9.4 ECCENTRIC JOINTS

Concentric riveted connections which carry no moment are assumed to be loaded evenly, i.e., the load is distributed equally to the rivets. This is approximately true, even when the rivets are in a single line the end fastėners (or fisrt fasteners) are not overloaded as might be expected. All fastened joints must be checked for the

- Shear value of the fasteners
- Bearing value of the fastener in the attached sheets

Where fastener clusters must carry moment load (M) as well as shear force (P) in members they must be investigated for combined loads.on the fastener clusters, as shown in Fig. 9.4.1. The solution of forces for a group fasteners subjected to moment is rather simple if the engineer assumes that the force on each fastener is proportional to its distance from the centroid. This is true of course only if the fasteners are all of the same size (also assume the same material).

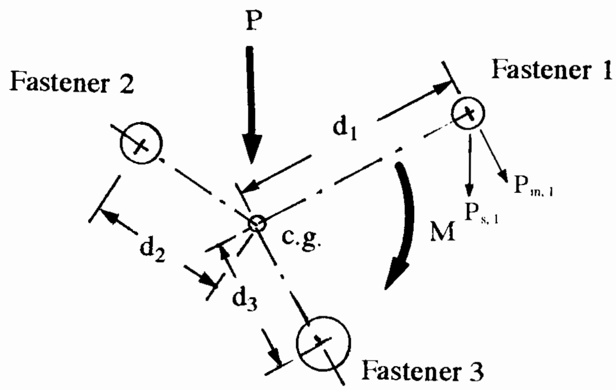


Fig. 9.4.1 Fastener Clusters

Shear load on given fastener 1, due to a concentrated load, P

$$P_{s,1} = P(\frac{A_1}{\Sigma A}) \qquad \text{Eq. 9.4.1}$$

'hear load on given fastener 1, due to moment , M,:

$$P_{m,1} = M(\frac{A_1 d_1}{\Sigma A d^2}) \qquad \text{Eq. 9.4.2}$$

where: A – Fastener area
d – Distance from centroid of the fastener clusters to given fastener

The above equations are based on the following:

- Fastener materials are the same
- Fastener bearing on the same material and thickness
- Fastener shear load assumes straight line distribution

Procedure for determining fastener shear loads is as follows:

(1) Find out fastener areas and their shear allowables.

(2) Determine centroid (c.g.) of the fastener clusters from a designated fastener:

X of c.g. – x-axis from a designated fastener
Y of c.g. – y-axis from a designated fastener

(3) Fastener shear load (see Eq. 9.4.1 and assume at fastener 1) due to a concentrated load, P:

$$P_{s,1} = P(\frac{A_1}{\Sigma A})$$

(Assume load, P, goes through the centroid of fastener clusters)

(4) Fastener shear load (see Eq. 9.4.2 and assume at fastener 1) due to moment, M,:

$$P_{m,1} = M(\frac{A_1 d_1}{\Sigma A d^2})$$

If areas of fasteners are equal and the fastener shear load (assume at fastener 1) due to moment , M,:

$$P_1 = M(\frac{d_1}{\Sigma d^2})$$

(5) Construct the vector diagrams for the loads on the assumed fastener 1 ($P_{s,1}$ and $P_{m,1}$) and measure the resultant vector ($P_{Sm,1}$).

(6) Margin of safety:

$$MS_{Shear} = \frac{\text{Fastener 1 shear allowable}}{P_{Sm,1}} - 1$$

or $$MS_{bearing} = \frac{\text{Sheet bearing allowable on fastener 1}}{P_{Sm,1}} - 1$$

Example 1 (Symmetrical Joint With Same Size Rivets):

Assume a symmetrical rivet pattern of five rivets of equal size ($D = \frac{3}{8}''$), carrying a shear load of $P_v = 10{,}000$ lbs. upward and a moment load of $M = 12{,}000$ in-lbs. acting as shown in Fig. A.

- For this case the centroid of the rivet joint about which moment load will take place is by inspection at the No. 5 rivet
- The center rivet cannot take any moment load because it has no lever arm, being at the centroid
- The four outer rivets are equal distance from the centroid and take equal moment loads
- The forces on the rivets are shown and the resultant forces on all rivets are different, being the resultants of the forces due to the vertical concentrated load of (P_V) and moment load (M)
- For the condition shown rivets No. 1 and No. 4 carry the greatest load and would be critical

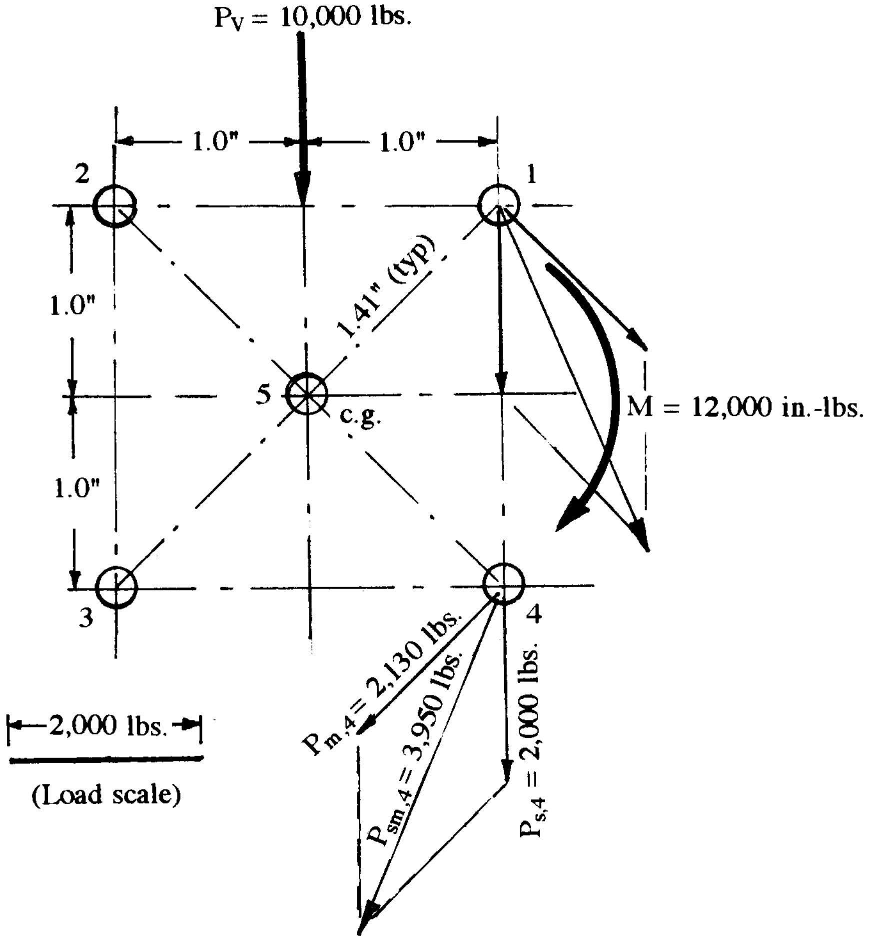


Fig. A Five Equal Sizes of Rivets ($D = \frac{3}{8}$") with Symmetrical Rivet Pattern

(Centroid of rivet group is at rivet No. 5 and therefore rivet No. 5 has no load from moment)

(1) Use D-rivet (2017-T31, F_{su} = 34 ksi) and rivet shear strength of dia. = $\frac{3}{8}$" is

P_{all} = 3,980 lbs. (Assume bearing is not critical)

(2) Centroid of rivet group is at rivet No. 5 and radial distance to each rivet effective in moment load is 1.41".

(3) Rivet shear load due to P_V = 10,000 lbs.:

$$P_s = \frac{10,000}{5} = 2,000 \text{ lbs./rivet}$$

(Assume P_V = 10,000 lbs. load goes through the centroid of the group of rivets)

(4) Rivet shear loads due to M = 12,000 in.-lbs.:

$$P_m = \frac{12,000}{4 \times 1.41} = 2,130 \text{ lbs./rivet}$$

(5) The final loads on the respective rivets are the vector resultants of the P_s = 2,000 lbs. and P_m = 2,130 lbs. loads; the max. resultant load is at rivets No. 1 and No. 4 and is $P_{sm,4}$ = 3,950 lbs. (by measuring the vector diagram):

(6) Margin of safety:

$$MS = \frac{P_{all}}{P_{sm,4}} - 1 = \frac{3,980}{3,950} - 1 = \underline{0.01} \qquad \text{O.K.}$$

Example 2 (Unsymmetrical Joint With Same Size Rivets):

Design the unsymmetrical rivet pattern shown in Fig. B to carry the external shear P_v = 10,000 lbs and moment M = 12,000 in-lbs. with equal size rivets ($D = \frac{3}{8}"$).

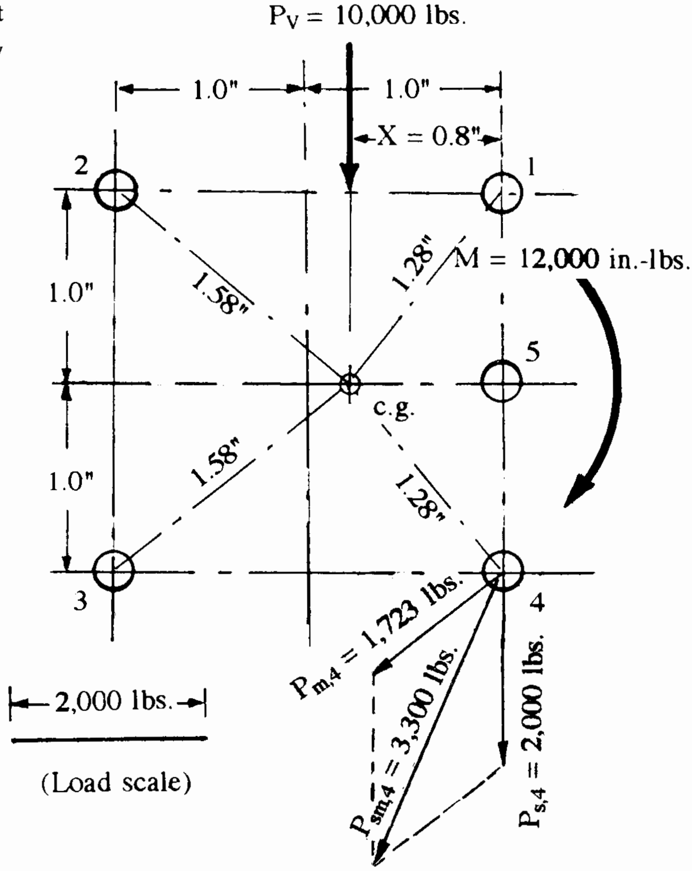


Fig. B Five Equal Sizes of Rivets ($D = \frac{3}{8}"$) with Unsymmetrical Rivet Pattern

(1) Use D-rivet (2017-T31, F_{su} =34 ksi) and rivet shear strength of dia. = $\frac{3}{8}"$ is

P_{all} = 3,980 lbs. (assume bearing strength is ample)

(2) Rivet shear load due to P_V = 10,000 lbs.:

$$P_s = \frac{10,000}{5} = 2,000 \text{ lbs./rivet}$$

(3) Assume each $\frac{3}{8}"$ rivet equal to 1.0 unit:

$$X = \frac{2(2 \times 1.0)}{5} \times 1.0 = 0.8"$$

(4) Rivet loads due to M = 12,000 in.-lbs. are:

Rivet No.	D (in.)	A (in.)	d (in.)	Ad^2
1	3/8	0.1105	1.28	0.181
2	3/8	0.1105	1.58	0.276
3	3/8	0.1105	1.58	0.276
4	3/8	0.1105	1.28	0.181
5	3/8	0.1105	0.8	0.071
				$\Sigma Ad^2 = 0.985$

Then the actual load on any rivet is equal to:

$$(Ad / \Sigma Ad^2)\, M$$

The load on river No. 4:

$$P_{m,4} = (0.1105 \times 1.28 / 0.985) \times 12{,}000 = 1{,}723 \text{ lbs.}$$

(5) The final loads on the respective rivers are the vector resultants of the $P_{s,4} = 2{,}000$ lbs. and $P_{m,4} = 1{,}723$ lbs. loads, the max. resultant load is at rivers No. 1 and No. 4 and is $P_{sm,4} = 3{,}300$ lbs. (by measuring the vector diagram):

(6) Margin of safety:

$$MS = \frac{P_{all}}{P_{sm,4}} - 1 = \frac{3{,}980}{3{,}300} - 1 = \underline{0.21} \qquad \text{O.K.}$$

Example 3 (Unsymmetrical Joint With Different Size Rivets):

Design a joint of unsymmetrical rivet pattern with unequal size rivets ($D = \frac{3}{8}''$ and $D = \frac{1}{4}''$) as shown in Fig. C to carry the vertical shear force $P_v = 10{,}000$ lbs and moment M = 12,000 in-lbs.

Note:
This type of joint design using differenct size of rivets is not recommended in new design except as rework or repair work

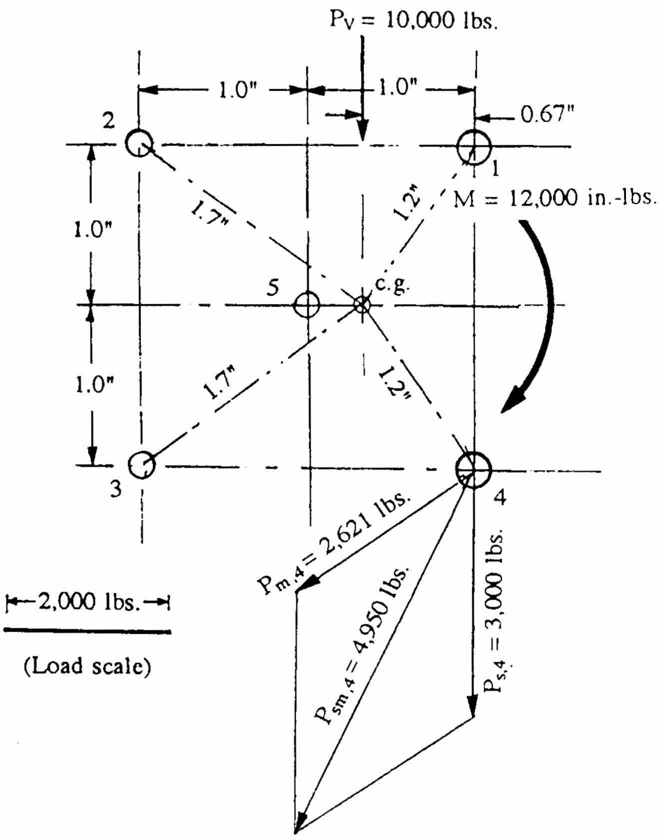


Fig. C Unequal Sizes of Rivets, $D = \frac{1}{4}''$ and $D = \frac{3}{8}''$, with Unsymmetrical Rivet Pattern

(1) Use E-rivet (material = 7075-T731, F_{su} = 43,000 psi) and assume bearing strength is ample:

Shear strength of rivets No. 2, 3, and 5 ($D = \frac{1}{4}$" and rivet area = 0.04908 in.^{2}) is

P_{all} = 2,230 lbs/rivet

Shear strength of rivets No. 1 and 4 ($D = \frac{3}{8}$" and rivet area = 0.1105 in.^{2}) is

P_{all} = 5,030 lbs/rivet

Rivet areas:

Area for $D = \frac{1}{4}$": $A_{\frac{1}{4}} = 0.0491$ in.^{2}

Area for $D = \frac{3}{8}$": $A_{\frac{3}{8}} = 0.1105$ in.^{2}

(2) Determine centroid (c.g.) of the fastener clusters from a designated fastener 1 and 4 (Y = 0 because it is symmetrical about x-axis):

$$X = \frac{2 \times (2 \times 0.0491) + 1 \times (1 \times 0.0491)}{2 \times 0.1105 + 3 \times 0.0491} = 0.67"$$

(3) Rivet shear load due to P_V = 10,000 lbs.:

On $D = \frac{1}{4}$" rivet:

$$P_{s,2} = P_{s,3} = P_{s,5} = \frac{10{,}000 \times 0.0491}{3 \times 0.0491 + 2 \times 0.1105} = 1{,}333 \text{ lbs.}$$

On $D = \frac{3}{8}$" rivet:

$$P_{s,1} = P_{s,4} = \frac{10{,}000 \times 0.1105}{3 \times 0.0491 + 2 \times 0.1105} = 3{,}000 \text{ lbs.}$$

(4) Rivet shear load due to M = 12,000 in.-lbs.:

The resisting moment for this rivet group which has a centroid at X = 0.67" and ΣAd^2 would be (put the solution into tabular form as follows):

Rivet No.	D (in.)	A (in.^{2})	d (in.)	Ad^2
1	$\frac{3}{8}$	0.1105	1.2	0.159
2	$\frac{1}{4}$	0.0491	1.7	0.142
3	$\frac{1}{4}$	0.0491	1.7	0.142
4	$\frac{3}{8}$	0.1105	1.2	0.159
5	$\frac{1}{4}$	0.0491	0.33	0.005
				$\Sigma Ad^2 = 0.607$

Then the actual load on any rivet is equal to:

$$\frac{Ad}{\Sigma Ad^2} M$$

The load on rivets No.1 and 4 ($D = \frac{3}{8}$"):

$$P_{m,1} = P_{m,4} = \frac{A_1 d_1}{\Sigma A d^2} M = \frac{0.1105 \times 1.2}{0.607} 12{,}000 = 2{,}621 \text{ lbs.}$$

The load on rivets No.2 and 3 ($D = \frac{1}{4}$"):

$$P_{m,2} = P_{m,3} = \frac{A_2 d_2}{\Sigma A d^2} M = \frac{0.0491 \times 1.7}{0.607} 12{,}000 = 1{,}650 \text{ lbs.}$$

(6) The final loads on the respective rivets are the vector resultants of the $P_{s,4} = 3{,}000$ lbs. and $P_{m,4} = 2{,}621$ lbs. loads; the max. resultant load is at rivets No.1 and No. 4 and is $P_{sm,4} = 4{,}950$ lbs (by measuring the vector diagram):

$$MS = \frac{P_{all}}{P_{s,m}} - 1 = \frac{5{,}030}{4{,}950} - 1 = 0.02 \qquad \text{O.K.}$$

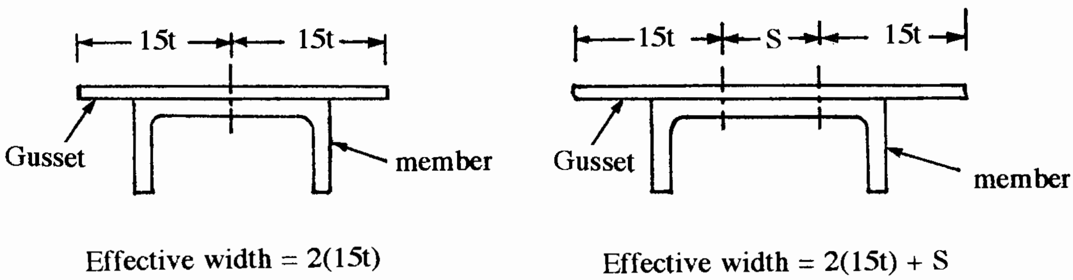
9.5 GUSSET JOINTS

Gusset joints are generally used on truss beams, e.g., spars, ribs, floor beams, etc. However, these types of structures are very seldom used in airframe primary construction today due to the problems of weight, cost, and repair difficulty except very special applications.

- The stress or load distribution in gusset joints is usually uncertain and complicated and there are no rules that will fit all cases
- Engineers must rely upon their judgment of the appearance of the gusset and the possibilities of peculiarities of stress distribution as well as upon computations

A single gusset plate (on one side) is generally used on airframe structures and the eccentricity between gusset and attached member will induce additional excessive stress (bending stress) on the gusset plate. The stress is obviously not "$\frac{P}{t}$" but it is not as bad as "$\frac{4P}{t}$" (see Fig. 9.1.2) and it is somewhere between them. Use following aluminum material design allowables (until the verification test data is available) are:

- In tension or compression – Use 40% F_{tu} of the material allowable as shown in Fig. 9.5.1
- In shear net section – Use 70% F_{tu} for thickness t = 0.125 in. and 50% F_{tu} for t = 0.04 in. as shown in Fig. 9.5.2



(a) Single row of fasteners (b) Double or staggered rows of fasteners

Fig. 9.5.1 Effective Width