# Fabry-Perot wavelength calibration

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# 1 Wavelength calibration from 2D interferograms

## 1.1 Models to fit 2D interferograms

### 1.1.1 Model to fit a single interferogram with a single line

One can use the Airy function in order to adjust the interferograms, in that case the spectrum only contains one line at $\lambda_c$ observed at order $p_c$:

$$I = C + I_0 \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2(\phi/2)} \tag{1}$$

where $\phi = 2\pi \frac{2ne\cos\theta}{\lambda_c}$.

In such a case, the parameters than have to be adjusted are

- $F$ the finesse, that might be known (user defined) and constrained within a range of $\sim 10\%$ (user defined);
- $I_0$ the intensity of the line that can be estimated at first order using the maximum $v_{max}$ and the minimum $v_{min}$ of the interferogram (in practice it might be more robust to use e.g. the 99th percentile and the 1st percentile to avoid artefacts in the data) and the finesse $F$:

$$I_0 = (v_{max} - v_{min}) \frac{1 + \frac{4F^2}{\pi^2}}{\frac{4F^2}{\pi^2}} \tag{2}$$

- $C$ the continuum, i.e. the background (either instrumental or due to the lamp spectrum), that can be estimated at first guess as the 99th percentile of the interferogram minus $I_0$ ($C = v_{max} - I_0$) and constrained at $\sim \pm 3\sigma$ where $\sigma$ is the standard deviation of the interferogram (user defined);
- $\theta$, which depends on:
  - the centre of the rings, which can be constrained within a circle of radius $\sim 50$ pixels (user defined).
  - $b = \frac{s_{pix}}{f_c}$ ($\theta = \arctan(br)$ where $r$ is the radius in pixel) which can be constrained within less than $\sim 10\%$ (user defined). Indeed:
    * $s_{pix}$ is the pixel size and might be known.
    * $f_c$ is the camera focal length and might be known as well.

  If $\lambda_c$ is observed in two orders ($p_c$ and $p_c - 1$), $b$ can be determined from the radius in pixels of the two rings ($r_{p_c}$ and $r_{p_c-1}$):

$$b^2 = \frac{p_c^2 - (p_c-1)^2}{r_{p_c-1}^2 (p_c-1)^2 - r_{p_c}^2 p_c^2} = \frac{2p_c - 1}{p_c^2 (r_{p_c-1}^2 - r_{p_c}^2) - 2p_c r_{p_c-1}^2 + r_{p_c-1}^2} \tag{3}$$

- the product $n \times e$, that can be estimated at first guess as

$$ne = \frac{\lambda_c p_c \sqrt{1 + b^2 r_{p_c}^2}}{2} \tag{4}$$

  and that can be constrained within a range $\pm \frac{\lambda_c}{4}$.

The wavelength $\lambda_c$ is the only parameter that does not need to be fitted (it depends on the calibration lamp).

**In principle, using only the geometry of the rings (centre and radius), one might be able to derive both $\theta$ and $ne$ which is sufficient to derive the phase map. The model fitting might just improve the accuracy on these parameters.**

#### 1.1.2 Model to fit several interferograms with a single line

In that case, let's consider that we have $l$ interferograms corresponding to some known channels $i$ not necessarily consecutive. The same model has to be used than in the case with only one interferogram and one line (cf. equation 1), however, for each interferogram the spacing changes. Therefore the parameters that need to be fitted remain the same than in the case with only one line (cf. section 1.1.1) but there are several spacings to adjust $ne_i$ (for each channel). In fact one may fit $ne$ ($e$ being the spacing for a reference channel) and $\delta e$:

$$ne_i = n(e + i\delta e)$$

Such a configuration enables to determine the parameter $\delta e$ from which will be deduced $\delta\lambda_s$.

#### 1.1.3 Model to fit a single interferogram with several lines

In that case, let's consider we observe $m$ lines with known wavelengths $\lambda_{cj}$, the model might be:

$$I = C + \sum_{j=1}^{m} I_j \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2(\phi_j/2)} \tag{5}$$

where $\phi_j = 2\pi \frac{2ne\cos\theta}{\lambda_{cj}}$.

The parameters that need to be fitted remain the same than in the case with only one line but there are several intensities to adjust $I_j$.

Such a configuration may help in determining the parameter $b$ in the case only one order for each line is observed.

#### 1.1.4 Model to fit several interferograms with several lines: general case

This case is the general case. We have $l$ interferograms corresponding to some known channels $i$ not necessarily consecutive with $m$ lines with known wavelengths $\lambda_{cj}$. It is a combination of the previous cases (cf. sections 1.1.2 and 1.1.3). The same model has to be used than in the case with only one interferogram and several line (cf. equation 5), however, for each interferogram the spacing changes. Therefore the parameters that need to be fitted remain the same than in the case with only one line (cf. section 1.1.1) but there are several intensities ($I_j$) and several spacings to adjust $ne_i$. In fact one may fit $ne$ and $\delta e$:

$$ne_i = n(e + i\delta e)$$

## 1.2 Wavelength map

Once the Airy function parameters are determined, the wavelength can be determined anywhere in the field for any order $p$ and in particular for the scanning order $p_s$:

$$\lambda = \frac{2ne}{p_s} \cos\theta \tag{6}$$

Therefore the scanning order $p_s$ has first to be estimated:

$$p_s = \text{round}\left(\frac{\lambda_c p_c}{\lambda_s}\right) \tag{7}$$

where $\lambda_s$ is the scanning wavelength.

## 1.3 Phase map

To compute the phase map from the previously determined wavelength map (i.e. $\lambda = f(r)$), the wavelength variation $\delta\lambda_s$ in the centre between two consecutive channels has to be known at order $p_s$. The phase map is therefore:

$$\frac{\lambda - \lambda_s}{\delta\lambda_s} \tag{8}$$

If one wants to have $\lambda_s$ in a given channel after calibration, an offset has to be applied. It depends on the interferogram used as calibration . If no offset is applied, $\lambda_s$ will be the wavelength of the channel corresponding to the calibration inteferogram after calibration.

In practice $\delta\lambda_s$ can be determined from the physical spacing difference $\delta e$ between two consecutive channels.

## 1.4 Determination of the wavelength variation between consecutive channels

In practice, $\delta e$ is either known (from a instrument control point of view) or determined using several (at least two) calibration interferograms (cf. sections 1.1.2 and 1.1.4).

There are two cases:

- The spacing for any channel $i$ is the same for the calibration and the observation. In that case, the spacing variation between consecutive channels for the observations $\delta e_s$ is the same as the one for the calibration $\delta e_c$:

$$\delta e_s = \delta e_c = \delta e \tag{9}$$

- The spacing for the observation is adjusted with respect to the spacing for the calibration to ensure the scan of the full spectral range both for observation and calibration. In that case, the spacing variation between consecutive channels for the observations $\delta e_s$ is different from the one for the calibration $\delta e_c$:

$$\delta e_s = \frac{\lambda_s \delta e_c}{\lambda_c} = \frac{\lambda_s \delta e}{\lambda_c} \tag{10}$$

The wavelength variation is then deduced following:

$$\delta\lambda_s = \frac{2n\delta e_s}{p_s} \tag{11}$$