

It can have different base values like: binary (base-2), octal (base-8), decimal (base 10) and hexadecimal (base 16), here the base number represents the number of digits used in that numbering system. As an example, in decimal numbering system the digits used are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Therefore the digits for binary are: 0 and 1, the digits for octal are: 0, 1, 2, 3, 4, 5, 6 and 7. For the hexadecimal numbering system, base 16, the digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

2. Binary numbers

Numbers that contain only two digit 0 and 1 are called Binary Numbers. Each 0 or 1 is called a Bit, from binary digit. A binary number of 4 bits is called a Nibble. A binary number of 8 bits is called a Byte. A binary number of 16 bits is called a Word on some systems, on others a 32-bit number is called a Word while a 16-bit number is called a Halfword.

Using 2 bit 0 and 1 to form

a binary number of 1 bit, numbers are 0 and 1

a binary number of 2 bit, numbers are 00, 01, 10, 11

a binary number of 3 bit, such numbers are 000, 001, 010, 011, 100, 101, 110, 111

a binary number of 4 bit, such numbers are 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111

Therefore, using n bits there are 2^n binary numbers of n bits

Each digit in a binary number has a value or weight. The LSB has a value of 1. The second from the right has a value of 2, the next 4, etc.,

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0

The binary equivalent for some decimal numbers are given below.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011

3. Number Base Conversions

3.1 Conversion of decimal number to any number system

Step 1 convert the integer part by doing successive division using the radix of asked number systems.

Step 2 convert the fractional part by doing successive multiplication using radix of asked number system

3.2 Conversion of decimal to binary number system

The radix of asked number system is 2

Convert 87_{10} to $()_2$

2	87	→ 1
2	43	→ 1
2	21	→ 1
2	10	→ 0
2	5	→ 1
2	2	→ 0
	1	

$(1010111)_2$

Convert $(14.625)_{10}$ decimal number to binary number

2	14
2	7-0
2	3-1
	1 MSB

$(1110)_2$

1st Multiplication Iteration

Multiply 0.625 by 2

$0.625 \times 2 = 1.25$ (Product) Fractional part=0.25 Carry=1 **(MSB)**

2nd Multiplication Iteration

Multiply 0.25 by 2

$0.25 \times 2 = 0.50$ (Product) Fractional part = 0.50 Carry = 0

3rd Multiplication Iteration

Multiply 0.50 by 2

$0.50 \times 2 = 1.00$ (Product) Fractional part = 1.00 Carry = 1 **(LSB)**

$(101)_2$

The binary number of $(16.625)_{10}$ is $(1110.101)_2$

3.3 Conversion of decimal to octal number system

The radix of asked number system is 8

Convert $(264)_{10}$ decimal number to octal number

33
8)264 ₁₀
24
24
24
0 → 0 (LSD)

4
8)33
32
1 → 1

0
8)4
0
4 → 4 (MSD)

$(410)_8$

The octal number of $(264)_{10}$ is $(410)_8$

Convert $(105.589)_{10}$ decimal number to octal number

$$\begin{array}{r} 13 \\ 8 \overline{) 105} \\ \underline{8} \\ 25 \\ \underline{24} \\ 1 \end{array} \quad \text{1 MSB}$$

$$\begin{array}{r} 1 \\ 8 \overline{) 13} \\ \underline{8} \\ 5 \end{array} \quad \rightarrow 5$$

$$\begin{array}{r} 0 \\ 8 \overline{) 1} \\ \underline{0} \\ 1 \end{array} \quad \rightarrow 1 \text{ LSB}$$

(151)

$$\begin{array}{r} 0.589 \\ \times 8 \\ \hline 4.712 \\ \times 8 \\ \hline 5.696 \\ \times 8 \\ \hline 5.568 \\ \times 8 \\ \hline 4.544 \end{array}$$

MSB ← 4 ← 5 ← 5 ← 4 ← LSB

(0.4554)

The octal number of $(105.589)_{10}$ is $(151.4554)_8$

3.4 Conversion of decimal to Hexadecimal number system

The radix of asked number system is 16

Convert $(1693)_{10}$ decimal number to Hexadecimal number

$$\begin{array}{ll} 1693/16 = 105 & \text{Reminder (13) D (LSB)} \\ 105/16 = 6 & \text{Reminder 9} \\ 6/16 = 0 & \text{Reminder 6 (MSB)} \end{array}$$

$$(1693)_{10} = (69D)_{16}$$

Convert $(1693.0628)_{10}$ decimal fraction to hexadecimal fraction $(?)_{16}$

$$\begin{array}{ll} 1693/16 = 105 & \text{Reminder (13) D (LSB)} \\ 105/16 = 6 & \text{Reminder 9} \\ 6/16 = 0 & \text{Reminder 6 (MSB)} \end{array}$$

(69D)

Multiply 0.0628 by 16

$$0.0628 \times 16 = 1.0048(\text{Product}) \quad \text{Fractional part} = 0.0048 \quad \text{Carry} = 1 \quad (\text{MSB})$$

Multiply 0.0048 by 16

$$0.0048 \times 16 = 0.0768(\text{Product}) \quad \text{Fractional part} = 0.0768 \quad \text{Carry} = 0$$

Multiply 0.0768 by 16

$$0.0768 \times 16 = 1.2288(\text{Product}) \quad \text{Fractional part} = 0.2288 \quad \text{Carry} = 1$$

Multiply 0.2288 by 16

$$0.2288 \times 16 = 3.6608(\text{Product}) \quad \text{Fractional part} = 0.6608 \quad \text{Carry} = 3 \quad (\text{LSB})$$

(.1013)

$$(1693.0628)_{10} = (69D.1013)_{16}$$

3.5 Conversion of any number system to decimal number system

In general the numbers can be represented as

$$N = A_{n-1}r_{n-1} + A_{n-2}r_{n-2} + \dots + A_1r^1 + A_0r^0 + A_{-1}r^{-1} + A_{-2}r^{-2} + \dots$$

Where n= number in decimal

A= digit

r= radix of number system

n= The number of digits in the integer portion of number

m= the number of digits in the fractional portion of number

3.6 Conversion of binary to decimal number system

Convert $(101.101)_2 = (?)_{10}$

$$101.101$$

$$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 1 \times 4 + 0 \times 2 + 1 \times 1 + 1 \times (1/2) + 0 \times (1/4) + 1 \times (1/8)$$

$$= 4 + 0 + 1 + (1/2) + 0 + (1/8)$$

$$= 5 + 0.5 + 0.125$$

$$= 5.625$$

$$\text{Therefore } (101.101)_2 = (5.625)_{10}$$

3.7 Conversion of octal to decimal number system

Convert $(123)_8 = (?)_{10}$

$$123_8 = 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 = 64 + 16 + 3 = 73$$

the decimal equivalent of the number 123_8 is 73_{10}

Convert $(21.21)_8 = (?)_{10}$

$$21.21$$

$$= 2 \times 8^1 + 1 \times 8^0 + 2 \times 8^{-1} + 1 \times 8^{-2}$$

$$= 2 \times 8 + 1 \times 1 + 2 \times (1/8) + 1 \times (1/64)$$

$$= 16 + 1 + (0.25) + (0.015625)$$

$$= 17 + 0.265625$$

$$= 17.265625$$

$$\text{Therefore } (21.21)_8 = (17.265625)_{10}$$

3.8 Conversion of hexadecimal to decimal number system

Convert $(EF.B1)_{16} = (?)_{10}$

$$= E \times 16^1 + F \times 16^0 + B \times 16^{-1} + 1 \times 16^{-2}$$

$$= 14 \times 16 + 15 \times 1 + 11 \times (1/16) + 1 \times (1/256)$$

$$= 224 + 15 + (0.6875) + (0.00390625)$$

$$= 239 + 0.6914$$

$$= 239.691406$$

$$\text{Therefore } (EF.B1)_{16} = (239.691406)_{10}$$

Convert $(0.9D9)_{16} = (?)_{10}$

$$= 0 \times 16^0 + 9 \times 16^{-1} + D \times 16^{-2} + 9 \times 16^{-3}$$

$$= 0 \times 1 + 9 \times (1/16) + 13 \times (1/256) + 9 \times (1/4096)$$

$$= 0 + (0.5625) + (0.050781) + (0.0021972)$$

$$= 0.6154782$$

= 0.6154782

3.9 Conversion of binary to octal number system

Convert $(101101001)_2$ to $()_8$

Divide the binary into group of three digits from LSB we will find the following pattern

101|101|001 Now writing the equivalent decimal number of each group we get 5 | 5 | 1 So the equivalent octal number is 551_8

Convert 11001100.101 to $()_8$

011|001|100. |101|

3 1 4 . 5

So the equivalent octal number is 314.5

3.10 Conversion of binary to hexadecimal number system

Convert 111100010 to $()_{16}$

Divide the binary into group of four digits from LSB

0001|1110|0010

Now writing the equivalent hexadecimal number of each group

1|E|2

So the equivalent Hexa decimal number is $1E2_{16}$

Convert 11000011001.101 to $()_{16}$

0110|0001|1001|.1010|

6 1 9 . A

So the equivalent Hexa decimal number is $619.A_{16}$

3.11 Conversion of octal number system to hexa decimal number system

Convert $(25)_8$ to $()_{16}$

First convert octal to binary

The binary equivalent of 25 is 010101

Divide the binary into group of four digits from LSB

0001|0101

1 5

So the equivalent Hexa decimal number is 15_{16}

3.12 Conversion of hexa decimal number system to octal number system

Convert $(1A.2B)_{16}$ to $()_8$

First convert hexadecimal to binary

The binary equivalent of 1A.2B is 00011010.00101011

Divide the binary into group of Three digits

011|010|.001|010|110

3 2 . 1 2 6

so the equivalent octal number is 32.126_8

4. COMPLEMENTS

In digital computers to simplify the subtraction operation and for logical manipulation complements are used. There are two types of complements for each radix system the radix complement and diminished radix complement. The first is referred to as the r 's complement and the second as the $(r-1)$'s complement.

r 's Complement

Given a positive number N in base r with an integer part of n digits, the r 's complement of N is defined as $r^n - N$ if $N \neq 0$ and 0 if $N = 0$