

In random samples of 12 from each of two normal populations, we found the following statistics. Test with $\alpha = .05$ to determine whether we can infer that the population means differ.

$$\begin{array}{ll} \bar{x}_1 = 74 & s_1 = 18 \\ \bar{x}_2 = 71 & s_2 = 16 \end{array}$$

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_1 : (\mu_1 - \mu_2) \neq 0$$

Equal-variances test statistic

Rejection region: $t < -t_{\alpha/2, v} = -t_{.025, 22} = -2.074$ or $t > t_{\alpha/2, v} = t_{.025, 22} = 2.074$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(74 - 71) - 0}{\sqrt{\left(\frac{(12-1)18^2 + (12-1)16^2}{12+12-2} \right) \left(\frac{1}{12} + \frac{1}{12} \right)}} = .43, \text{ p-value} = .6703. \text{ There is not}$$

enough evidence to infer that the population means differ.

Random sampling from two normal populations produced the following results. Can we infer at the 5% significance level that μ_1 is greater than μ_2 ?

$$\begin{array}{lll} \bar{x}_1 = 412 & s_1 = 128 & n_1 = 150 \\ \bar{x}_2 = 405 & s_2 = 54 & n_2 = 150 \end{array}$$

$$H_0 : (\mu_1 - \mu_2) \leq 0$$

$$H_1 : (\mu_1 - \mu_2) > 0$$

Unequal-variances test statistic

$$v = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}} = 200.4 \text{ (rounded to 200)}$$

Rejection region: $t > t_{\alpha, v} = t_{.05, 200} = 1.653$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{(412 - 405) - 0}{\sqrt{\left(\frac{128^2}{150} + \frac{54^2}{150}\right)}} = .62, \text{ p-value} = .2689. \text{ There is not enough evidence}$$

to infer that μ_1 is greater than μ_2 .

Who spends more on their vacations, golfers or skiers? To help answer this question, a travel agency surveyed 15 customers who regularly take their spouses on either a skiing or a golfing vacation. The amounts spent on vacations last year are shown here. Can we infer that golfers and skiers differ in their vacation expenses at 10% significance level?

Golfer	2,450	3,860	4,528	1,944	3,166	3,275
	4,490	3,685	2,950			
Skier	3,805	3,725	2,990	4,357	5,550	4,130

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_1 : (\mu_1 - \mu_2) \neq 0$$

Two-tail F test: $F = 1.04$, $p\text{-value} = .9873$; use equal-variances test statistic

Rejection region: $t < -t_{\alpha/2, v} = -t_{.05, 13} = -1.771$ or $t > t_{\alpha/2, v} = t_{.05, 13} = 1.771$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(3,372 - 4,093) - 0}{\sqrt{\left(\frac{(9-1)755,196 + (6-1)725,778}{9+6-2} \right) \left(\frac{1}{9} + \frac{1}{6} \right)}} = -1.59, p\text{-value} = .1368. \text{ There is}$$

not enough evidence to infer a difference between the two types of vacation expenses.

Many people use scanners to read documents and store them in a Word (or some other software) file. To help determine which brand of scanner to buy, a student conducts an experiment wherein eight documents are scanned by each of the two scanners in which he is interested. He records the number of errors made by each. These data are listed here. Can he infer that Brand A (the more expensive scanner) is better than Brand B at 5% significance level?

Document	1	2	3	4	5	6	7	8
Brand A	17	29	18	14	21	25	22	29
Brand B	21	38	15	19	22	30	31	37

$$H_0 : \mu_D \geq 0$$

$$H_1 : \mu_D < 0$$

Rejection region: $t < -t_{\alpha, v} = -t_{.05, 7} = -1.895$

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}} = \frac{-4.75 - 0}{4.17 / \sqrt{8}} = -3.22, \text{p-value} = .0073. \text{ There is enough evidence to infer that the}$$

Brand A is better than Brand B.

In a preliminary study to determine whether the installation of a camera designed to catch cars that go through red lights affects the number of violators, the number of red-light runners was recorded for each day of the week before and after the camera was installed. These data are listed here. Can we infer that the camera reduces the number of red-light runners at 5% significance level?

Day	Sunday	Monday	Tuesday	Wednesday
Before	7	21	27	18
After	8	18	24	19

Day	Thursday	Friday	Saturday
Before	20	24	16
After	16	19	16

$$H_0 : \mu_D = 0$$

$$H_1 : \mu_D > 0$$

$$\text{Rejection region: } t > t_{\alpha, v} = t_{.05, 6} = 1.943$$

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}} = \frac{1.86 - 0}{2.48 / \sqrt{7}} = 1.98, \quad \text{p-value} = .0473. \text{ There is enough evidence to infer that the}$$

camera reduces the number of red-light runners.

An operations manager who supervises an assembly line has been experiencing problems with the sequencing of jobs. The problem is that bottlenecks are occurring because of the inconsistency of sequential operations. He decides to conduct an experiment wherein two different methods are used to complete the same task. He measures the times (in seconds). The data are listed here. Can he infer that the second method is more consistent than the first method at 5% significance level?

Method 1 8.8 9.6 8.4 9.0 8.3 9.2 9.0 8.7 8.5 9.4

Method 2 9.2 9.4 8.9 9.6 9.7 8.4 8.8 8.9 9.0 9.7

$$H_0 : \sigma_1^2 / \sigma_2^2 \geq 1$$

$$H_1 : \sigma_1^2 / \sigma_2^2 < 1$$

Rejection region: $F < F_{1-\alpha, v_1, v_2} = 1 / F_{\alpha, v_2, v_1} = 1 / F_{0.05, 9, 9} = 1 / 3.18 = .314$

$F = s_1^2 / s_2^2 = .1854 / .1893 = .98$, p-value = .4879. There is not enough evidence to infer that the second method is more consistent than the first method.

Many stores sell extended warranties for products they sell. These are very lucrative for store owners. To learn more about who buys these warranties a random sample of a store's customers who recently purchased a product for which an extended warranty was available was drawn. Among other variables each respondent reported whether they paid the regular price or a sale price and whether they purchased an extended warranty.

Can we conclude at the 10% significance level that those who paid the regular price are more likely to buy an extended warranty?

	Regular price	Sale price
Sample size	229	178
Number who bought extended warranty	47	25

$$H_0 : (p_1 - p_2) \leq 0$$

$$H_1 : (p_1 - p_2) > 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.205 - .140)}{\sqrt{.177(1-.177)\left(\frac{1}{229} + \frac{1}{178}\right)}} = 1.70, \text{ p-value} = P(Z > 1.70) = 1 - .9554$$

$$= .0446.$$

There is enough evidence to conclude that those who paid the regular price are more likely to buy an extended warranty.

The process that is used to produce a complex component used in medical instruments typically results in defective rates in the 40% range. Recently, two innovative processes have been developed to replace the existing process. Process 1 appears to be more promising, but it is considerably more expensive to purchase and operate than process 2. After a thorough analysis of the costs, management decides that it will adopt process 1 only if the proportion of defective components it produces is more than 8% smaller than that produced by process 2. In a test to guide the decision, both processes were used to produce 300 components. Of the 300 components produced by process 1, 33 were found to be defective, whereas 84 out of the 300 produced by process 2 were defective. Conduct a test using a significance level of 1% to help management make a decision.

$$H_0 : (p_1 - p_2) \geq -.08$$

$$H_1 : (p_1 - p_2) < -.08$$

$$\text{Rejection region: } z < -z_{\alpha} = -z_{.01} = -2.33$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{(.11 - .28) - (-.08)}{\sqrt{\frac{.11(1-.11)}{300} + \frac{.28(1-.28)}{300}}} = -2.85, \text{ p-value} = P(Z < -2.85) = 1 - .9978 = .0022.$$

There is enough evidence to conclude that management should adopt process 1.

In attempt to understand the frustrations of driving in large cities an experiment was conducted. At a red light in a busy intersection the lead car hesitated for 2 seconds. The researchers recorded several variables and whether the driver in the following car honked his or her horn. The following tables were created from the recorded data. Conduct all tests at the 5% significance level.

Driver in following car was female	Lead car driver using cell phone	Lead car driver not using cell phone
Honked horn	18	27
Did not honk horn	77	162

Is there enough evidence to infer that women drivers in a following car are more likely to honk when the lead car driver is using a cell phone?

$$H_0 : (p_1 - p_2) \leq 0$$

$$H_1 : (p_1 - p_2) > 0$$

$$\text{Rejection region: } z > z_{\alpha} = z_{.05} = 1.645$$

$$\hat{p}_1 = \frac{18}{95} = .1895 \quad \hat{p}_2 = \frac{27}{189} = .1429 \quad \hat{p} = \frac{18 + 27}{95 + 189} = .1585$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.1895 - .1429)}{\sqrt{.1585(1-.1585)\left(\frac{1}{95} + \frac{1}{189}\right)}} = 1.02, \text{p-value} = P(Z > 1.02) = 1 - .8461 = .1539.$$

There is not enough evidence to infer that women drivers in a following car are more likely to honk when the lead car driver is using a cell phone.