Implementation of Freedman Inequality in IBM Qiskit

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Objectives :-

Creating a circuit in IBM Qiskit analogous to the setup performed in a physical lab to prove violation of Freedman Inequality . Learn and understand about quantum circuits , quantum entanglement , hidden variables , local theory , rotational invariance , no-enhancement assumption . Learn Qiskit code and its libraries . Understand research paper published by American Journal of Physics.

Using the results from Qiskit to calculate δ (if positive then violates the freedman inequality) .

Introduction:-

Local realism infers that physical processes occurring at one location should not instantaneously influence events at another distant location.

The assumption of local realism imposes constraints, such as Bell inequalities, on quantities obtained from measurements. In recent years, various tests of local realism have gained popularity in undergraduate laboratories, giving students the exciting opportunity to experimentally contradict this philosophical assumption.

Freedman's inequality is a special case of the Clauser-Horne inequality.The calculations required to test Freedman's inequality are correspondingly simpler and the theory is less abstract.

Mathematical Lemma's Used:-

If real numbers x1, x2, y1, y2, X, and Y satisfy

(1)

$0 \le x\ 1 < X$,

(1a)

$0 \le x2 \le X$,

(1b)

$0 \le Y1 \le Y$,

(1c)

$0 \le Y2 \le Y$,

(1d)

and

$U = x1y1 - x1y2 + x2y1 + x2y2 - Yx2 - Xy1$,

(2)

then it can be shown that

$-XY \le U \le 0$.

(3)

The Part of The Proof where local hidden variables is assumed :-

We specify that photon 1 travels to polarizer A, and photon 2 travels to polarizer B. If we assume that photon 1 is unaffected by polarizer B, and photon 2 is unaffected by polarizer A, then $p_{12}(\lambda,a,b)$ may be factored as

$$p_{12}(\lambda, a, b) = p_1(\lambda, a)p_2(\lambda, b). \tag{9}$$

Equation (9) assumes *locality*, which means, in this case, that the measurement of one photon is unaffected by the other photon's polarizer. Equation (9) relies also on the hidden-variables assumption. In fact, the dependence on a common variable λ is what allows the outcomes of the two measurements to be written as independent events.[22] λ characterizes both photons. In a local hidden-variables theory, λ is the (only) explanation for any correlations in the measurements of the photon pairs.

Freedman's Inequality Formula :-

$$\delta \equiv \left| \frac{N(22.5°) - N(67.5°)}{N_0} \right| - \frac{1}{4} \leq 0,$$

If valid assumptions are made in the derivation of Freedman's inequality, δ must be nonpositive. If measurement contradicts this requirement, then one or more of the assumptions must be incorrect.

A violation of Freedman's inequality thus must contradict the equation, which expresses locality in the context of a hidden-variables theory. The most obvious conclusion is that nonlocality is a fact of nature, and the two entangled photons maintain some kind of connection.

Snippet of Our Code in Qiskit Jupyter Lab:-

[9]:
```
from qiskit import QuantumCircuit, Aer, execute
from qiskit.visualization import plot_histogram
import numpy as np
import matplotlib.pyplot as plt
from math import sqrt

# Function to run the Bell test experiment
def run_bell_test_experiment(angle_a, angle_b):
    qc = QuantumCircuit(2, 2)
    qc.h(0)
    qc.cx(0, 1)
    qc.ry(2 * angle_a, 0)
    qc.ry(2 * angle_b, 1)
    qc.measure(0, 0)
    qc.measure(1, 1)
    backend = Aer.get_backend('qasm_simulator')
    job = execute(qc, backend, shots=1024)
    result = job.result()
    counts = result.get_counts(qc)
    return counts

# Function to calculate N(ϕ)
def calculate_N(counts):
    coincidences = counts.get('00', 0) + counts.get('11', 0)
    anti_coincidences = counts.get('01', 0) + counts.get('10', 0)
    return coincidences - anti_coincidences

# Fixed angle_b
angle_b = np.deg2rad(67.5)

# Lists to hold results and plotting data
violation_angles = []
satisfaction_angles = []
theta_values = []
normalized_delta_values = []

# Run experiment and check Freedman's inequality over a range of angles
for degree in np.arange(0, 90.25, 0.25):
```

```python
# Run experiment and check Freedman’s inequality over a range of angles
for degree in np.arange(0, 90.25, 0.25):
    angle_a = np.deg2rad(degree)

    counts_a = run_bell_test_experiment(angle_a, angle_b)
    counts_b = run_bell_test_experiment(angle_b, angle_a)

    N_a = calculate_N(counts_a)
    N_b = calculate_N(counts_b)

    delta = abs(N_a - N_b) / N_b - 0.25

    theta_values.append(degree)
    normalized_delta_values.append(delta)

    # Print delta values and degrees
    print(f"Degree: {degree}, Delta: {delta}")

    if delta <= 0:
        satisfaction_angles.append(degree)
    else:
        violation_angles.append(degree)

# Print angle lists
print(f"Angles where Freedman's inequality is satisfied: {satisfaction_angles}")
print(f"Angles where Freedman's inequality is violated: {violation_angles}")

# Normalize delta values
sum_of_squares = sum(delta**2 for delta in normalized_delta_values)
norm_factor = sqrt(sum_of_squares)
normalized_delta_values = [delta / norm_factor for delta in normalized_delta_values]

# Plot the graph with adjusted y-axis limits
plt.scatter(theta_values, normalized_delta_values, label='Normalized Delta vs Theta'
plt.xlabel('Theta (degrees)')
plt.ylabel('Normalized Delta')
plt.title('Graph of Normalized Delta vs Theta')

# Set y-axis limits to control spacing

plt.legend()
plt.grid(True)
plt.show()
```

Results:

```
Angles where Freedman's inequality is satisfied: [0.0, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.25, 3.5, 3.75, 4.0, 4.25, 4.5, 4.75, 5.0, 5.25,
6.25, 6.5, 6.75, 7.0, 7.25, 7.5, 7.75, 8.0, 8.25, 8.5, 8.75, 9.0, 9.25, 9.5, 9.75, 10.0, 10.25, 10.5, 10.75, 11.0, 11.25, 11.5, 11.75, 12.0, 12.25, 12.5, 12.75, 13.0, 13.
14.0, 14.25, 14.5, 14.75, 15.0, 15.25, 15.5, 15.75, 16.0, 16.25, 16.5, 16.75, 17.0, 17.25, 17.5, 17.75, 18.0, 18.25, 18.5, 18.75, 19.0, 19.25, 19.5, 19.75, 20.0, 20.25, 2
21.75, 22.0, 22.25, 22.75, 23.75, 24.0, 24.25, 25.25, 26.0, 26.25, 27.0, 27.25, 27.5, 27.75, 28.0, 28.25, 28.5, 29.0, 29.25, 29.5, 29.75, 30.0, 30.25, 30.5, 30.75, 31.0,
0, 32.25, 32.5, 32.75, 33.0, 33.25, 33.5, 33.75, 34.25, 34.5, 34.75, 35.0, 35.25, 35.5, 35.75, 36.0, 36.25, 36.5, 36.75, 37.0, 37.25, 37.5, 37.75, 38.0, 38.25, 38.5, 38.7
9.5, 39.75, 40.0, 40.25, 40.5, 40.75, 41.0, 41.25, 41.5, 41.75, 42.0, 42.25, 42.5, 42.75, 43.0, 43.25, 43.5, 43.75, 44.0, 44.25, 44.5, 44.75, 45.0, 45.25, 45.5, 45.75, 46
46.75, 47.0, 47.25, 47.5, 47.75, 48.0, 48.25, 48.5, 48.75, 49.0, 49.25, 49.5, 49.75, 50.0, 50.25, 50.5, 50.75, 51.0, 51.25, 51.5, 51.75, 52.0, 52.25, 52.5, 52.75, 53.0, 5
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68.25, 68.5, 68.75, 69.0, 69.25, 69.5, 69.75, 70.0, 70.25, 70.5, 70.75, 71.0, 71.25, 71.5, 71.75, 72.0, 72.25, 72.5, 72.75, 73.0, 73.25, 73.5, 73.75, 74.0, 74.25, 74.5, 7
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2.5, 82.75, 83.0, 83.25, 83.5, 83.75, 84.0, 84.25, 84.5, 84.75, 85.0, 85.25, 85.5, 85.75, 86.0, 86.25, 86.5, 86.75, 87.0, 87.25, 87.5, 87.75, 88.0, 88.25, 88.5, 88.75, 89
89.75, 90.0]
Angles where Freedman's inequality is violated: [21.25, 21.5, 22.5, 23.0, 23.25, 23.5, 24.5, 24.75, 25.0, 25.5, 25.75, 26.5, 26.75, 28.75, 31.5, 34.0]
```

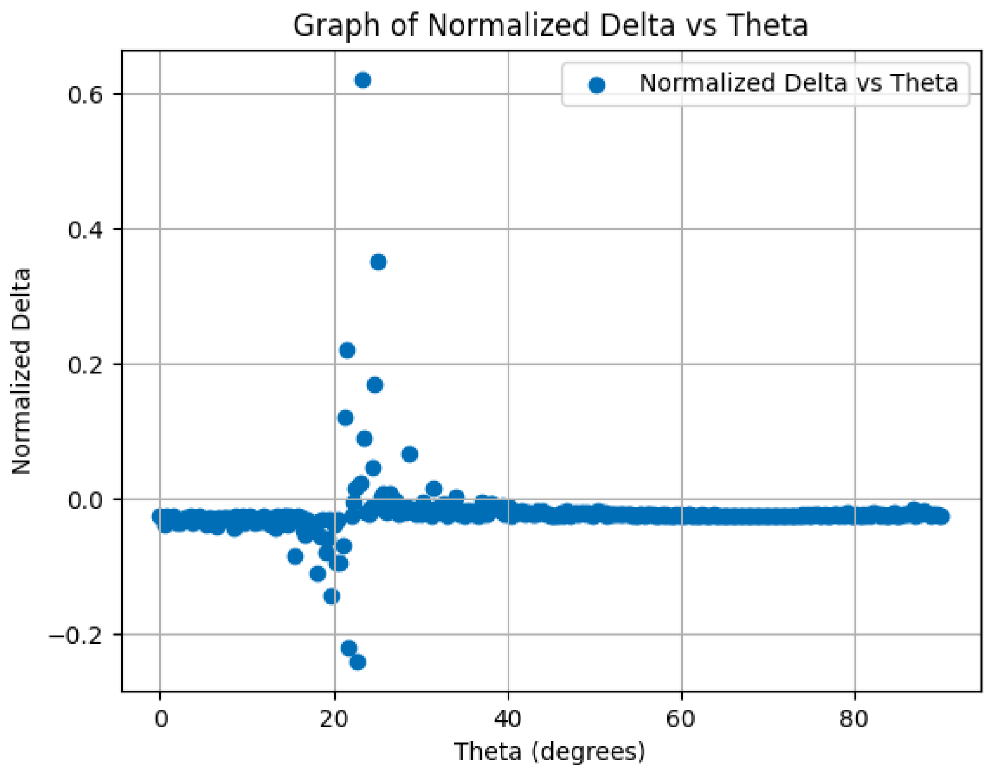

Conclusion:

In our code we had kept the second fixed at 67.5 degrees . Clearly from the graph we can see that the value of δ is positive at 22.5 degrees(and angles near its vicinity) , hence the inequality is getting violated .

Thus the constraint imposed by local realism is incorrect .Nonlocality is a fact of nature, and the two entangled photons maintain some kind of connection.

Cited References & Resources:

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