- Test each solution set. Whichever gives the greatest (or the smallest) value is the maximum (or minimum) point
  - Lagrangian is a necessary but ***not a sufficient*** condition

Solving the toy example by Lagrange multiplier

- Maximize $5x_1x_2$ subject to $2x_1 + x_2 = 100$

$f(x_1, x_2) = 5x_1x_2,\ g(x_1, x_2) = 2x_1 + x_2 - 100$

$\mathscr{L}(x_1, x_2, \lambda_1) = 5x_1x_2 + \lambda_1(2x_1 + x_2 - 100)$

$$\begin{cases} \dfrac{\partial \mathscr{L}(x_1, x_2, \lambda_1)}{\partial x_1} = 5x_2 + 2\lambda_1 := 0 \\ \dfrac{\partial \mathscr{L}(x_1, x_2, \lambda_1)}{\partial x_2} = 5x_1 + \lambda_1 := 0 \\ \dfrac{\partial \mathscr{L}(x_1, x_2, \lambda_1)}{\partial \lambda_1} = 2x_1 + x_2 - 100 := 0 \end{cases}$$

$\Rightarrow (x_1, x_2, \lambda_1) = (25, 50, -125)$???

Generalized Lagrange multiplier

- Lagrange multipliers is generalized to include the ***inequality constraints*** under the Karush–Kuhn–Tucker (KKT) condition
- Standard form problem

  Minimize $f(\mathbf{x})$ subject to $g_i(\mathbf{x}) \leq 0\ (i = 1, \ldots, p)$ and $h_j(\mathbf{x}) = 0\ (j = 1, \ldots, m)$
  - If the task is to maximize $f(\mathbf{x})$, transform the problem into minimize $-f(\mathbf{x})$
- Lagrangian

$5b + 2c = 0 \qquad b = \dfrac{-2c}{5}$

$5a + c = 0 \qquad a = \dfrac{-c}{5}$

$2a + b = 100$

$\dfrac{-4c}{5} = 100$

$c = \cancel{100}^{25} \times -\dfrac{5}{\cancel{4}} = -125$