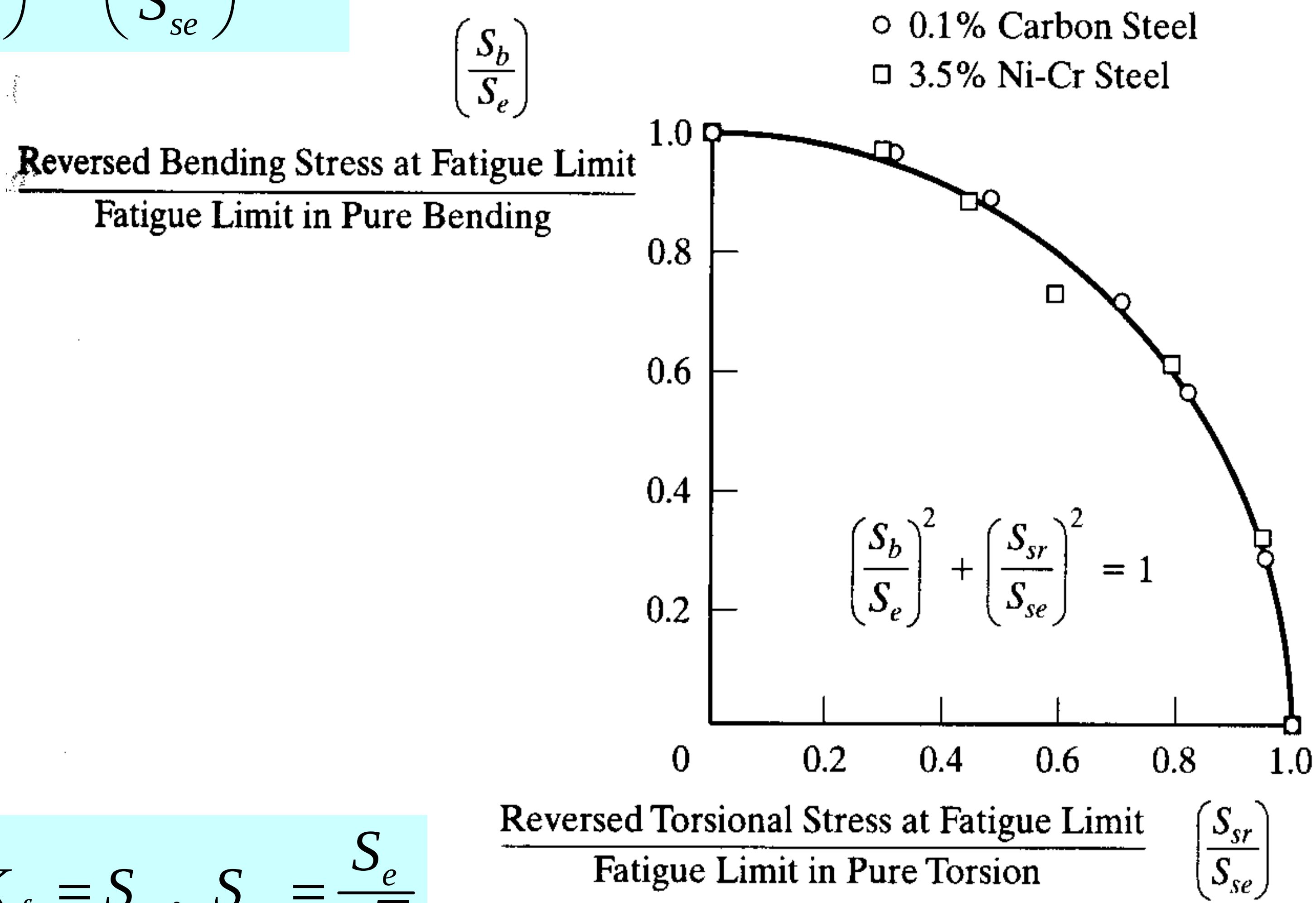
$$\text{Let} \quad \tau_r K_{ft} = S_{sr}, \quad S_{se} = \frac{S_e}{\sqrt{3}} \quad \text{and} \quad N_{fs} = 1$$

$$\Rightarrow \quad \left( \frac{S_b}{S_e} \right)^2 + \left( \frac{S_{sr}}{S_{se}} \right)^2 \le 1$$

圓方程式

$$\left(\frac{S_b}{S_e}\right)^2 + \left(\frac{S_{sr}}{S_{se}}\right)^2 \le 1$$

<u>實驗測試結果：</u>




where $\tau_r K_{ft} = S_{sr}$, $S_{se} = \dfrac{S_e}{\sqrt{3}}$

# EXAMPLE 3-7

**Problem Statement:**

A revolving shaft with machined surface carries a bending moment of 3,000,000 Nmm and a torque of 9,000,000 Nmm with ± 20% fluctuation. The stress concentration factor for bending and torsion is equal to 1.35. The material has a yield strength of 620 MPa, and an endurance limit of 300 MPa. Design a shaft of minimum diameter that will safely handle these loads if the factor of safety is 2.0.

$$S_{yp} = 620 \text{ MPa}$$

$$S_e = 300 \text{ MPa}$$

$$K_{fb} = 1.35$$

$$K_{ft} = 1.35$$

$$N_{fs} = 2.0$$

$$\left(\sigma_{av}+\sigma_r K_{fb}\left(\frac{S_{yp}}{S_e}\right)\right)^2+3\left(\tau_{av}+\tau_r K_{ft}\left(\frac{S_{yp}}{S_e}\right)\right)^2\le\left(\frac{S_{yp}}{N_{fs}}\right)^2$$

$$\sigma_{av}=0$$

$$\sigma_r=\frac{Mc}{I}=\frac{3,000,000\times 79/2}{\pi(79)^4/64}=62.0\ \ \text{MPa}$$

$$\tau_{av}=\frac{Tr}{J}=\frac{9,000,000\times 79/2}{\pi(79)^4/32}=93.0\ \ \text{MPa}$$

$$\tau_r=93.0\times 20\%=18.6\ \ \text{MPa}$$

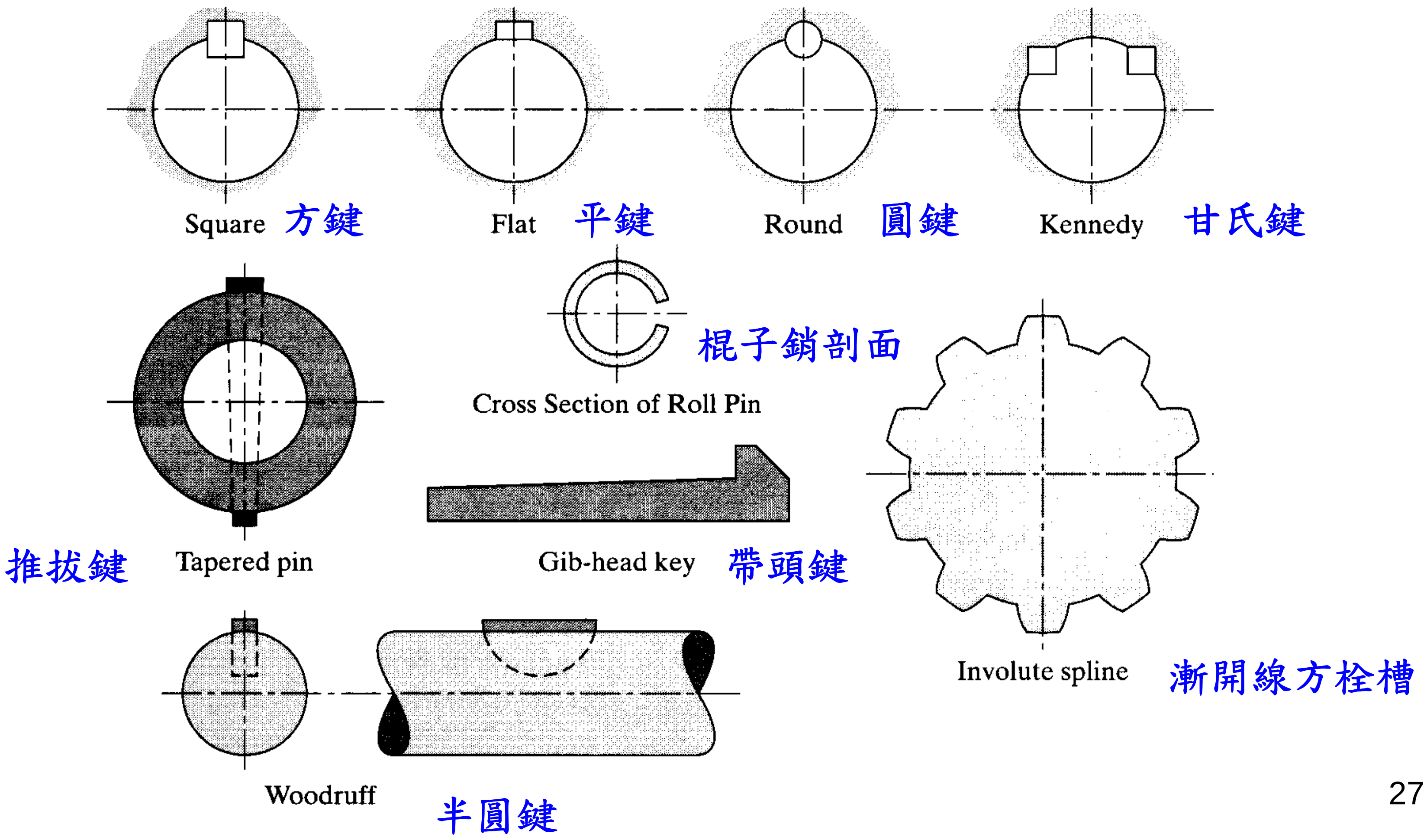
$$\left(0+62.0\times 1.35\left(\frac{620}{300}\right)\right)^2+3\left(93.0+18.6\times 1.35\left(\frac{620}{300}\right)\right)^2\le\left(\frac{620}{2.0}\right)^2$$

$$9.28\times 10^4\le 9.61\times 10^4$$

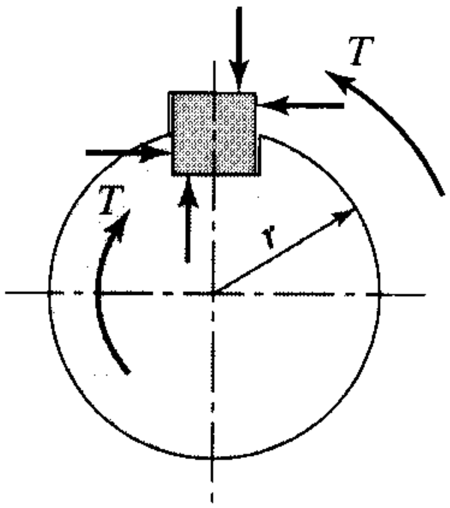
不等式成立，安全

# *Keys*

***Shafts and hubs are usually fastened together by means of keys.***

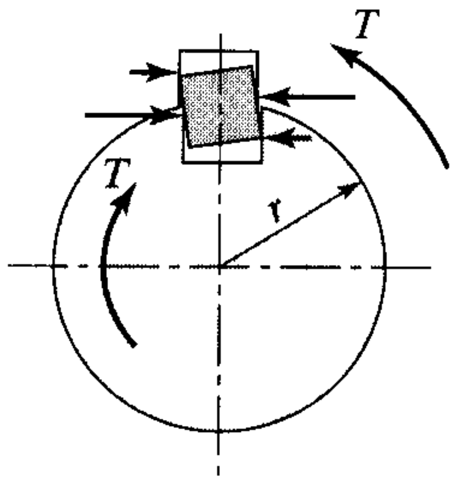


緊配合的鍵槽




(a) Forces on key which fits tightly top and bottom.

鬆配合的鍵槽




(b) Forces acting on loosely fitted key.

$$T = Fr$$

# EXAMPLE 3-8

**Problem Statement:**

A 80 mm diam shaft is made from material with a yield point value of 400 MPa. A 22 $\times$ 22 mm key of material with a yield point value of 360 MPa is to be used. Let $t_{yp} = 0.5\ s_{yp}$. The factor of safety is equal to 2. Find the required length of key based on the torque value of the gross shaft.

**For the shaft, working stress:**

$$\sigma = \frac{\sigma_{yp}}{N_{fs}} = \frac{400}{2} = 200 \text{ MPa}$$

$$\tau = \frac{\tau_{yp}}{N_{fs}} = \frac{200}{2} = 100 \text{ MPa}$$

**For the key, working stress:**

$$\sigma = \frac{\sigma_{yp}}{N_{fs}} = \frac{360}{2} = 180 \text{ MPa}$$

$$\tau = \frac{\tau_{yp}}{N_{fs}} = \frac{180}{2} = 90 \text{ MPa}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi}{32} \times (80)^4 = 4,021,248 \text{ mm}^4$$

**Torque in the shaft** $$T = \frac{\tau J}{r} = \frac{100 \times 4,021,248}{40} = 10,053,120 \ \text{ Nmm}$$

**Force at the shaft surface** $$F = \frac{T}{r} = \frac{10,053120}{40} = 251,328 \ \text{ N}$$

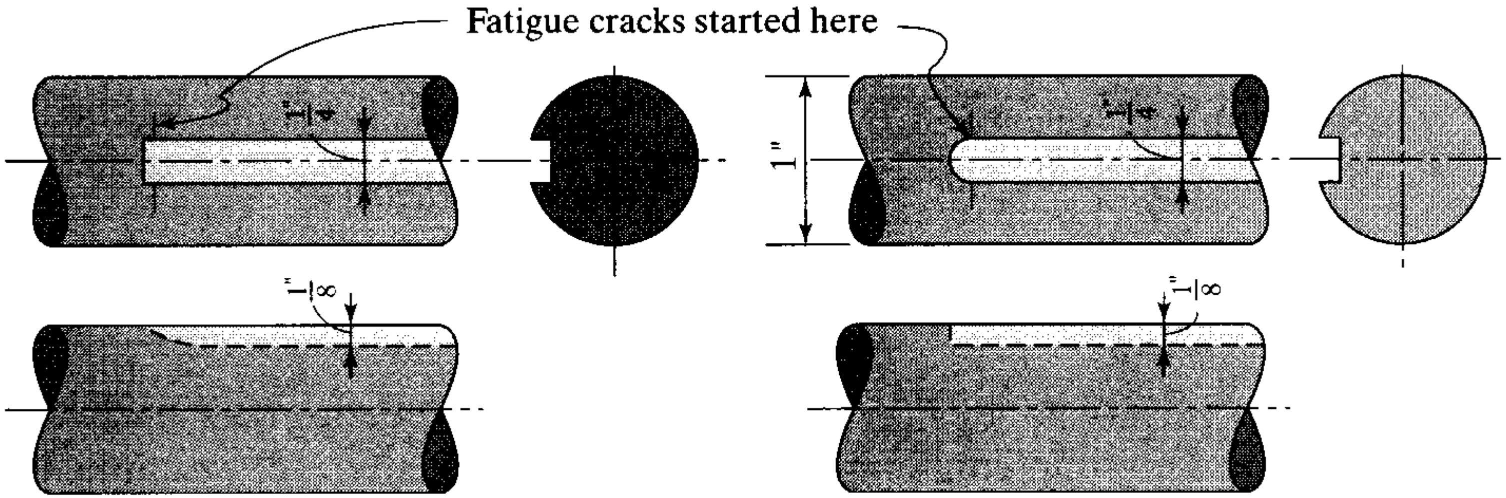
## For the length of the key:

**Based on bearing on shaft** $$l = \frac{251,328}{200 \times (11)} = 114.24 \text{ mm}$$

**Based on bearing on key** $$l = \frac{251,328}{180 \times (11)} = 126.93 \text{ mm}$$

**Based on shear in key** $$l = \frac{251,328}{90 \times (22)} = 126.93 \text{ mm}$$

# 3-6 STRESS CONCENTRATION




**Sled-runner keyway**
**滑入式鍵槽**

**Profile keyway**
**輪廓式鍵槽**

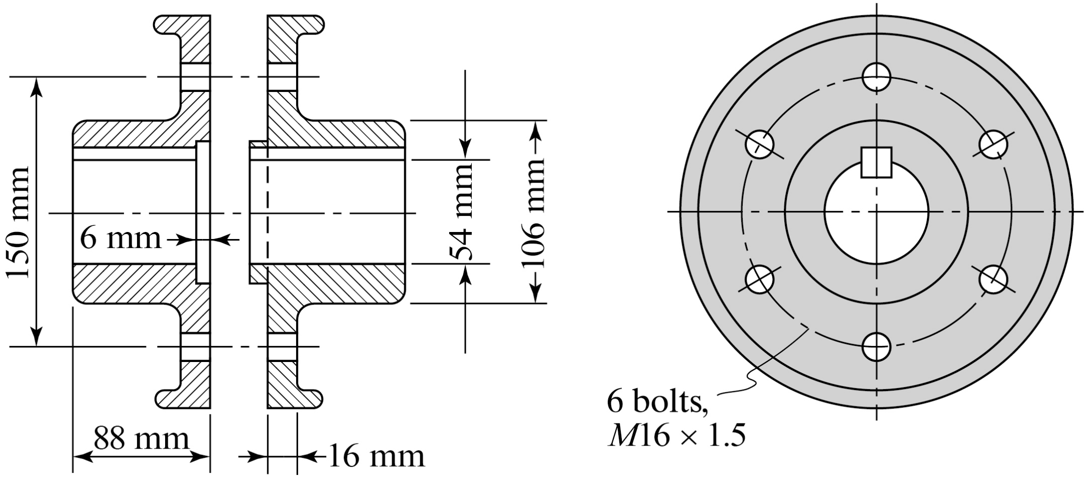
***For fluctuating loads, the fatigue stress concentration factor is***

$$K_f = \frac{\text{endurance limit for plain specimen}}{\text{endurance limit with keyway or hole}}$$

**TABLE 3-2** FATIGUE STRESS CONCENTRATION FACTORS IN BENDING FOR SHAFTS WITH KEYWAYS BASED ON SECTION MODULUS OF FULL AREA.

| Material | Stress Concentration Factor for Sled-Runner Keyway | Stress Concentration Factor for Profile Keyway |
|---|---|---|
| Chrome–nickel (about SAE 3140) $S_{ult}$ = 714 Mpa(103,500 psi) $S_{yp}$ = 483 Mpa(70,000 psi) $S_e$ = 400 Mpa(58,000 psi) | 1.6 | 2.07 |
| Medium-carbon steel (about SAE 1045) $S_{ult}$ = 552 Mpa(80,000 psi) $S_{yp}$ = 310 Mpa(45,000 psi) $S_e$ = 255 Mpa(37,000 psi) | 1.32 | 1.61 |

# 3-7 COUPLINGS

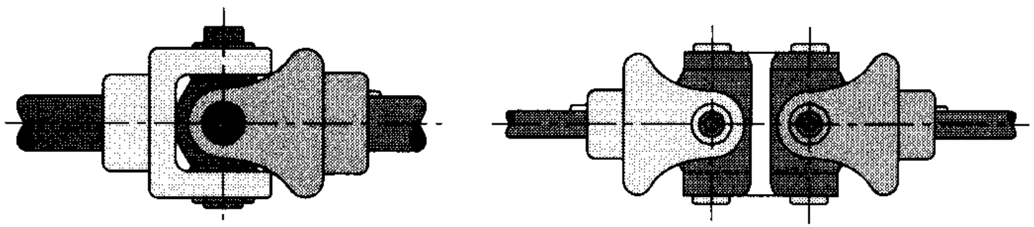



Sled-runner keyway

Profile keyway

**Solid coupling**

**Figure 3-12** Solid coupling.



(a)

(b)

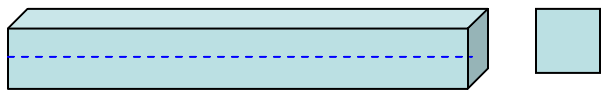
**Universal joint coupling**

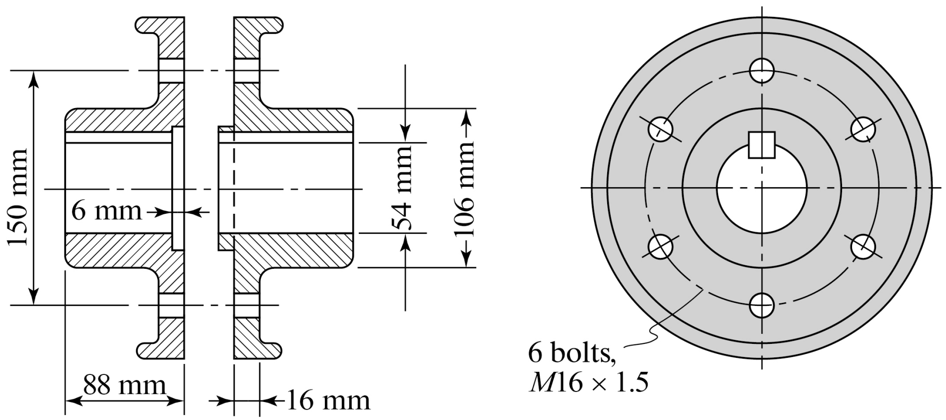
# EXAMPLE 3-9

## Problem Statement:

For the coupling shown in Fig. 3-12, the key is 12 × 12 mm. The shaft carries a steady load of 40 kW at 150 rpm. For all parts, $\sigma_{yp}$ = 420 MPa, and $\tau_{yp}$ = 210 MPa. Find the following stresses and the $F_s$ based on the yield point.

(a) Shear and bearing in key.
(b) Shear in bolts.
(c) Bearing on bolts in flange.
(d) Shear in flange at hub.






Sled-runner keyway    Profile keyway

**Figure 3-12** Solid coupling.

**(a)**

$$T = \frac{9550\text{kW}}{n} = \frac{9,550,000 \times 40}{150} = 2,546,667 \ \text{ Nmm}$$

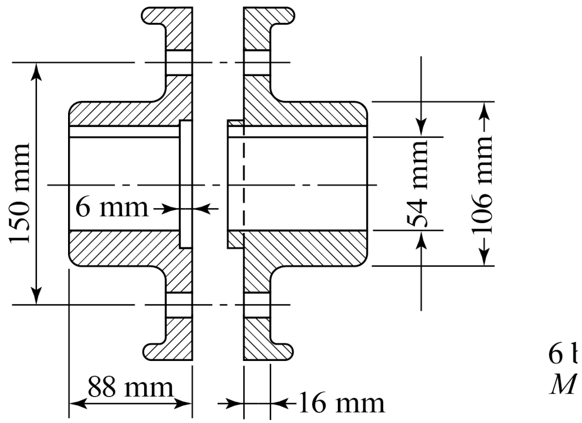
**Tangential force at shaft surface** $$F = \frac{T}{r} = \frac{2,546,667}{54/2} = 94,321 \ \text{ N}$$

**Compressive stress** $$\sigma_c = \frac{94,321}{(12/2)\times(88-6)} = 191.7 \text{ MPa}$$

**Factor of safety in bearing** $$N_{fs} = \frac{420}{191.7} = 2.19$$

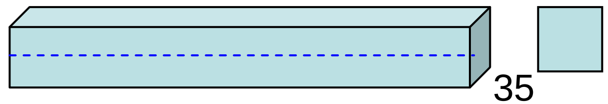
**Shear stress in the key** $$\tau = \frac{94,321}{12\times(88-6)} = 95.8 \text{ MPa}$$

**Factor of safety in shear** $$N_{fs} = \frac{210}{95.8} = 2.19$$




Sled-runner keyway

**Figure 3-12** Solid cc



**(b)**

**Force at the bolt circle**

$$F = \frac{2,546,667}{150/2} = 33,955.6 \ \ \text{N}$$

**Shear stress in the bolts**

$$\tau = \frac{33,955.6}{6 \times \frac{\pi (16)^2}{4}} = 28.1 \text{ MPa}$$

**Factor of safety**

$$N_{fs} = \frac{210}{28.1} = 7.46$$

**(c)**

**Compressive stress in the bolts**

$$\sigma = \frac{33,955.6}{6 \times 16 \times 16} = 22.1 \text{ MPa}$$

**Factor of safety**

$$N_{fs} = \frac{420}{22.1} = 19.0$$

## (d)

**Force at edge of the hub** $$F = \frac{2,546,667}{106/2} = 48.050 \ \text{N}$$

**Shear stress** $$\tau = \frac{48,050}{106\pi \times 16} = 9.1 \ \text{MPa}$$

**Factor of safety** $$N_{fs} = \frac{210}{9.1} = 23.3$$

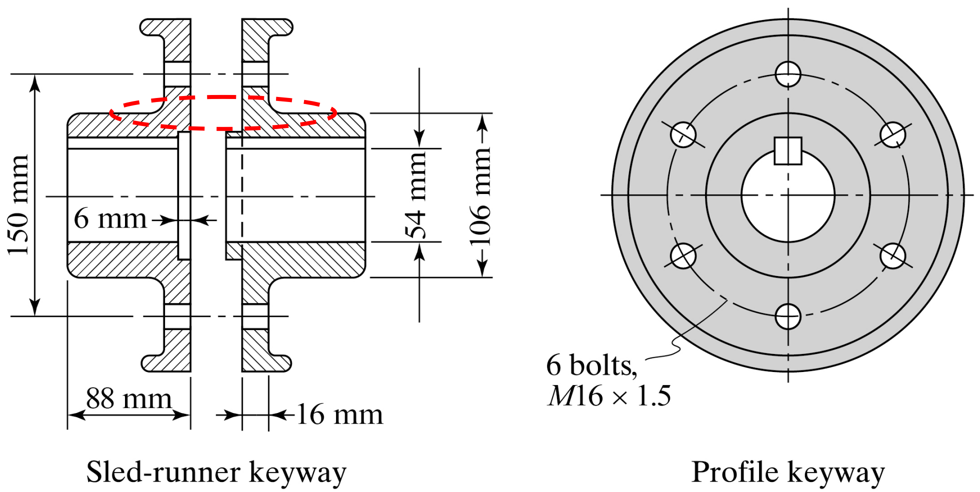



**Figure 3-12** Solid coupling.

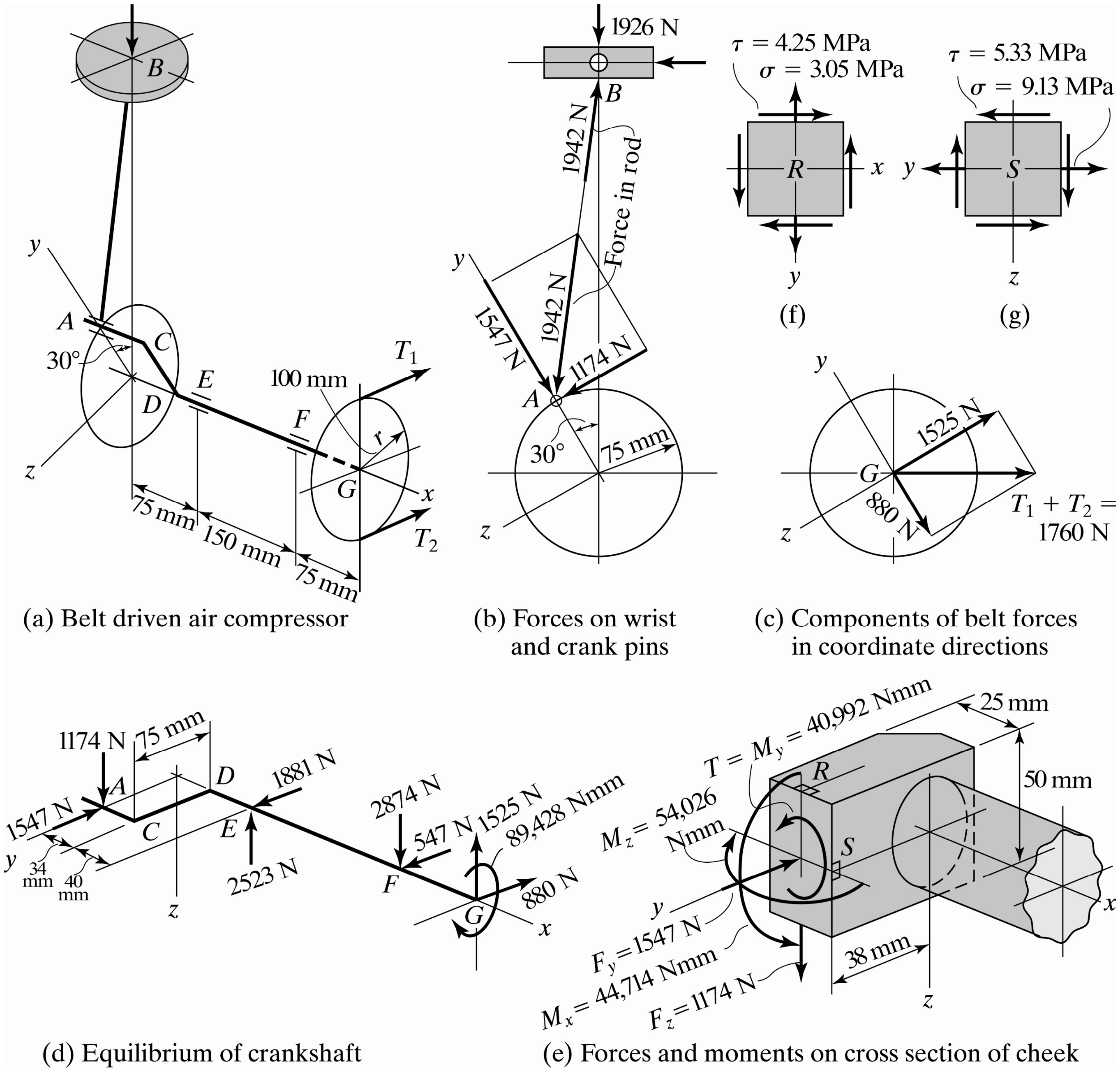
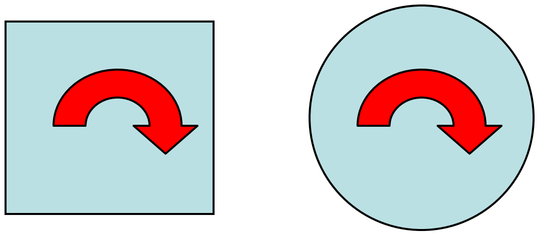



**Figure 3-16** Forces and moments for crankshaft. Example 3-14.

# 3-14 TORSION OF NONCIRCULAR SHAFT

***The theory of torsion for noncircular cross section is complicated because the assumptions that are valid for circular shaft do not apply. Cross sections are no longer plane and perpendicular to the shaft axis after twisting; rather, they are warped.***



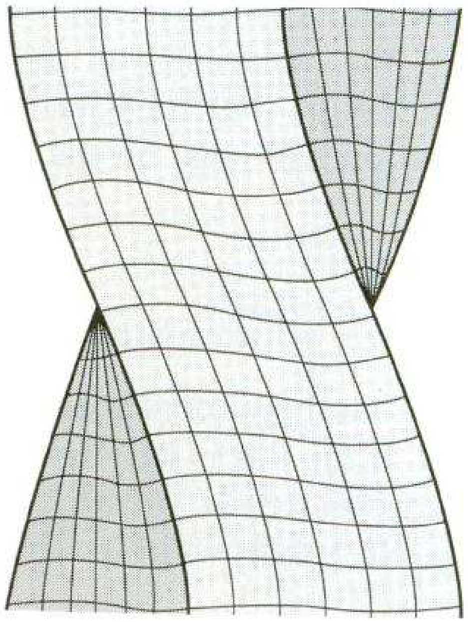


**Figure 3-22** Rectangular bar in torsion.

- the membrane may have a steep slope, indicating a high stress at the internal corners of a keyway as shown in Fig. 3-23(a). The membrane analogy indicates that this stress concentration can be reduced by rounding off the bottom corners.

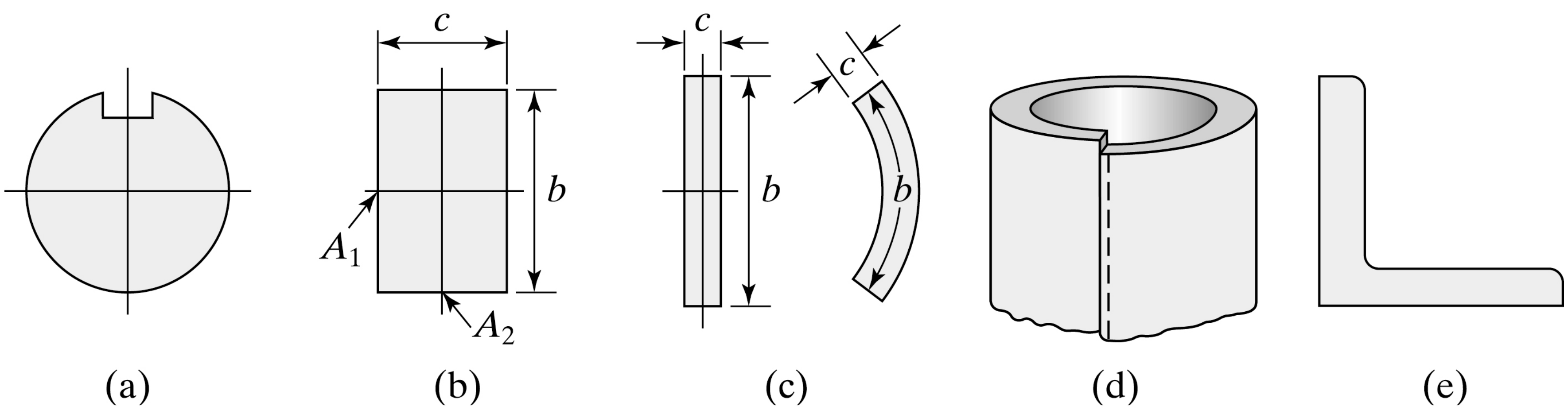
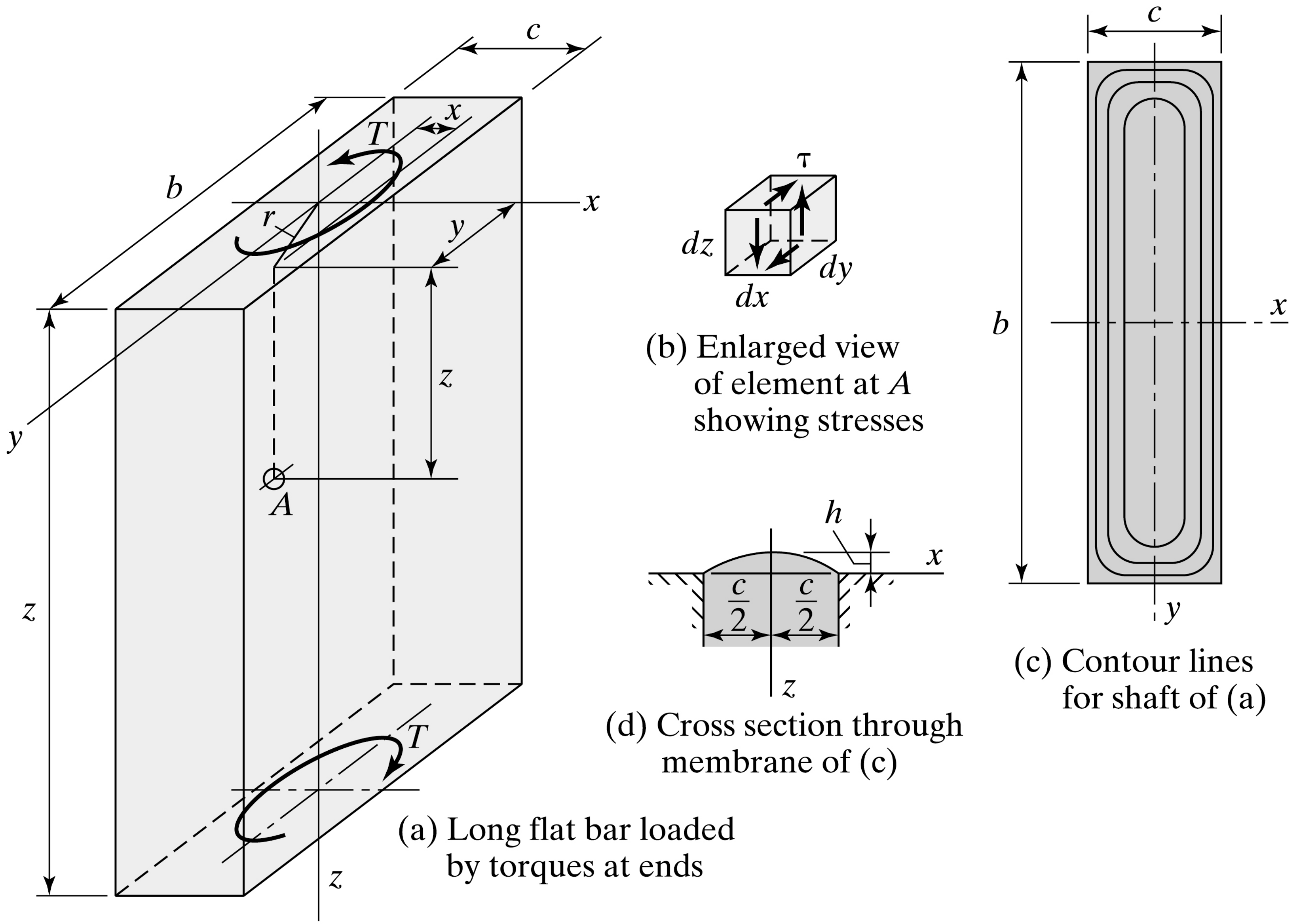



**Figure 3-23** Typical cross sections of bars loaded in torsion.




(a) Long flat bar loaded by torques at ends

(b) Enlarged view of element at $A$ showing stresses

(c) Contour lines for shaft of (a)

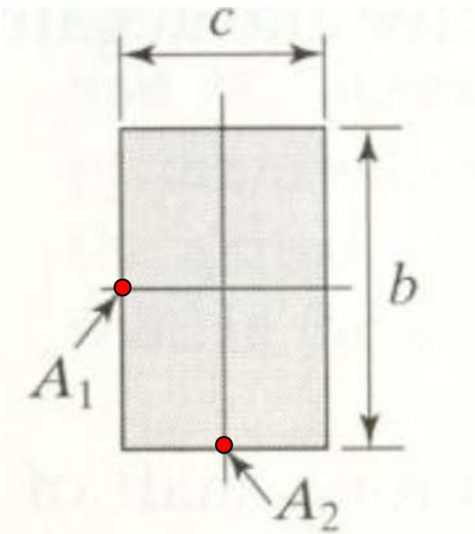
(d) Cross section through membrane of (c)

**Figure 3-24** Torsion of thin, wide, rectangular shaft.

# 3-16 Torsion of Rectangular Bars

**TABLE 3-3** CONSTANT FOR TORSION OF RECTANGULAR BARS

| $b/c$ | 1.00 | 1.20 | 1.50 | 1.75 | 2.00 | 2.50 | 3.00 | 4.00 | 5.00 | 6.00 | 8.00 | 10.00 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_1$ | 0.208 | 0.219 | 0.231 | 0.239 | 0.246 | 0.258 | 0.267 | 0.282 | 0.291 | 0.299 | 0.307 | 0.312 | 0.333 |
| $\alpha_2$ | 0.208 | 0.235 | 0.269 | 0.291 | 0.309 | 0.336 | 0.355 | 0.378 | 0.392 | 0.402 | 0.414 | 0.421 | … |
| $\beta$ | 0.1406 | 0.166 | 0.196 | 0.214 | 0.229 | 0.249 | 0.263 | 0.281 | 0.291 | 0.299 | 0.307 | 0.312 | 0.333 |




$$\tau = \frac{T}{\alpha_1 bc^2}$$ for point $A_1$ （長邊中點應力較大）

$$\tau = \frac{T}{\alpha_2 bc^2}$$ for point $A_2$

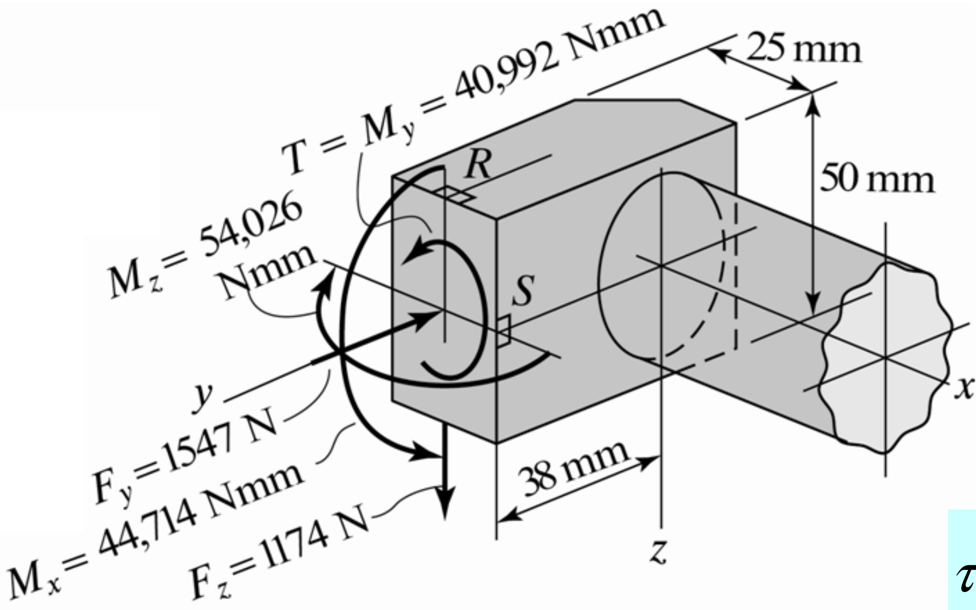
注意單位!!

$$\theta_1 = \frac{T}{\beta G bc^3}$$ angular rotation per inch

# EXAMPLE 3-17

**Problem Statement:**

Find the stresses at points R and S at the center of the sides of the cheek for the cross section midway between points C and D in Fig. 3-16.



(e) Forces and moments on cross section of cheek

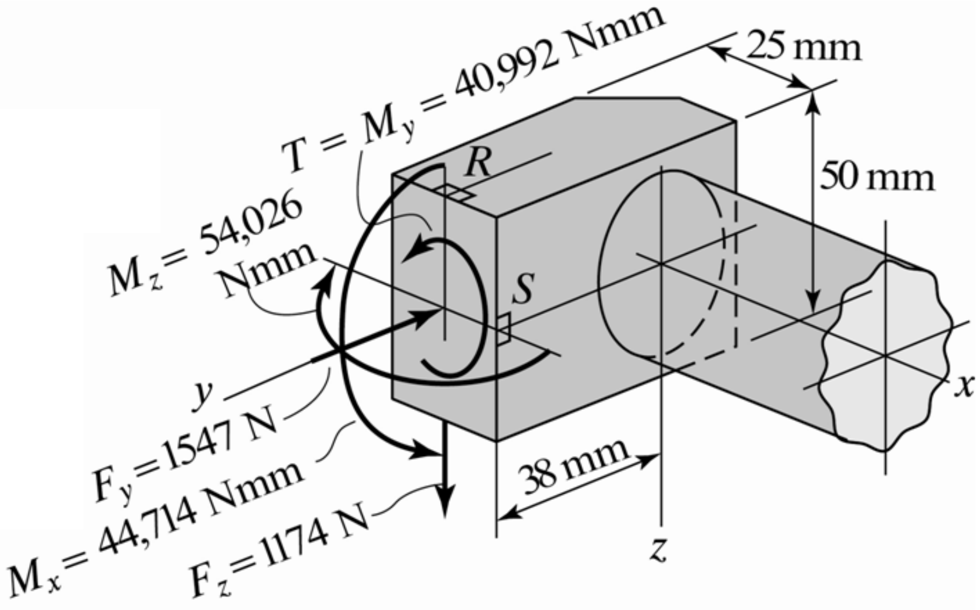
$$b/c = 2 \quad \Rightarrow \quad \begin{cases} \alpha_1 = 0.246 \\ \alpha_2 = 0.309 \\ \beta = 0.229 \end{cases}$$

**For the element at R**

$$\sigma_R = -\frac{P}{A} + \frac{6M_x}{bh^2} = -\frac{1547}{25\times 50} + \frac{6\times 44{,}714}{25\times 50^2} = 3.05 \;\; \text{MPa}$$

$$\tau_R = \frac{T}{\alpha_2 bc^2} = \frac{40{,}992}{0.309\times 50\times 25^2} = 4.245 \;\; \text{MPa}$$

$$\theta = \frac{T}{\beta G bc^3} = \frac{40{,}992}{0.229\times 79{,}300\times 50\times 25^3} = 2.88\times 10^{-6} \quad \text{rad/mm}$$



(e) Forces and moments on cross section of cheek

**For the element at S**

$$\sigma_S = -\frac{P}{A} + \frac{6M_z}{bh^2}$$
$$= -\frac{1547}{25\times 50} + \frac{6\times 54,026}{50\times 25^2} = 9.13 \ \text{MPa}$$

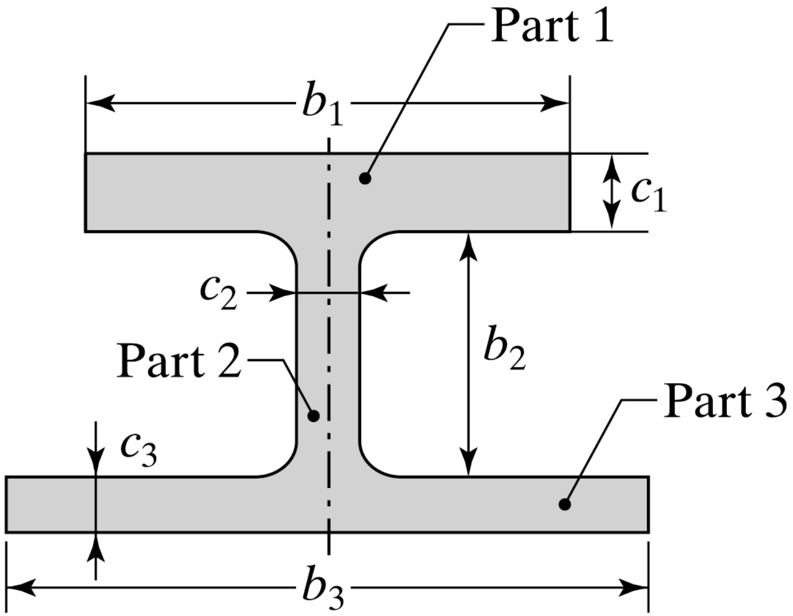
$$\tau_S = -\frac{3V}{2A} + \frac{T}{\alpha_1 bc^2}$$
$$= -\frac{3\times 1174}{2\times 50\times 25} + \frac{40,992}{0.246\times 50\times 25^2}$$
$$= -1.41 + 5.33$$
$$= 3.92 \ \text{MPa}$$

$$b/c = 2 \quad \Rightarrow \begin{cases} \alpha_1 = 0.246 \\ \alpha_2 = 0.309 \\ \beta = 0.229 \end{cases}$$

# 3-17 Composite Sections

$$\theta_1 = \frac{T}{\beta G b c^3}$$

假設受扭矩T作用後，單位長度扭轉角為$\theta_1$




**Figu**[...]ion loaded in to[...]

$$T_1 = \theta_1 G \beta' b_1 c_1^3$$
$$T_2 = \theta_1 G \beta'' b_2 c_2^3$$
$$T_3 = \theta_1 G \beta''' b_3 c_3^3$$

$$T = \theta_1 G \left( \beta' b_1 c_1^3 + \beta'' b_2 c_2^3 + \beta''' b_3 c_3^3 \right)$$

$$\tau_1 = \frac{T_1}{\alpha_1 b_1 c_1^2} = \frac{\theta_1 G \beta' c_1}{\alpha_1}$$

$$\tau_1 = \frac{T \beta' c_1}{\alpha_1 \left( \beta' b_1 c_1^3 + \beta'' b_2 c_2^3 + \beta''' b_3 c_3^3 \right)}$$

$$\theta_1 = \frac{T}{G \left( \beta' b_1 c_1^3 + \beta'' b_2 c_2^3 + \beta''' b_3 c_3^3 \right)}$$

## TABLE 3-4 STRESS CONCENTRATION FACTOR $K_t$ FOR STRUCTURAL ANGLE AND THIN-WALLED SQUARE TUBE

| *r/c* | 0.125 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|---|---|
| Angle | 2.72 | 2.00 | 1.63 | 1.57 | 1.56 | 1.57 | 1.60 |
| Square Tube | 2.46 | 1.70 | 1.40 | 1.25 | 1.14 | 1.25 | 1.07 |

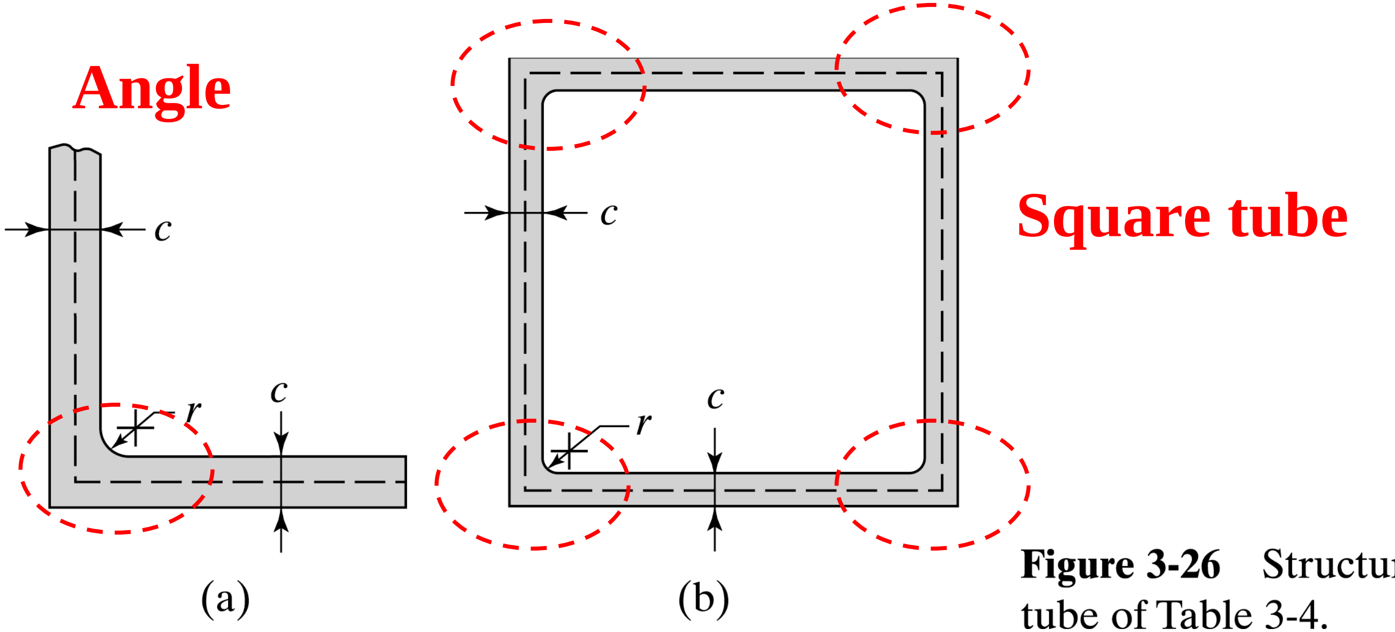



**Figure 3-26** Structural angle and square tube of Table 3-4.

# EXAMPLE 3-18

**Problem Statement:**

外緣長度

Find the torque that a 750 mm piece of 50 × 50 × 9 mm angle iron can carry if the maximum shearing stress on the fillet is to be 84 MPa. Radius of the fillet is 6 mm and G = 79,300 MPa. Find the angular deformation sustained with such loading.

$$\frac{r}{c} = \frac{6}{9} = 0.67 \quad \Rightarrow \quad K_t = 1.59$$

內插

developed center line: $b = 50 + 50 - 9 = 91$

$$\frac{b}{c} = \frac{91}{9} = 10.11 \quad \Rightarrow \quad \begin{array}{l} \alpha_1 = 0.312 \\ \beta = 0.312 \end{array}$$

$$\tau = K_t \frac{T}{\alpha_1 bc^2} = 1.59 \times \frac{T}{0.312 \times 91 \times 9^2} = 84 \quad \Rightarrow \quad T = 121,496 \ \text{Nmm}$$

$$\theta = \theta_1 l = \frac{Tl}{\beta Gbc^3} = \frac{121,496 \times 750}{0.312 \times 79,300 \times 91 \times 9^3} = 0.05552 \ \text{rad}$$

# 3-18 THIN-WALLED TUBE

$$T = \int \tau r c \left( r d\theta \right) = 2\tau c \int \frac{r^2}{2} d\theta = 2\tau c A \quad \Rightarrow \quad \tau = \frac{T}{2Ac}$$

$$\theta_1 = \frac{T}{GJ} = \frac{T}{4A^2G} \sum \frac{a}{c}$$

$$J = \frac{4A^2}{\int_0^L \frac{ds}{c}}$$

由應變能導出

$$U = \int dU = \int \frac{\tau^2}{2G}(cdsdx)$$
$$= \int \frac{\tau^2 c^2}{2G}\left(\frac{ds}{c}dx\right) = \frac{\tau^2 c^2 L}{2G}\int \frac{ds}{c}$$
$$= \frac{T^2 L}{8GA^2}\int \frac{ds}{c}$$

$$W = \frac{T\varphi}{2} = U = \frac{T^2 L}{8GA^2}\int \frac{ds}{c}$$

$\theta_1$ = rotation angle per unit length

$a$ = length of center line around the wall

$c$ = width of the wall

# EXAMPLE 3-19

**Problem Statement:**

(a) Find the torque that the steel hollow cross section of Fig. 3-27(a) can carry at a shearing stress of 70 MPa. Find the angular deformation $\theta_1$ per mm of axial length.

(b) Do the same for the cross section of Fig. 3-27(b), where the wall is not continuous.

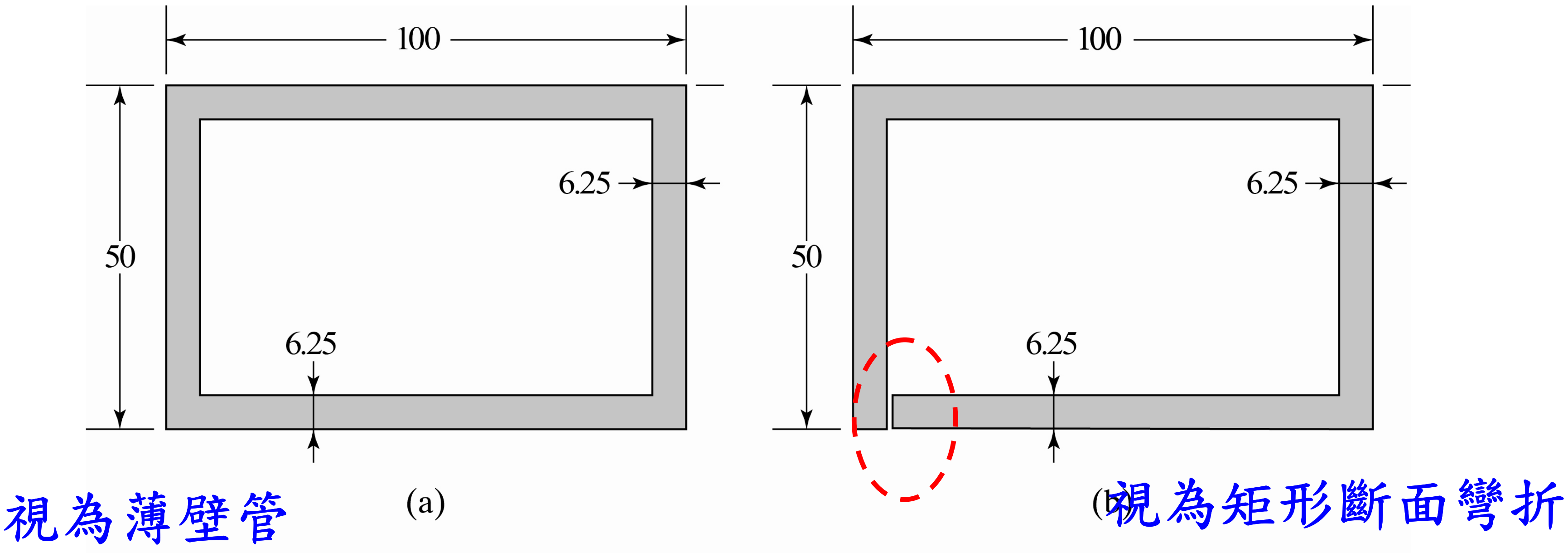



**Figure 3-27** Composite cross sections loaded in torsion. Example 3-19.

**(a)**

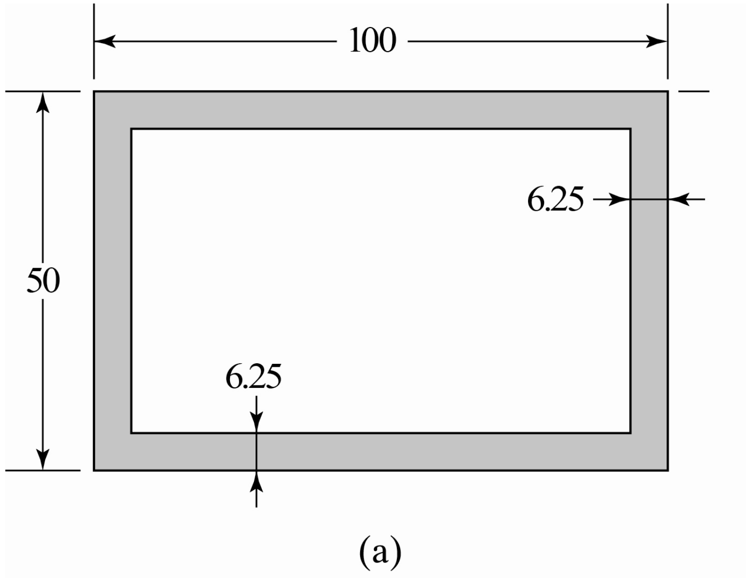



**Figure 3-27** Composite cross sectio

$$\tau = \frac{T}{2Ac} \quad \Rightarrow \quad T = 2\tau Ac$$

$$T = 2\tau Ac = 2\times 70\times\left(43.75\times 93.75\right)\times 6.25 = 3,588,867 \quad \text{Nmm}$$

$$a = 2\times 43.75 + 2\times 93.75 = 275 \quad \text{mm}$$

$$\theta_1 = \frac{T}{4A^2G}\frac{a}{c} = \frac{3,588,867}{4\times\left(43.75\times 93.75\right)^2\times 79,300}\cdot\frac{275}{6.25} = 2.69\times 10^{-5} \quad \text{rad/mm}$$

**(b)**

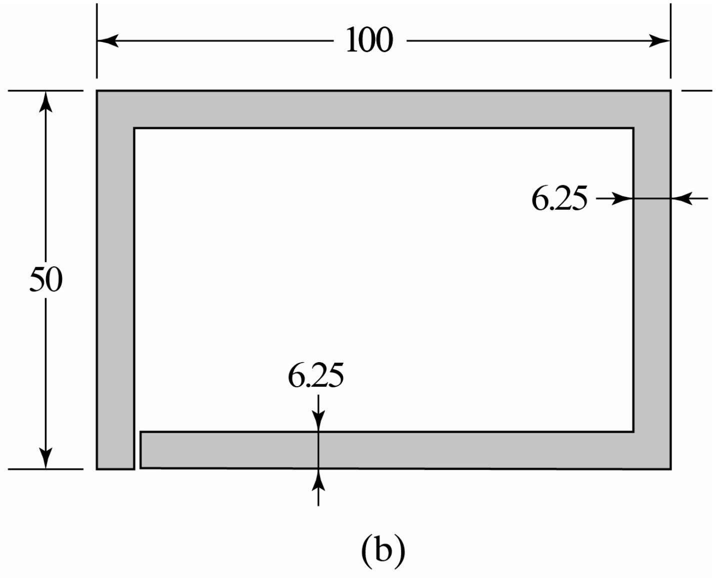



**Figure 3-27** Composite cross sections loaded in torsion. Example 3-19.

$$T = 0.333bc^2\tau = 0.333\times 275\times (6.25)^2 \times 70 = 250,400 \quad \text{Nmm}$$

$$b = 2\times 43.75 + 2\times 93.75 = 275 \quad \text{mm}$$

$$\theta_1 = \frac{T}{0.333Gbc^3} = \frac{250,400}{0.333\times 79,300\times 275\times (6.25)^3} = 14.1\times 10^{-5} \quad \text{rad/mm}$$

# Selected Exercises

**CH2: 3, 14, 16, 33, 62, 66**

**CH3: 1, 12, 19, 23, 51, 54**