Constructor University Bremen

CO-520-B

Signals & Signals Lab

Fall Semester 2023

Lab Experiment 5 - AM Modulation

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Prelab - Amplitude Modulation

Problem 1 - Single frequency Amplitude Modulation

1. **Derive an expression describing the modulation index m as a function of the modulation envelope, (use A_{min} and A_{max}!).**

We know that,

$$A_{max} = A_c + A_m \text{ and } A_{min} = A_c - A_m$$

where A_m is the amplitude of the modulation signal and A_c is the amplitude of the carrier signal.

$$A_m = \frac{A_{max} - A_{min}}{2}$$
$$A_c = \frac{A_{max} + A_{min}}{2}$$

We also know that the modulation index, which is a measure of amplitude variation about an unmodulated carrier, is given by:

$$m = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

2. **Derive an expression describing the ratio of the total sideband power to the total power $r_P = P_s/P_{tot}$ in the modulated wave delivered to a load resistor. Express the ratio in terms of the modulation index.**

The carrier power and side power are given by the following formulae:

$$P_c = \overline{(A_c \cos \omega_c t)^2} = \frac{{A_c}^2}{2}$$
$$P_s = \frac{1}{2}\overline{x^2(t)} = \frac{1}{2}\overline{(A_m \cos \omega_m t)^2} = \frac{{A_m}^2}{4}$$
$$P_{tot} = P_c + P_s$$

$$P_{tot} = \frac{{A_c}^2}{2} + \frac{{A_m}^2}{4} = \frac{\frac{{A_m}^2}{4}}{\frac{{A_m}^2}{4}}\left(\frac{{A_c}^2}{2} + \frac{{A_m}^2}{4}\right) = \frac{{A_m}^2}{4}\left(\frac{2{A_c}^2}{{A_m}^2} + 1\right) = P_s\left(\frac{2}{m^2} + 1\right)$$

$$P_{tot} = P_s\left(\frac{(2 + m^2)}{m^2}\right)$$
$$\frac{P_s}{P_{tot}} = \frac{m^2}{2 + m^2}$$
$$r_p = \frac{m^2}{2 + m^2}$$

3. **Calculate the ratio r_P assuming a modulation index of 100%.**

$$m = 100\% = 1$$

$$r_P = \frac{1^2}{2+1^2} = \frac{1}{3}$$

4. **A carrier $V_C(t)$=5cos(20000πt) is modulated by a signal $V_m(t)$=2+cos(2000πt). Calculate the ratio r_P. How would you change the input signals to maximize the side band to total power ratio?**

Since there is an offset in the modulation signal,

$$P_c = \frac{(A_c+V_{off})^2}{2R} = \frac{49}{2R}$$

$$P_s = \frac{A_m^2}{4R} = \frac{1}{4R}$$

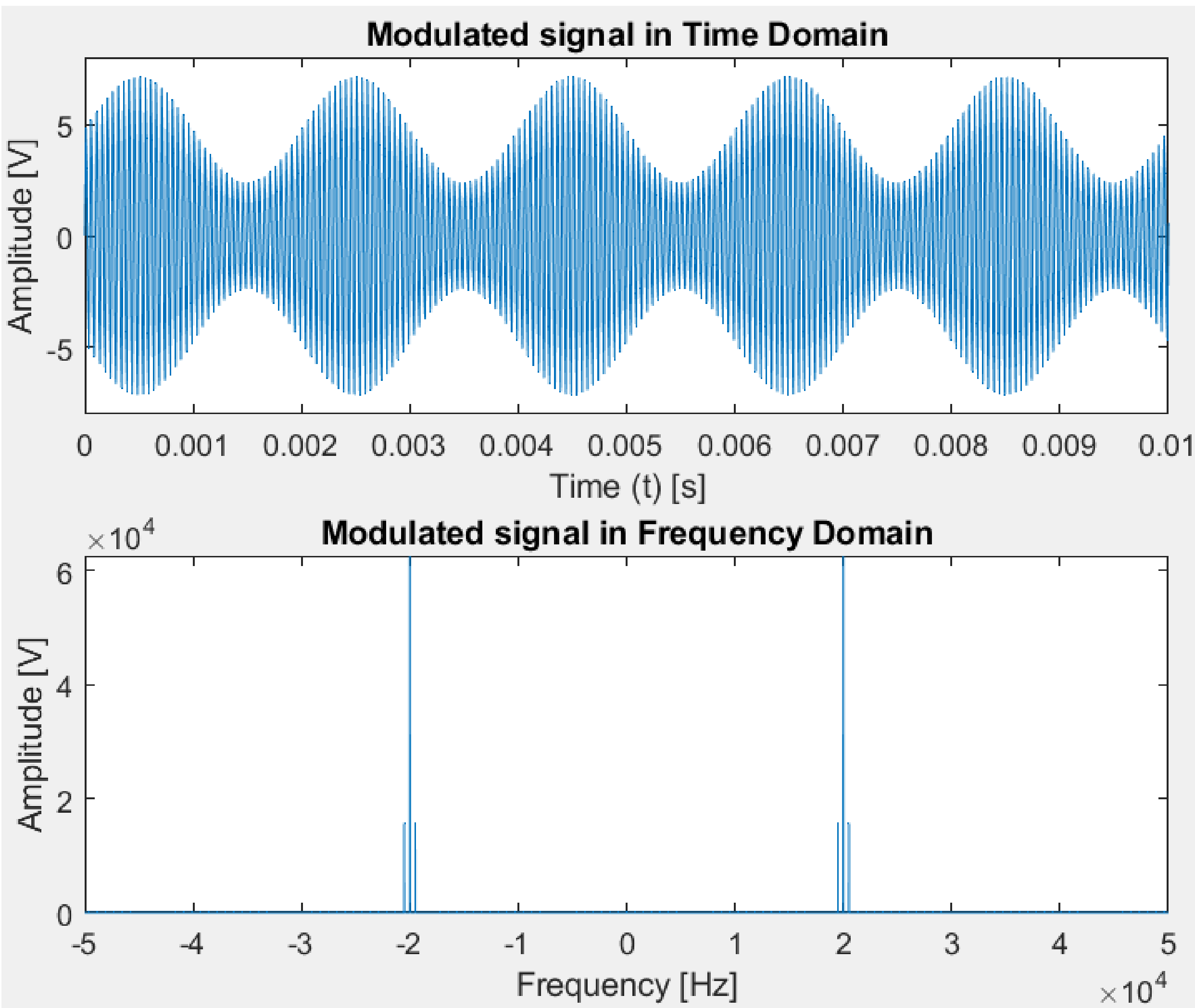
$$r_P = \frac{\frac{1}{4R}}{\frac{98}{4R}+\frac{1}{4R}} = \frac{1}{99}$$

The DC offset contributes to $P_{tot,}$ and to maximize the ratio, it needs to be reduced.

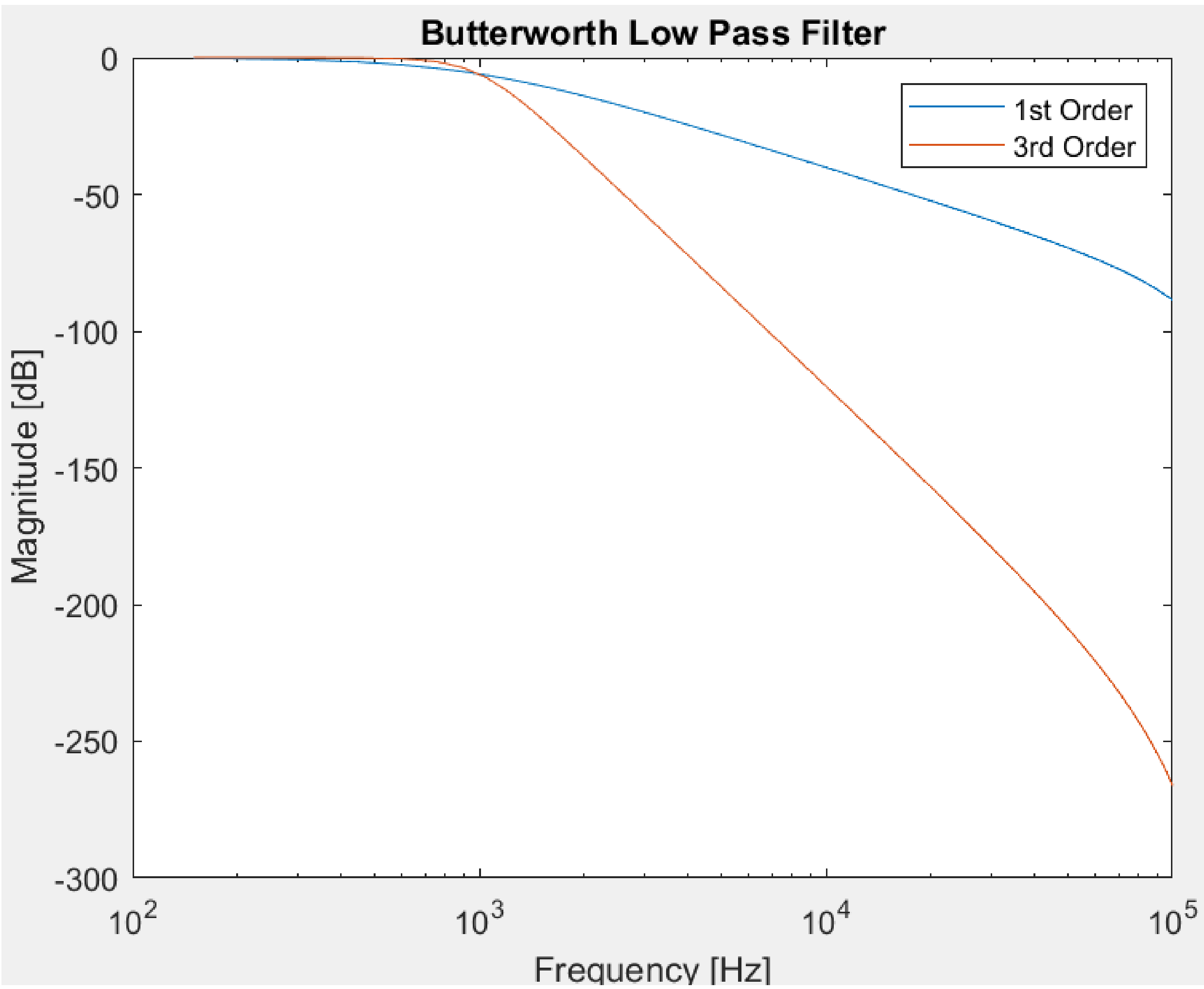
Problem 2 - Amplitude Demodulation

Use MatLab to generate an AM modulated signal with a carrier frequency f = 20 KHz and a_m = 5 V. Modulate with a sine wave f = 500 Hz. Use a modulation index m = 50%. Simulate the demodulation of this AM signal with Matlab. Show the following steps:

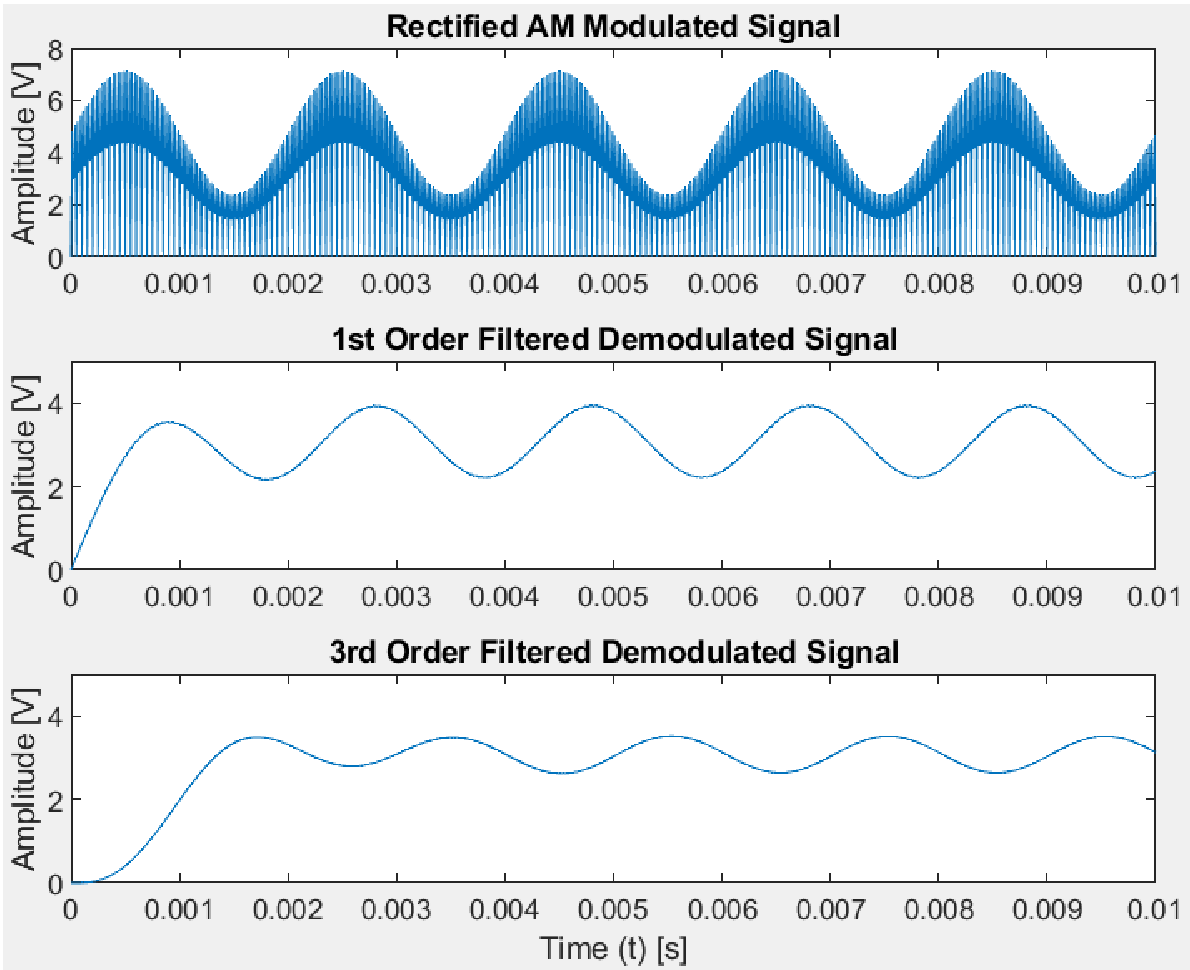
1. **Plot the modulated signal in the time and frequency domain.**



2. **Design a first and a third order low pass filter (butterworth filter) to demodulate the signal. The cut-off frequencies of the filters should be 1 KHz. Plot the Bode diagram of these filters for a frequency range from 100 Hz to 100 KHz to verify the function.**



3. **Rectify the AM modulated signal and apply the 1. order low pass filter to the rectified signal. Plot the rectified and the demodulated signal. Change the order of the filter from 1. to 3. Plot the demodulated signal.**



4. **Why is it better to use a higher-order filter for the demodulation of the signal?**

The demodulated signal exhibits a significantly smoother and sharper profile when filtered with a 3rd-order filter compared to a 1st-order filter. This observation highlights the effectiveness of higher-order filters in minimizing the residual effects of the carrier signal. In contrast, lower-order filters, as seen in the MATLAB simulation, tend to retain some carrier signal, resulting in a less smooth output. The benefits of using a higher-order filter become evident as it diminishes the effects carried over from the carrier signal. The MATLAB simulation demonstrates that higher-order filters contribute to achieving a more sharply filtered and distortion-reduced demodulated signal compared to their lower-order counterparts.

5. Attache the full Matlab script

```
%Plot the modulated signal in time and frequency domain
Fs=100000;                    %given values
Fc=20000;
Fm=500;
N=25000;
m=0.5;
t=0:1/Fs:(N*(1/Fs)-(1/Fs)); %domains
f=(-Fs/2):Fs/N:Fs/2-Fs/N;
wc=2*pi*Fc;                   %functions
wm=2*pi*Fm;
car=5*sin(wc*t);
mod=sin(wm*t);
x=(1+m*mod).*car;
x_fft=fft(x);
x_fft=fftshift(abs(x_fft));
figure (1);                   %plotting
subplot(2,1,1);
plot(t,x);
xlabel('Time (t) [s]');
ylabel('Amplitude [V]');
title('Modulated signal in Time Domain');
xlim([0,0.01]);
ylim([-8,8]);
subplot(2,1,2);
plot(f,x_fft);
xlabel('Frequency [Hz]');
ylabel('Amplitude [V]');
```

```
title('Modulated signal in Frequency Domain');

%Designing first and third order butterworth filters
fs=300000;
fc=1000;
[b1,a1]=butter(1,(fc/(fs/2)));
[b3,a3]=butter(3,(fc/(fs/2)));
[h1,f1]=freqz(b1,a1,1000,fs);
[h3,f3]=freqz(b3,a3,1000,fs);
hdB1=20*log10(h1.*conj(h1));
hdB3=20*log10(h3.*conj(h3));
figure(2);
semilogx(f1, hdB1);
hold on;
semilogx(f3,hdB3);
title('Butterworth Low Pass Filter');
xlabel('Frequency [Hz]');
ylabel('Magnitude [dB]');
xlim([100,100000]);
legend('1st Order','3rd Order');

%Rectifying the AM signal
abs_x=abs(x);
figure(3);
subplot(3,1,1);
plot(t,abs_x);
title('Rectified AM Modulated Signal');
```

```
xlim([0,0.01]);
ylim([0,8]);
ylabel('Amplitude [V]');
%1st order filter
x1 = filter(b1, a1, abs_x);
subplot(3,1,2);
plot(t, x1);
title('1st Order Filtered Demodulated Signal');
ylabel('Amplitude [V]');
xlim([0, 0.01]);
ylim([0,5]);
%3rd order filter
x3 = filter(b3, a3, abs_x);
subplot(3,1,3);
plot(t, x3);
title('3rd Order Filtered Demodulated Signal');
xlabel('Time (t) [s]');
ylabel('Amplitude [V]');
xlim([0,0.01]);
ylim([0,5]);
```

Experimental Set-up and Results

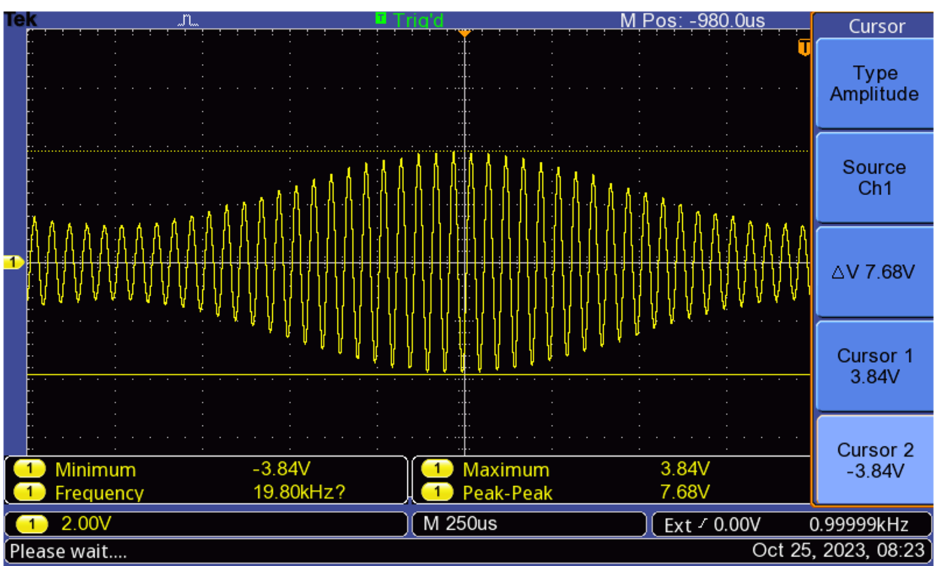
Part 1 - AM Modulated Signals in Time Domain

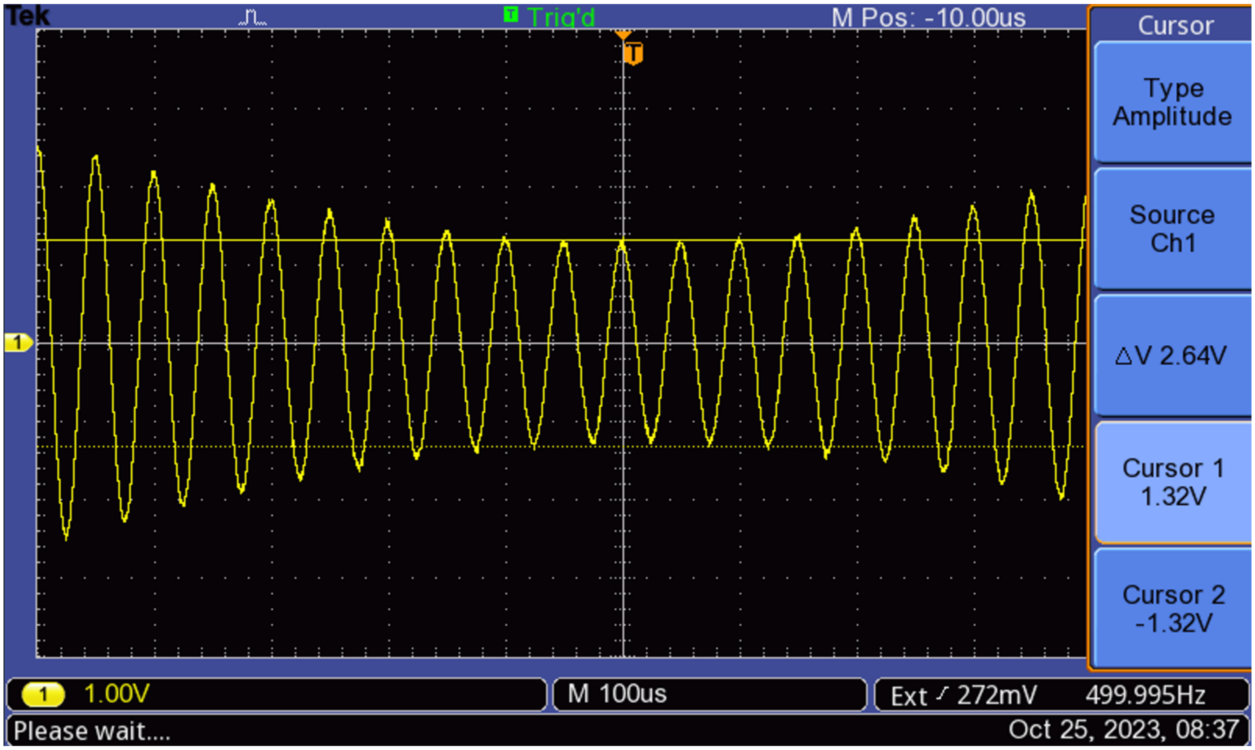
Experimental Setup and Procedure

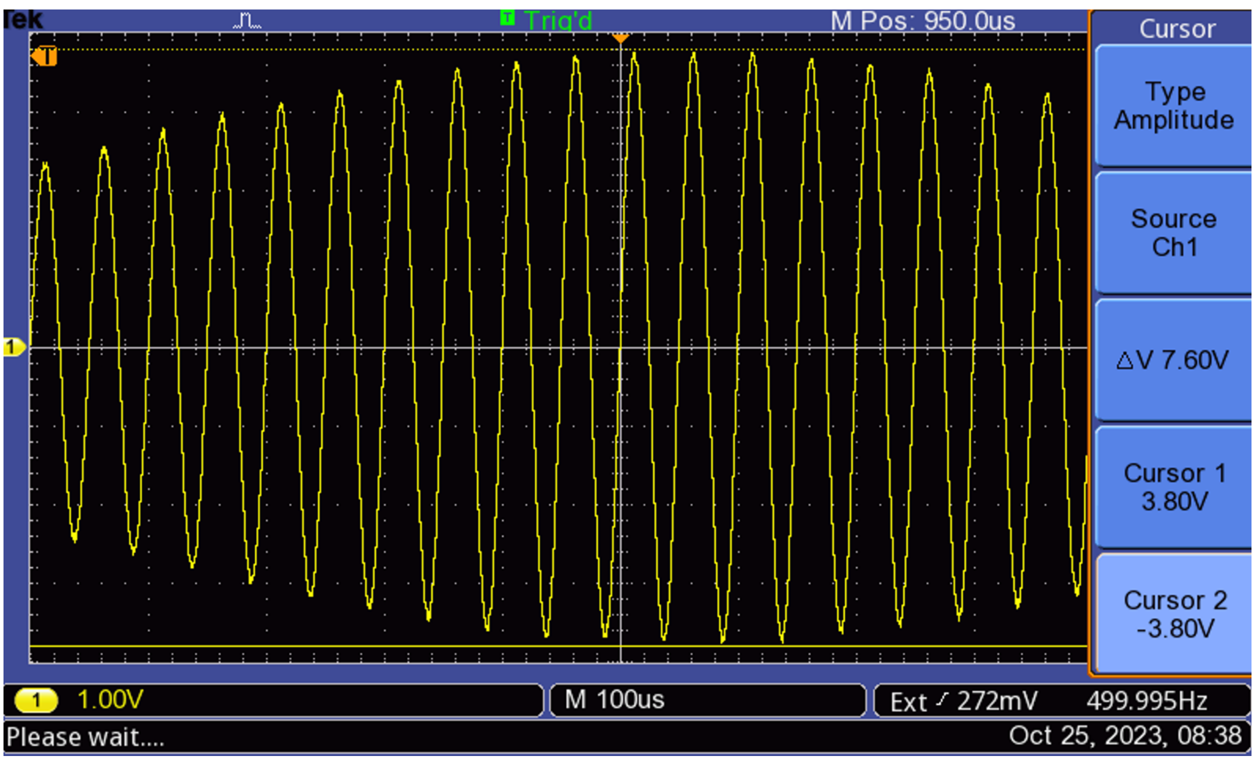
1. The AM signal is generated by the function generator. Settings of the function generator:
    - ➢ Signal Shape = Sine
    - ➢ Modulation = AM
    - ➢ Carrier frequency = 20 kHz
    - ➢ Carrier Amplitude = 10 V_{PP}
    - ➢ Modulation Frequency = 500 Hz
    - ➢ Modulation index = 50%
2. Connect the function generator to the oscilloscope. Measure the frequency and the amplitude properties of the modulated signal and obtain the modulation index. Take hardcopies.
3. Repeat step 1 by setting the modulation index to 70%. Take hardcopies.
4. Adjust the modulation index to be 120% and observe the effect on the AM signal. Take a hardcopy.

Results

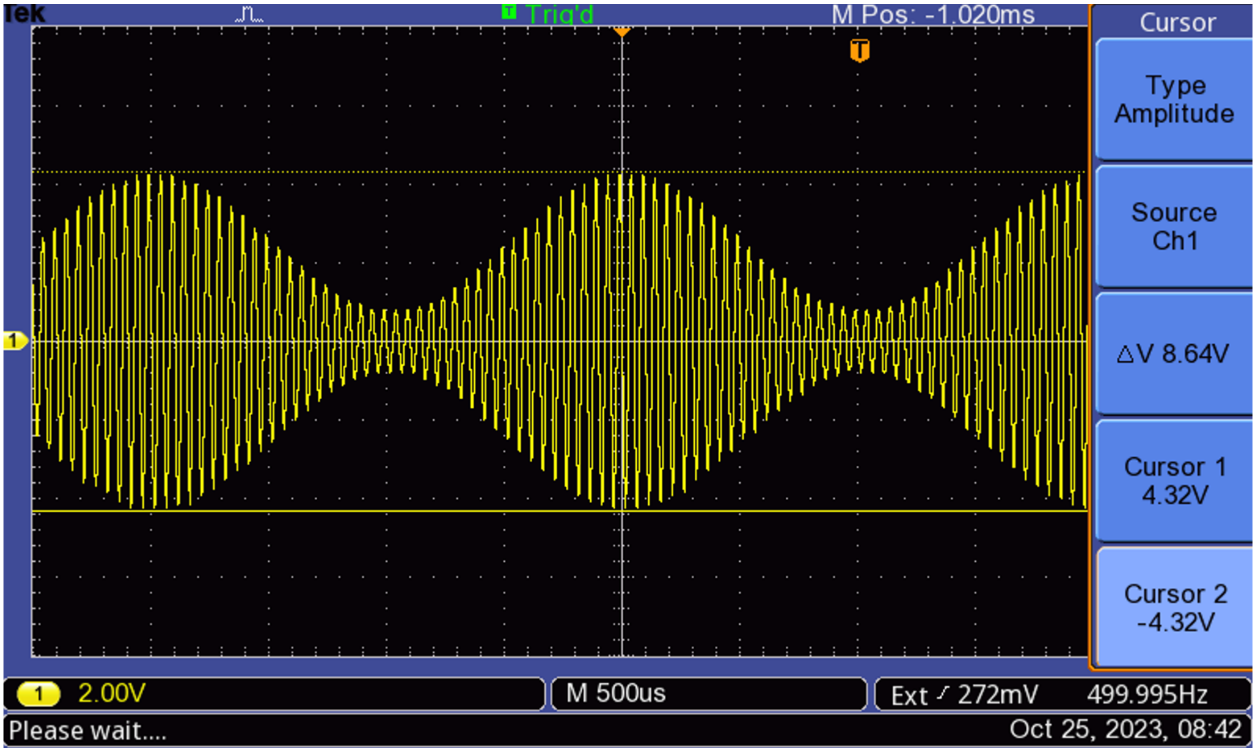
The results for 50% modulation index are shown below:

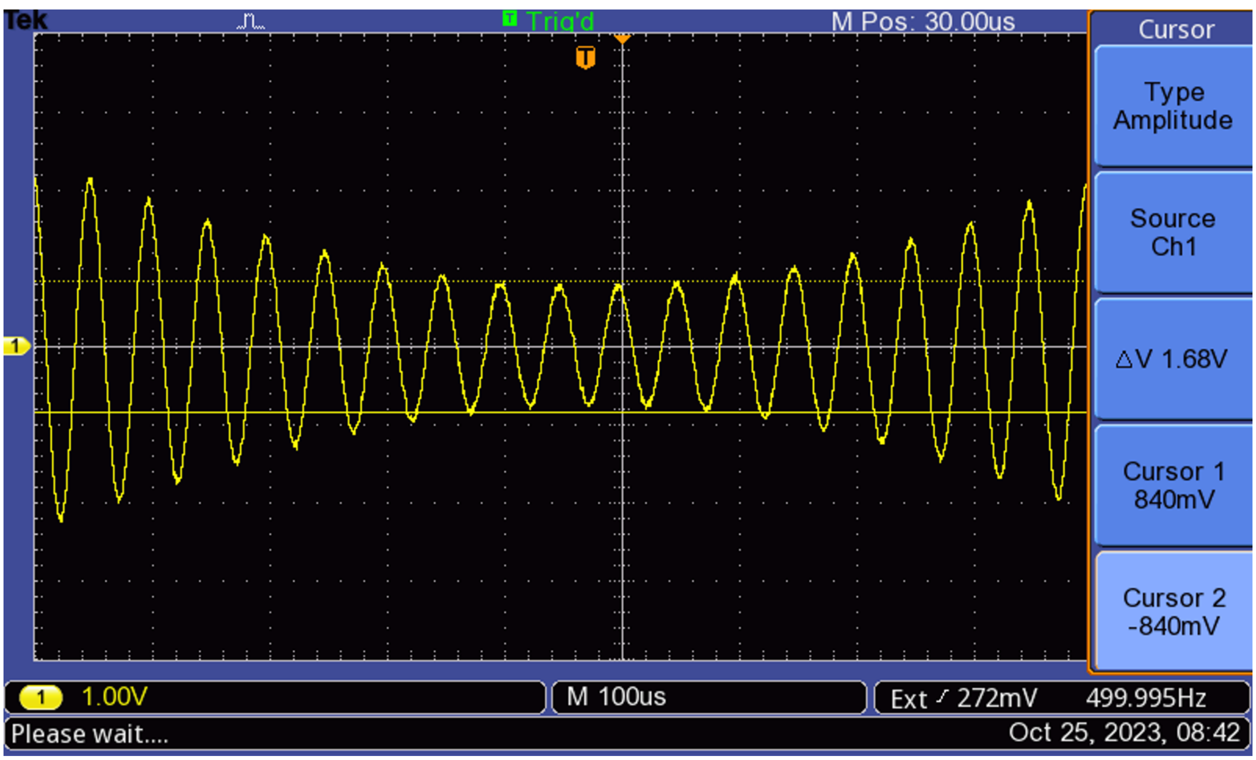


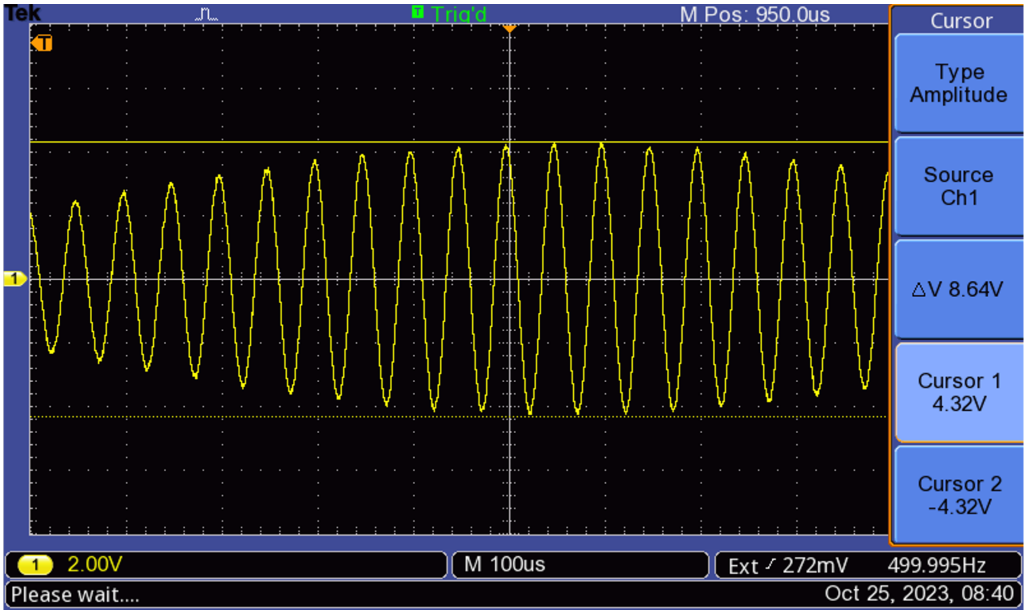

Tek
Trig'd
M Pos: -10.00us
Cursor
Type
Amplitude
Source
Ch1
ΔV 2.64V
Cursor 1
1.32V
Cursor 2
-1.32V
1 1.00V
M 100us
Ext ⸝ 272mV
499.995Hz
Please wait....
Oct 25, 2023, 08:37


Tek
Trig'd
M Pos: 950.0us
Cursor
Type
Amplitude
Source
Ch1
ΔV 7.60V
Cursor 1
3.80V
Cursor 2
-3.80V
1 1.00V
M 100us
Ext ⸝ 272mV
499.995Hz
Please wait....
Oct 25, 2023, 08:38

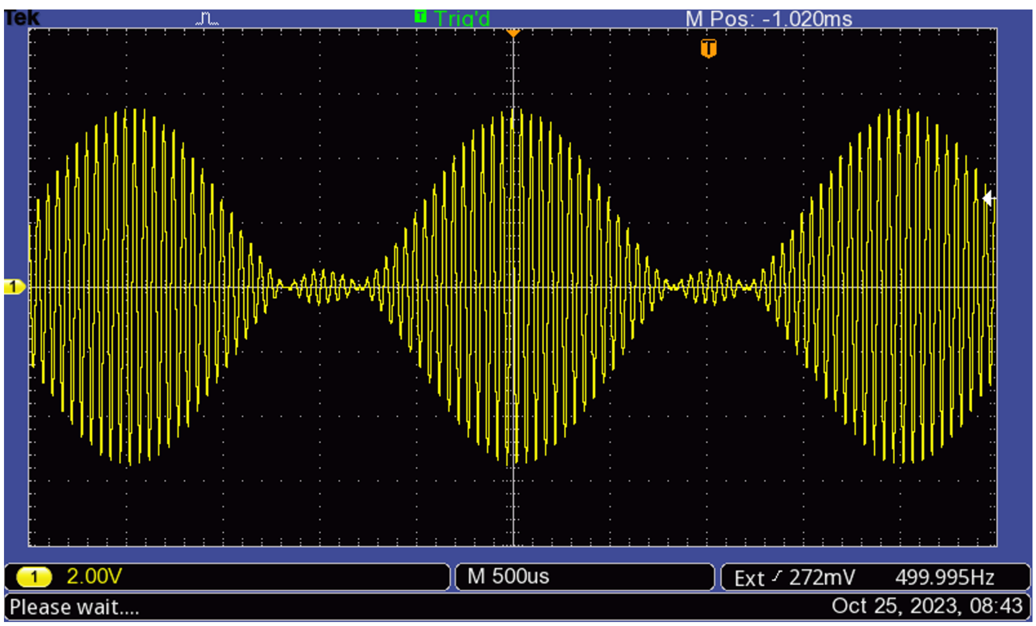
The results for 70% modulation index are shown below:







The result for 120% modulation index is shown below:



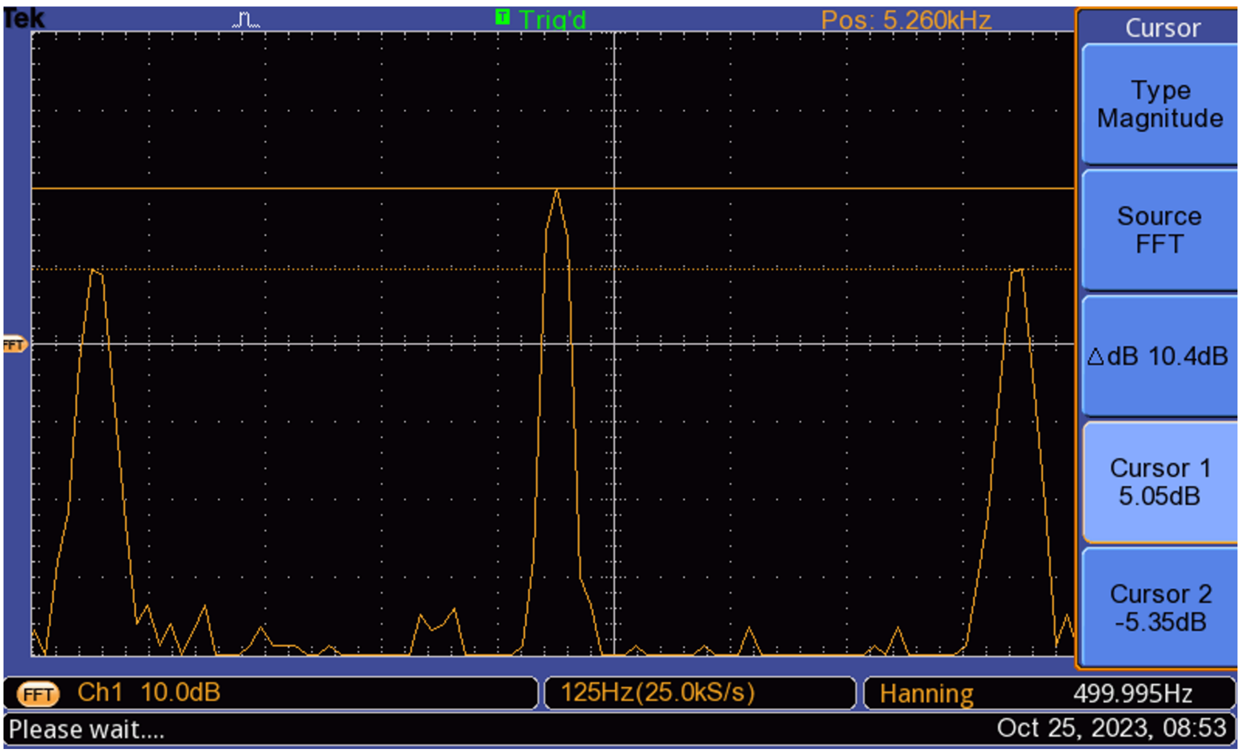
Modulation Index	A_{max} (V)	A_{min} (V)	Frequency (kHz)
50%	7.60	2.64	19.8
70%	8.64	1.68	20
120%	–	–	–

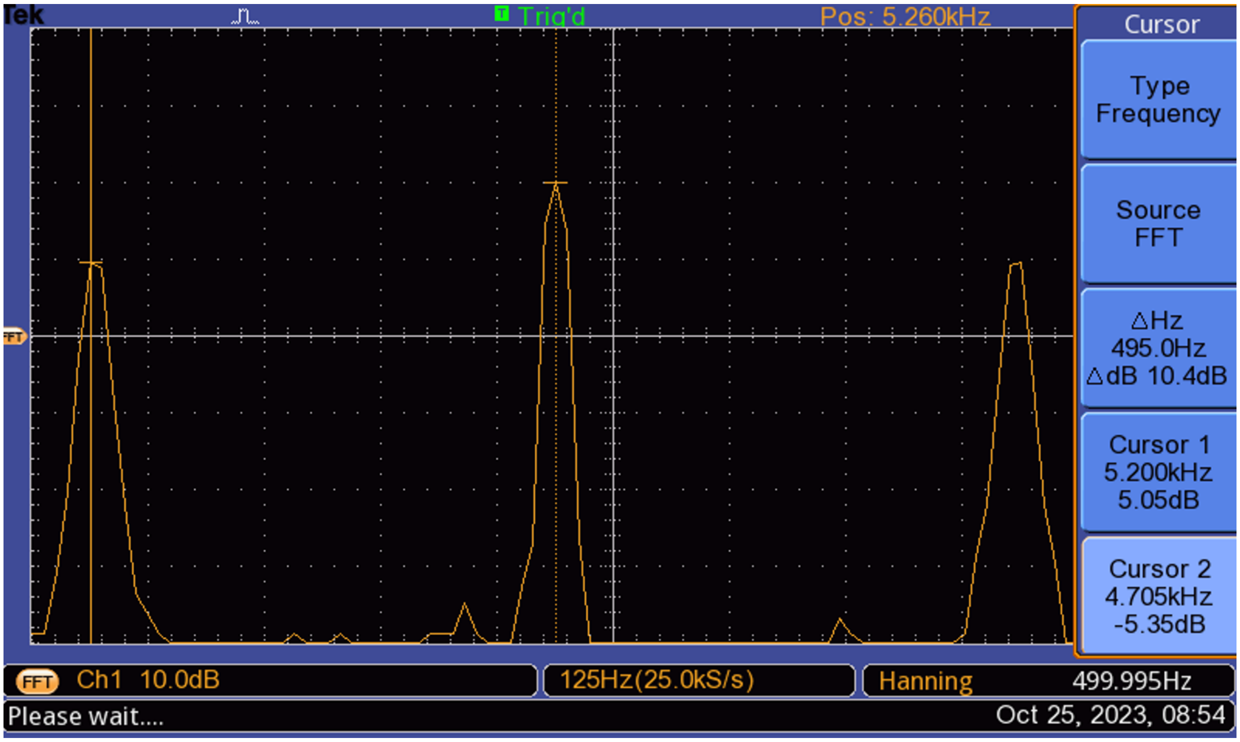
Part 2 - AM Modulated Signals in Frequency Domain

Experimental Setup and Procedure

Use the same setup as before. Set the modulation index at the function generator to 70%. At the oscilloscope display the amplitude modulated signal in frequency domain. (FFT!). Use the cursors to measure the magnitudes and the frequencies. Take hardcopies!

Results

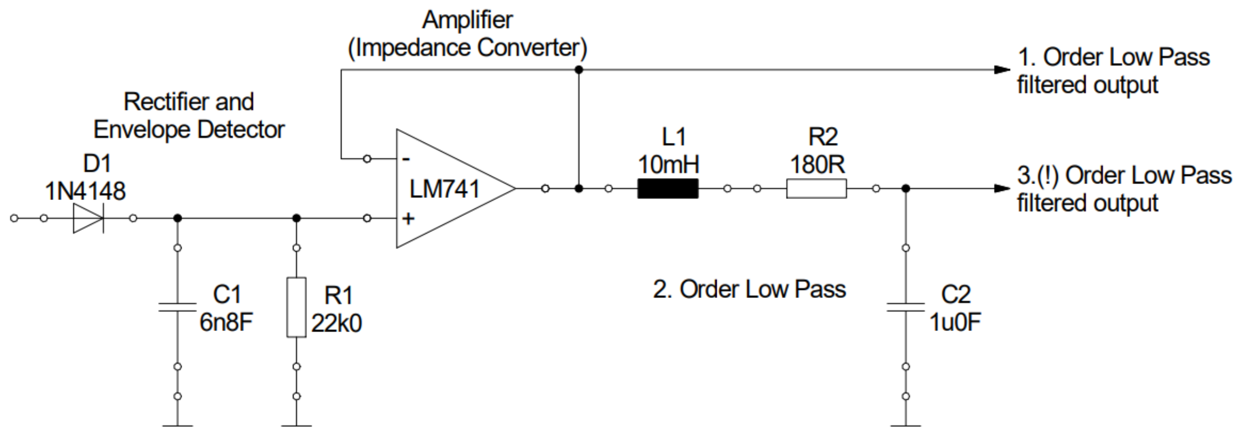


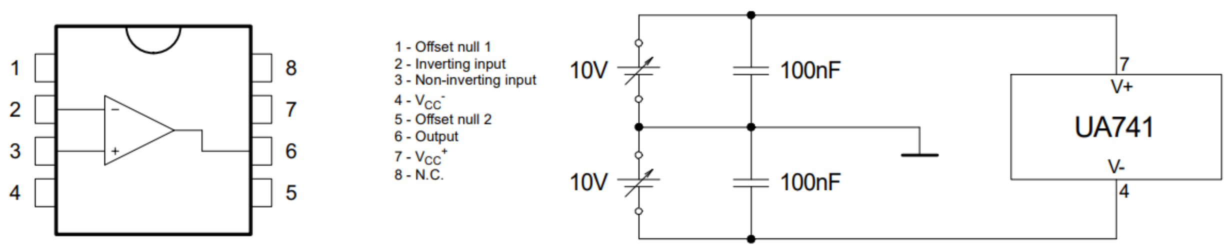


Peaks	Frequency (kHz)	Magnitude (dB)
Carrier	5.20	5.05
Side Band 1	4.71	-5.35
Side Band 2	5.69	-5.35

Part 3 - Demodulation of a Message Signal

Experimental Setup and Procedure

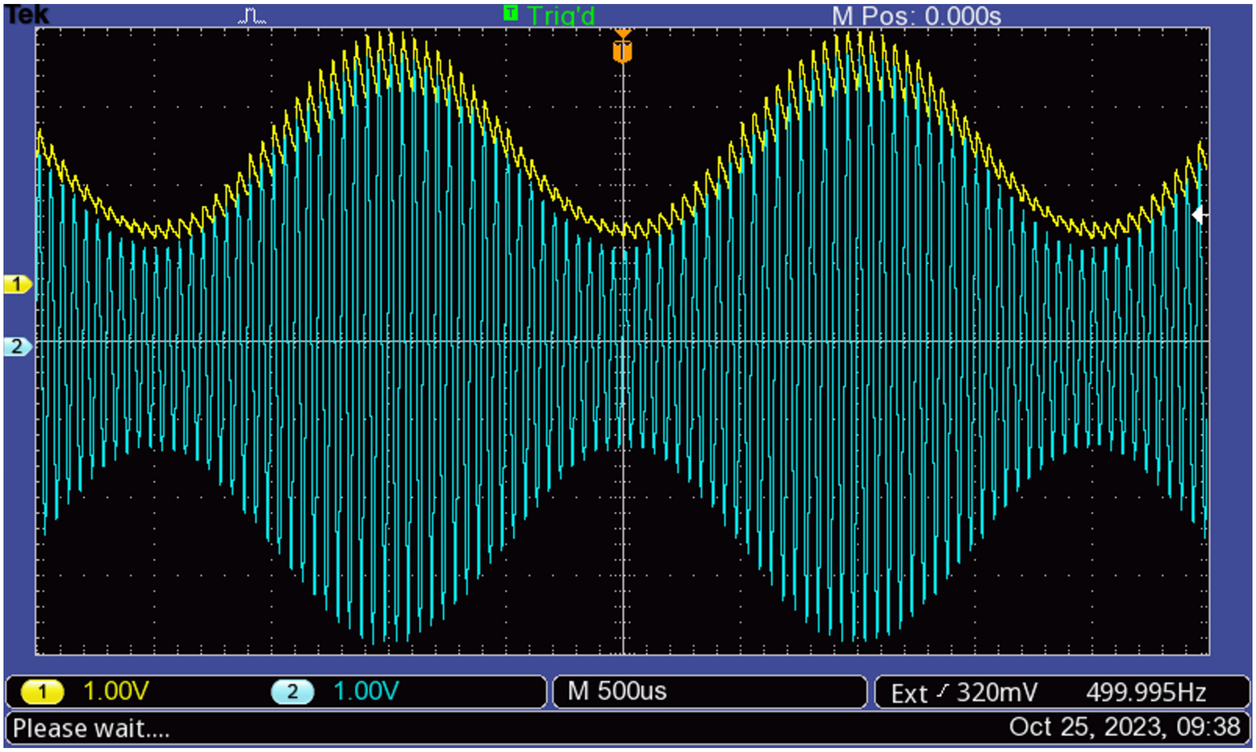




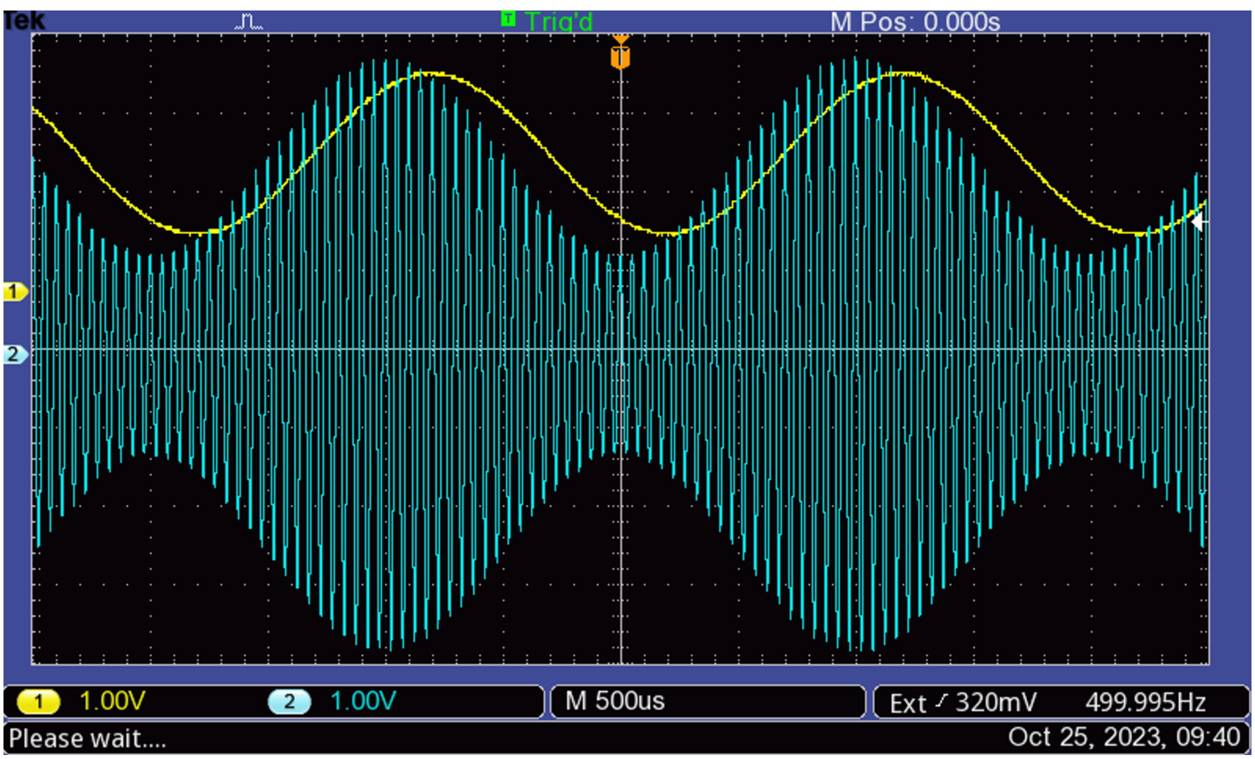
1. Build both the demodulation and supply circuits with the op-amp.
2. Connect the function generator to the input of the demodulating circuit. Use the following settings:
   - ➢ Signal Shape = Sine
   - ➢ Modulation = AM
   - ➢ Carrier frequency = 20 KHz
   - ➢ Carrier Amplitude = 10 V_{PP}
   - ➢ Modulation Frequency = 500 Hz
   - ➢ Modulation index = 50%
3. Display the AM modulated signal together with the 1st order filter output. Take a hardcopy.
4. Display the AM modulated signal together with the 3rd order filter output. Take a hardcopy.
5. Measure the amplitude of the demodulated signal at the 3rd order output.
6. Take FFT of the signal at the 3rd order filter output. Take hardcopies.

Results

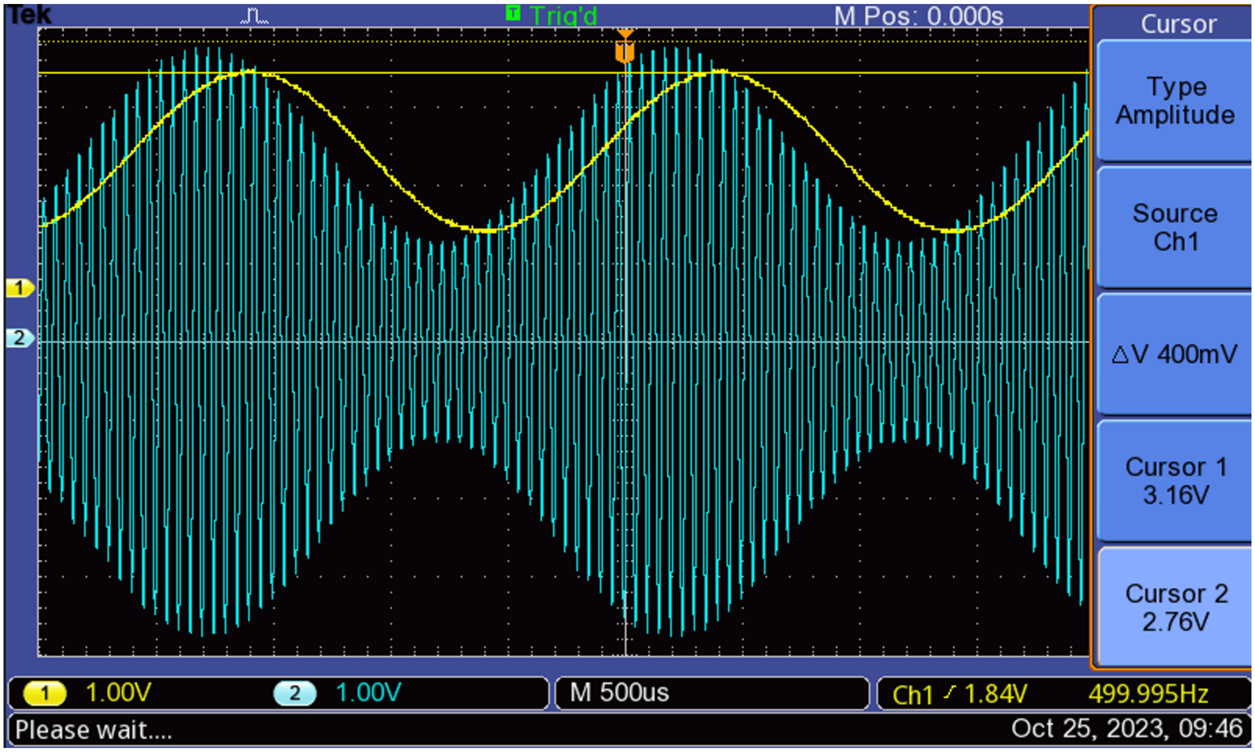
AM modulated signal together with the 1st order filter output:



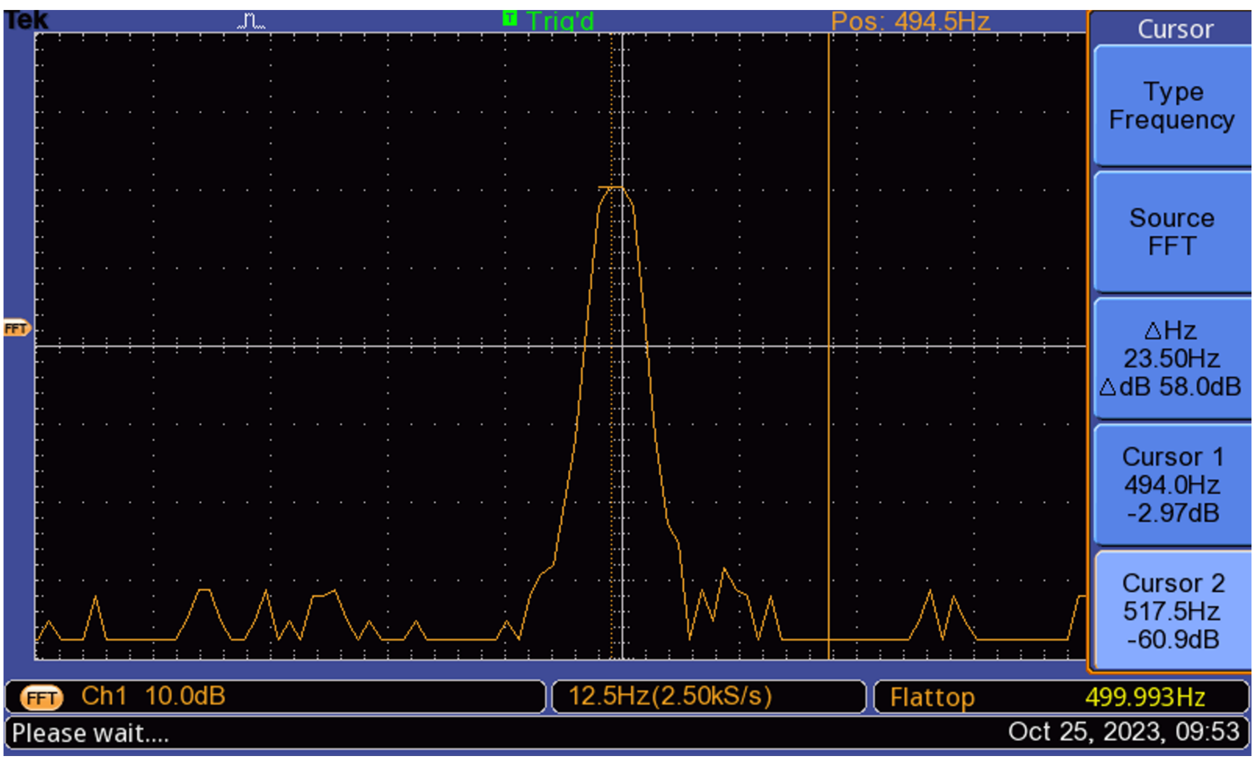
AM modulated signal together with the 3rd order filter output:



Measurement of the amplitude of the demodulated signal at the 3rd order output:



FFT of signal at 3rd order filter output:



Evaluation

Problem 1 - AM Modulated Signals in Time Domain

What is the relation between the modulation index and the relative magnitudes of the frequency components?

As shown in the prelab, the relation is given by:

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Where A_{max} is the maximum amplitude and A_{min} is the minimum amplitude of the modulated signal.

Calculate the modulation index using the measurements! Compare to the index you have used to generate the AM signal.

Plugging in our experimental results into the formula above gives:

Modulation Index	A_{max} (V)	A_{min} (V)	Calculated m
50%	7.60	2.64	48%
70%	8.64	1.68	67%

In both instances, the calculated modulation index values closely align with, albeit slightly lower than, the modulation index applied via the signal generator. For a modulation index set at 50%, the experimental value measured is 48%, while at a modulation index of 70%, the experimental value registers at 67%. The minimal discrepancy between the calculated and experimental values falls within an acceptable range. This slight deviation could be attributed to potential errors introduced by the signal generator, the resolution limitations of the oscilloscope, or precision errors in measurements taken using cursors.

Discuss the effects and the disadvantages of using a modulation index greater than 100

When the modulation index is greater than 100%, leading to overmodulation, information loss occurs during the process. The signal's amplitude exceeds that of the carrier, causing an increase in the ratio of sideband power to the total power. This overmodulated state distorts the modulated signal, making it challenging to recover the original signal. Demodulating an overmodulated signal results in the removal of part of the envelope, contributing to the loss of essential information. Hence, it is crucial to maintain the modulation index below 100% to prevent overmodulation and ensure the faithful reconstruction of the original signal during demodulation.

Problem 2 - AM Modulated Signals in Frequency Domain

How does the spectrum look like in theory? Compare to the experiment!

Theoretically, the spectrum of the modulated signal consists of 3 peaks: a peak at the carrier frequency, and two peaks at the carrier frequency plus or minus the modulating frequency.

Since the carrier frequency is 20kHz and the modulation frequency is 500Hz, the carrier peak should be at 20kHz with side band peaks at 19.5kHz and 20.5kHz. During the experiment, a mistake was made and the carrier frequency was instead set to 5kHz. As such, we should expect sideband peaks at 4.5kHz and 5.5kHz.

However, we measured the carrier peak at 5.20kHz and the side bands at 4.71 and 5.69kHz.

For the magnitudes, we observed a carrier peak at 5.05dB and sideband peaks at -5.35dB.

Theoretically,

$$dBA_c = 20\log\left(\frac{A_{c,rms}}{\sqrt{2}}\right) = 7.96\ dB$$

$$dBA_m = 20\log\left(\left(\frac{1}{\sqrt{2}}\right)\left(\frac{A_{m,rms}}{2}\right)\right) = -12.04\ dB$$

So the experimental magnitudes are quite different from theory.

Does the function generator generate a DSB or DSB-SC AM signal?

A DSB-SC AM signal is a double-sideband suppressed carrier AM signal. However, our FFT shows a peak at the carrier frequency. Therefore, the function generator is producing a DSB AM signal.

How does changing the carrier frequency affect the AM spectrum?

Changing the carrier frequency will cause the entire spectrum to shift in the frequency domain. Increasing the carrier frequency will shift the spectrum to the right while decreasing the carrier frequency will shift the spectrum to the left.

How does changing the message frequency affect the AM spectrum?

Changing the message frequency will change the distance between the peaks on the spectrum. Increasing the message frequency will take the peaks further apart from each other, while decreasing the message frequency will bring the peaks closer together.

Determine the modulation index m using the measured values.

$$20\log\left(\frac{A_{c,rms}}{\sqrt{2}}\right) = 5.05 \text{ dB}$$

$$20\log\left(\frac{1}{\sqrt{2}}\left(\frac{A_{m,rms}}{2}\right)\right) = -5.35$$

$$m = \frac{A_{m,rms}}{A_{c,rms}}$$

Solving the above, we get $m = 60.3990\%$

We see an error of $\frac{70-60.399}{70} = 14\%$, which is quite high. This could be a result of various things, including the precision error while taking data, instrumental error, and error in carrier frequency (used 5 kHz instead 20kHz).

Problem 3 - Demodulation of a Message Signal

Compare the 1. and 3. order filter output signal with the message signal.

While comparing the oscilloscope outputs of the first and third-order filters, notable differences become evident. The first-order filter exhibits no phase change in its output, yet it displays distortion and noise, indicating the persistence of some carrier signal in the demodulated output. On the other hand, the third-order filter, while demonstrating a noticeable phase change, produces a smoother and sharper output. This suggests effective filtering of the carrier signal components, rendering the demodulated signal of higher quality compared to the first-order filter output.

Compare the measured signals with the MatLab results. What are the differences between simulation and measurement?

The experimental results closely mirror the outcomes obtained through MATLAB simulations. Distortion is evident in the output of the 1st-order filter in both MATLAB and oscilloscope measurements, with MATLAB displaying a smoother output, likely due to its idealized scenario. Similarly, the 3rd-order filter's MATLAB output is smoother than the 1st-order filter, aligning with experimental findings. Notably, a discernible phase shift is observed in the experimental results for the 3rd-order filter, whereas the simulation's phase shift is less noticeable. This suggests a degree of divergence between the simulated and measured signals, emphasizing the influence of real-world factors on the observed phase shift.

Conclusion

The experiments delved into the principles of Amplitude Modulation (AM) and Demodulation, focusing on modulated sinusoidal signals in both the time and frequency domains. The demodulation process through 1st and 3rd order filters was a crucial aspect of the investigation.

In the initial phase, the oscilloscope revealed an AM signal generated by the signal generator, allowing the determination of the modulation index. Deviations in modulation index values were noted, potentially linked to internal resistance in the function generator or external resistances from wiring. The oscilloscope's resolution limitations introduced errors in amplitude and frequency measurements. Maintaining the modulation index below 100% was highlighted as crucial to prevent signal loss during retracing.

Shifting to the frequency domain examination, the observation indicated that the carrier was not suppressed, confirming the generation of a Double Sideband (DSB) AM signal. Modulation index determination from peak magnitudes closely approximated the expected theoretical value. Altering the carrier frequency induced shifts in the spectrum, and modifying the message signal affected sideband spacing relative to the central peak of the message carrier.

The final experiment focused on signal demodulation using first and third-order low-pass filters, revealing the superiority of higher-order filters in producing a distinctly demodulated output. First-order demodulation displayed traces from the carrier signal in the output.

In summary, the experiments successfully achieved their goal of providing a comprehensive understanding of AM signal analysis and the demodulation process through filters of varying orders. The oscilloscope plots visually confirmed the signal characteristics.

Valuable conclusions were drawn, including the identification of three peaks in the modulated signal spectrum, the cautions against exceeding a 100% modulation index, and the superiority of higher-order filters in producing better-quality demodulated outputs. Deviations in experimental data were explained by equipment limitations and errors in precision during measurements with the cursor.

References

Pagel, Uwe. *CO-520-B Signals and Systems Lab Manual*. 2023.

Appendix

Prelab - FM Modulation

Problem 1 - Frequency Modulator

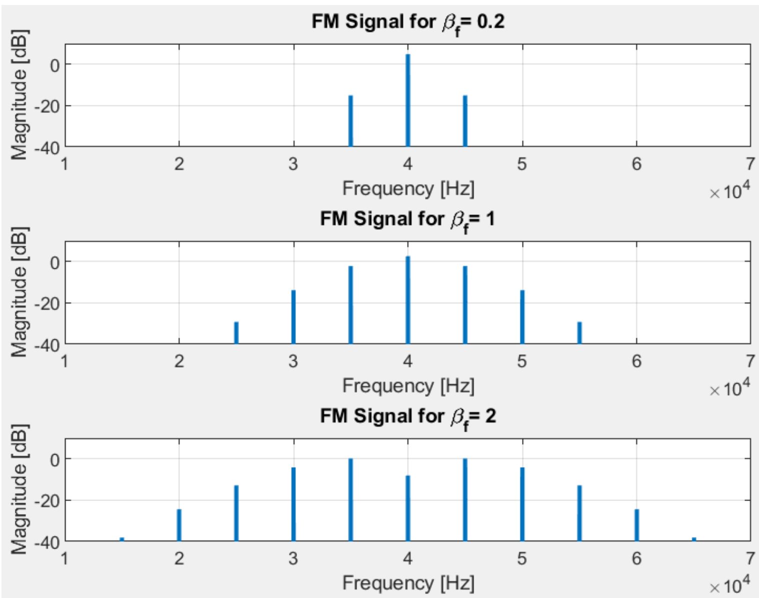
A sinusoidal modulation signal, m(t) = 4cos(8000πt) is applied to an FM modulator that has a frequency deviation constant K_f = 10kHz/V. Compute the frequency deviation and the frequency modulation index.

Frequency deviation $\Delta f = K_f \cdot A_m = 10\frac{kHz}{V} \cdot 4V = 40kHz$

Frequency modulation index $\beta_m = \frac{\Delta f}{f_m} = \frac{2\pi\Delta f}{\omega_m} = \frac{2\pi(40000)}{(8000\pi)} = 10$

Problem 2 - FM signal in the frequency domain

Plot a frequency-modulated signal in the frequency domain. The signal exhibits 2.5V peak carrier amplitude, 40 kHz carrier frequency, and 5 kHz modulation frequency. Vary βf between 0.2, 1, and 2. Display the magnitudes in dB$_{rms}$! Calculate the bandwidth using Carlsons rule. Tabulate the peak magnitudes inside the bandwidth from the three plots.




```
Ac = 2.5;
Fc = 40000;
Fm = 5000;
t_min = 30*10^-3; % minimal signal length
d = [1, 5, 10];
B = d/5;
Fs = Fc * 8; % Sampling frequency
F_nyq = Fs/2; % Nyquist frequency
Ts = 1/Fs * (2^nextpow2(t_min * Fs));
t = 0: 1/Fs : Ts - 1/Fs;
for i = 1:length(B)
wc = 2*pi*Fc ;
wm = 2*pi*Fm;
Afm = Ac*cos(wc*t + B(i)*sin(wm*t));
Lt = length(t);
fm_fft = 2*abs(fft(Afm))/Lt;
fm_fft = fm_fft(1:end/2);
f = linspace(0, F_nyq, length(fm_fft));
subplot(3,1,i);
log_fm = 20 * log10 (fm_fft/sqrt(2));
plot (f, log_fm, 'LineWidth', 2);
xlabel('Frequency [Hz]');
ylabel('Magnitude [dB]');
ylim ([-40, 10]); xlim ([1*10^4, 7*10^4]);
title(['FM Signal for \beta_f= ',num2str(B(i))])
grid on;
end
```

$$B_T = 2f_m(\beta_f + 1)$$

$$\beta_f = 0.2 \Rightarrow B_T = 12kHz$$

$$\beta_f = 1 \Rightarrow B_T = 20kHz$$

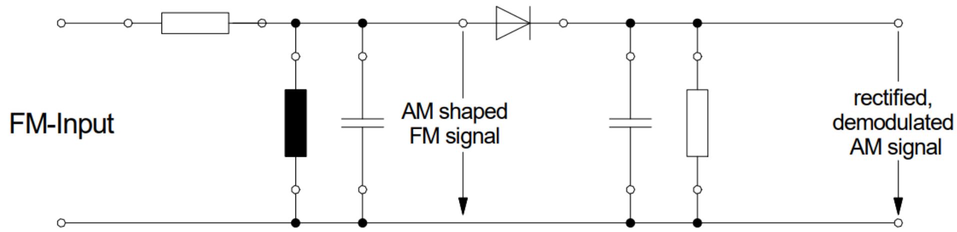
$$\beta_f = 2 \Rightarrow B_T = 30kHz$$

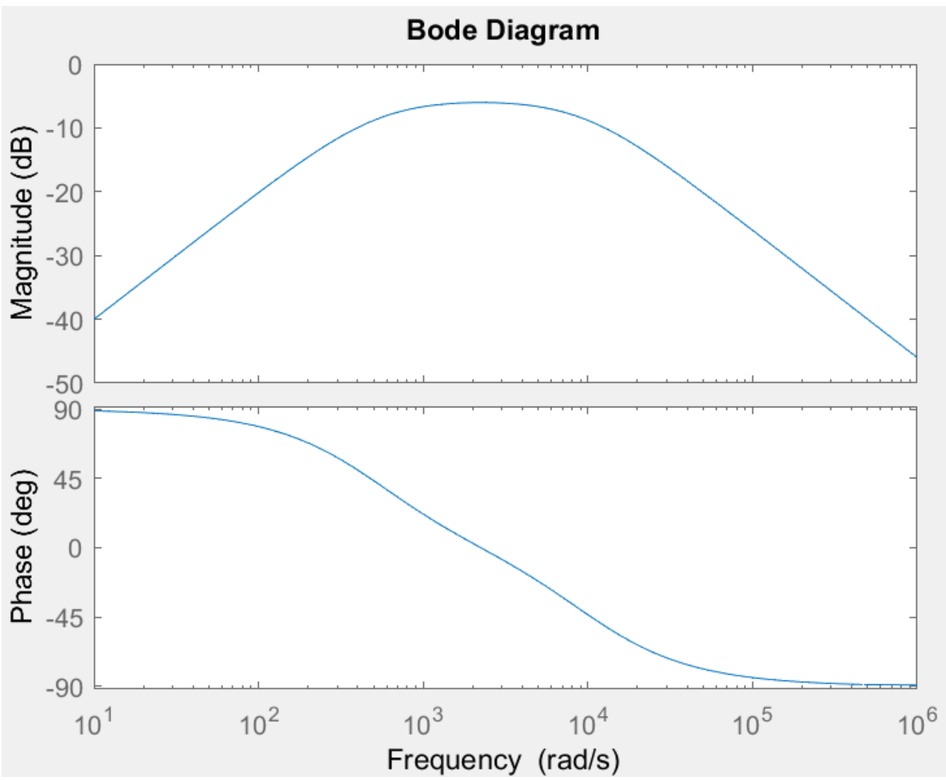
The peak magnitudes from the plots are:

β_f	Frequency (kHz)										
	15	20	25	30	35	40	45	50	55	60	65
0.2	–	–	–	–	-15	5	-15	–	–	–	–
1	–	–	-29	-14	-2	3	-2	-14	-29	–	–
2	-38	-24	-13	-4	0	-8	0	-4	-13	-24	-38

Problem 3 - Frequency demodulation

A simple slope detector circuit is shown below. The circuit can be used as an FM demodulation circuit. Provide a bode diagram and explain the operation principle of the circuit.






```
R=100;
C=10^-6;
L=10^-1;
s=tf('s');
ZR=R;
ZC=1/(s*C);
ZL=s*L;
Zr=1/ZR;
Zc=1/ZC;
Zl=1/ZL;
ZO=Zl+2*Zc+Zr;
Zo=1/ZO;
H=Zo/(Zo+ZR);
bode(H);
```

The input circuit, functioning as a resonance circuit with bandpass characteristics, behaves as a second-order filter due to its two reactive components. Utilizing the linear segment within the cutoff region, particularly from the high-pass portion, is crucial, and minimizing damping is essential. By placing the center frequency of the Frequency Modulation (FM) within the slope, the filter's output mimics that of an Amplitude Modulated (AM) signal. The second part of the circuit serves as both a higher-order filter and a rectifier, functioning as an envelope detector. Its primary role is to extract the message signal from the AM-modulated output generated by the input circuit.