Chapter 5: Mixtures and Alligations

Theory

Mixtures are of generally of two types. When two different ingredients are mixed together, it is known as simple mixture. Eg. a mixture of water and milk.

When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Allegation Rule

The rule of allegation is generally applied to

1. Find the mean or average value of mixture when the price of two or more ingredients which may be mixed together and the proportion in which they are mixed is given.
2. Find the proportion in which the ingredients at given prices must be mixed to get mixture of a given price.

Important

1. The word Allegation literally means linking. The rule takes its name in working out questions on mixture.
2. Allegation method is applied for percentage value, ratio, rate, prices, speed etc. and not for absolute values.

The Allegation rule states that,

"When different quantities of same or different ingredients, of different cost (value) are mixed together to produce a mixture of a mean cost (value), the ratio of their quantities are inversely proportional to the differences in their cost (value) from the mean cost (value).

Quantity of Cheaper/Quantity of Dearer = (Cost of Dearer - Mean Price)/(Mean Price - Cost of Cheaper).

We can use the allegation rule in following manners:

1. **To find the ratio** in which two ingredients A and B costing Rs. C_1 per kg and Rs. C_2 per kg be mixed in order to get a mixture costing Rs. C per kg.

   Quantity of A/Quantity of B = $(C_2 – C)/(C – C_1)$

2. **To find the cost price(s)** of mixture (mean price) when two ingredients A and B are mixed with quantity q_1 and q_2 and cost C_1 and C_2 respectively.

   C = cost of mixture = $(C_1 q_1 + C_2 q_2)/(q_1 + q_2)$

   When more than two ingredients are mixed

   C = cost of mixture = $(C_1 q_1 + C_2 q_2 + C_3 q_3 + … + C_n q_n)/(q_1 + q_2 + q_3 + … + q_n)$

Important Results

1. Suppose a container contains M units of mixture of 'A' and 'B'. From this, R units of mixture is taken out and replaced by an equal amount of ingredient B only. This process (of taking out and replacing it) is repeated 'n' times, then, after 'n' operations, Amount of 'A' left/Amount of 'A' originally present = $(1 – (R/M))^n$ and Amount of 'B' left = M – Amount of 'A' left

2. When the container contains only ingredient 'A' of x_0 unit. From this, x_n unit is taken out and replaced by an equal amount of ingredient 'B'. This process is repeated n times; after n operations,

    Amount of 'A' left/Amount of 'A' originally present = $(1 - (x_n/x_0))^n$ this implies

    Amount of 'A' left = $x_0(1 - (x_n/x_0))^n$ and

    Amount of 'A' left/Amount of 'B' left = $(1 - (x_n/x_0))^n/[1 - (1 - (x_n/x_0))^n]$

3. If n containers of equal size are filled with a mixture of A and B. The ratio of A and B in container is as follows: $a_1:b_1, a_2:b_2, a_3:b_3, \ldots, a_n:b_n$. If the contents of all the n containers are emptied into a single vessel, then proportion of A and B in it is given by $a_1/(a_1 + b_1) + a_2/(a_2 + b_2) + a_3/(a_3 + b_3) + \ldots + a_n/(a_n + b_n): b_1/(a_1 + b_1) + b_2/(a_2 + b_2) + b_3/(a_3 + b_3) + \ldots + b_n/(a_n + b_n)$.

Important Results

1. If a can contains x units of a liquid initially and y units of the liquid is replaced by another liquid for n times, then the amount of the first liquid in the can = $x[1 - (y/x)]^n$ units.

2. To find ratio when three ingredients of quantities q_1, q_2, q_3 and cost C_1, C_2, C_3 are mixed to form a mixture of cost C.

    $$q_1:q_2:q_3 = (C_2 - C)(C_3 - C):(C_3 - C)(C_1 - C):(C_1 - C)(C_2 - C)$$

3. When mixture 1 has ingredients (A and B) in a : b and mixture 2 has the same ingredients (A and B) in x : y. Now, M units of mixture 1 and N units of mixture 2 are mixed to form a resultant mixture with ingredient (A and B) in $q_A : q_B$.

Case I: To find $q_A : q_B$

   $q_A : q_B = M[a/(a + b)] + N[x/(x + y)]/[b/(a + b)] + N[y/(x + y)]$

   And, amount of A in the resultant mixture = $[q_A/(q_A + q_B)] \times (M + N)$ and

amount of B = $[q_B/(q_A + q_B)] \times (M + N)$

Case II: To find M and N

Quantity of 1st Mixture/Quantity of 2nd Mixture = Q_1/Q_2

= $[x/(x + y)] - [q_A/(q_A + q_B)]/[q_A/(q_A + q_B)] - [a/(a + b)]$

Then amount of 1st mixture in the resultant mixture = $M = [Q_1/(Q_1 + Q_2)] \times (M + N)$ and amount of 2nd mixture in the resultant mixture = $N = [Q_2/(Q_1 + Q_2)] \times (M + N)$

Class Work

1) In what ratio the rice of inferior quality (price – Rs. 15/kg) should be mixed with the rice of superior quality (price – Rs. 25/ kg) so that the seller will gain Rs. 2 after selling the rice for Rs. 20/kg.

   a) 7:3   b) 6:5   c) 5:3   d) 7:6

2) Find the price of a particular type of wheat in Rs./kg if 10 kg of it is mixed with 20 kg of another type of wheat (price – Rs. 18/kg) so that the seller earns nothing after selling the mixed wheat for Rs. 15/kg.

   a) Rs. 9/kg   b) Rs. 8/kg   c) Rs. 10/kg   d) Rs. 11/kg

3) A milkman mixes 1 liter of water to 'x' liters of milk and sells the mixture for Rs. 20/liter. If the price of pure milk is Rs 25/liter then find the value of 'x'.

   a) 4   b) 3   c) 2   d) 1

4) The owner of a wine shop gains Rs. 25 after selling one bottle of the mixture of water and wine in the proportion 1:5 for Rs. 100. Find the price of one bottle of pure wine.

   a) Rs. 90   b) Rs. 80   c) Rs. 85   d) Rs. 95

5) A builder uses cement formed by mixing an 2 kg of inferior quality cement of price Rs. 80/kg to 3 kg of superior quality cement of price Rs. 120/kg. Find the price in Rs./kg of the cement that the builder uses should be considered for costing purposes.

   a) Rs. 104/kg   b) Rs. 100/kg   c) Rs. 110/kg   d) Rs. 95/kg

6) A fruit seller mixes strawberries of two types in the ratio 1:3. The quantity of better quality strawberries is more in the mixture. He sells the mixture of strawberries for Rs. 100/kg and there by gains Rs. 20. Find the price of the strawberries of inferior quality in Rs./kg if the price of the strawberries of the superior quality is Rs. 90/kg.

   a) Rs. 50/kg   b) Rs. 55/kg   c) Rs. 60/kg   d) Rs. 45/kg

7) A tank contains 100 liters of petrol. 10% of petrol was replaced with diesel. This process was repeated four times. Find the amount of petrol remaining in the tank.

   a) 65.61 liters   b) 65 liters   c) 70 liters   d) None of these

8) A reservoir contains 1000 liters of liquid A. Alternately 10% and 20% liters of liquid was replaced with water. Find the amount of liquid A left in the tank if such alternate replacements were carried out two times.

   a) 518.4 liters   b) 525 liters   c) 750 liters   d) None of these

9) A vessel contains 100 liters of mixture of water and alcohol. The ratio of alcohol is to water is 1:4. Another vessel contains 200 liters of mixture of water and alcohol. The ratio of water is to alcohol

is 1:4. The mixtures from both the vessels are poured into a third vessel. Find the ratio of alcohol is to water in the third vessel.

   a) 3:2   b) 2:3   c) 3:1   d) 1:3

10) A vessel contains liquid A and water in the ratio 4:1 Another vessel contains liquid A and water in the ratio 1:4 In what proportion the mixture from the first vessel should be mixed with the mixture from second vessel so that the net ratio of the amount of liquid A is to amount of water becomes 7:3

   a) 5:1   b) 1:2   c) 3:1   d) 3:2

11) A vessel contains 100 liters of pure milk. 20% of the liquid in the vessel is now replaced with water. This procedure was carried out three times. Find the amount of milk in the vessel.

   a) 51.2 liters   b) 50 liters   c) 60 liters   d) None of these

12) The weight of an empty bottle is $1/6^{th}$ the weight of the bottle full of water. The certain percentage of the water is removed from the bottle full of water and the bottle was weighed. The weight of the bottle turned out to be the $1/3^{rd}$ the weight of the bottle full of water. Find the percentage of the water removed.

   a) 80%   b) 75%   c) 70%   d) 85%

13) A vessel contains 'x' liters of pure acid. 10% of acid is replaced with water. This process was repeated once more. The difference in the quantity of acid in the vessel and the quantity of water in the vessel now is 62 liters. How much the vessel hold?

   a) 100 liters   b) 125 liters   c) 80 liters   d) 90 liters

14) A cup of coffee contains 1 part of pure milk and 3 parts of water (with dissolved coffee powder). How much part of the water (with dissolved coffee powder) must be replaced with milk so that the resulting mixture is half milk and half water?

   a) One-third   b) One-fourth   c) One-fifth   d) One-sixth

15) 2 liters of water is added to 8 liters of 25% solution of alcohol in water. What is the strength of alcohol now?

   a) 20 %   b) 19 %   c) 18 %   d) 21 %

16) In what proportion a salt at Rs. 3.2 per kg be mixed with salt at 2.9 per kg so that the mixture will be worth Rs. 3.08 per kg.

   a) 3:2   b) 2:3   c) 3:4   d) 4:3

17) How many kg of tea selling at Rs. 5.2 a kg be mixed with tea selling at Rs. 4.4 a kg to make mixture of 15 kg. at Rs. 73.2?

    a) 9 kg   b) 8 kg   c) 7 kg   d) 10 kg

18) A dealer mixes varieties of grains at Rs. 12 a kg and Rs. 30 a kg so that he can gain 10% by selling the resulting mixture at Rs.16.5 a kg. Find the proportion in which the grains are mixed?

    a) 5:1   b) 1:5   c) 2:3   d) 3:2

19) A milk seller buys milk at Rs. 6 a liter and after adding water he sells it at Rs. 7.2 a liter and thus gains 60%. What is the percentage of water in the mixture?

    a) 25 %   b) 24 %   c) 23 %   d) 26 %

20) What should be the proportion in which the water must be mixed with milk to gain 33(1/3)% by selling the mixture at cost price?

    a) 1:3   b) 2:3   c) 3:1   d) 3:2

Home Work

1) Rs. 198.5 were divided among 300 children. Each boy gets Rs. 1 and each girl gets Rs. 0.5. Find the number of boys.

   a) 94   b) 93   c) 91   d) 92

2) The average salary per head of all officers and workers is Rs. 140. The average salary of 12 officers is Rs. 800 and the average salary per head of the rest is Rs. 120. What is the number of workers in the factory?

   a) 396   b) 392   c) 400   d) 404

3) From a cask full of alcohol 4 liters of alcohol is drawn and the cask is filled with water. Again four liters of the mixture is drawn and the cask is again filled with water. The quantity of alcohol now left in the cask to that of water in it is 36:13.Find the capacity of the cask.

   a) 28 liters   b) 25 liters   c) 30 liters   d) 32 liters

4) Mr. X has 100 kg of tea, a part of which he sells at a loss of 5% and the rest at cost price. In the whole transaction, he gets a loss of 3%. What is the quantity of tea which he sells at loss of 5%?

   a) 60 kg   b) 50 kg   c) 55 kg   d) 65 kg

5) Two casks of capacity 30 liters and 25 liters are filled with mixture of alcohol and water. The proportion water is to alcohol in two casks being respectively 5:1 and 4:1 If the contents of the cask are mixed and 5 liters of water is added in sufficient large vessel, what will be the proportion alcohol and water in the resulting mixture?

   a) 1:6   b) 6:1   c) 2:3   d) 3:2

6) A man Y stole wine from a bottle of sherry that contained 30% of alcohol and replaced what he had stolen by wine containing 12% of spirit. The bottle was then of 18% strength only. How much of bottle did the man steal?

   a) 2/3rd b) 1/3rd c) 1/4th d) 2/5th

7) Metal A, B, C, and D are melted together in the ratio 2:6:7:8 and formed into a mass of an alloy. Cost of 1 kg of metal D is Rs. 15, of a kg of metal C is 20% more than that of metal D, of metal A 20 times that metal C. If the quantity melted was sold at Rs. 66 for profit of 10%, what is the price of metal B in rupees per kg.?

   a) Rs. 69 per kg   b) Rs. 70 per kg   c) Rs. 72 per kg   d) Rs. 71 per kg

8) A person owns certain number of cows and certain number of hens. The head count of all the cows and hens is 90 while total number of legs is 224. What is the number of cows and the number of hens owned by the person?

   a) 22 cows, 68 hens   b) 20 cows, 70 hens

   c) 22 cows, 70 hens   d) 20 cows, 68 hens

9) Metal A is 19 times as heavy as water and metal B is 9 times as heavy as water. In what ratio must these metals be mixed so that the mixture is 15 times as heavy as water?

   a) 3:2   b) 2:3   c) 1:3   d) 3:1

10) In a mixture of 72 liters of alcohol and water mixed in the ratio 11:1, find the amount of water that should be added such that alcohol and water are in the ratio 9:1

   a) 1(1/3) liters   b) 2(1/3) liters   c) 3(1/3) liters   d) 4(1/3) liters

Answer Keys:

Answers for the above questions is option a.