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Development and implementation of engineering software for  
buckling strength assessment of plated marine structures by the  
recommended practices of leading classification societies

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Master Thesis

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## Statement of Originality

This thesis contains no material that has been accepted for the award of any other degree or diploma in any other university. To the best of my knowledge, this thesis contains no material previously published or written by another person, except where reference is made in the text.

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Development and implementation of engineering software for buckling strength assessment of plated marine structures by the recommended practices of leading classification societies

Master thesis

B.Sc. Eng. Vasil Yordanov  
Matriculation Number: 31132007

**The aim of this thesis is** to develop and implement engineering software for assessment the buckling strength of plated marine structures by the recommended practices of leading classification societies.

The following workpackages should be processed:

1. Literature survey about buckling, reliability and design of marine structures
2. Literature survey about recommended practices for buckling assessment of leading classification societies
3. Implementation of algorithms for buckling strength assessment presented by the classification societies
4. Implementation of algorithms for automatic buckling assessment of compound marine structures using elastic stress results from DNV Software - Sesam

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Adviser: Dipl. Eng. Plamen Stoykov

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## Preface

This thesis has been submitted as a partial fulfillment of the requirements for the Bulgarian Master Degree. The work was performed at Keppel Fels Baltech - Deepwater Technology Group in Varna, Bulgaria during the period March 2012 to June 2012. Assoc. Prof. Hristo Trendafilov, the Technical University of Varna, and Eng. Plamen Stoykov, Keppel Fels Baltech - Deepwater Technology Group, supervised this work.

Special thanks to Hristo Trendafilov, Plamen Stoykov and Lyudmil Stoev for giving me the opportunity to combine my daily work at Keppel Fels Baltech with a Master Degree study at the Technical University of Varna, for their guidance and for all our inspiring and motivational discussions.

Thanks to all the people and institutions that in some way have collaborated with me on various subjects of the present Master study. Especially, I would like to thank Eng. Stanimir Iliev and Eng. Martin Gospodinov for teaching me how to use DNV SESAM and Patran-Pre software.

Vasil Yordanov  
Varna, June 2012

## **Abstract**

Buckling is one of the major concerns in structural design. Structural elements subjected to a compressive load may experience instability far before reaching their yielding stress. Marine structures are exposed to continuously changing, various external loads which combination may result in a severe structural failure.

This Master Thesis consist of C++ implementation of the recommended practices for buckling assessment evaluation of leading classification societies such as “Det Norske Veritas” and “American Bureau of Shipping” in form of a cross-platform computer application which has to be used as a buckling filter in the stage of global strength analysis of marine structures.

**Keywords:** Buckling, Structural Design, Elastic Stability, Marine Structures

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# Abbreviations

<b>DNV</b>	Det Norske Veritas: Norwegian Classification Society
<b>ABS</b>	American Bureau of Shipping: US Classification Society
<b>FEM</b>	Finite Element Method
<b>FEA</b>	Finite Element Analysis
<b>BLS</b>	Buckling Limit State
<b>ULS</b>	Ultimate Limit State

# Chapter 1

## Introduction

### 1.1 General

In the recent years the development of floating and grounded offshore structures has been significant. Not only the dimensions of the structures have been increasing, but the operational depth tend always to increase as well as the development of new working sites and routes such as the Arctic offshore areas. Furthermore, it becomes more common that Marine Structures operate at sites with harsh weather conditions. Therefore, the issues of serviceability, more detail investigation of the environmental loads and the ultimate strength of the structure, in particular with respect to the buckling strength of the structure are very important to the research carried out in the offshore structures area.

A subject which seems to be somehow neglected in the past is the role of the structural arrangement with respect to the buckling and collapse of the marine structures due to the combined loading applied on the structure during the design of a offshore unit. Despite of yielding and fatigue failures the buckling failure strongly depends not only of the stress distribution but of the geometrical and topological configuration of the structure as well.

Nowadays the structural marine engineers go through extensive programs of structural and hydrodynamic calculations before constructing a vessel, as well as confirming the structural design with various classification rules. Besides supplying the engineers with exact solutions about the actual stress condition of the structure it is foreseen that future developments of structural assessment software will be able to give advice about the geometrical and topological arrangement of the structure with respect to the buckling and the ultimate strength of the structure.

### 1.2 Background

The structural designer of offshore units always pay attention to the information and guidance provided by the classification societies, as well as the previous experience of owners and designers. But when it comes to designing a prototype vessel or a vessel for not so well explored working site it is obligatory that the structure has to be designed with a minimum level of uncertainty, as well as reasonable safety factor with respect to the economical point of view.

Satisfying these requirements imposes the use of advanced numerical software for determination of wide range of environmental loads caused by the immense combinations of different weather conditions, as well as another advanced numerical software for investigation of the stress distributions in a structure which had been modeled in advance. Knowing that the buckling and ultimate strength of a given structure strongly depends of its geometrical and topological arrangement it means that in case when the structure has insufficient buckling or ultimate strength, the structure has to be rearranged and analyzed again,

which in most cases consumes a long time, because the whole procedure has to be repeated, see Figure 1.1.

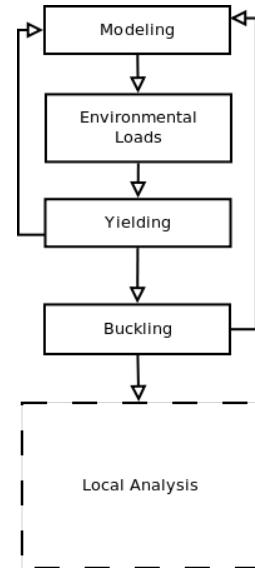


Figure 1.1: Global Analysis Procedure

### 1.3 Scope of This Master Thesis

The scope of the present Master Thesis is to address to the problems connected with the development of simplified automated buckling assessment software, which also can help the designer to choose the most appropriate structural configuration with respect to the buckling and ultimate strength. The main emphasis will be on the problems connected with providing a procedure for simplified buckling analysis and the development of the software tool required for applying this procedure.

## Chapter 2

# Buckling Analysis of Offshore Structures

### 2.1 General

The major building blocks of the ship hull are the stiffened plates. A plate panel can be part of a box girder, plated deck, plated shell etc. An example of a stiffened plate panel is shown on Figure 2.1a.

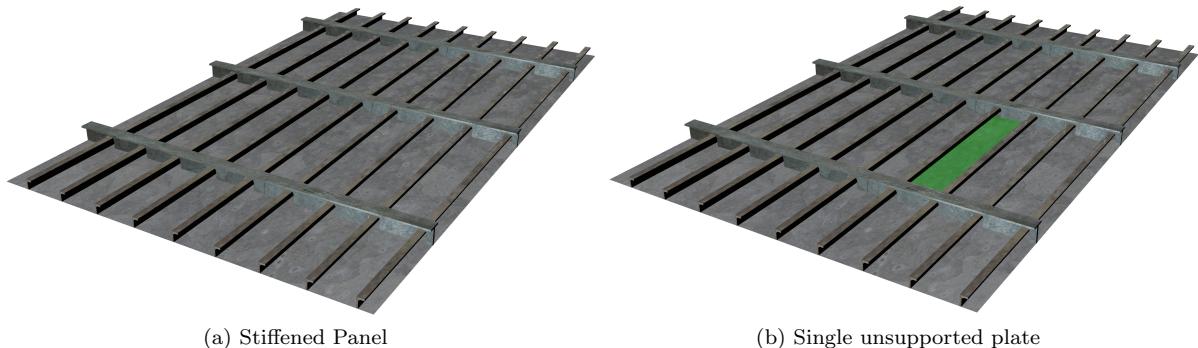


Figure 2.1: Basic building blocks

The plating is supported by a system of secondary supporting members such as stiffeners, which are supported by a system of primary supporting members such as girders. The strength the plates is most important since many kind of loads act directly on them, especially water pressure and longitudinal bending. In this case it is very important the failure modes of the plates to be considered and investigated in details see Figure 2.2. The scope of this thesis is to be developed an automatized procedure for checking each unsupported plate see Figure 2.1b for buckling failure according to different Classification Societies Recommended Procedures.

The recommended practices implemented in this thesis are presented in Appendix D, in particular with respect to the plate structural stability.

### 2.2 Numerical Methods

In this section are presented different numerical methods which can be applied for buckling assessment of marine and offshore structures, which are used nowadays. These methods will not be discussed in details,

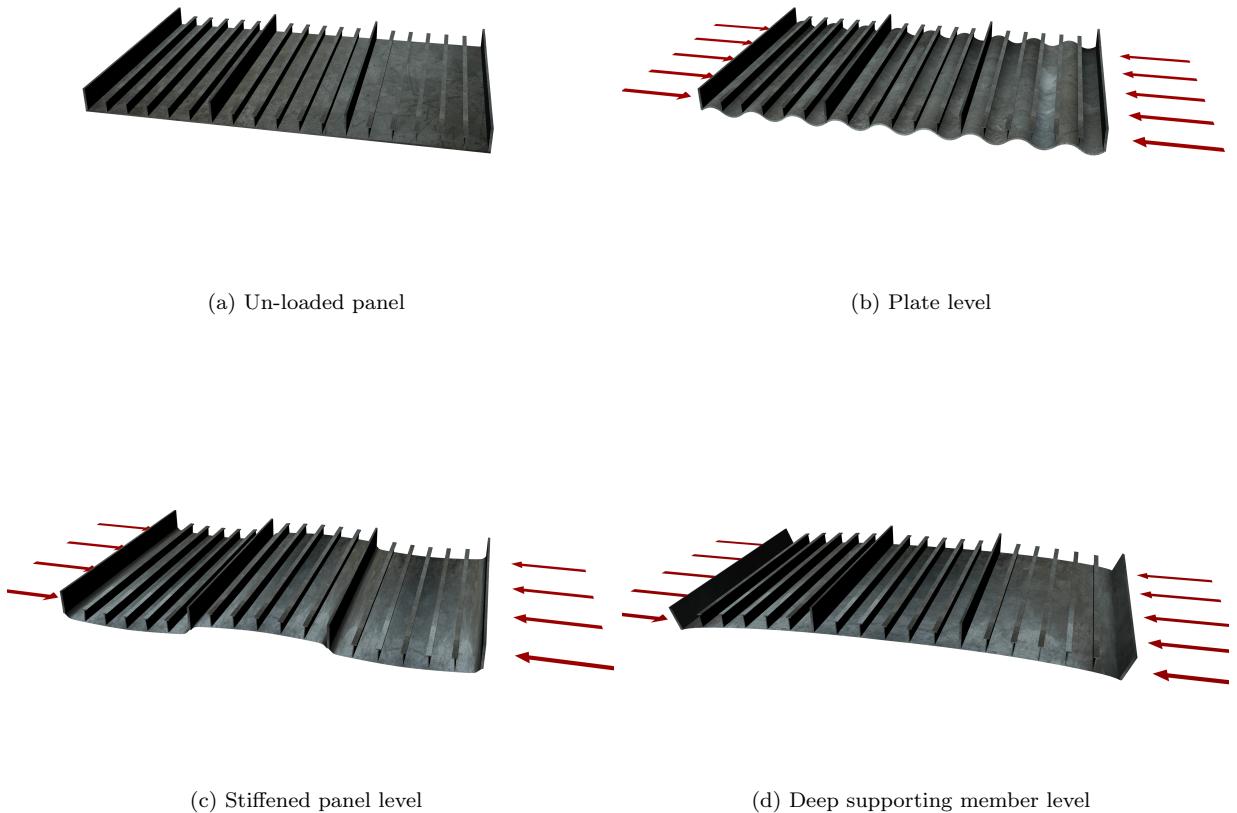


Figure 2.2: Failure modes of stiffened panel

because they are beyond the scope of this thesis, and are presented only for thoroughness of the thesis. In addition it is not reasonable and effective to be applied on a complete offshore construction such as a semi-submersible or a jack-up platform, because they are very time consuming, as well as the results are not presented in an appropriate format. These methods are suitable only for small structures. They require fine mesh representation of the entire structure in order the obtained results to be calculated with more accuracy, however it is always possible some of the buckling modes to be undetected. In addition some of the FEM software applications can work only with one load-case at the same time.

### 2.2.1 Finite Difference Method for Plate Buckling [8]

By replacing the derivatives in the governing differential equations of plate buckling by ordinary finite differences, difficult stability problems involving arbitrary configurations, boundary conditions and loadings can be solved numerically. With sufficient fine subdivision, the method yields acceptable accuracy.

The ordinary finite difference method in combination with Stodola-Vianello's iteration, offer usable approaches to most of the elastic stability problems that cannot be solved analytically.

### 2.2.2 Finite Element Method and Gridwork for Plate Buckling [8]

Compared to the finite difference method the finite elements and gridwork methods are more versatile, since they can cope with variable loads and boundary conditions along with irregular plate geometries.

The finite element method appears to be a flexible and powerful tool in the stability analysis of plates of arbitrary shape and boundary conditions. Various investigators have extended the FEM for prediction the buckling behavior of plane and stiffened plates in the plastic range. This technique, like most of the numerical methods, requires, however, high-speed electronic computers of considerable storage capacity.

## 2.3 Available Software

The scope of the work reported in the present section is to identify commercially available software for calculation of plate buckling modes. The source of the information is mainly the Internet.

Rule-Based	Semi-analytical	FEM
OSAP	DNV – PULS	Ansys
STIPLA		CSC - Elmer
DNV – Platework		ABAQUS
Genie – Plate Code-check		MSC – Nastran
		Code – Aster
		DNV – SESTRA

Table 2.1: Existing Buckling Assessment Software

All the software listed above, except OSAP and STIPLA, work with geometrical or finite element models of the structure of interest. However, OSAP and STIPLA are usually used as a simple code-checking worksheets, where the dimensions, the properties and the loadings are entered manually and as a result the user can only see the buckling unity-check. Whereas the FEM and the semi-analytical software is much more accurate and as a result the user can see the actual deformed shape of every individual member of the structure graphically.

## 2.4 Concluding Remarks

From an analysis of existing software for buckling analysis, the following conclusions can be drawn:

1. Concerning the software performance, due to the usage of highly sophisticated non-linear numerical methods, the overall time consumed for buckling analysis of complex structures on desktop computers is a lot for a single run.
2. All the existing software requires the finite element model of the structure to be modeled in details and to be topologically correct, which also requires a lot of time for 3D modeling

From all the listed above it is obvious that these software is inappropriate to be applied in the stage of initial design of offshore structures, when the structural details are not yet specified and it is not possible a detailed finite element method to be prepared.

Thus a buckling assessment software tool which can be used in the stage of initial design would be in a great help of the marine structural engineers. Such a software application should be able to execute the following tasks:

1. Working with roughly modeled structures
2. Working with topologically independent geometries

3. It is also recommended the usage of simplified buckling assessment methods, such as the published by the Classification Societies

## Chapter 3

# Development of Buckling Assessment Software

### 3.1 General

This chapter describes the developments of the current buckling assessment software that have been carried out:

- It is described which environmental loads are used.
- It is also described which stresses are used for the buckling calculation.
- It is also described how the different structural arrangements are being checked for buckling failure at the same time, without their actual modeling.
- In addition it is described how the developed procedure has to be executed, in particular with respect to usage of DNV – SESAM software package.

### 3.2 Environmental loads [2]

Even though the determination of the environmental loads is beyond the scope of this thesis, the philosophy for their obtaining is presented just for thoroughness of the review.

The calculation of wave loads and load combinations is the first step in marine structural design. This step involves making predictions of the worst seas in which the offshore unit could encounter within its lifetime, this assumption is made with accordance with the scatter diagram, see Table 3.1, of the future installation site of the offshore unit, as the ultimate goal is the design value of the ship's response to be determined.

For the prediction of the design ship's response may be used the so called “design wave method”. This method assumes that the largest waves appear in the most severe stationary sea-state, which the ship is likely to encounter. Thus these wave values are used as the design values of the ship, along with a couple of less severe sea-states. This method may not be considered to be accurate, because a larger wave may be encountered in a less severe sea-state. However it is less time-consuming and is the preferred method unless a more accurate determination of the design values is required.

The second method requires that all possible sea-states, which the ship is likely to encounter in its lifetime, be evaluated. A complete analysis of all the sea-states is carried out and the different sea-states are weighted to the likelihood of being encountered by the ship. This method is computationally more expensive but is a more realistic analysis.

$H_s$ (m)	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	sum
1	59	403	1061	1569	1634	1362	982	643	395	232	132	74	41	22	12	7	4	2	2	8636
2	9	212	1233	3223	5106	5814	5284	4102	2846	1821	1098	634	355	194	105	56	30	16	17	32155
3	0	8	146	831	2295	3896	4707	4456	3531	2452	1543	901	497	263	135	67	33	16	15	25792
4	0	0	6	85	481	1371	2406	2960	2796	2163	1437	849	458	231	110	50	22	10	7	15442
5	0	0	0	4	57	315	898	1564	1879	1696	1228	748	398	191	84	35	13	5	3	9118
6	0	0	0	0	3	39	207	571	950	1069	885	575	309	142	58	21	7	2	1	4839
7	0	0	0	0	0	2	27	136	347	528	533	387	217	98	37	12	4	1	0	2329
8	0	0	0	0	0	0	0	2	20	88	197	261	226	138	64	23	7	2	0	1028
9	0	0	0	0	0	0	0	0	2	15	54	101	111	78	39	14	4	1	0	419
10	0	0	0	0	0	0	0	0	0	2	11	30	45	39	22	8	2	1	0	160
11	0	0	0	0	0	0	0	0	0	0	2	7	15	16	11	5	1	0	0	57
12	0	0	0	0	0	0	0	0	0	0	0	1	4	6	5	2	1	0	0	19
13	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	1	0	0	0	6
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
sum	68	623	2446	5712	9576	12799	14513	14454	12849	10225	7256	4570	2554	1285	594	263	117	52	45	100001

Table 3.1: Wave Scatter Diagram, Representative from the Northern North Sea ( Faltinsen, 1990 )

Once the method to be used is chosen, and the design wave load is determined, the ship's required structural strength may be evaluated. Nowadays the actual calculation for the design environmental loads is done using non-linear hydrodynamic software such as SESAM – WADAM module.

### 3.3 Global Response and Working Loads

Based on the design Environmental loads discussed in section 3.2 the ship's global response and the design loads have to be determined for every single load-case. Nowadays this calculation is done using linear static FEM analysis software, such as SESAM – SESTRA module.

Later the actual stress results for all design waves (load-cases) are scanned for the minimum values of  $\sigma_x$ ,  $\sigma_y$  and the maximum absolute maximum of  $\tau_{xy}$ , in addition these extremal results are later combined with the static load-case for obtaining the working stresses for the so called “buckling load-case”.

### 3.4 Buckling assessment

The method which is proposed in this thesis is tended to be applied on roughly modeled structures, which has to meet the following requirements:

1. All the plated and shell structures such as decks, shells, web frames, bulkheads, platforms etc. has to be modeled correctly
2. All the cut-outs in the plated structures should be defined
3. Some of the secondary supporting members such as stiffeners may be grouped to a single secondary supporting member with a equivalent section area, see Figure 3.2.
4. The finite element mesh size may be taken equal to the girder spacing in the corresponding area

The listed requirements are compulsory in order the result stress plot to be evaluated correctly, because this stress response and more specific the decomposed membrane stresses, see Figure 3.1, will be used later for the evaluation of the buckling strength of the structure.

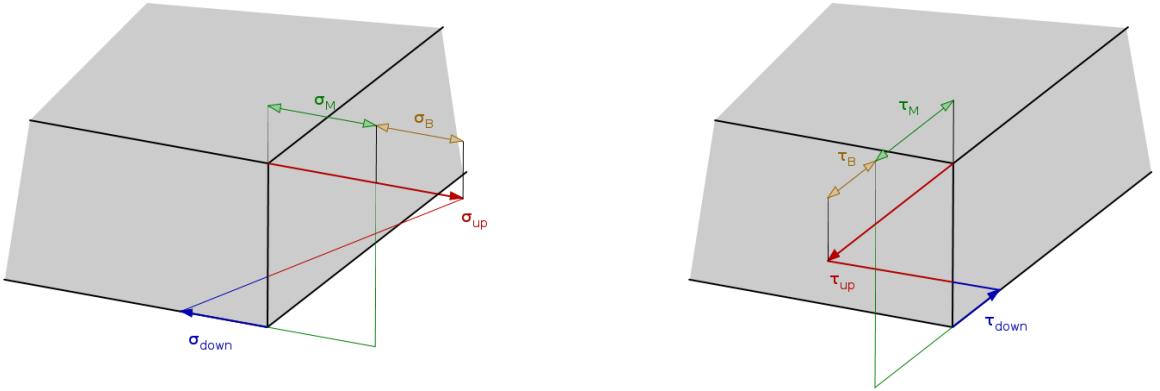


Figure 3.1: Decomposed Stresses for 4 node shell element

On Figure 3.1  $\sigma_M = \frac{\sigma_{up} + \sigma_{down}}{2}$  and  $\sigma_B = \frac{\sigma_{up} - \sigma_{down}}{2}$  are the decomposed membrane and bending stresses.

The actual method diagram is shown on Figure 3.3. Using this approach at the position of each finite element, each of the user inputted typical panels is placed and subjected to the local conditions. More specific the physical properties of the finite element are assigned on the typical panel and the latter is

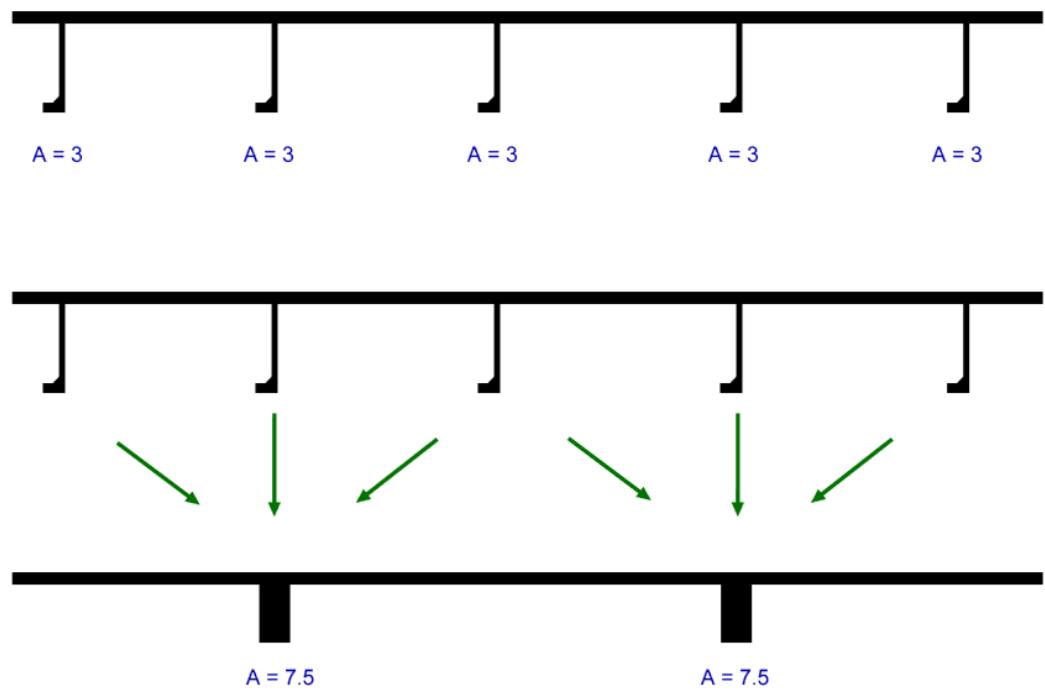


Figure 3.2: Stiffener reduction approach

subjected to the averaged membrane stresses calculated for the current finite element, then the buckling mode can be calculated. Doing this for all finite elements provides information about the buckling modes in each finite element for the entire finite element model.

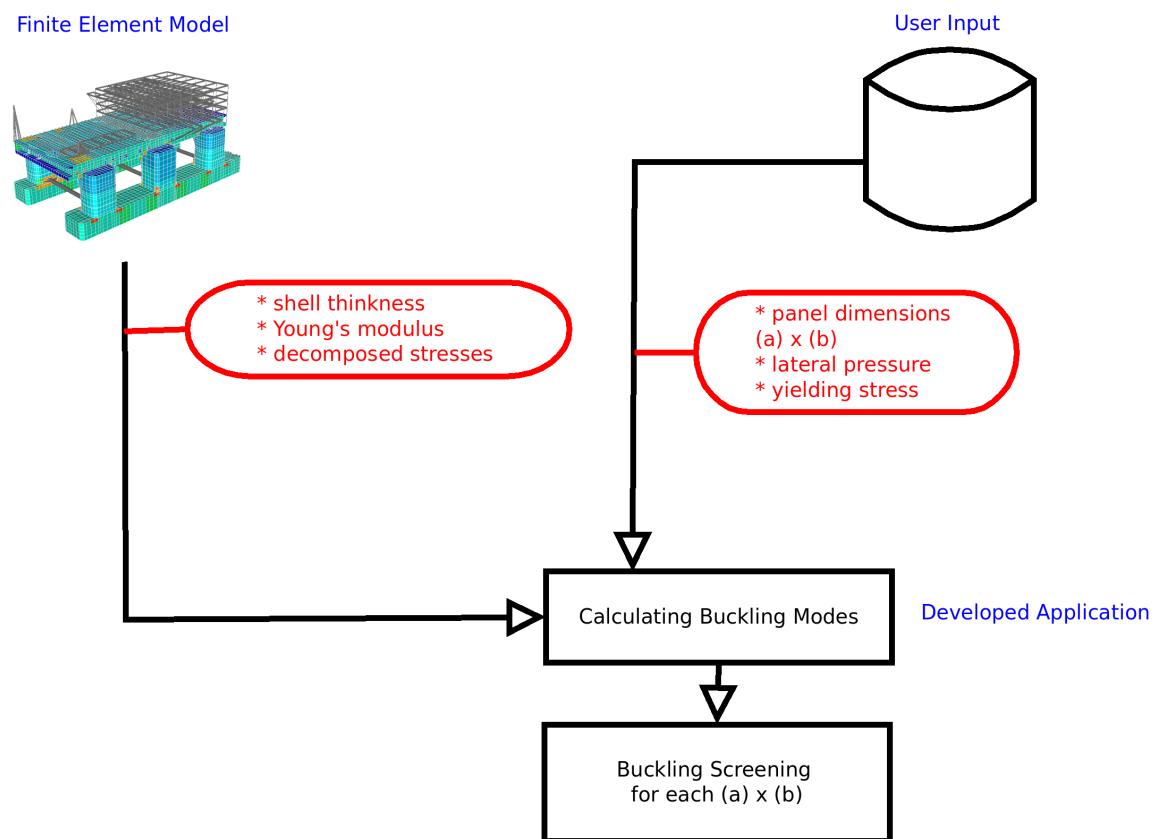


Figure 3.3: Buckling assessment diagram

## Chapter 4

# Analysis and Postprocessing of Buckling Results

### 4.1 Analysis

In this section will be presented a short comparison between the interaction equations of the different implemented buckling assessment methods.

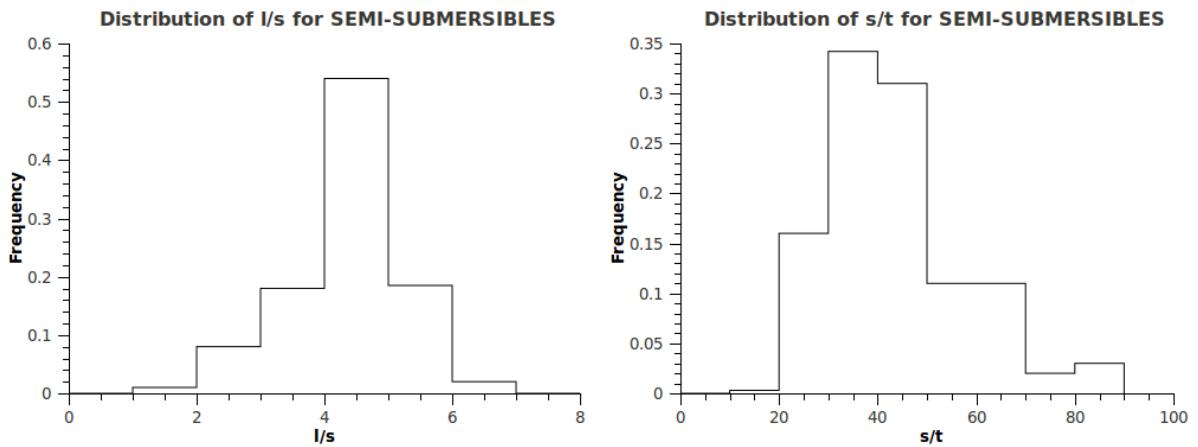


Figure 4.1: Statistical Data for Semi-Submersibles

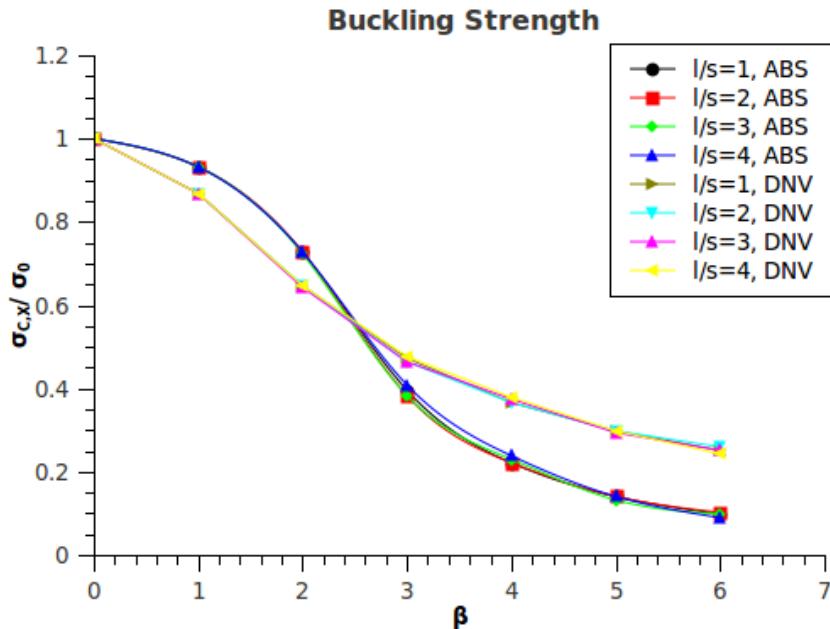


Figure 4.2: Longitudinal Buckling Strength comparison between ABS and DNV rules

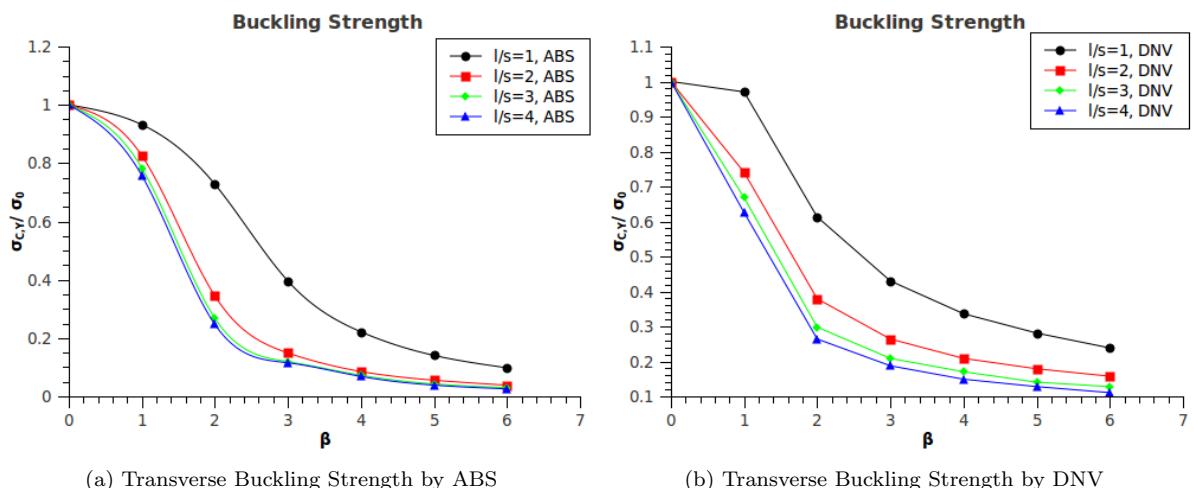


Figure 4.3: Transverse Buckling Strength comparison between ABS and DNV rules

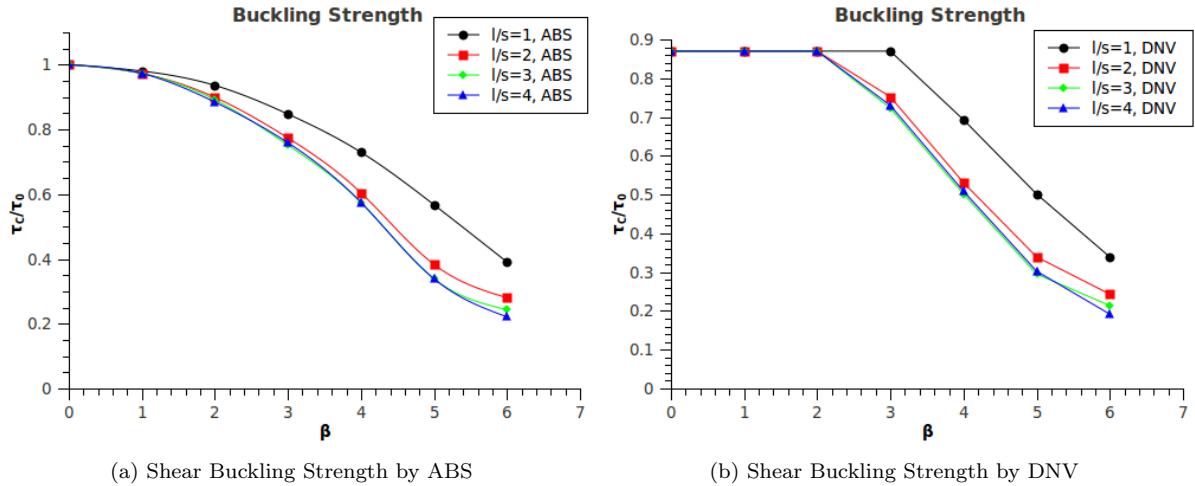


Figure 4.4: Shear Buckling Strength comparison between ABS and DNV rules

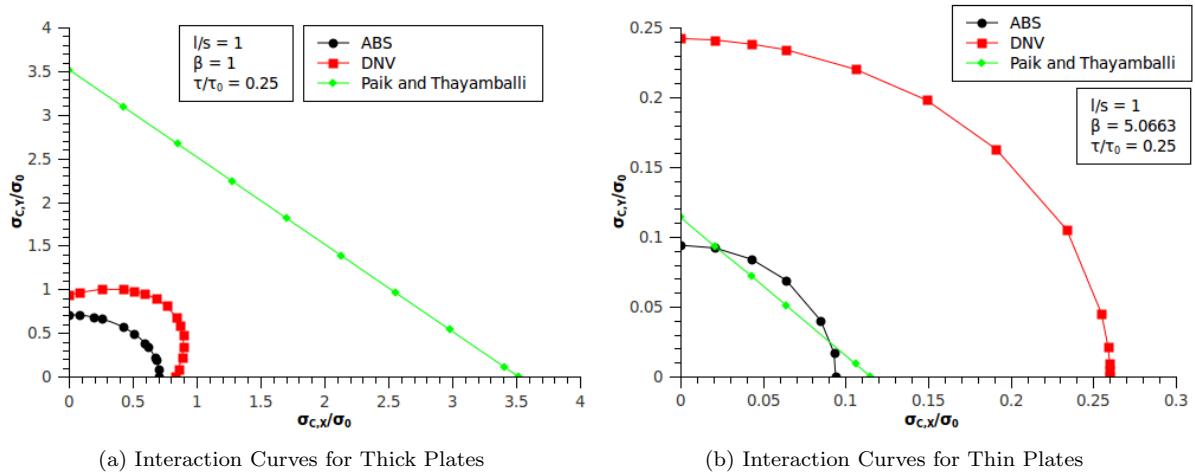


Figure 4.5: Interaction Curves Comparison for Square Plates

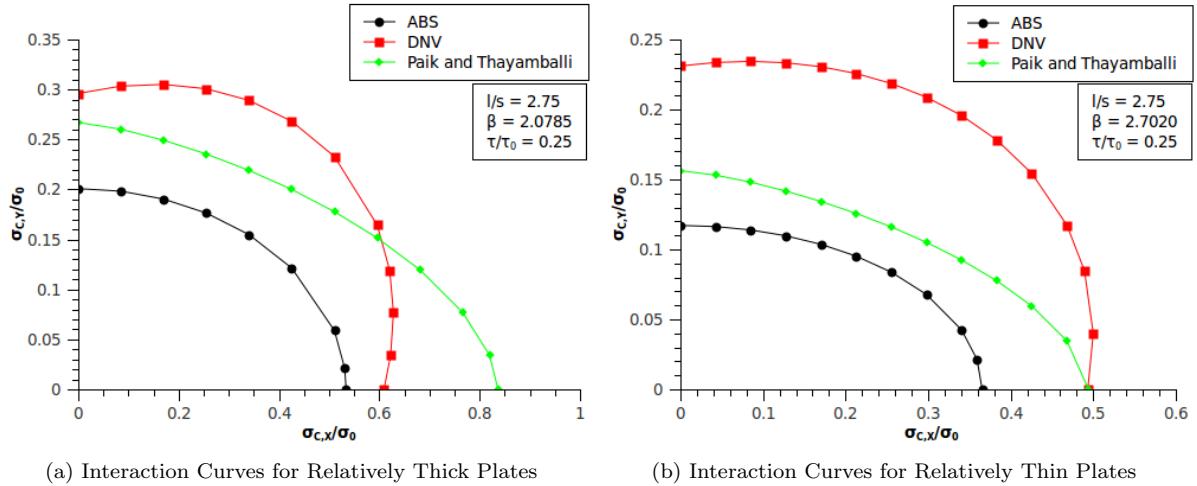


Figure 4.6: Influence of the Plate Slenderness over the Interaction Curves for Plates with Aspect Ratio 2.75

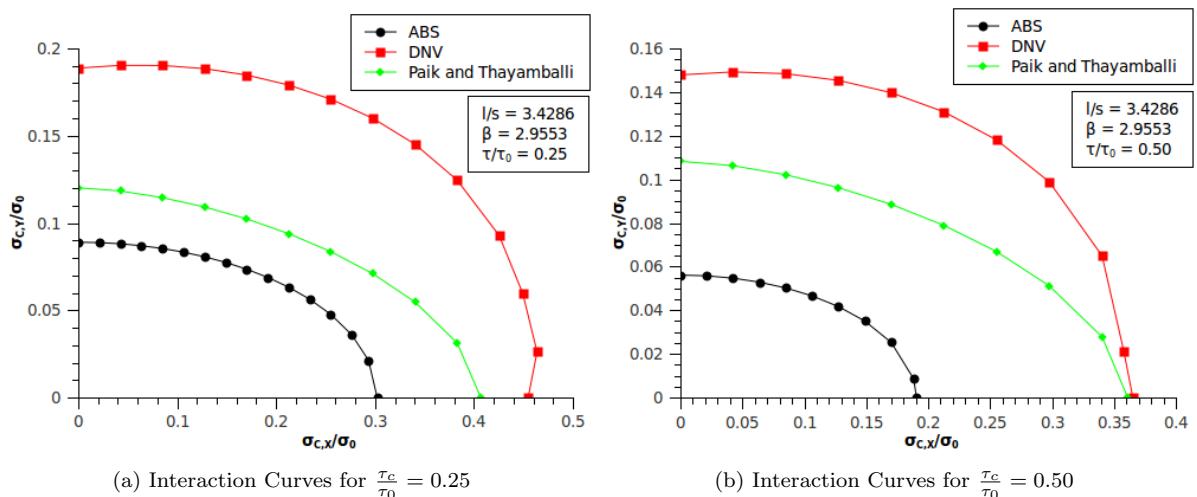


Figure 4.7: Influence of the Shear Stress over the Interaction Curves for Plates with Aspect Ratio 3.4286

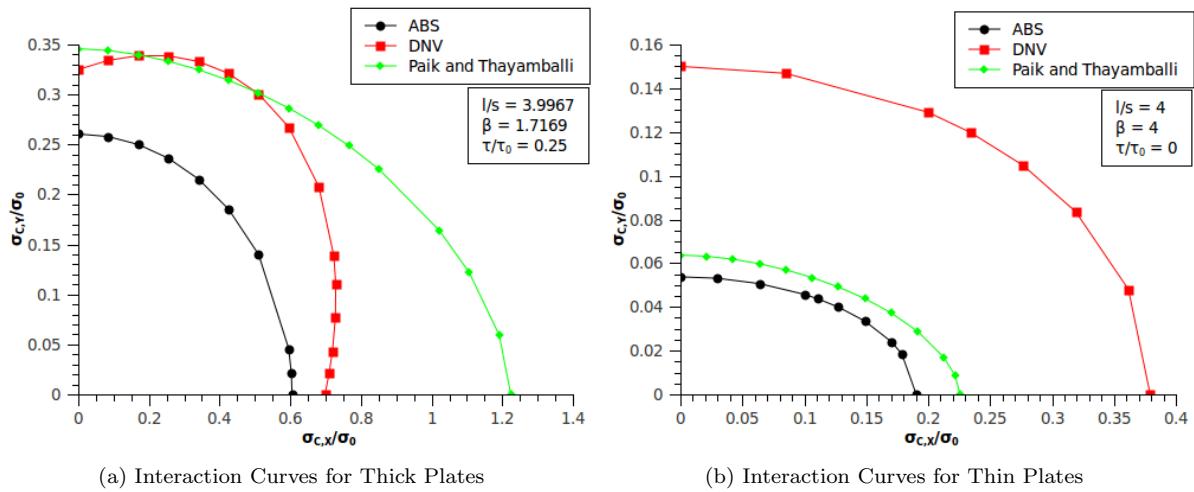


Figure 4.8: Influence of the Plate Slenderness over the Interaction Curves for Plates with Aspect Ratio 4

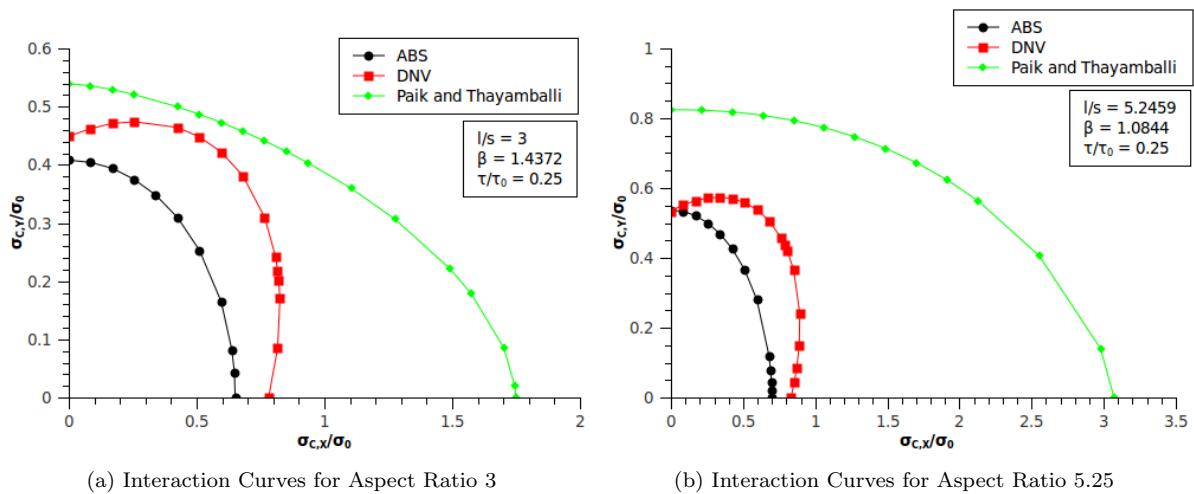


Figure 4.9: Influence of the Plate Aspect Ratio over the Interaction Curves for Plates with slenderness near 1

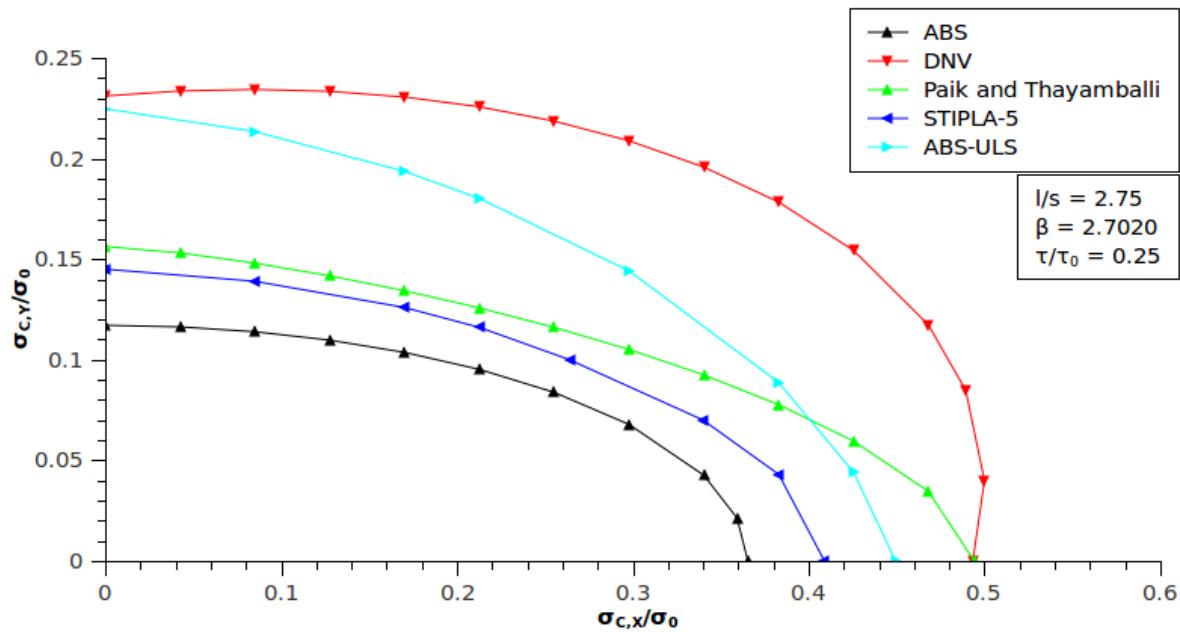


Figure 4.10: More detailed comparison for plates with aspect ratio 2.75

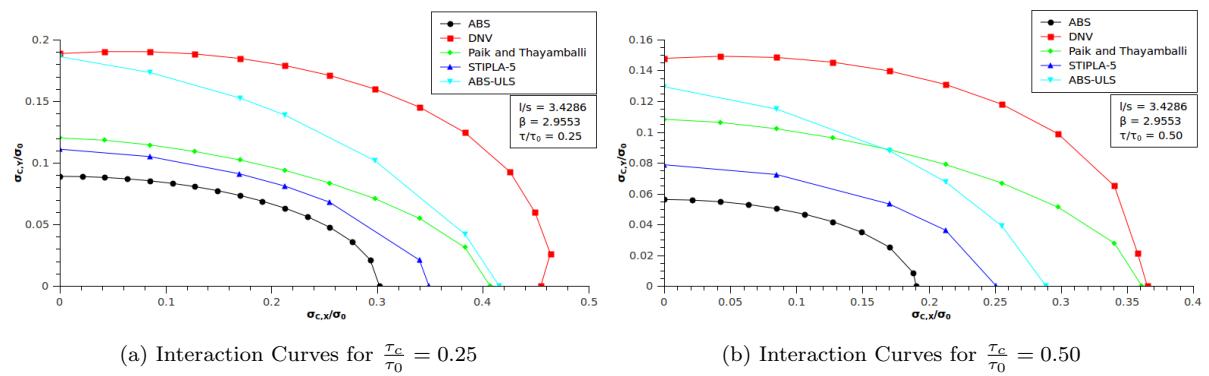


Figure 4.11: More detailed analysis of the influence of the Shear Stress over the Interaction Curves for Plates with Aspect Ratio 3.4286

## 4.2 Results Validation

During the development of the engineering software, a considerable number of test cases were investigated and the results were compared with other software applications which had already been approved by the Classification Societies.

No	$\ell$	$s$	$t$	$\sigma_0$	$E$	$\nu$	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	UC – app	UC – OSAP
No	m	m	mm	MPa	GPa	–	MPa	MPa	MPa	–	–
1	2438	610	12	235	210	0.3	13.24	8.09	23.62	0.078	0.078
2	2438	610	14	235	210	0.3	16.69	26.47	21.76	0.142	0.142
3	2438	610	14	235	210	0.3	25.81	38.41	16.2	0.232	0.232
4	2438	610	14	235	210	0.3	24.04	20.22	33.66	0.181	0.181
5	2438	610	12	235	210	0.3	9.94	-19.73	28.56	0.174	0.174
6	2438	610	14	235	210	0.3	8.49	-2.27	24.34	0.06	0.06
7	2438	610	14	235	210	0.3	13.17	3.16	25.18	0.069	0.069
8	2438	610	14	235	210	0.3	24.56	52.83	25.06	0.425	0.425
9	2438	610	12	235	210	0.3	6.3	32.42	29.14	0.326	0.326
10	2438	610	12	235	210	0.3	8.5	48.55	34.82	0.66	0.66
11	2438	610	12	235	210	0.3	9.15	61.63	39.67	1.024	1.024
12	2438	610	12	235	210	0.3	10.53	65.51	39.99	1.14	1.14
13	2438	610	12	235	210	0.3	10.99	49.42	30.3	0.652	0.652
14	2438	610	12	235	210	0.3	11.09	61.87	33.45	0.987	0.987
15	2438	610	12	235	210	0.3	11.1	70.45	35.79	1.261	1.261
16	2438	610	12	235	210	0.3	12.77	73.66	38.27	1.387	1.386
17	2438	610	19	235	210	0.3	15.56	22.91	34.89	0.146	0.146
18	2438	610	19	235	210	0.3	21.37	16	22.29	0.074	0.074
19	2438	610	19	235	210	0.3	10.63	54.38	21.83	0.204	0.204
20	2438	610	19	235	210	0.3	17.23	14.71	22.01	0.065	0.065
21	2438	610	19	235	210	0.3	17.38	49.89	16.54	0.167	0.167
22	2438	610	10	235	210	0.3	36.55	0.06	32.77	0.188	0.188
23	2438	610	10	235	210	0.3	37.94	4.61	37.12	0.237	0.237
24	2438	610	10	235	210	0.3	50.46	19.39	44.37	0.528	0.528
25	2438	610	12	235	210	0.3	60.78	19.38	45.03	0.448	0.448
26	2438	610	12	235	210	0.3	67.04	18.04	45.72	0.478	0.478
27	2438	610	12	235	210	0.3	38.5	28.86	33.92	0.369	0.369
28	2438	610	12	235	210	0.3	29.71	25.84	37.95	0.334	0.334
29	2438	610	19	235	210	0.3	14.03	71.37	30.1	0.36	0.36
30	2438	610	12	235	210	0.3	48.81	5.83	33.41	0.223	0.223
31	2438	610	19	235	210	0.3	10.6	2.21	21.06	0.044	0.044
32	2438	610	12	235	210	0.3	40.19	15.54	32.89	0.233	0.233
33	2438	610	19	235	210	0.3	11.98	25.42	18.89	0.072	0.071
34	2438	610	10	235	210	0.3	35.14	34.31	30.02	0.719	0.719
35	2438	610	10	235	210	0.3	23.04	24.47	19.68	0.353	0.353
36	2438	610	12	235	210	0.3	27.75	98.84	49.19	2.495	2.495
37	2438	610	12	235	210	0.3	46.5	115.33	47.37	3.34	3.359
38	2438	610	12	235	210	0.3	37.45	86.35	46.24	1.969	1.969
39	2438	610	12	235	210	0.3	57.84	91.37	59.55	2.397	2.397
40	2438	610	12	235	210	0.3	38.6	93.78	61.4	2.44	2.44
41	2438	610	12	235	210	0.3	41.05	82.25	55.31	1.916	1.916
42	2438	610	12	235	210	0.3	19.53	78.85	48.2	1.661	1.662
43	2438	610	12	235	210	0.3	19.76	93.16	44.18	2.184	2.184
44	2438	610	12	235	210	0.3	26.6	79.23	47.53	1.683	1.683
45	2438	610	12	235	210	0.3	29.27	85.44	42.42	1.876	1.876
46	2438	610	12	235	210	0.3	34.21	90.38	48.13	2.138	2.138

Table 4.1: Comparison between the BLS results from OSAP and the developed application

### 4.3 Post-processing

The results shown in Appendix C are obtained for a Semi-Submersible unit. The Elastic Stress FEM analysis is performed for the following environmental conditions shown in Table 4.2, 4.3 and 4.4.

No	1	2	3	4	5	6	7	8	9	10
Direction $\theta$ [deg]	0.00	22.50	30.00	45.00	67.50	90.00	112.50	135.00	157.50	180.00

Table 4.2: Wave Directions used for the Global Analysis

Frequencies					
No	$\omega$ [ $sec^{-1}$ ]	No	$\omega$ [ $sec^{-1}$ ]	No	$\omega$ [ $sec^{-1}$ ]
1	0.20	10	0.70	19	1.40
2	0.30	11	0.75	20	1.50
3	0.35	12	0.80	21	1.60
4	0.40	13	0.85	22	1.70
5	0.45	14	0.90	23	1.80
6	0.50	15	1.00	24	1.90
7	0.55	16	1.10	25	2.00
8	0.60	17	1.20		
9	0.65	17	1.30		

Table 4.3: Wave Frequencies used for the Global Analysis

No	Wave Direction $\theta$	Frequency $\omega$
1	0.00 [deg]	0.80 [ $s^{-1}$ ]
2	45.00 [deg]	0.65 [ $s^{-1}$ ]
3	67.50 [deg]	0.70 [ $s^{-1}$ ]
4	90.00 [deg]	0.55 [ $s^{-1}$ ]
5	90.00 [deg]	0.60 [ $s^{-1}$ ]
6	90.00 [deg]	0.70 [ $s^{-1}$ ]
7	90.00 [deg]	0.70 [ $s^{-1}$ ]
8	90.00 [deg]	0.75 [ $s^{-1}$ ]
9	90.00 [deg]	0.80 [ $s^{-1}$ ]
10	90.00 [deg]	0.90 [ $s^{-1}$ ]
11	112.50 [deg]	0.70 [ $s^{-1}$ ]
12	135.00 [deg]	0.75 [ $s^{-1}$ ]
13	135.00 [deg]	0.85 [ $s^{-1}$ ]
14	180.00 [deg]	0.75 [ $s^{-1}$ ]

Table 4.4: Design Waves used for the Global Analysis

In Appendix C.2 are presented the results for the upper deck of the semi-submersible shown in Appendix C.1.

In Appendix C.3 is presented a mating barge which is designed to transport the upper hull of a semi-submersible. The buckling results are presented for the mating barge as well as the upper hull of the semi-submersible. For this study are used the most conservative rules of ABS – BLS.

## Chapter 5

# Conclusions and Recommendations for Future Work

### 5.1 Conclusions

The aim of the present Master thesis was to address the problems connected with the development of buckling assessment software applied in the stage of initial design of marine structures, with main emphasis on the problems of applying a buckling assessment algorithm on a simplified model of the actual structure.

For buckling assessment algorithms, similarly to the buckling code-check software applications, were implemented *DNV RP-C201 – October 2010, ABS - Guide for Buckling and Ultimate Strength Assessment for Offshore Structures – February 2012* as well as a regression algorithm published by Paik and Thayamballi in [7]. These procedures are simple, reliable and especially sufficient for the evaluation of the buckling strength of offshore structures.

A part of the Master thesis was devoted to the application, comparison and the result validation of the various algorithms. The results which can be obtained by the tool are demonstrated with examples, and a study concerning the behavior of the different procedures was shown.

The main contribution of this Master thesis is that it has been shown that the buckling assessment algorithms can be applied to a simplified structures, where the structural arrangement is not yet specified. In addition the proposed methodology gives information about dozens of different structural arrangements with a single study, which assuredly reduces the required time for structural design, increases the reliability of the designed structure and makes the optimization of the structural arrangement more easy and available.

### 5.2 Recommendations for Future Work

A central and important issue for further study is to examine the possibilities of implementing the following additional features:

- Implementing algorithms for buckling check of entire stiffened panels ( plate panels including supporting stiffeners )
- Implementing algorithms for buckling check of plate panels with linear varying compression
- Implementing algorithms for buckling check of curved and cylindrical shells and stiffened panels.

Another logical development of the software can be addressed to automated design and optimization of the structural arrangement of marine and offshore structures:

- The procedure can be modified in order to choose the most appropriate panel in geometrical, topological and strength point of view for a specified region.
- Another option is a new algorithm for optimal typical panel tessellation to be developed in order a complete structural arrangement to be obtained automatically.

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- [14] “*Buckling Strength of Plated Structures*” Det Norske Veritas 2002
- [15] “*SESAM Input Interface File*” Det Norske Veritas 1996
- [16] “*SESAM Interface File*” Det Norske Veritas 1996
- [17] “*SESAM User Manual - SESTRA*” Det Norske Veritas 2007
- [18] “*SESAM User Manual - Xtract*” Det Norske Veritas 2011

## Appendix A

# Technical Specifications

In this appendix is provided all meaningful technical information about this thesis. Starting with a short description of the relevant file formats and finishing with a detailed explanation of reading, manipulating and writing the data.

### A.1 Xtract List File Format - \*.TXT

This file is exported from the Xtract application its structure is described in Table A.2. It is done so, because of the fact that the scaling of the design waves and their further scanning and combining is done in Xtract using the results from the SESTRA Linear Analysis. The developed application uses three such files. One for the minimum of  $\sigma_x$ , one for the minimum of  $\sigma_y$  and one for the absolute maximum of  $\tau_{xy}$ . These three stresses are used for the actual buckling check of every element from the construction.

Superelement ID	Element No	Nodes			
		stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
ID	No	stress	stress	stress	stress
⋮					
ID	No	stress	stress	stress	stress

Table A.1: Xtract List File format

### A.2 SESAM Interface Input File - T\*.FEM

This file is normally exported from Patran-Pre and it contains all node, element, material and mass information for the structure in a form of a ASCII formated command-sequential file. Its structure is really complex and it is fully described in [15]. The main purpose of this file is to provide us all thicknesses and the material properties of the finite elements. This information is later combined with the information from the Xtract List File in order to provide all data for the buckling calculation.

The buckling screening results are also provided in this file format. This is so because the SESAM Post-Processor - Xtract can open and visualize single T\*.FEM files.

### A.3 Buckling Results Riles - \*.CSV

These files are generated from the developed application in order to provide more detail information about the buckling state of any single finite element. This is a simple comma-separated file with the following structure:

Element No	Unity Check	Result	stress			Thickness	Young's Modulus
No	UC	Yes/No	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	t	E
No	UC	Yes/No	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	t	E
No	UC	Yes/No	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	t	E
No	UC	Yes/No	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	t	E
:							
No	UC	Yes/No	$\sigma_x$	$\sigma_y$	$\tau_{xy}$	t	E

Table A.2: Buckling Results File Structure

This file can easily be opened in Excel and manipulated in any matter without any difficulty.

### A.4 Performance

The performance tests were made on Microsoft Windows Vista with Intel Core 2 Duo T6400 CPU.

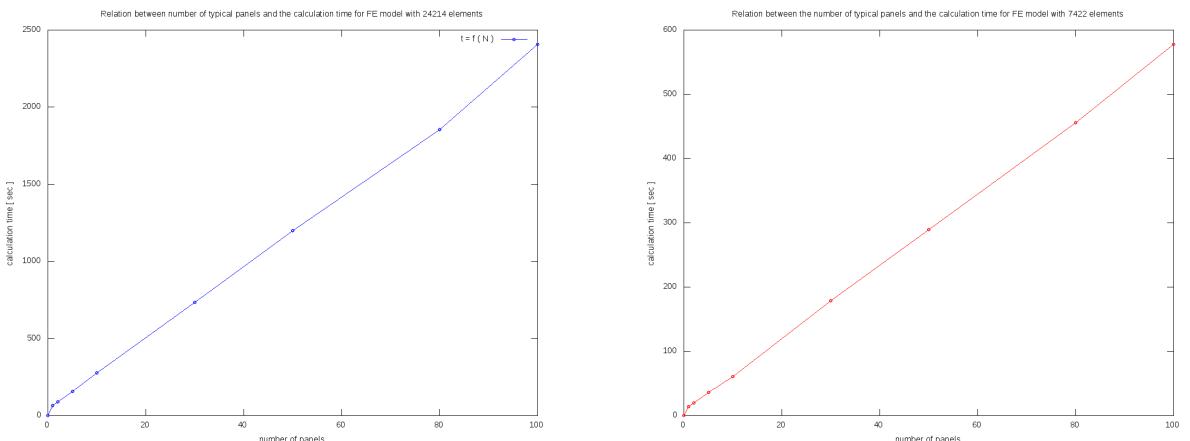


Figure A.1: Results from the performance tests of the developed application

## Appendix B

# The use of software

### B.1 List of files

These are the files required from the application:

**sigma-m-x.txt** This is a standard Xtract list file which is intended to content the normal stress in x direction for all selected elements and the chosen load-case.

**sigma-m-y.txt** This is a standard Xtract list file which is intended to content the normal stress in y direction for all selected elements and the chosen load-case.

**tau-m-xy.txt** This is a standard Xtract list file which is intended to content the tangential stress in xy direction for all selected elements and the chosen load-case.

**T\*.FEM** This is a standard SESAM interface input file, which contains all required geometrical and material data for the selected super-element.

### B.2 Entry data

**Lateral Pressure** By this field the user can apply lateral pressure to all elements. This is necessary because in the T\*.FEM file this information is not included neither in any of the Xtract list files.

**Yield Strength** By this field the user can define the yield strength for all elements. This is necessary because this information is not available in the SESAM Interface Input File /T\*.FEM/.

**Classification Society** By this field the user can choose the recommended practice by which the actual buckling check will be executed.

**Typical Panel - Length and Width** By this field the user can control the dimensions and the count of the typical panels, which have to be checked for buckling. There is no limit for the count or the dimensions of the panels.

### B.3 Running the application

The application is developed to be platform independent so it can be executed natively under Linux, Windows and MAC OS. This is possible because it had been developed using Qt4 for C++. The application has its own graphical user interface which is completely compatible with all platforms. The GUI is user-friendly and all user-controlled fields are already explained in B.2

## Appendix C

# Test Cases Results

In this appendix are provided all test cases which had been investigated during the development of this thesis. The information listed here is only in form of stress plots and buckling screening, exported from Xtract application.

### C.1 SSDT

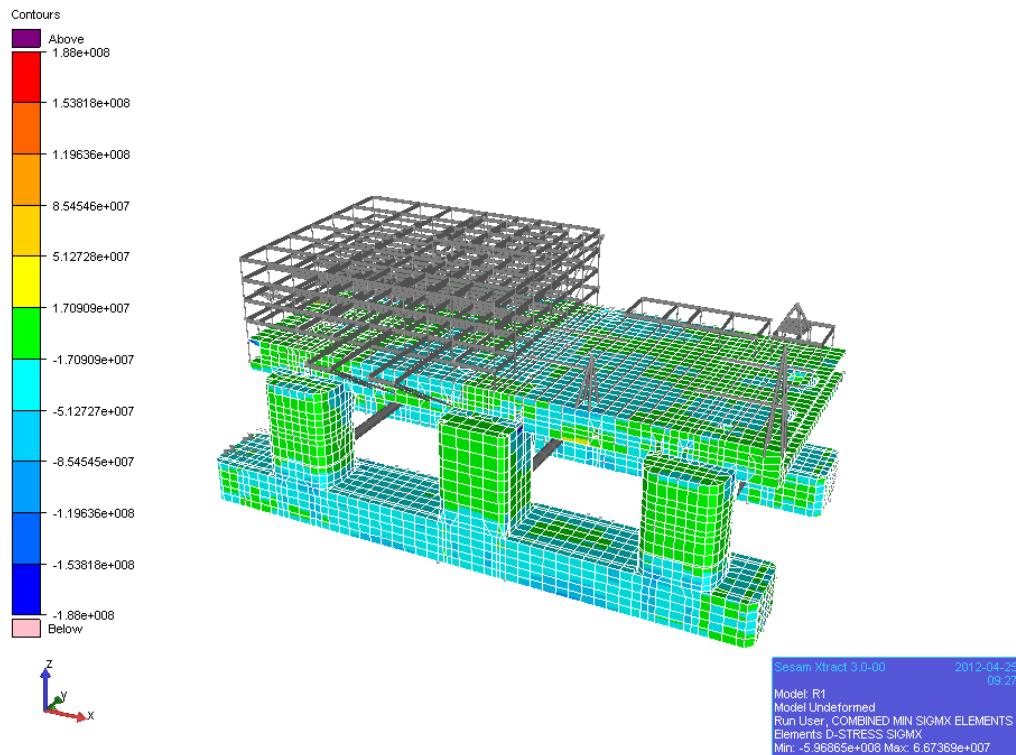
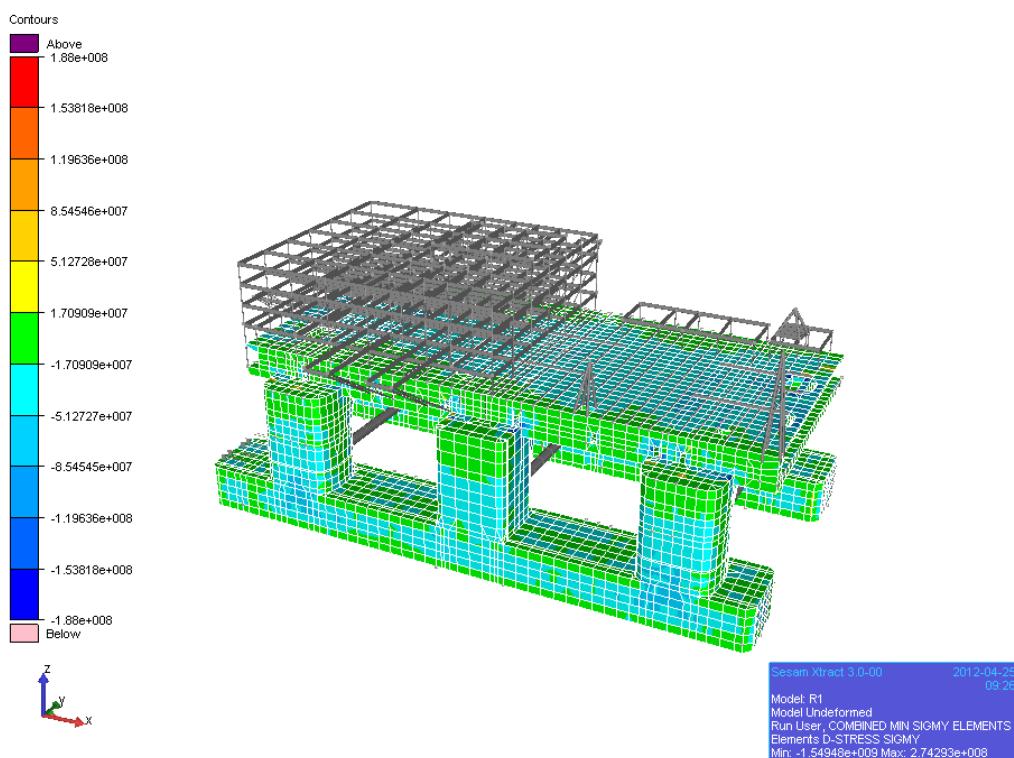
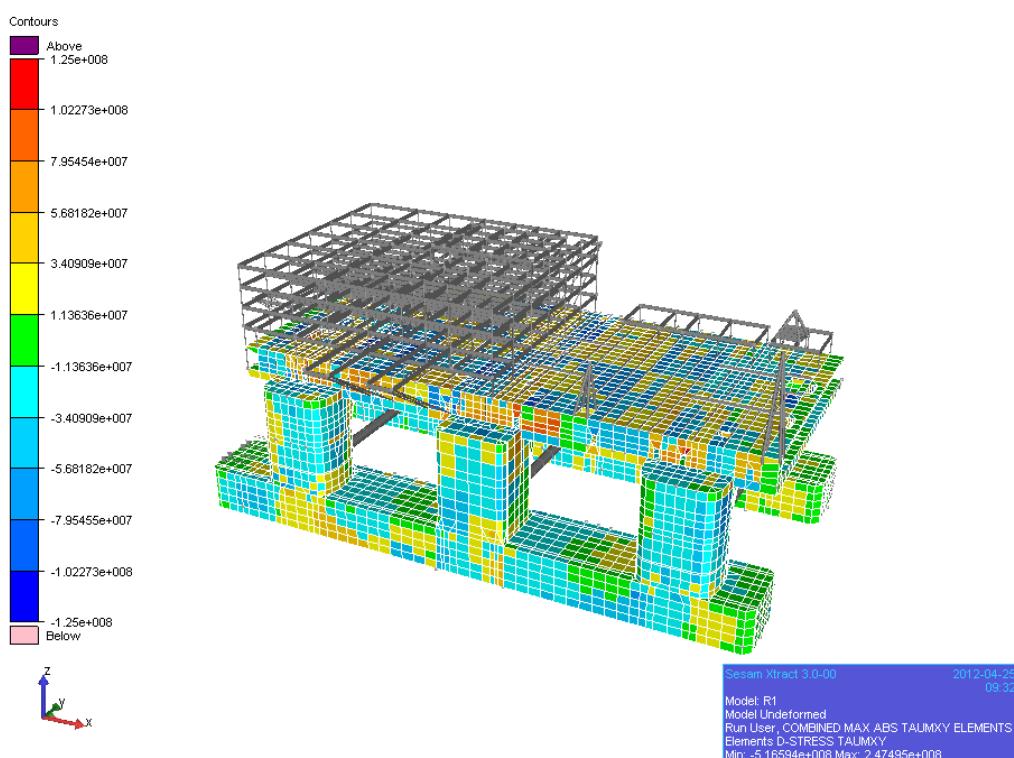


Figure C.1: Stress plot of the minimal  $\sigma_x$

Figure C.2: Stress plot of the minimal  $\sigma_y$ Figure C.3: Stress plot of the absolute maximum of  $\tau_{xy}$

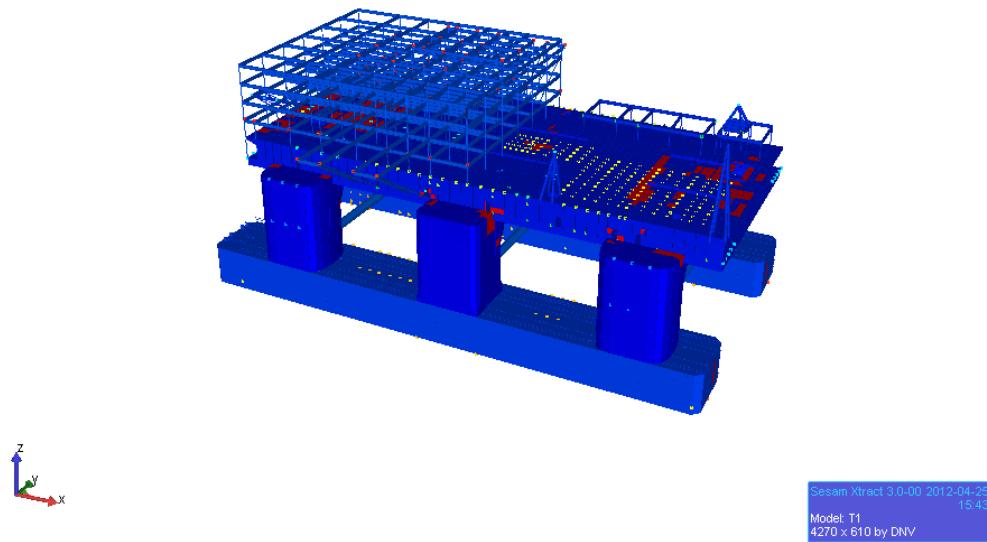


Figure C.4: Buckling Screening of critical panel with dimensions 4270 x 610 by the recommended practice of DNV

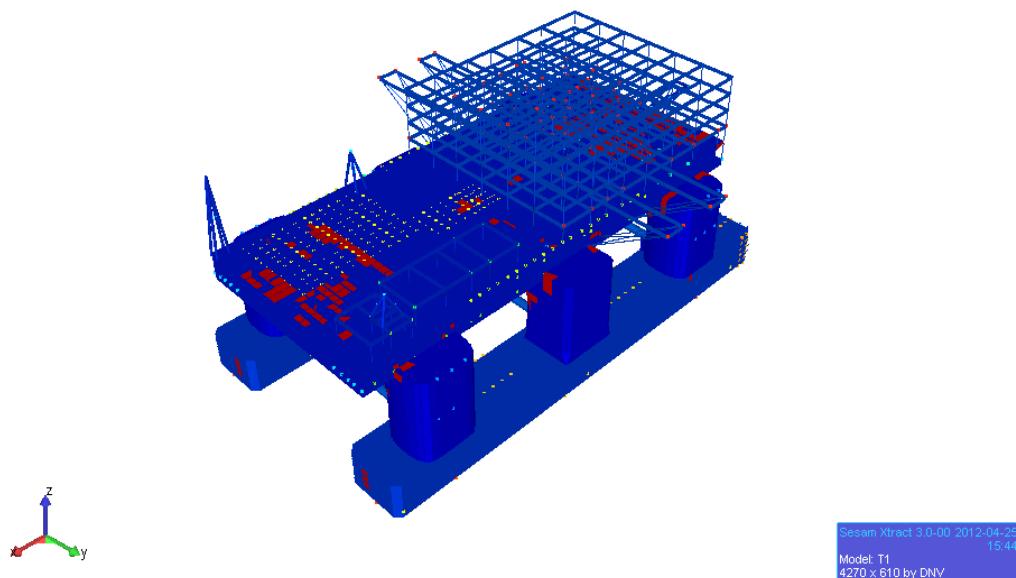
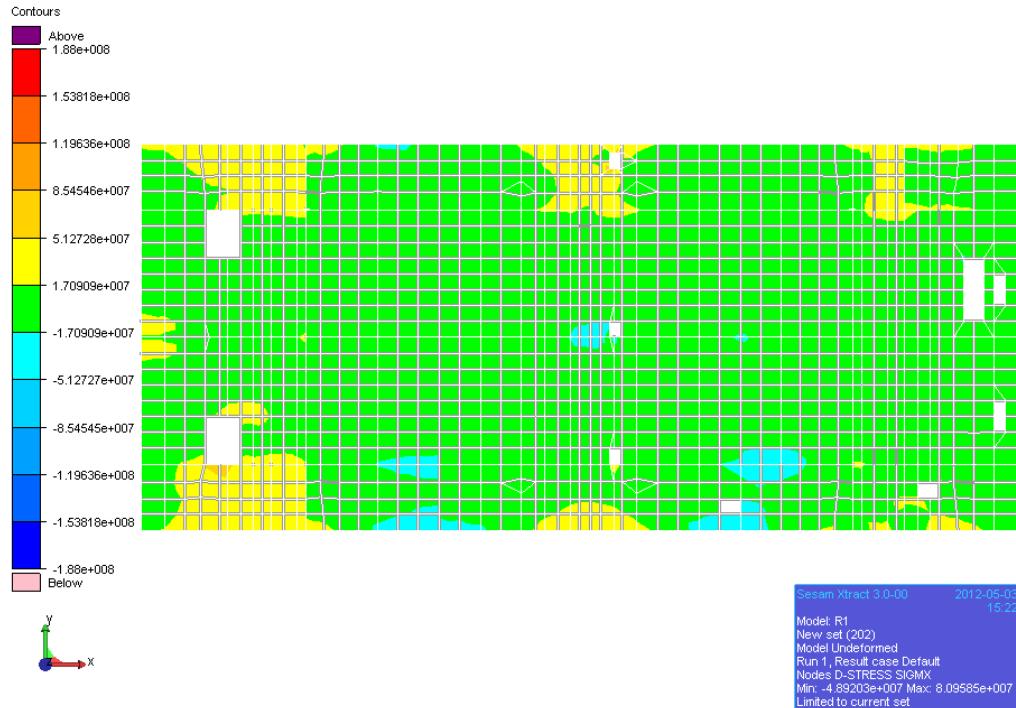
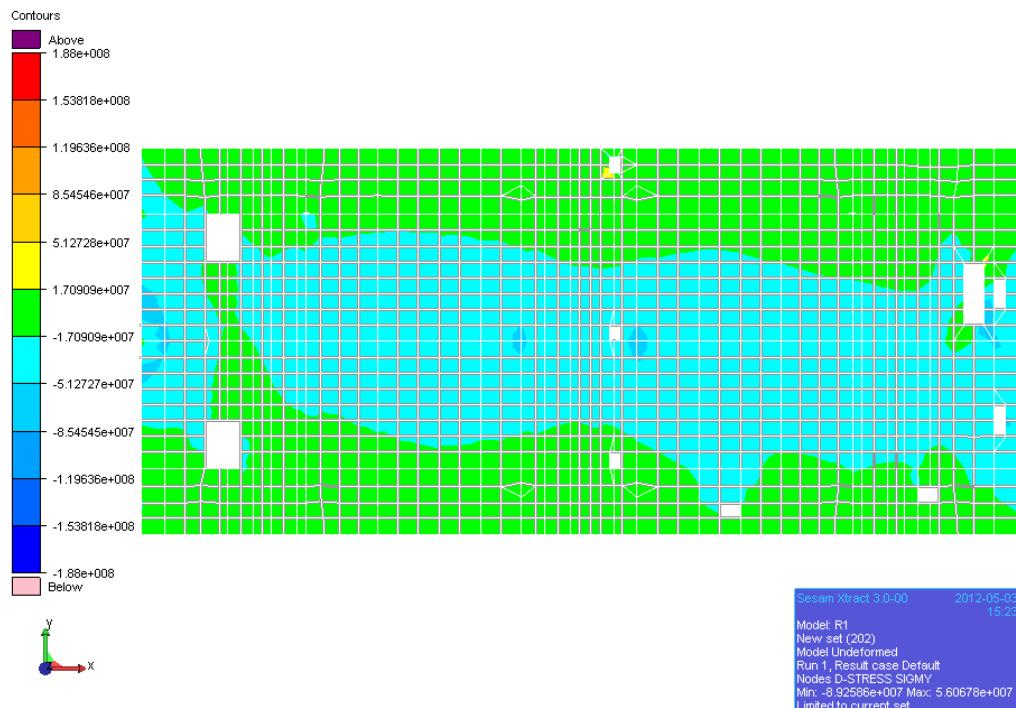


Figure C.5: Buckling Screening of critical panel with dimensions 4270 x 610 by the recommended practice of DNV

## C.2 SSDT Upper Deck

Figure C.6: Stress plot of the minimal  $\sigma_x$ Figure C.7: Stress plot of the minimal  $\sigma_y$

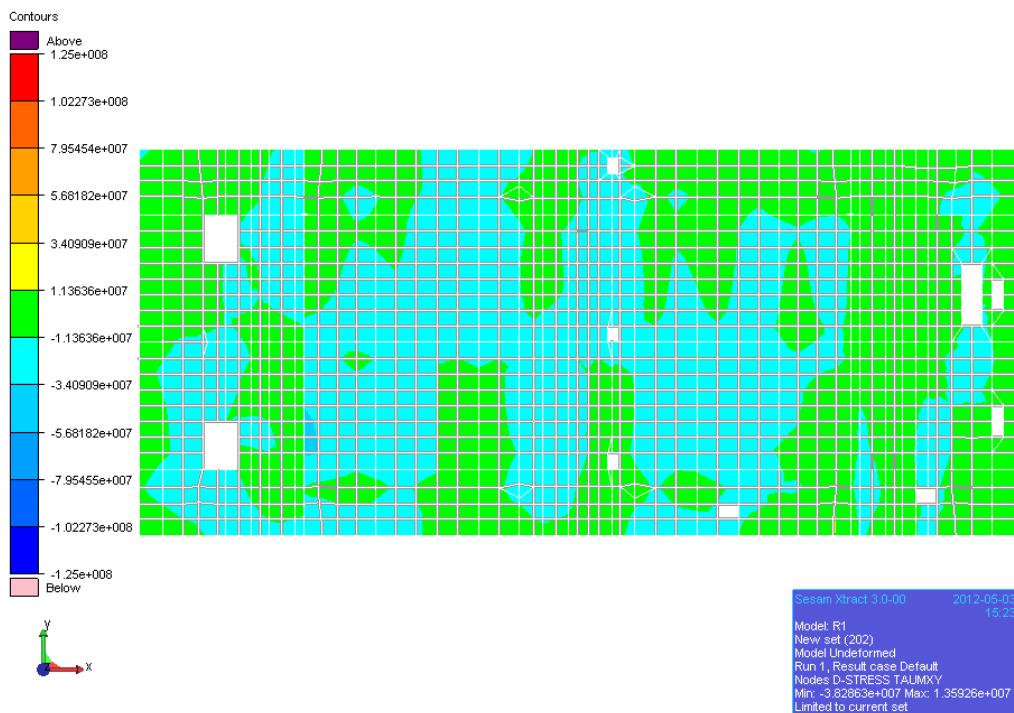
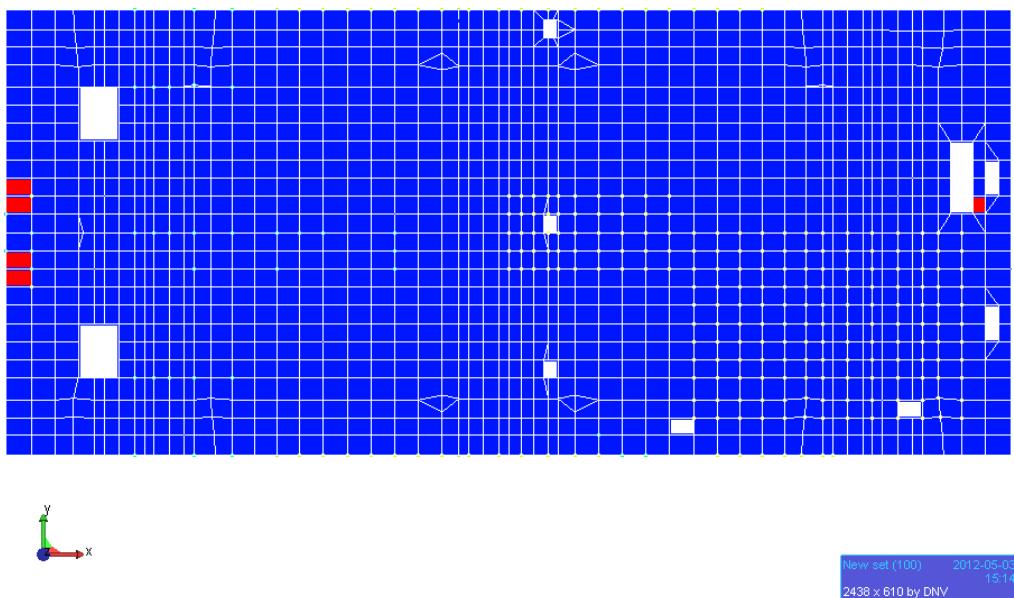
Figure C.8: Stress plot of the absolute maximum of  $\tau_{xy}$ 

Figure C.9: Buckling Screening for critical panel 2438 x 610 by DNV

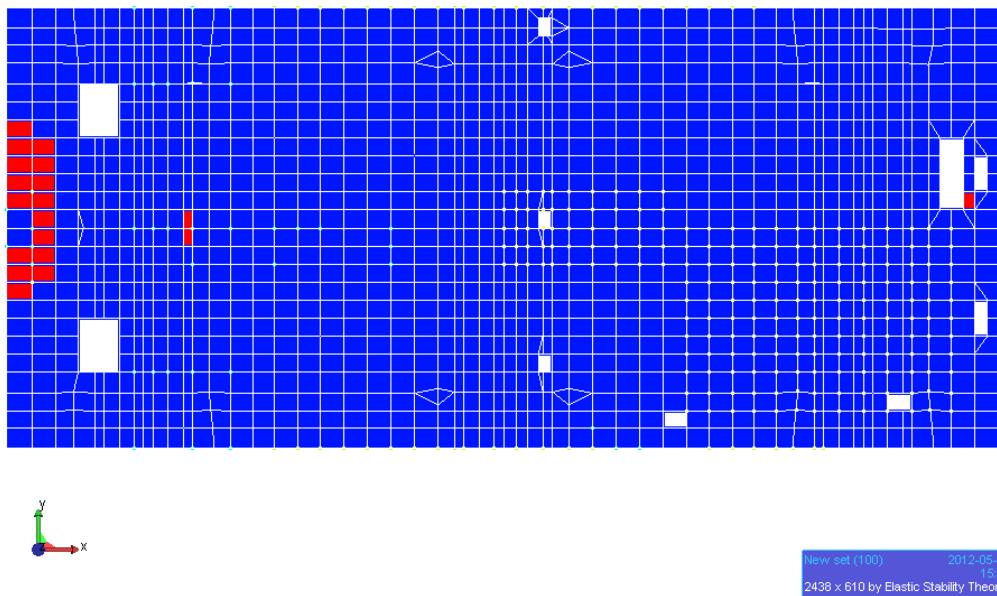


Figure C.10: Buckling Screening for critical panel 2438 x 610 by Paik and Thayambali + Fujikubo correction

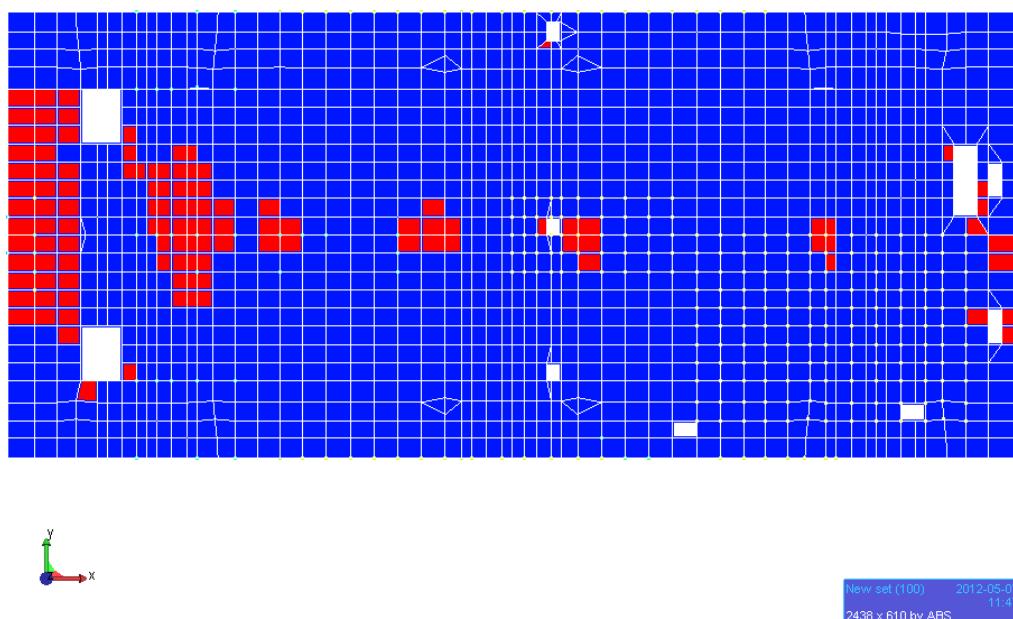


Figure C.11: Buckling Screening for critical panel 2438 x 610 by ABS – BLS

### C.3 Mating Barge

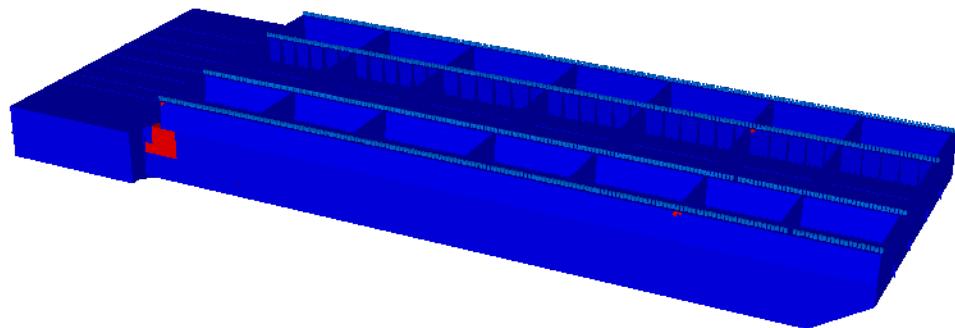


Figure C.12: Buckling Screening for critical panel 3000 x 700 by ABS – BLS

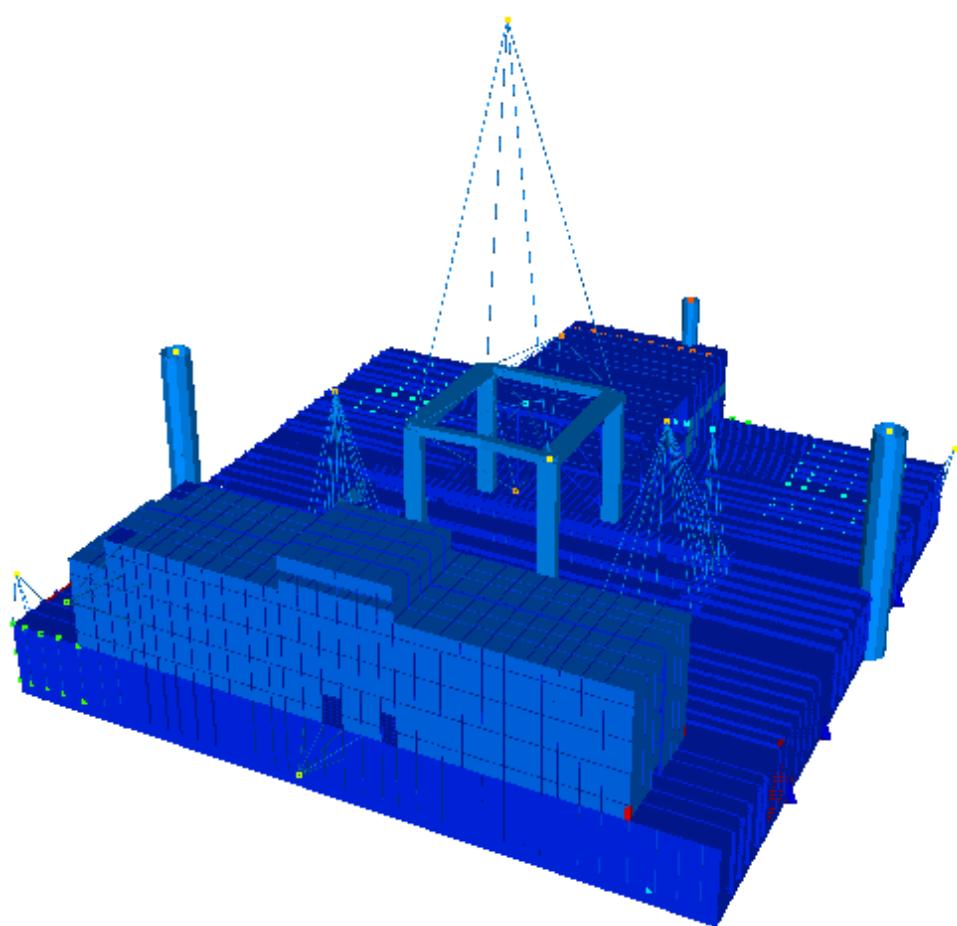


Figure C.13: Buckling Screening for critical panel 2500 x 500 by ABS – BLS

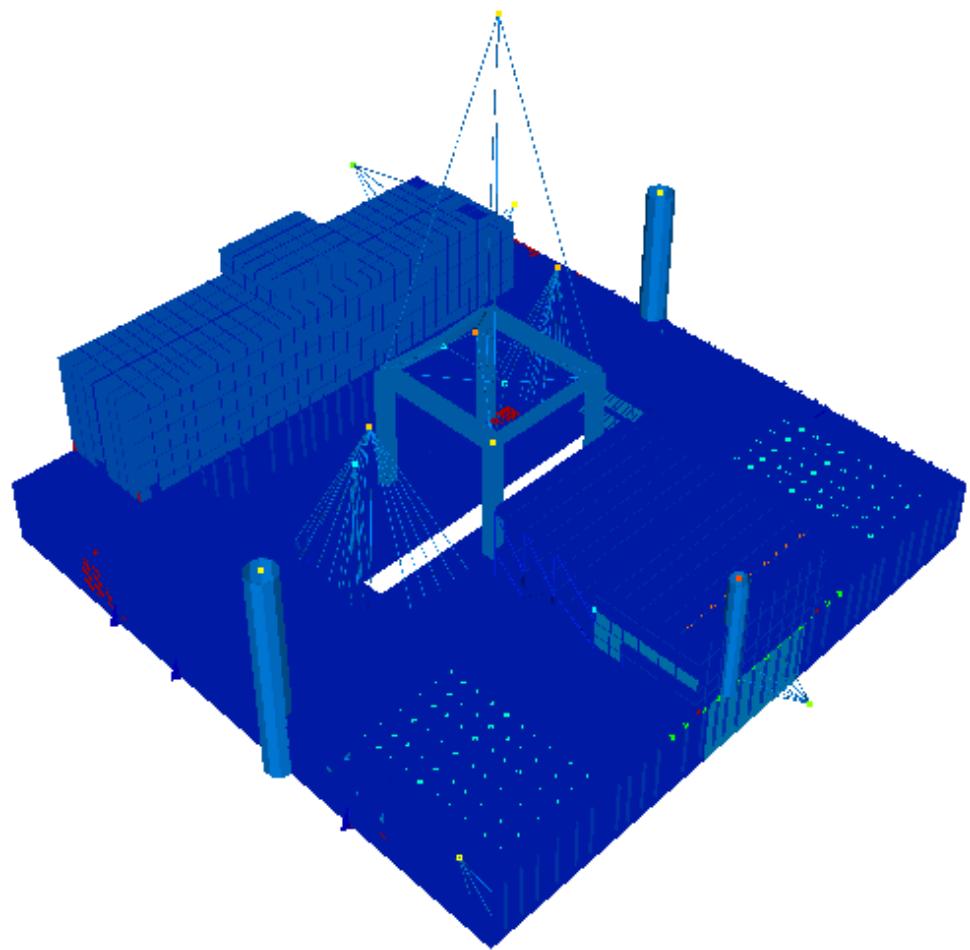


Figure C.14: Buckling Screening for critical panel 2500 x 500 by ABS – BLS

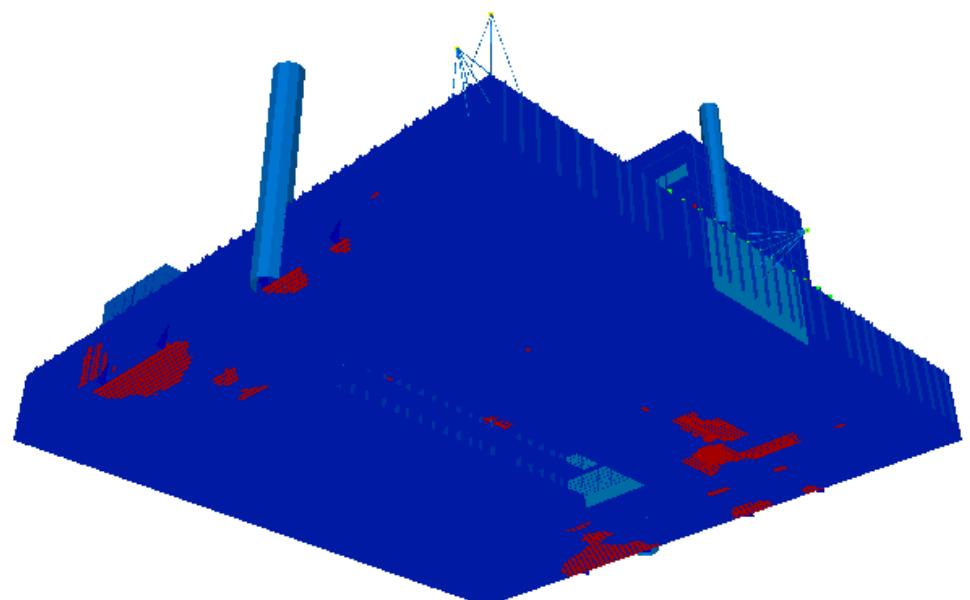


Figure C.15: Buckling Screening for critical panel 2500 x 500 by ABS – BLS

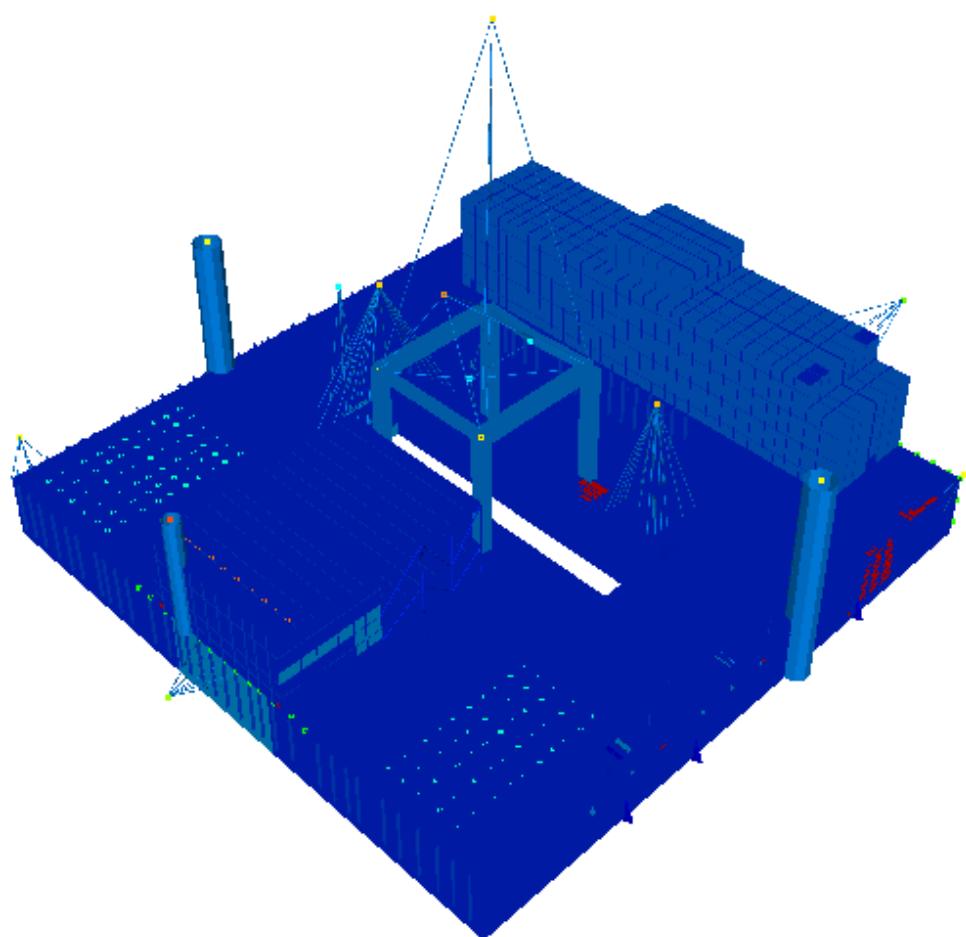


Figure C.16: Buckling Screening for critical panel 2500 x 500 by ABS – BLS

## Appendix D

# Theory Background

### D.1 Analytical solutions

The equilibrium equation for plate subjected to in-plane uniform compression is given by:

$$\nabla^4 w = \frac{1}{D} \left( q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right) \quad (\text{D.1})$$

Where the plate stiffness is given by:

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (\text{D.2})$$

and

$$\nabla^4 = (\nabla^2)^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 \quad (\text{D.3})$$

The quantities,

$$\begin{aligned} N_x &= \sigma_x t \\ N_y &= \sigma_y t \\ N_{xy} &= \tau_{xy} t \end{aligned} \quad (\text{D.4})$$

are the membrane stress resultants.

#### D.1.1 Buckling of un-stiffened plates under longitudinally uniform compression

The equilibrium condition for un-stiffened plate subjected only to longitudinally uniform compression can be reduced to:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_x \frac{\partial^2 w}{\partial x^2} = 0 \quad (\text{D.5})$$

The solution will be searched under this form:

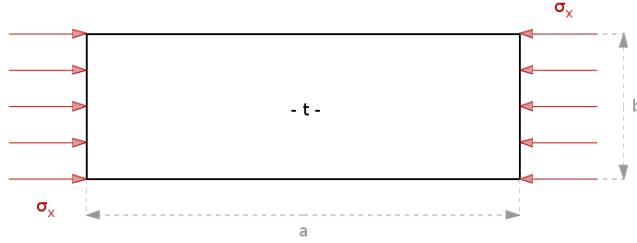


Figure D.1: Simply supported plate under longitudianl uniform compression

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (\text{D.6})$$

Finally, the solution for the critical load is expressed as:

$$N_{x,cr} = k_{\ell} \frac{\pi^2 D}{b^2} \quad (\text{D.7})$$

and the solution for the critical stress is expressed as:

$$\sigma_{x,cr} = k_{\ell} \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t}{b} \right)^2 \quad (\text{D.8})$$

where, according to [4] and [7], the buckling stress coefficient can be expressed as:

$$k_{\ell} = \left( \frac{mb}{a} + \frac{n^2 a}{mb} \right)^2 \approx 4 \quad (\text{D.9})$$

### D.1.2 Buckling of un-stiffened plates under transverse uniform compression

The equilibrium condition for un-stiffened plate subjected only to longitudinally uniform compression can be reduced to:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_y \frac{\partial^2 w}{\partial y^2} = 0 \quad (\text{D.10})$$

The solution will be searched under this form:

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (\text{D.11})$$

Finally, the solution for the critical load is expressed as:

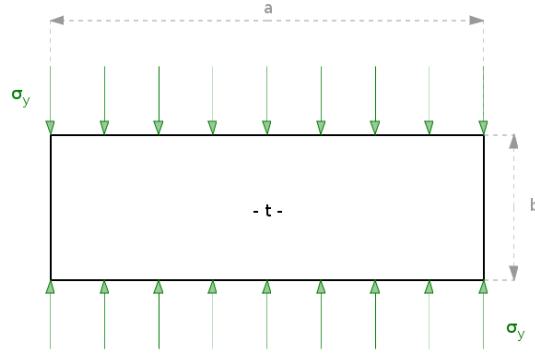


Figure D.2: Simply supported plate under transverse uniform compression

$$N_{y,cr} = k_t \frac{\pi^2 D}{a^2} \quad (\text{D.12})$$

and the solution for the critical stress is expressed as:

$$\sigma_{y,cr} = k_t \frac{\pi^2 E}{12(1-v^2)} \left(\frac{t}{b}\right)^2 \quad (\text{D.13})$$

where, according to [4] and [7], the buckling stress coefficient can be expressed as:

$$k_t = \left[ 1 + \left( \frac{b}{a} \right)^2 \right]^2 \quad (\text{D.14})$$

### D.1.3 Buckling of un-stiffened plates with shear

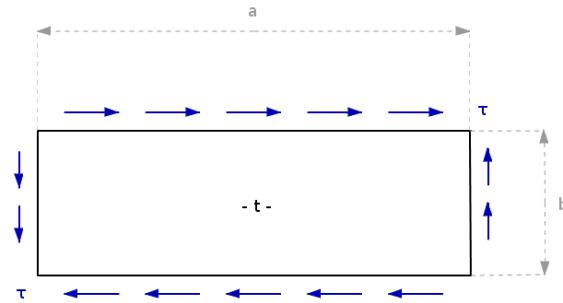


Figure D.3: Simply supported plate with shear

The equilibrium condition for un-stiffened plate subjected only to longitudinally uniform compression can be reduced to:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0 \quad (\text{D.15})$$

The solution will be searched under this form:

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (\text{D.16})$$

Finally, the solution for the critical load is expressed as:

$$N_{xy,cr} = k_s \frac{\pi^2 D}{b^2} \quad (\text{D.17})$$

and the solution for the critical stress is expressed as:

$$\tau_{xy,cr} = k_s \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t}{b} \right)^2 \quad (\text{D.18})$$

where, according to [4] and [7], the buckling stress coefficient can be expressed as:

$$k_s = 4 \left( \frac{b}{a} \right)^2 + 5.34 \quad (\text{D.19})$$

#### D.1.4 Buckling of un-stiffened plates under biaxially uniform compression with shear and lateral pressure

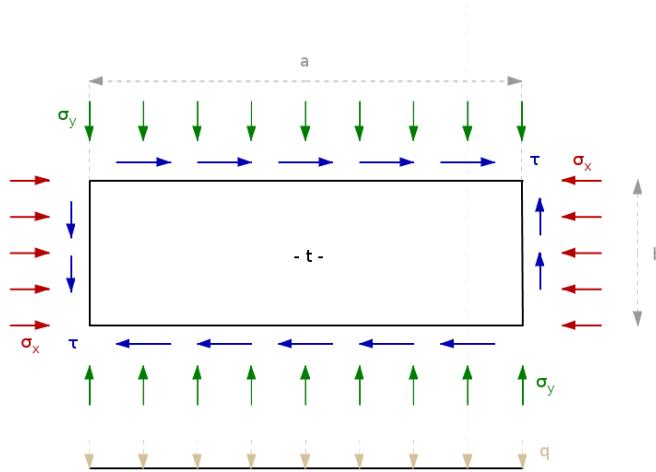


Figure D.4: Simply supported plate under biaxially uniform compression with shear and lateral pressure

The equilibrium condition in this case is:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} + q = 0 \quad (\text{D.20})$$

For this situation it is more convenient, a method which is not strictly analytical to be used. In [7] are proposed correction factors for the critical stresses to account the effect of the lateral pressure, which are obtained by a regression analysis of a FEM solutions for long plates, as follows:

	$\frac{a}{b} < 2$	$\frac{a}{b} \geq 2$
$C_{px} =$	1	$1 + \frac{1}{576} \left( \frac{qb^4}{Et^4} \right)^{1.6}$
$C_{py} =$	1	$1 + \frac{1}{160} \left( \frac{b}{a} \right)^{0.95} \left( \frac{qb^4}{Et^4} \right)^{1.75}$

Table D.1: Tabulated values for  $C_{px}$  and  $C_{py}$

where in Table D.1  $C_{px}$  is the correction factor for  $\sigma_{x,cr}$  and  $C_{py}$  is the correction factor for  $\sigma_{y,cr}$

With these correction factors now is much faster and easier to take into account the lateral pressure influence over the buckling state of a simply supported rectangular plate.

According to [7], under a total of four types of multiple-load components, the elastic plate buckling interaction criterion may be given as a function of the individual elastic buckling stress components, which should take into account the effect of the lateral pressure by Table D.1 as follows:

$$\Gamma_B = \left[ \frac{\sigma_x}{\sigma_{x,cr} \left( 1 - \left( \frac{\tau_{xy}}{\tau_{xy,cr}} \right)^{\alpha_{11}} \right)} \right]^{\alpha_1} + \left[ \frac{\sigma_y}{\sigma_{y,cr} \left( 1 - \left( \frac{\tau_{xy}}{\tau_{xy,cr}} \right)^{\alpha_{12}} \right)} \right]^{\alpha_2} \leq 1 \quad (\text{D.21})$$

where

	$1 \leq \frac{a}{b} \leq \sqrt{2}$	$\frac{a}{b} > \sqrt{2}$
$\alpha_1 =$	1	$0.0293 \left( \frac{a}{b} \right)^3 - 0.3364 \left( \frac{a}{b} \right)^2 + 1.5854 \left( \frac{a}{b} \right) - 1.0596$
$\alpha_2 =$	1	$0.0049 \left( \frac{a}{b} \right)^3 - 0.1183 \left( \frac{a}{b} \right)^2 + 0.6153 \left( \frac{a}{b} \right) + 0.8522$

Table D.2: Tabulated values for  $\alpha_1$  and  $\alpha_2$

	$1 \leq \frac{a}{b} \leq 3.2$	$\frac{a}{b} > 3.2$
$\alpha_{11} =$	$0.6153 \left( \frac{a}{b} \right) + 1.082$	1

Table D.3: Tabulated values for  $\alpha_{11}$

	$1 \leq \frac{a}{b} \leq 2$	$2 < \frac{a}{b} \leq 6$	$\frac{a}{b} > 6$
$\alpha_{12} =$	$0.10 \left( \frac{a}{b} \right) + 1.90$	$0.70 \left( \frac{a}{b} \right) + 0.70$	4.90

Table D.4: Tabulated values for  $\alpha_{12}$

in eq. D.21  $\Gamma_B$  will be considered as “Unity Factor” for the plate buckling mode.

## D.2 Det Norske Veritas - Recommended practice

This section presents supplementary information to better explain the criteria that are implemented from the *Buckling Strength of Plated Structures* [14].

Note that all linear units are in [ mm ], all pressure and stress units are in [  $\frac{N}{mm^2}$  ]

### D.2.1 Buckling of un-stiffened plates under longitudinally uniform compression

the reduced plate slenderness is obtained as follows:

$$\bar{\lambda}_p = 0.525 \frac{s}{t} \sqrt{\frac{f_y}{E}} \quad (\text{D.22})$$

the buckling factor for stresses in x-direction is obtained as follows:

	$\bar{\lambda}_p \leq 0.673$	$\bar{\lambda}_p > 0.673$
$C_x =$	1	$\frac{\bar{\lambda}_p - 0.22}{\bar{\lambda}_p^2}$

Table D.5: Tabulated values for  $C_x$

the designed buckling resistance may be calculated as:

$$\sigma_{x,Rd} = C_x \frac{f_y}{\gamma_m} \quad (\text{D.23})$$

where  $f_y$  is the characteristic yield strength of the plate and  $\gamma_m = 1.15$  is the material factor

### D.2.2 Buckling of un-stiffened plates under transverse uniform compression

$$h_a = 0.05 \frac{s}{t} - 0.75 \quad \text{but} \quad h_a \geq 0 \quad (\text{D.24})$$

the reduction factor due to lateral load  $k_p$  may, in lieu of more accurate results, be calculated as:

	$p_{sd} \leq 2 \left( \frac{s}{t} \right)^2$	$p_{sd} > 2 \left( \frac{s}{t} \right)^2$
$k_p =$	1	$1 - h_a \left[ \frac{p_{sd}}{f_y} - 2 \left( \frac{t}{s} \right)^2 \right]$ but $k_p \geq 0$

Table D.6: Tabulated values for  $k_p$

$$\lambda_c = 1.1 \frac{s}{t} \sqrt{\frac{f_y}{E}} \quad (\text{D.25})$$

the geometric parameter  $\mu$  is obtained as follows:

$$\mu = 0.21 (\lambda_c - 0.2) \quad (\text{D.26})$$

	$\lambda_c \leq 0.2$	$0.2 < \lambda_c < 2$	$\lambda_c \geq 2$
$\kappa =$	1	$\frac{1}{2\lambda_c^2} \left( 1 + \mu + \lambda_c^2 - \sqrt{(1 + \mu + \lambda_c^2)^2 - 4\lambda_c^2} \right)$	$\frac{1}{2\lambda_c^2} + 0.07$

Table D.7: Tabulated values for  $\kappa$ 

$$\sigma_{y,R} = \left[ \frac{1.3t}{l} \sqrt{\frac{E}{f_y}} + \kappa \left( 1 - \frac{1.3t}{l} \sqrt{\frac{E}{f_y}} \right) \right] f_y k_p \quad (\text{D.27})$$

the design buckling  $\sigma_{y,Rd}$  resistance of a plate under transverse compression may be found from:

$$\sigma_{y,Rd} = \frac{\sigma_{y,R}}{\gamma_M} \quad (\text{D.28})$$

### D.2.3 Buckling of un-stiffened plates with shear

the boundary dependent constant  $k_l$  is calculated as follows:

$$k_l = 5.34 + 4 \left( \frac{s}{l} \right)^2 \quad (\text{D.29})$$

$$\lambda_w = 0.795 \frac{s}{t} \sqrt{\frac{f_y}{E k_l}} \quad (\text{D.30})$$

	$\lambda_w \leq 0.8$	$0.8 < \lambda_w \leq 1.2$	$\lambda_w > 1.2$
$C_\tau =$	1	$1 - 0.625 (\lambda_w - 0.8)$	$\frac{0.9}{\lambda_w}$

Table D.8: Tabulated values for  $C_\tau$ 

shear buckling strength of a plate can be calculated as:

$$\tau_{Rd} = \frac{C_\tau}{\gamma_m} \frac{f_y}{\sqrt{3}} \quad (\text{D.31})$$

#### D.2.4 Buckling of un-stiffened plates under biaxially uniform compression with shear

the interaction factor  $c_i$  may be calculated as follows:

	$\sigma_{x,Sd} \leq 0$ or $\sigma_{y,Sd} \leq 0$	$\sigma_{x,Sd} > 0$ and $\sigma_{y,Sd} > 0$
		$\frac{s}{t} \leq 120$
$c_i =$	1	$1 - \frac{s}{120t}$
		0

Table D.9: Tabulated values for  $c_i$

the critical axial stress  $\sigma_{x,Rd}$  is obtained as follows:

	$\sigma_{x,Sd} > 0$	$\sigma_{x,Sd} < 0$
$\sigma_{x,Rd} =$	$C_x \frac{f_y}{\gamma_m}$	$\frac{f_y}{\gamma_m}$

Table D.10: Tabulated values for  $\sigma_{x,Rd}$

the critical axial stress  $\sigma_{y,Rd}$  is obtained as follows:

	$\sigma_{y,Sd} > 0$	$\sigma_{y,Sd} < 0$
$\sigma_{y,Rd} =$	$\frac{\sigma_{y,R}}{\gamma_M}$	$\frac{f_y}{\gamma_m}$

Table D.11: Tabulated values for  $\sigma_{y,Rd}$

	$\lambda_w \leq 0.8$	$0.8 < \lambda_w \leq 1.25$	$\lambda_w > 1.25$
$C_{\tau e} =$	1	$1 - 0.8 (\lambda_w - 0.8)$	$\frac{1}{\lambda_w^2}$

Table D.12: Tabulated values for  $C_{\tau e}$

the critical shear stress  $\tau_{xy,Rd}$  is obtained by the following criteria:

	$\sigma_{y,Sd} > 0$	$\sigma_{y,Sd} \leq 0$
$\tau_{xy,Rd} =$	$\frac{C_\tau f_y}{\gamma_m \sqrt{3}}$	$\frac{C_{\tau e} f_y}{\gamma_m \sqrt{3}}$

Table D.13: Tabulated values for  $\tau_{xy,Rd}$

The interaction equation between the in-plane compressive loads and the shear load is as follows:

$$\left( \frac{\sigma_{x,Sd}}{\sigma_{x,Rd}} \right)^2 + \left( \frac{\sigma_{y,Sd}}{\sigma_{y,Rd}} \right)^2 - c_i \left( \frac{\sigma_{x,Sd}}{\sigma_{x,Rd}} \right) \left( \frac{\sigma_{y,Sd}}{\sigma_{y,Rd}} \right) + \left( \frac{\tau_{xy,Sd}}{\tau_{xy,Rd}} \right)^2 \leq 1.0 \quad (\text{D.32})$$

where  $\sigma_{x,Sd}$ ,  $\sigma_{y,Sd}$ ,  $\tau_{xy,Sd}$  are the design loads,  $c_i$  is the interaction factor

### D.3 American Bureau of Shipping - Recommended practice

This section presents supplementary information to better explain the criteria that are implemented from the *ABS Buckling Guide [13]*.

Note that all linear units are in [ cm ], all pressure and stress units are in [  $\frac{N}{cm^2}$  ]

#### D.3.1 Buckling of un-stiffened plates under longitudinally uniform compression

The boundary dependent constant  $k_\ell$  is obtained by following equation:

$$k_\ell = C_1 \frac{8.4}{\kappa + 1.1} \approx 4 \quad (\text{D.33})$$

where  $\kappa$  is the ratio of the edge stress ( for uniform compression  $\kappa = 1$  ) and  $C_1$ <sup>1</sup> is a coefficient which represents the boundary conditions (  $C_1 = 1$  for plate elements, plate panels between flat bars or bulb plates and  $C_1 = 1.1$  for panels between angles or tee stiffeners )

The elastic buckling stress  $\sigma_{E,x}$  is obtained by the following equation:

$$\sigma_{E,x} = k_\ell \frac{\pi^2 E}{12(1-v^2)} \left(\frac{t}{s}\right)^2 \quad (\text{D.34})$$

The critical buckling stress  $\sigma_{C,x}$  is obtained by the following criteria:

	$\sigma_{E,x} \leq P_r \sigma_0$	$\sigma_{E,x} > P_r \sigma_0$
$\sigma_{C,x} =$	$\sigma_{E,x}$	$\sigma_0 \left[ 1 - P_r (1 - P_r) \frac{\sigma_0}{\sigma_{E,x}} \right]$

Table D.14: Tabulated values for  $\sigma_{C,x}$

#### D.3.2 Buckling of un-stiffened plates under transverse uniform compression

The boundary dependent constant  $k_t$  is obtained by following equation:

$$k_t = C_2 \left( 1 + \frac{1}{\alpha^2} \right)^2 (1.675 - 0.675\kappa) \quad (\text{D.35})$$

where  $\kappa$  is the ratio of the edge stress ( for uniform compression  $\kappa = 1$  ),  $C_2$ <sup>2</sup> is a coefficient which represents the boundary conditions (  $C_2 = 1$  for plate elements,  $C_2 = 1.1$  plate panels between flat bars or bulb plates and  $C_2 = 1.2$  for plate panels between angles or tee stiffeners ) and  $\alpha = \frac{\ell}{s}$  is the aspect ratio of the plate

The elastic buckling stress  $\sigma_{E,y}$  is obtained by the following equation:

$$\sigma_{E,y} = k_t \frac{\pi^2 E}{12(1-v^2)} \left(\frac{t}{s}\right)^2 \quad (\text{D.36})$$

The critical buckling stress  $\sigma_{C,x}$  is obtained by the following criteria:

<sup>1</sup>In order to achieve the most conservative results for the elastic buckling stress,  $C_1 = 1$

<sup>2</sup>In order to achieve the most conservative results for the elastic buckling stress,  $C_2 = 1$

	$\sigma_{E,y} \leq P_r \sigma_0$	$\sigma_{E,y} > P_r \sigma_0$
$\sigma_{C,y} =$	$\sigma_{E,y}$	$\sigma_0 \left[ 1 - P_r (1 - P_r) \frac{\sigma_0}{\sigma_{E,y}} \right]$

Table D.15: Tabulated values for  $\sigma_{C,x}$ 

### D.3.3 Buckling of un-stiffened plates with shear

The boundary dependent constant  $k_s$  is obtained by following equation:

$$k_s = \left[ 4 \left( \frac{b}{a} \right)^2 + 5.34 \right] C_1 \quad (\text{D.37})$$

The elastic buckling stress  $\tau_E$  is obtained by the following equation:

$$\tau_{E,xy} = k_s \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t}{s} \right)^2 \quad (\text{D.38})$$

The critical buckling stress  $\tau_C$  is obtained by the following criteria:

	$\tau_{E,xy} \leq P_r \tau_0$	$\tau_{E,xy} > P_r \tau_0$
$\tau_{C,xy} =$	$\tau_{E,xy}$	$\tau_0 \left[ 1 - P_r (1 - P_r) \frac{\tau_0}{\tau_{E,xy}} \right]$

Table D.16: Tabulated values for  $\tau_C$ 

where  $P_r$  is the proportional linear elastic limit of the structure ( for steel  $P_r = 0.6$  ),  $\tau_0$  is the shear strength of the plate  $\tau_0 = \frac{\sigma_0}{\sqrt{3}}$  and  $\sigma_0$  is the specified minimum yield point of plate

### D.3.4 Buckling of un-stiffened plates under biaxially uniform compression with shear

The interaction equation between the in-plane compressive loads and the shear load is as follows:

$$\left( \frac{\sigma_x}{\eta \sigma_{C,x}} \right)^2 + \left( \frac{\sigma_y}{\eta \sigma_{C,y}} \right)^2 + \left( \frac{\tau_{xy}}{\eta \tau_{C,xy}} \right)^2 \leq 1.0 \quad (\text{D.39})$$

where  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$  are the compressive stresses applied on the plate;  $\sigma_{C,x}$ ,  $\sigma_{C,y}$ ,  $\tau_{C,xy}$  are the critical buckling stresses obtained by Table D.14, D.15 , D.16;  $\eta$  is the allowable utilization factor obtained by Table D.17

Load Conditions	Environmental events	Allowable utilization factor
Static Loading	Operational gravity loads and the weight of the unit	$\eta = 0.6$
Combined Loading	Static Loads combined with relevant environmental loads	$\eta = 0.8$

Table D.17: Tabulated values for  $\eta$

## Appendix E

# Buckling Assessment Procedure

In this Appendix is presented the procedure which has to be applied for obtaining the buckling results. A few comments can be made:

- The stresses can be exported from a single super-elements as well as from a set of super-elements pre-handled with PRESEL.
- When the calculation is performed a pair of result FEM files is created. In the first file all the panels are with the same orientation as the finite elements, and in the second file all the panels are rotated perpendicularly.

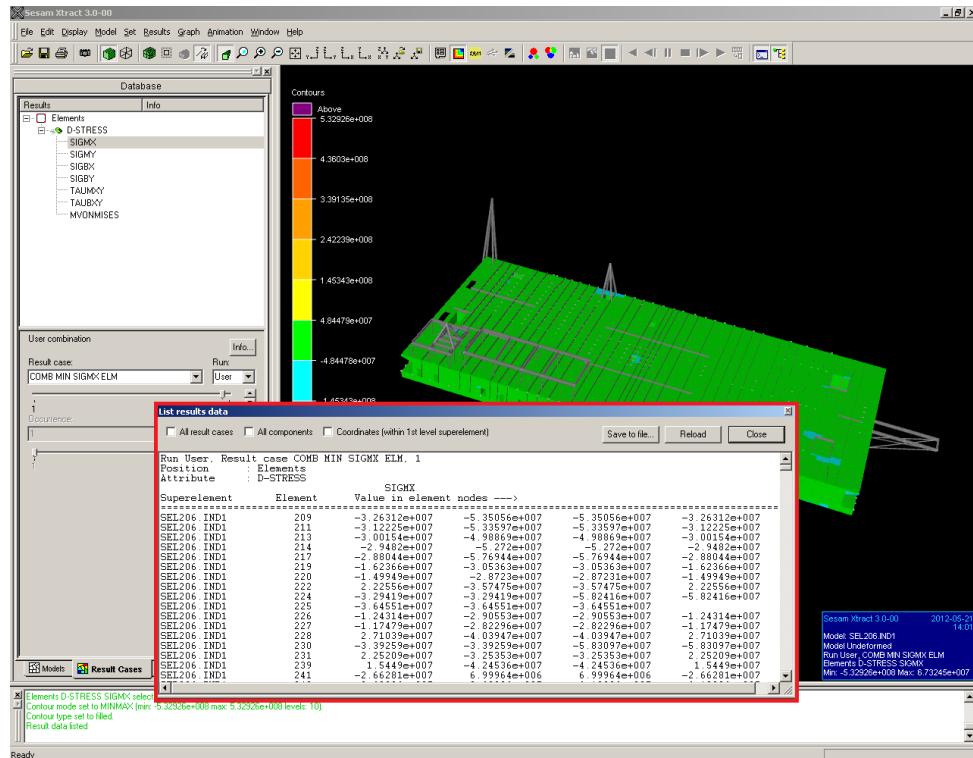
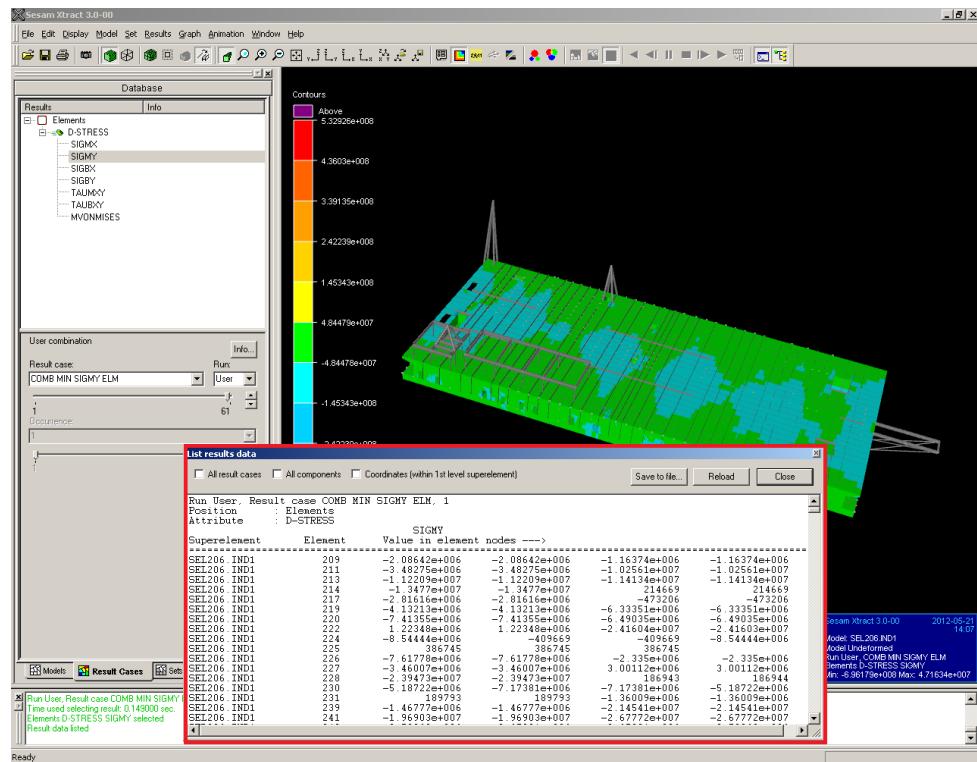
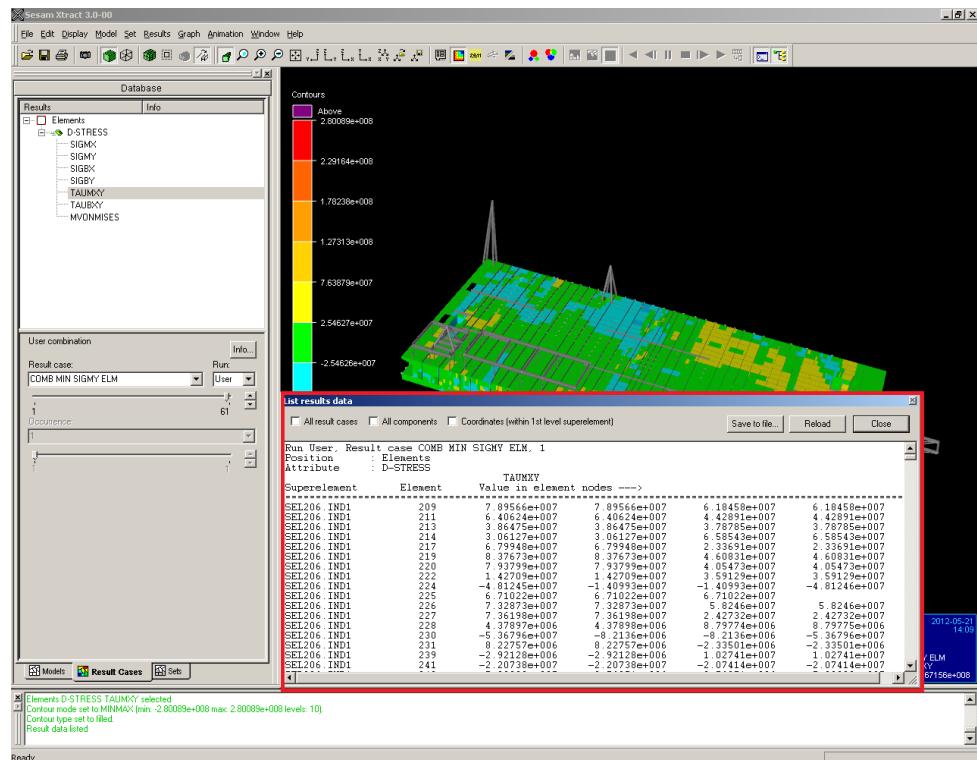


Figure E.1: Export the combined minimum  $\sigma_x$  for all finite elements

Figure E.2: Export the combined minimum  $\sigma_y$  for all finite elementsFigure E.3: Export the combined absolute maximum  $\tau_{xy}$  for all finite elements

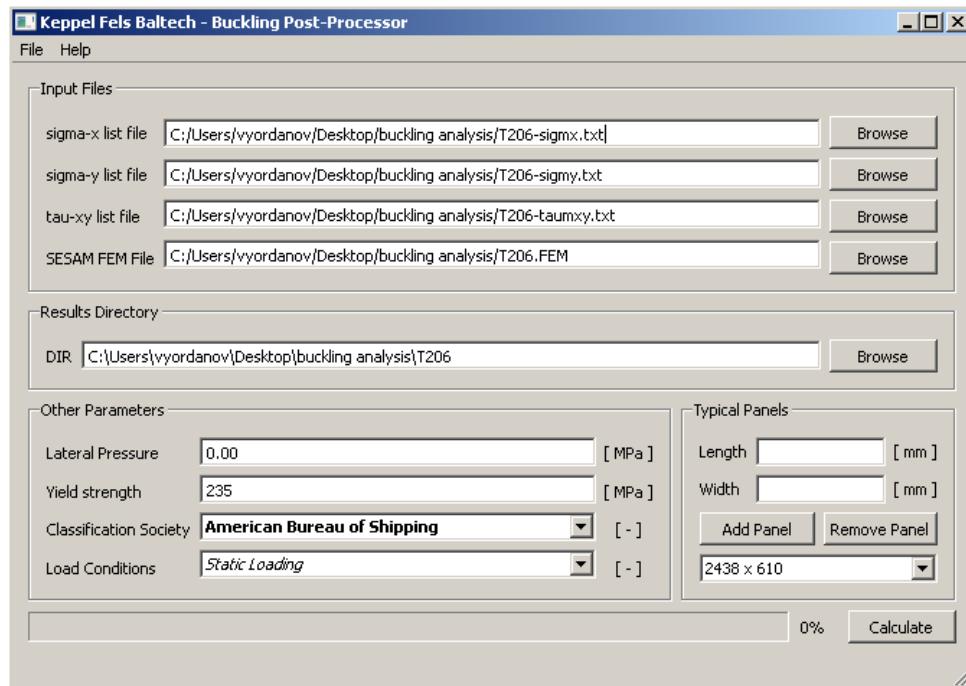
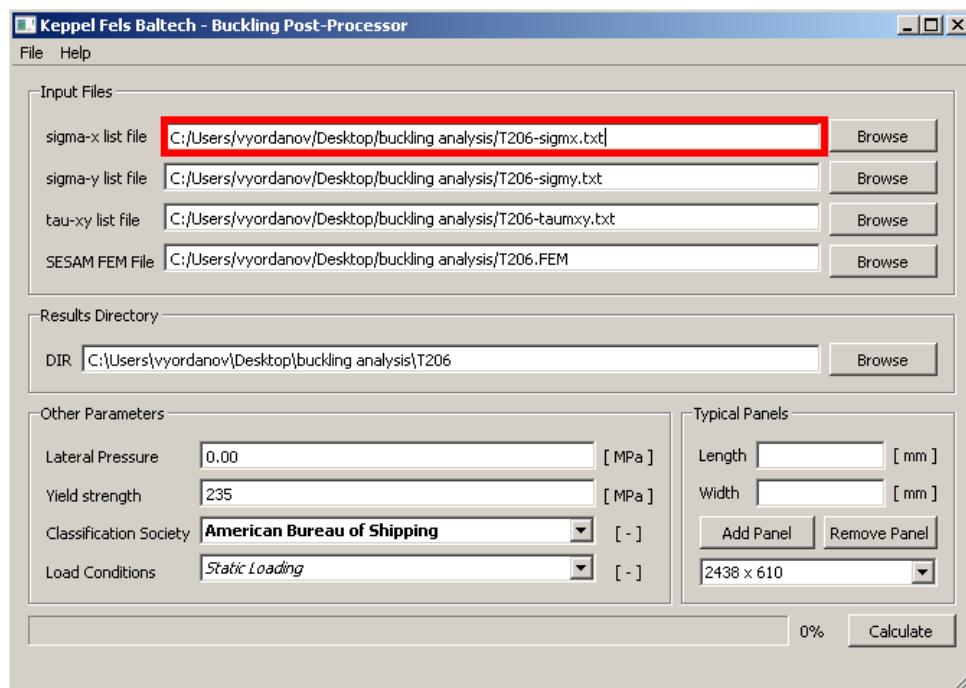
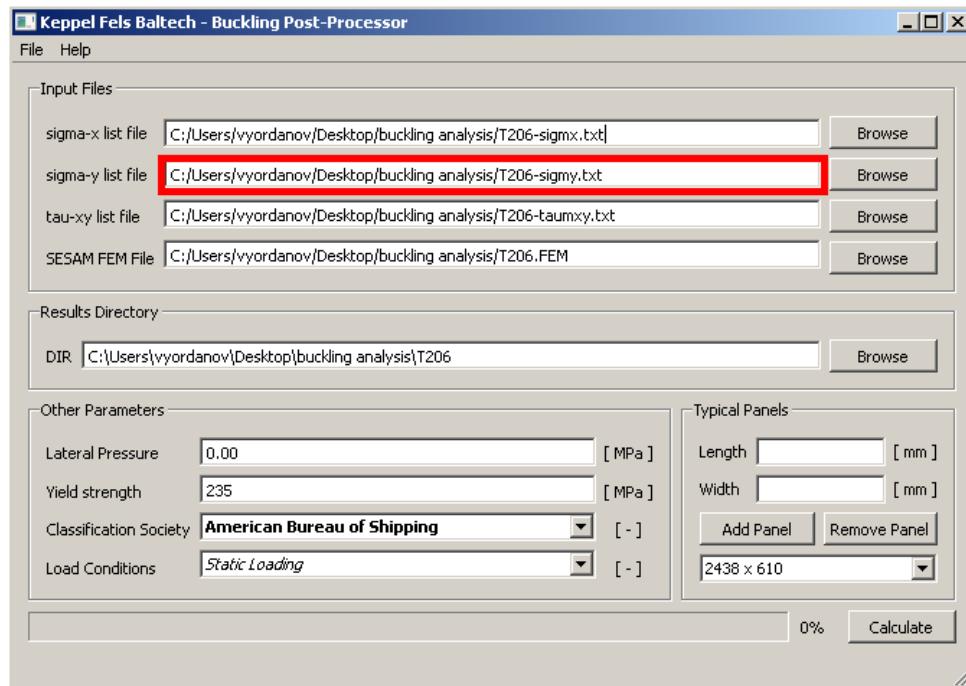
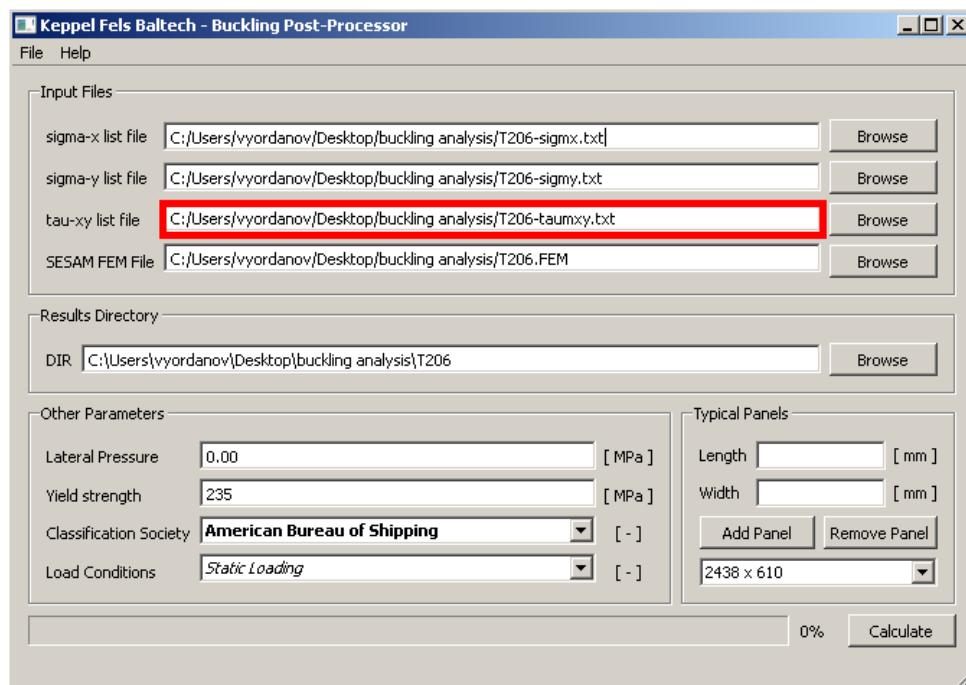


Figure E.4: Open the application named Buckling Post-Processing

Figure E.5: Specify the path to the previously exported  $\sigma_x$  list file

Figure E.6: Specify the path to the previously exported  $\sigma_y$  list fileFigure E.7: Specify the path to the previously exported  $\tau_{xy}$  list file

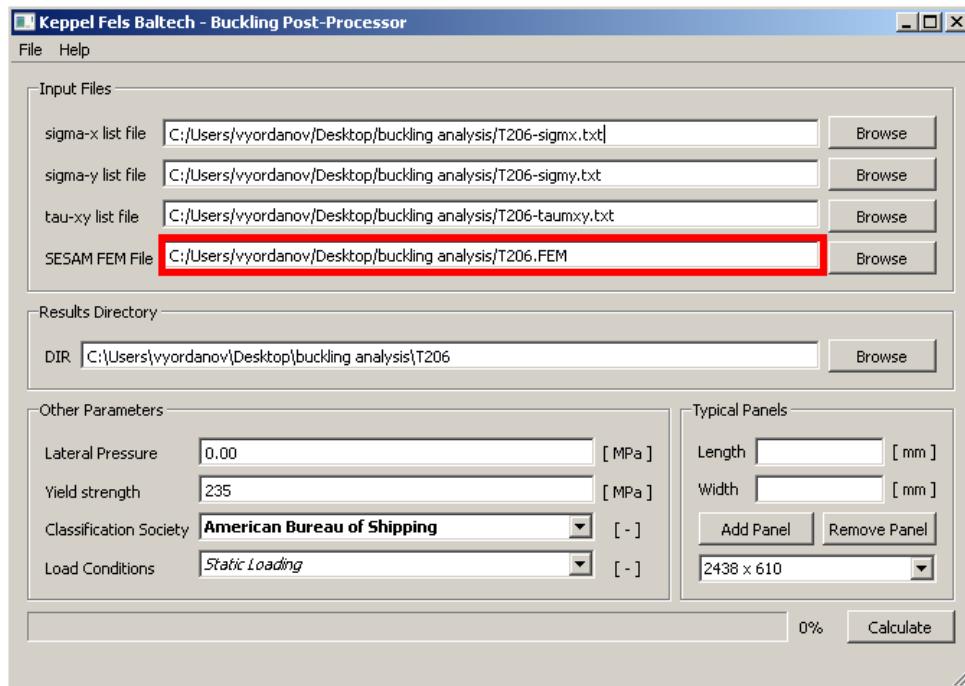


Figure E.8: Specify the path to the PATRAN-PRE FEM file of the structure of interest

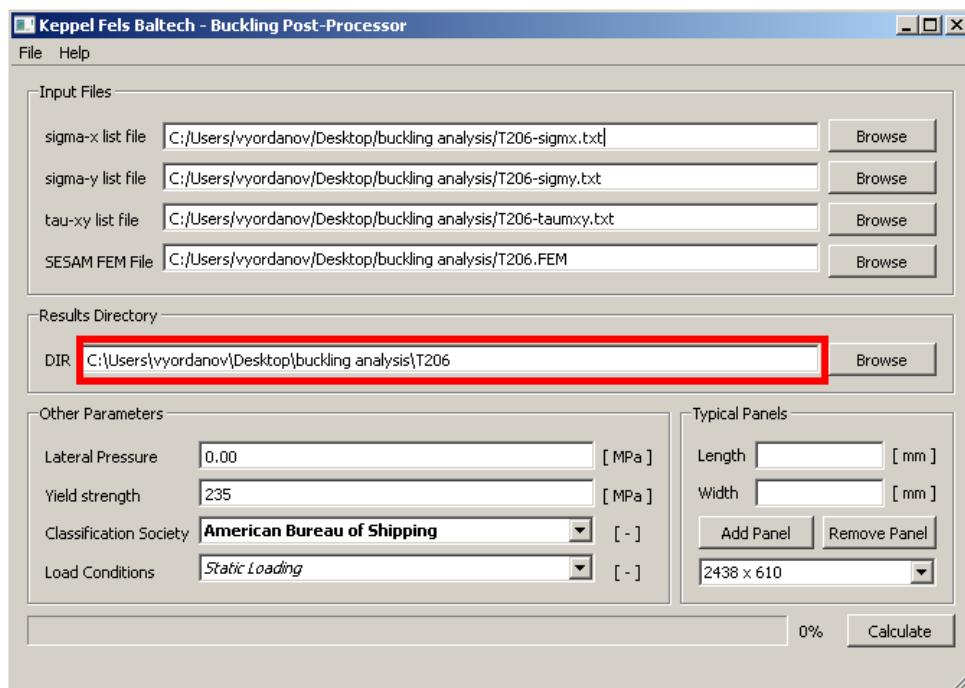


Figure E.9: Specify the path of the directory intended to store the results

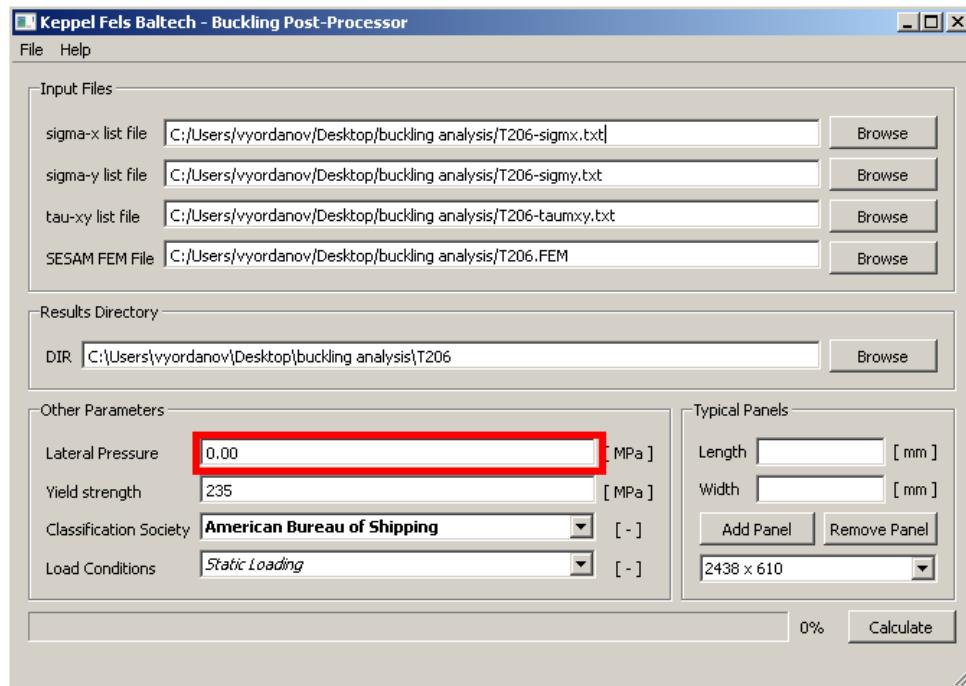


Figure E.10: Apply lateral pressure to all typical panels

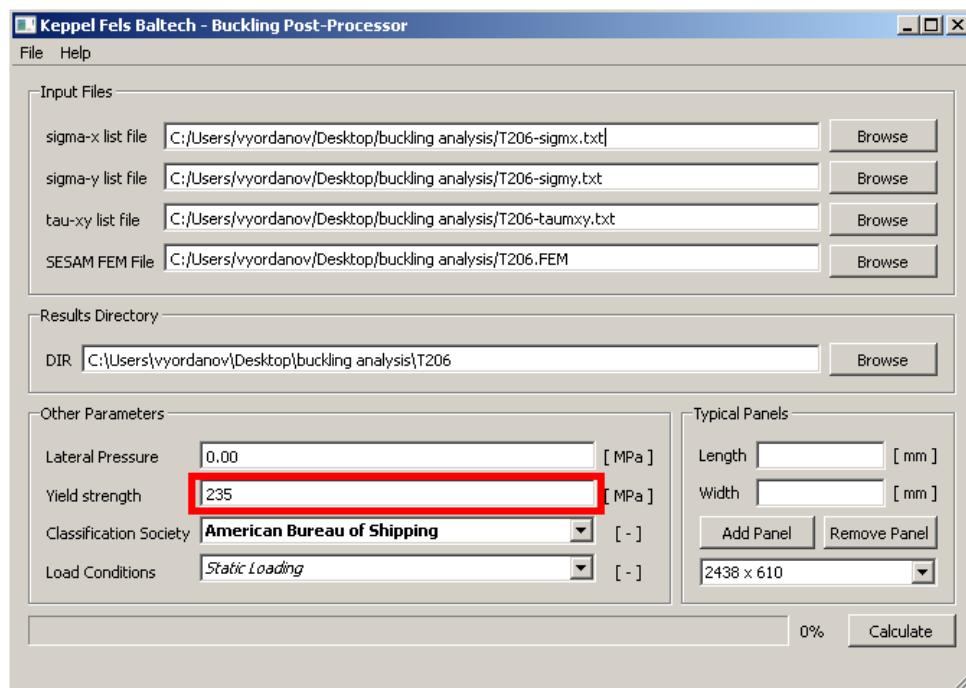


Figure E.11: Define the material yield point

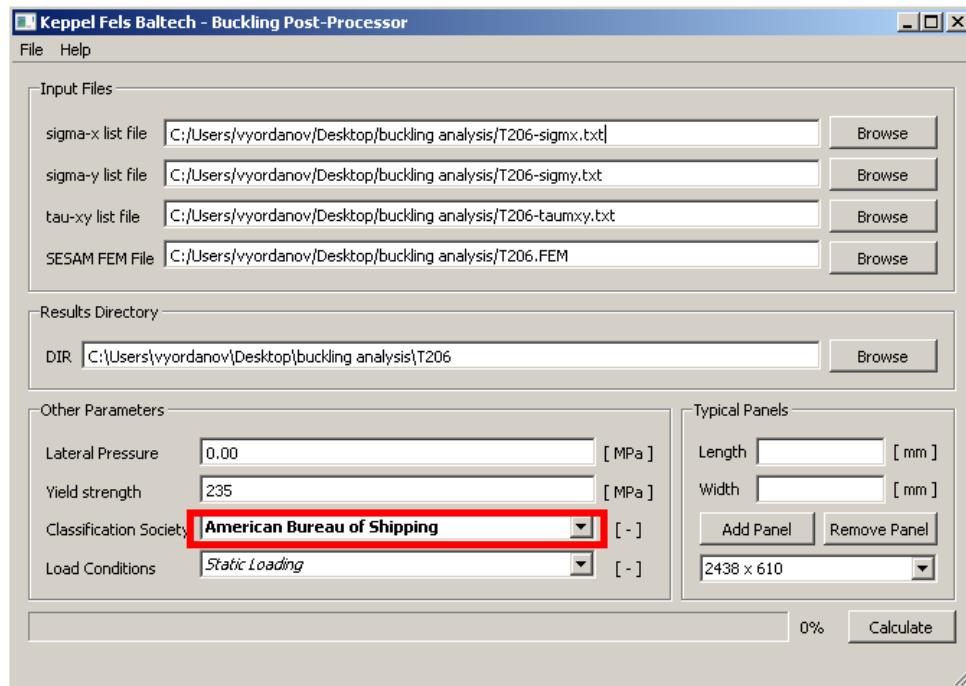


Figure E.12: Select panel check algorithm / criteria

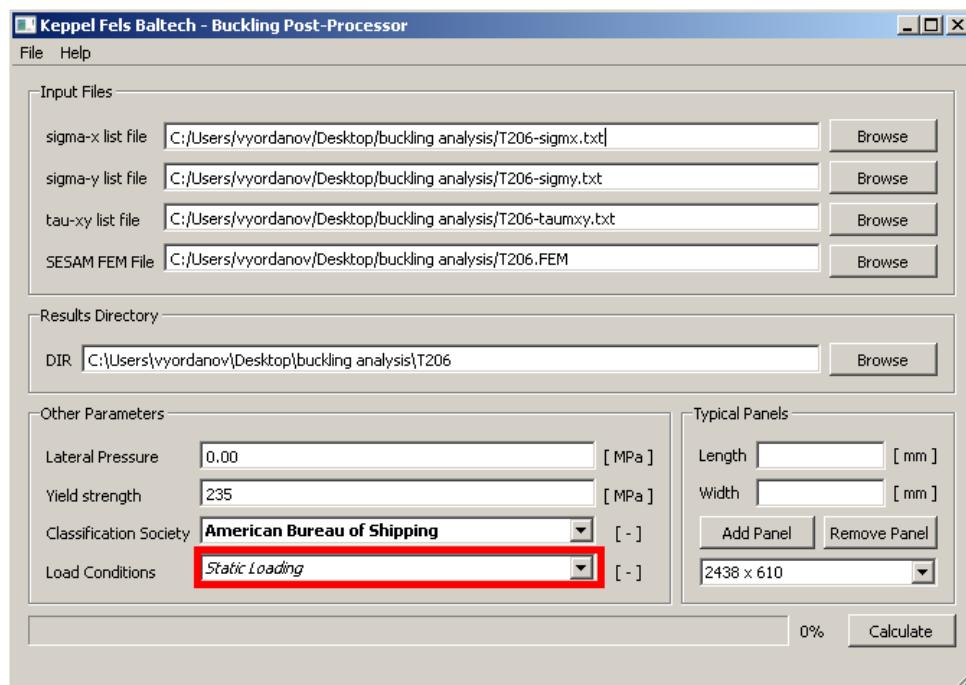


Figure E.13: Define the loading conditions [ static / combined ]

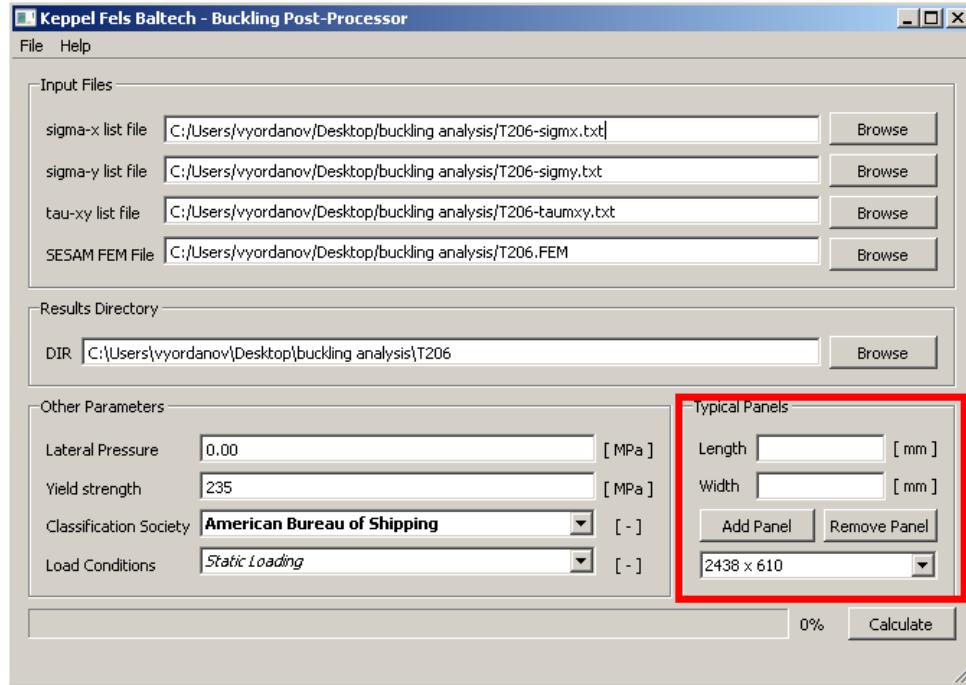


Figure E.14: Add the dimensions of all typical panels which have to be checked for buckling or ultimate strength

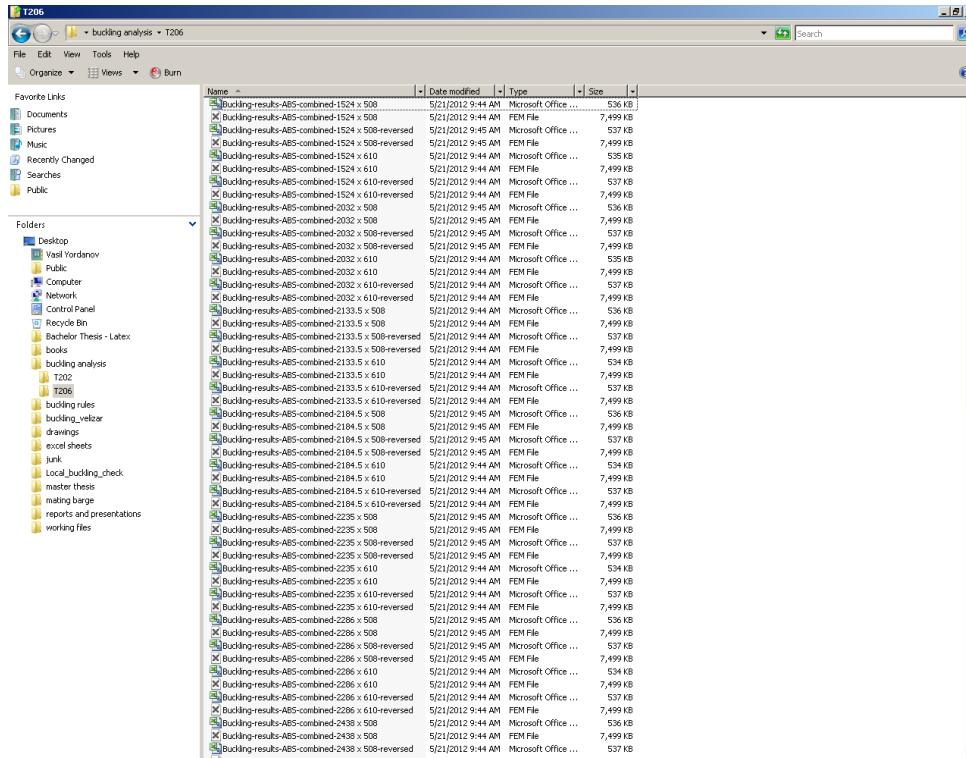


Figure E.15: Open the result folder and investigate the results

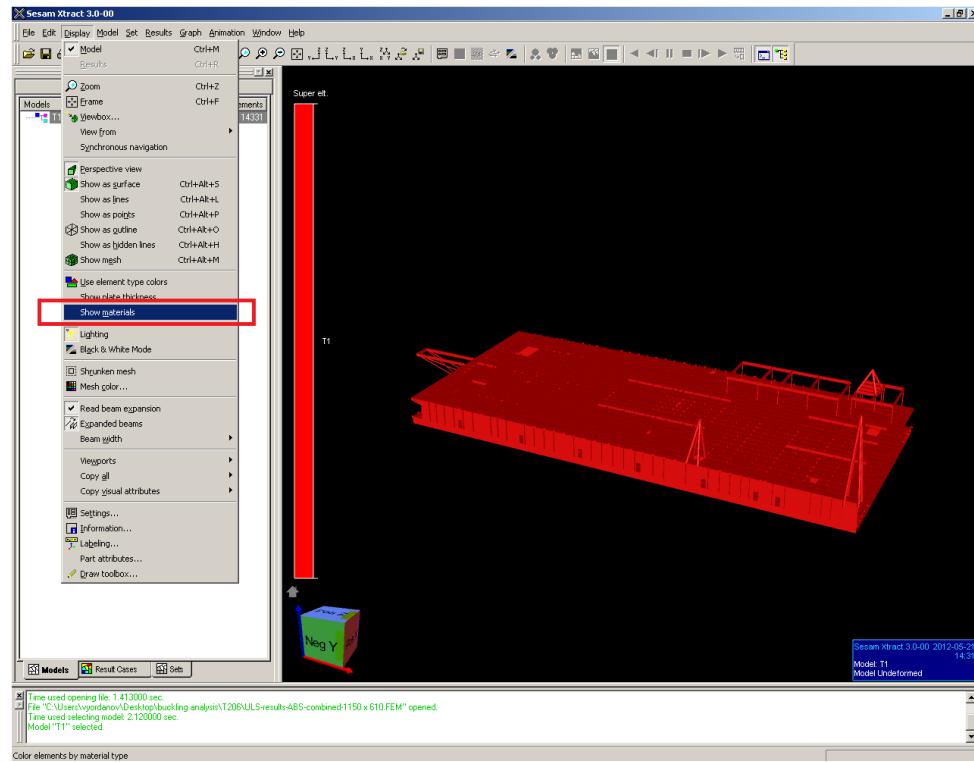


Figure E.16: When opened a model select “Show Materials”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Element Number	Unitycheck	Result	SIGMAX	SIGMY	TAUMXY	Thickness	Young Modulus									
2	209	0.79922379	0	-4366400	-1625000	6563600	0.0127	2.0E+11									
3	211	0.63101521	0	-42291100	-6869425	87270400	0.0127	2.0E+11									
4	213	0.76683554	0	-41101150	-1131150	93065000	0.0127	2.0E+11									
5	214	0.63630829	0	-41101000	-681185.5	7718600	0.0127	2.0E+11									
7	217	0.63450946	0	-43294000	-10446700	90612275	0.0127	2.0E+11									
9	219	0.63450946	0	-23388450	-5232600	90612275	0.0127	2.0E+11									
8	220	0.77256366	0	-21658975	-6951950	87026700	0.0127	2.0E+11									
9	222	0.39962252	0	-674590	-4476945	44321100	0.0079375	2.0E+11									
10	224	0.34863190	0	-45591750	-447704.5	51316800	0.0127	2.0E+11									
11	226	0.69911690	0	-2074350	-4976390	82939100	0.0127	2.0E+11									
12	227	0.6643931	0	-19988750	-229475	81176300	0.0127	2.0E+11									
13	228	0.25252938	0	-6645400	-11801178	26815200	0.0079375	2.0E+11									
14	230	0.3036269	0	-46171800	-6180515	46065400	0.0127	2.0E+11									
15	231	0.089646351	0	-4607200	-585146.5	26183800	0.0079375	2.0E+11									
16	239	0.24687543	0	-18502300	-1143635	20012200	0.0079375	2.0E+11									
17	241	0.26168013	0	-20733750	-1233750	8459200	0.0127	2.0E+11									
18	242	0.24046395	0	-20733750	-610495	43026200	0.0127	2.0E+11									
19	243	0.28180568	0	-22784200	-1809708.8	51398000	0.0127	2.0E+11									
20	245	0.87955349	0	-7761615	-22949300	47437500	0.0079375	2.0E+11									
21	246	0.25750957	0	-23979300	-423925	48421700	0.0127	2.0E+11									
22	249	0.22781249	0	-12068350	-11465245	23027100	0.0079375	2.0E+11									
23	250	0.23163569	0	-27121050	-4920135	44638000	0.0127	2.0E+11									
24	251	0.070897246	0	-8228300	-476272.5	22481700	0.0079375	2.0E+11									
25	252	0.23403392	0	-1143360	-1197256	21937000	0.0079375	2.0E+11									
26	263	0.83991217	0	-24538350	-2688130	90578300	0.0127	2.0E+11									
27	264	0.26216801	0	-6649075	-945484.5	45038500	0.0079375	2.0E+11									
28	265	0.78803659	0	-43036250	-49716	28534000	0.0127	2.0E+11									
29	266	0.26216801	0	-34211600	-3691200	28534000	0.0127	2.0E+11									
30	267	0.26173397	0	-26384075	-669430	45027000	0.0079375	2.0E+11									
31	271	0.20646224	0	-26334650	-1411006	42472700	0.0127	2.0E+11									
32	372	6.8947308	1	-273080.5	-78380750	205468000	0.0079375	2.0E+11									
33	373	0.73277756	0	-62433390	-221450	76302900	0.0127	2.0E+11									
34	374	0.8642520	0	-39961790	-18270195	86244600	0.0127	2.0E+11									
35	375	0.22342437	0	-31006125	-12133120	40150625	0.0127	2.0E+11									
36	377	6.4556693	1	-3265460	-75798300	21291100	0.0079375	2.0E+11									
37	378	0.7553371	0	-461537400	-303157.5	77894200	0.0127	2.0E+11									
38	379	0.74791918	0	-65688200	-6600760	75847400	0.0127	2.0E+11									
39	380	5.5639638	1	-7123090	-70310560	20753600	0.0079375	2.0E+11									
40	382	0.24345407	0	-67029700	-5506634	32396400	0.0127	2.0E+11									
41	383	4.4000000	1	-171685	-4855600	14761500	0.0079375	2.0E+11									
42	384	2.69579	1	-228000	-4855600	14761500	0.0079375	2.0E+11									
43	385	0.16365798	0	-324000	-4295085	32729600	0.0127	2.0E+11									
44	459	6.7713696	1	-285925	-77731500	18352300	0.0079375	2.0E+11									
45	460	6.7995151	0	-6519950	-7579680	78846700	0.0127	2.0E+11									
46	461	6.9771637	1	-285925	-78975950	15789500	0.0079375	2.0E+11									
47	462	0.80852645	0	-66747000	-18258350	75933400	0.0127	2.0E+11									

Figure E.17: Open the corresponding result file and investigate the numerical results