Testing the Validity of the Central Limit Theorem applied to Exponential Distribution

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Problem Definition

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is $\frac{1}{\lambda}$ and the standard deviation is also $\frac{1}{\lambda}$. Set $\lambda = 0.2$ for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should:

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

Introduction

According the to Central Limit Theorem (CLT) the sum of random variables (from unknown distribution with mean μ and variance σ) is also a random variable which follows the normal distribution with mean $\bar{X} = \mu$ and variance $s = \frac{\sigma}{n}$.

This is what we are going to test in this simulation

Investigation

We base our numerical experiment on the exponential distribution which has probability density function (pdf):

$$f(x;\lambda) = \lambda e^{-\lambda x}$$

In our case $\lambda = 0.2$ which results in mean $\mu = \frac{1}{\lambda} = \frac{1}{0.2} = 5$ and standard deviation $\sigma = \frac{1}{\lambda} = \frac{1}{0.2} = 5$

Before starting the simulation we need to define some general variables:

```
library(ggplot2)    # loading the plotting library

rate = 0.2          # rate of the exponential distribution
mean = 1/rate       # theoretical mean of the exponential distribution
sd = 1/rate         # theoretical standard deviation of the exponential distribution
n = 40              # number of sample variables from each distribution
num.sim = 1000      # number of distributions
binwidth = 0.2      # parameter used to scale the width of the histogram

sim = NULL          # initializing the vector of sample varaibles
mns = NULL          # initializing the vector of sample means
sds = NULL          # initializing the vector of sample standard deviations
```

Now we will continue by simulating 1000 exponential distributions with $\lambda = 0.2$ from which we will each time take only 40 random variables and compute their mean and standard deviation:

```
for (i in 1 : num.sim) {
    set.seed(i)
    sim = rexp(n = n, rate = rate)  # drawing n samples from the exponential distribution
    mns = c(mns, mean(sim))         # appending the sample mean to the vector of sample means
    sds = c(sds, sd(sim))           # appending the sample standrad deviation to the vector of standrd
}

df = data.frame(mns,sds)            # arranging the sample means and standard deviations to a data.fram
```

One we have the data in our hands we can calculate the mean of the sample means and sample standard deviations:

```
sim.mean = mean(df$mns)  # calculating the mean of sample means
sim.sd = mean(df$sds)    # calculating the mean of sample variances
```

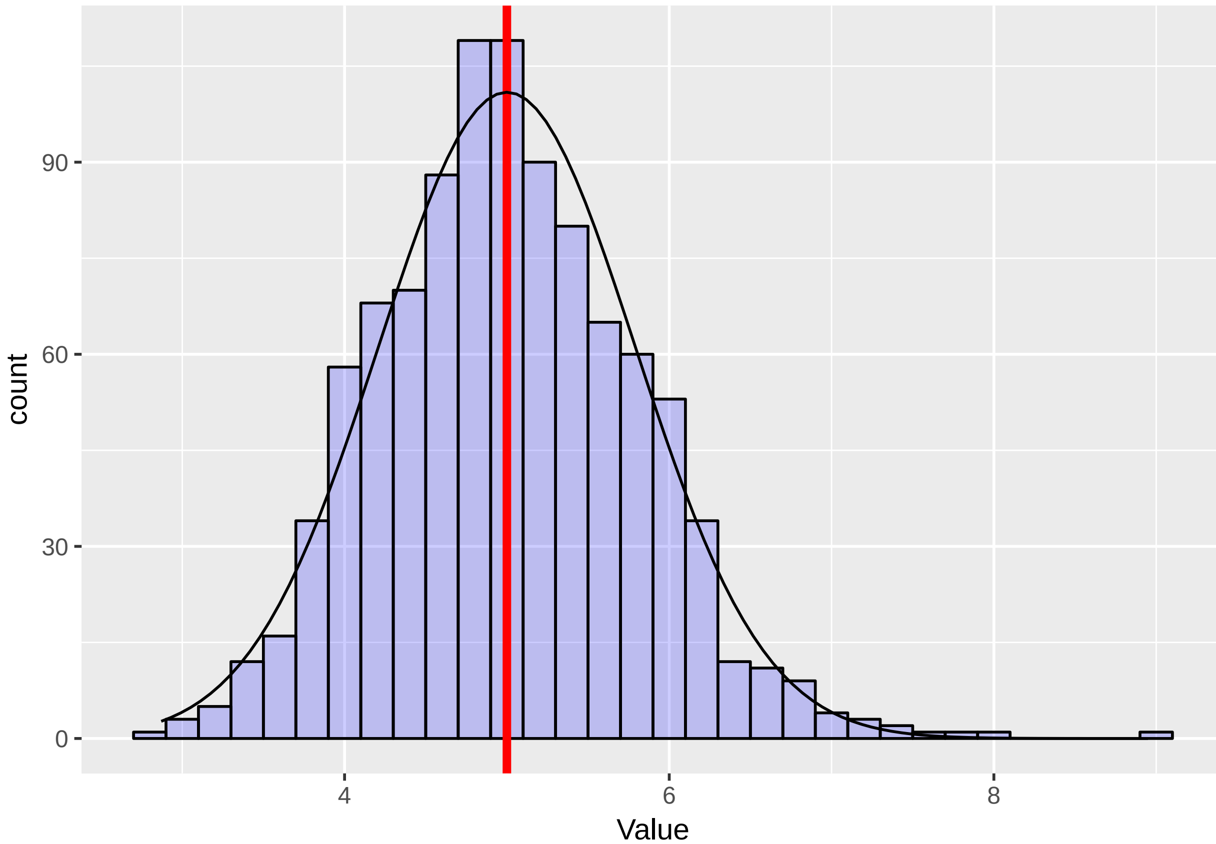
from our simulation we got sim.mean $= 5.002$ (compared with $\mu = 5$) and sim.sd $= 4.848$ (compared with $\sigma = 5$) which is pretty close.

However lets stop with all this writting and look at some graphs. Here is everything in only a single graph:

```
# plotting the variable with the simulated means
p = qplot(df$mns,
      geom="histogram",
      xlab = "Value",
      fill=I("blue"),
      col=I("black"),
      alpha=I(.2),
      binwidth=binwidth)

# plotting the theoretical mean as a vertical red line
# fitting a normal distribution N(mean, sd/sqrt(n)) to the histogram

p + geom_vline(xintercept = mean, size = 1.5, col="red") +
    stat_function(
        fun = function(x, mean, sd, n, bw){
            dnorm(x = x, mean = mean, sd = sd) * n * bw
        },
        args = c(mean = mean, sd = sd/sqrt(n), n = num.sim, bw = binwidth))
```



Some explanations. The bars represent the frequency plot of our sample means (histogram), the red line represends the theoretical mean of the exponential distribution and the black curve is representing a normal distribution with mean μ and standard deviation $\frac{\sigma}{n}$.

Conclusions

From the graph shown above we can conduct that in fact the CLT works and the distribution of sample means follows a normal distribution. As a little bonus here is a graph of the sample variances:

```
# plotting the variable with the simulated means
p = qplot(df$sds,
      geom="histogram",
      xlab = "Value",
      fill=I("green"),
      col=I("black"),
      alpha=I(.2),
      binwidth=binwidth)

# plotting the theoretical mean as a vertical red line
# fitting a normal distribution N(mean, sd/sqrt(n)) to the histogram

p + geom_vline(xintercept = mean, size = 1.5, col="red") +
    stat_function(
        fun = function(x, mean, sd, n, bw){
            dnorm(x = x, mean = mean, sd = sd) * n * bw
        },
```

```
       args = c(mean = mean, sd = sd/sqrt(n), n = num.sim, bw = binwidth))
```