1.

$$P = P_0 + tV$$
$$(10,1,0.5) = (0,0,0) + tV$$
$$\therefore tV = (10,1,0.5)$$
$$V = \left(\frac{10}{|V|}, \frac{1}{|V|}, \frac{0.5}{|V|}\right), where\ |V| = \sqrt{10^2 + 1^2 + 0.5^2} = \sqrt{101.25}$$
$$V = (0.994,0.099,0.050)$$
$$\therefore P = (0,0,0) + t(0.994,0.099,0.050)$$

$$|P - C|^2 - r^2 = 0$$
$$|P_0 + tV - C|^2 - r^2 = 0$$

$solve\ at^2 + bt + c = 0$

$$where\ a = |V|^2 = 1, b = 2V \cdot (P_0 - C), c = |P_0 - C|^2 - r^2$$
$$and\ P_0 = (0,0,0), V = (0.994,0.099,0.050), C = (20,0,0), r = 2$$
$$\therefore t^2 - 39.76t + 396 = 0$$
$$\Delta = b^2 - 4ac = 39.76^2 - 4(1)(396) = -3.1424 < 0$$

Therefore the ray does not intersect with the red ball and pixel p will be white.

c)

Let N be the old normal vector and N' be the new normal vector.

In the new coordinate system, let new N' be the new X axis and the rotational axis be the new Z axis.

$$X' = N' = (1,1,1), \qquad \widehat{X'} = \frac{X'}{|X'|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$
$$Z' = N \times N' = (0,-1,1), \qquad \widehat{Z'} = \frac{Z'}{|Z'|}\left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$
$$Y' = Z' \times X' = (-2,1,1), \qquad \widehat{Y'} = \frac{Y'}{|Y'|} = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

The overall transformation matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 10 - \frac{10}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{20}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore p' = Mp = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 10 - \frac{10}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{20}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 1 \\ 0.5 \\ 1 \end{bmatrix} = [10.866 \quad 0.612 \quad -0.354 \quad 1]$$

Ray equation of p':

$$P = P_0 + tV$$

$$(10.866, 0.612, -0.354) = (0,0,0) + tV$$

$$V = \left(\frac{10.866}{|V|}, \frac{0.612}{|V|}, \frac{-0.354}{|V|}\right), where\ |V| = \sqrt{10.866^2 + 0.612^2 + (-0.354)^2} = \sqrt{118.570}$$

$$V = (0.998, 0.056, -0.033)$$

$$\therefore P = (0,0,0) + t(0.998, 0.056, -0.033)$$

$solve\ at^2 + bt + c = 0$

$$where\ a = |V|^2 = 1, b = 2V \cdot (P_0 - C), c = |P_0 - C|^2 - r^2$$

$$and\ P_0 = (0,0,0), V = (0.998, 0.056, -0.033), C = (20,0,0), r = 2$$

$$\therefore t^2 - 39.92t + 396 = 0$$

$$\triangle = b^2 - 4ac = 39.76^2 - 4(1)(396) = 9.6064 > 0$$

Therefore the ray intersects with the red ball and pixel p will be red.

d)

The original image is trivially a perfect circle, and after it is projected on the plane then:

$$C = \begin{bmatrix} 10 \\ \frac{10}{20}(2)\cos t \\ \frac{10}{20}(2)\sin t \end{bmatrix} = \begin{bmatrix} 10 \\ \cos t \\ \sin t \end{bmatrix}$$

Then apply the transformation matrix from 1c)

$$C' = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 10 - \frac{10}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{20}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ \cos t \\ \sin t \\ 1 \end{bmatrix} = \begin{bmatrix} 10 + \frac{\sin t + \cos t}{\sqrt{3}} \\ \frac{\sin t + \cos t}{\sqrt{6}} \\ \frac{\sin t - \cos t}{\sqrt{2}} \\ 1 \end{bmatrix}$$

2.

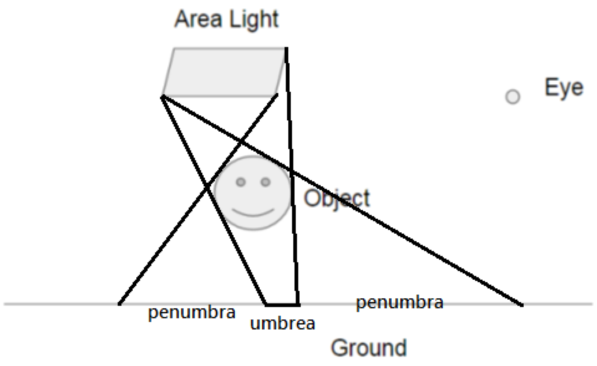
Image created by ray tracing looks fake:

- Jagged edges
- Hard shadows
- Everything in focus
- Objects completely still
- Surfaces perfectly shiny
- Glass perfectly clear

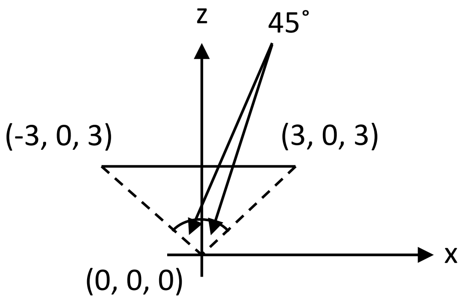
To get motion blur, we need to distribute the rays over time. As an object moves, it will get hit by different camera rays. Finally the moving objects should get averaged with the environment.

To get depth of field, we need to generate multiple rays for each pixel and distribute them across the surface of the lens. We can put a lens between the image plane and the object. Then we can distribute the ray and average the obtained color values of those distributed rays.

3.



b)



As the shadow mapping is a square with side length 6 and at z = 3, the maximum cone angle for the spotlight would obviously be 90°

4.

The four major types are Optical, Magnetic, Inertial trackers, Mechanical. Kinect is based on depth sensor which is a new technology recently, therefore it does not belong to the above four major types.

Blended Motion = A*(1-s) + B*s (where 0<s<1). First we generate a motion in between, then gradually shift from motion A to motion B. Finally concatenate the two motions that is to blend the two ends by gradually shifting s from 0 to 1.

The two categories are rendering individual particles, or rendering the surface of the entire liquid. Rendering individual particles works well for modelling both waterfalls and spray.

We can model the cloth by mass-spring models, which are particles connected with springs. Mass will be pulled by the interconnecting springs.

5.

The first one is minimal lag. It is the time from when the user moves until the movement is reflected in the environment. If it takes too long, then it is difficult to control and causes problems for many interaction techniques.

The second one is time management. Application must adapt computations to the amount of time available to produce best interaction in available time. Otherwise the update rate cannot be in uniform rate.

The third one is portability. Applications must be able to deal with different devices as there is no standard device set. Since 3D devices may produce different coordinate system, the software must be able to interpret this data and the software architecture should isolate application from the device details.

Augmented Reality augments the real world scene. User maintains a sense of presence in real world which needs a mechanism to combine virtual and real worlds.

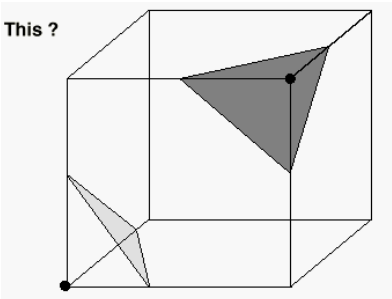
On the other hand, Virtual Reality provides a totally immersive environment. Visual senses, such as aural and proprioceptive senses, are under control of the system.

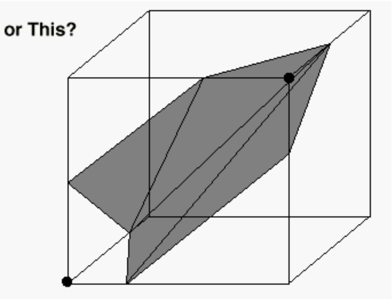
6.

The six steps are:

1. Create a cube
2. Classify each vertex
3. Build an index
4. Get edge list
5. Interpolate triangle vertices
6. Calculate and interpolate normals

Sometimes ambiguity cases will result in holes:





2D-Textured Object-aligned slicing is to draw the volume as a stack of 2D textures which uses Bilinear Interpolation in hardware. It decomposes the volume into axis-aligned slices and texture stack is swapped based on the one that is the closest to viewpoint. 3D-Textured View-aligned slicing is to intersect slicing planes with bounding box, then sort resulting vertices in (counter) clockwise order and finally construct polygon primitive from centroid as triangle fan.

2D-Textured Object-aligned slicing is fast and simple, but 3 time memory consumption as data needs to be replicated along 3 principle directions. Moreover a change in view point will result in stack swap, causing image popping artifacts and lag while downloading new textures. The sampling distance changes with viewpoint thus there will be intensity variations as camera moves. In contrast, 3D-Textured View-aligned slicing is slower but much more memory efficient. It also facilitates consistent sampling distance.