(B) CASE I – AXIAL LOAD ($\alpha = 0°$)

The lug failure modes for this load case are shown in Fig. 9.8.4.

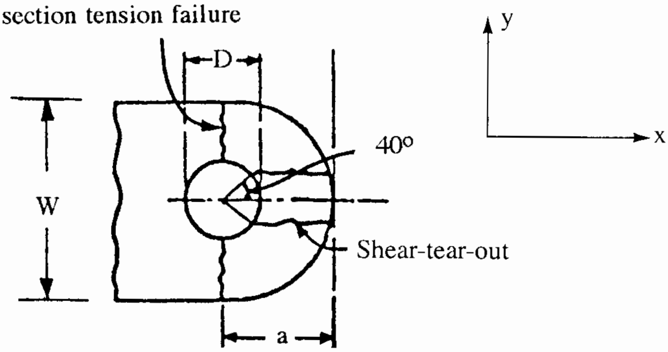


Fig. 9.8.4 Lug Tension and Shear-tear-out Failures

(a) Shear-bearing failure:

Failure consists of shear tear-out of the lug along a 40° angle on both sides of the pin (see Fig. 9.8.4) while bearing failure involves the crushing of the lug by the pin bearing. The ultimate load for this type of failure is given by the equation:

$$P_{bru} = k_{br} F_{tux} A_{br} \qquad \text{Eq. 9.8.1}$$

where: P_{bru} – Ultimate load for shear tear-out and bearing failure
k_{br} – Shear-bearing efficiency factor from Fig. 9.8.5
A_{br} – Projected bearing area ($A_{br} = Dt$)
D – Pin diameter or bushing outside diameter D_b (if bushing is used)
t – Lug thickness
F_{tux} – Ultimate tensile stress in x-direction

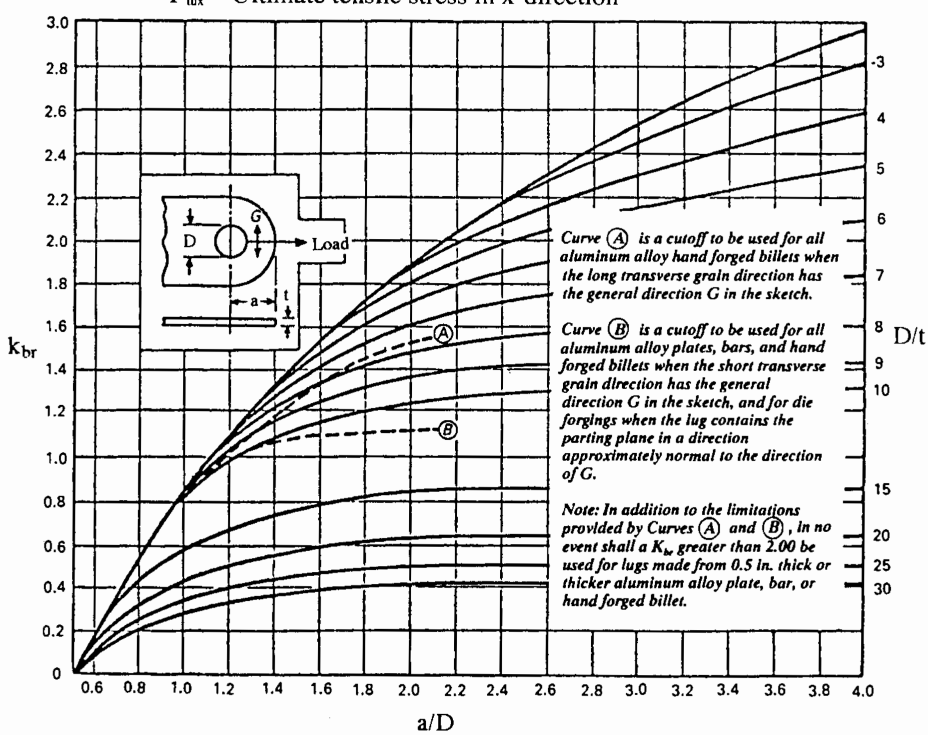


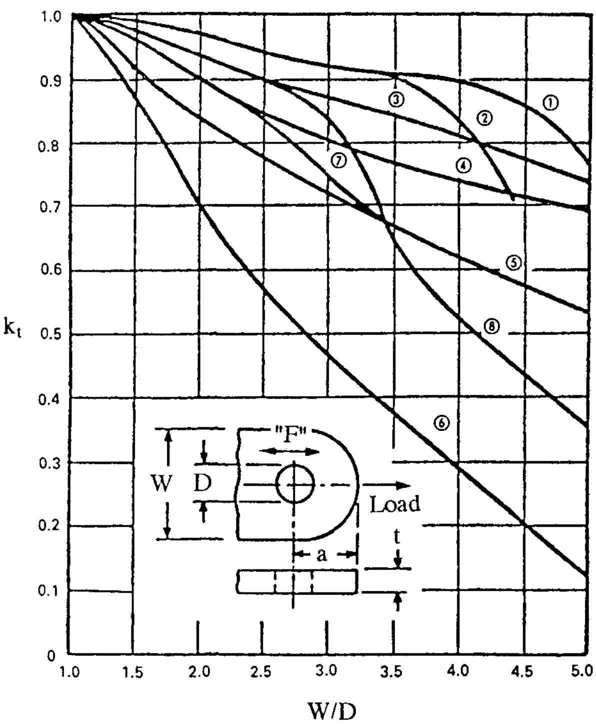
Fig. 9.8.5 Shear-Bearing Efficiency Factor, k_{br}

(b) Tension failure:

Tensile failure is given by:

$$P_{tu} = k_t F_{tux} A_t \qquad \text{Eq. 9.8.2}$$

where: P_{tu} – Ultimate load for tension failure
k_t – Net tension efficiency factor from Fig. 9.8.6
F_{tux} – Ultimate tensile stress of the lug material in x-direction
A_t – Minimum net section for tension $[A_t = (W - D)t]$
W – Width of the lug



Legend L, LT and ST indicate grain in direction F in sketch:
Aluminum designation:
14S-2014
24S-2024
75S-7075

Curve ①
4130 steel
14S-T6 and 75S.T6 plate ≤ 0.5 in (L, LT)
75S-T6 bar and extrusion (L)
14S-T6 hand forged billet ≤ 144 sq in (L)
14S-T6 and 75S-T6 die forgings (L)

Curve ②
14S-T6 and 75S-T6 plate > 0.5 in, ≤ 1in
75S-T6 extrusion (LT, ST)
75S-T6 hand forged billet ≤ 36 sq in (L)
14S-T6 hand forged billet > 144 sq in (L)
14S-T6 hand forged billet ≤ 36 sq in (LT)
14S-T6 and 75S-T6 die forgings (LT)

Curve ③
24S-T6 plate (L, LT)
24S-T4 and 24S-T42 extrusion (L, LT)

Curve ④
24S-T4 plate (L, LT)
24S-T3 plate (L, LT)
14S-T6 and 75S-T6 plate > 1 in (L, LT)
24S-T4 bar (L, LT)
75S-T6 hand forged billet > 36 sq in (L)
75S-T6 hand forged billet ≤ 16 sq in (LT)

Curve ⑤
75S-T6 hand forged billet > 16 sq in (LT)
14S-T6 hand forged billet > 36 sq in (LT)

Curve ⑥
Aluminum alloy plate, bar, hand forged billet, and die forging (ST)
75S-T6 bar (LT)

Curve ⑦
18-8 stainless steel, annealed

Curve ⑧
18-8 stainless steel, full hard. Note: for $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ hard, interpolate between Curves ⑦ and ⑧

Fig. 9.8.6 Lug Efficiency Factor for Tension, k_t

(c) Yield failure – lug:

Lug yield load attributable to shear-bearing is given by:

$$P_y = C\left(\frac{F_{tyx}}{F_{tux}}\right)(P_u)_{min} \qquad \text{Eq. 9.8.3}$$

where: P_y – Yield load
C – Yield factor from Fig. 9.8.7
F_{tyx} – Tensile yield stress of lug material in load direction
F_{tux} – Ultimate tensile stress of lug material in load direction
$(P_u)_{min}$ – The smaller P_{bru} (Eq. 9.8.1) or P_{tu} (Eq. 9.8.2)

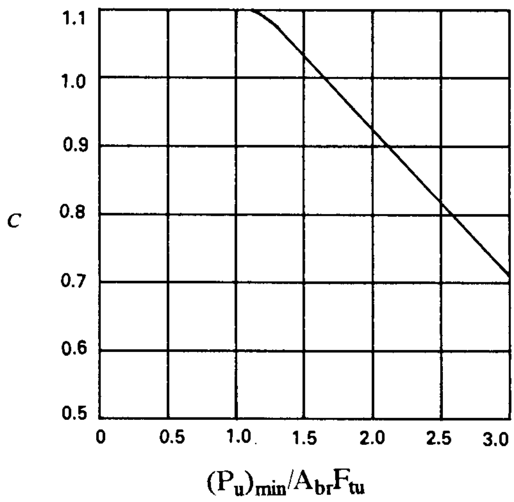


Fig. 9.8.7 Yield Factor, C

(d) Yield failure – bushing (if used):

Bushing yield bearing load attributable to shear-bearing is given by:

$$P_{bry} = 1.85\, F_{cy}\, A_{brb} \qquad \text{Eq. 9.8.4}$$

where: P_{bry} – Bushing yield bearing load
F_{cy} – Compression yield stress of bushing material
A_{brb} – The smaller of the bearing areas of bushing on pin or bushing on lug (the latter may be the smaller as a result of external chamfer on the bushing).

(e) Pin shear-off failure:

Pin single shear-off failure is given by:

$$P_{p,s} = F_{su}\left(\frac{\pi D^2}{4}\right) \qquad \text{Eq. 9.8.5}$$

where: $P_{p,s}$ – Ultimate load for pin shear-off failure
F_{su} – Ultimate shear stress of the pin material

In most design cases, the lug design is in double shear:

$$P_{p,s} = 2F_{su}\left(\frac{\pi D^2}{4}\right) \qquad \text{Eq. 9.8.6}$$

or $P_{p,s} = 2P_{s,all}$ Eq. 9.8.7

where: $P_{s,all}$ – Allowable ultimate single shear load from Fig. 9.8.8

(f) Pin bending failure:

If the pin used in the lug is too small, the pin can bend enough to precipitate failure in the lug because, as the pin bends, the stress distribution acting on the inner side of the lug tends to peak rather than form an even distribution as shown in Fig. 9.8.9.

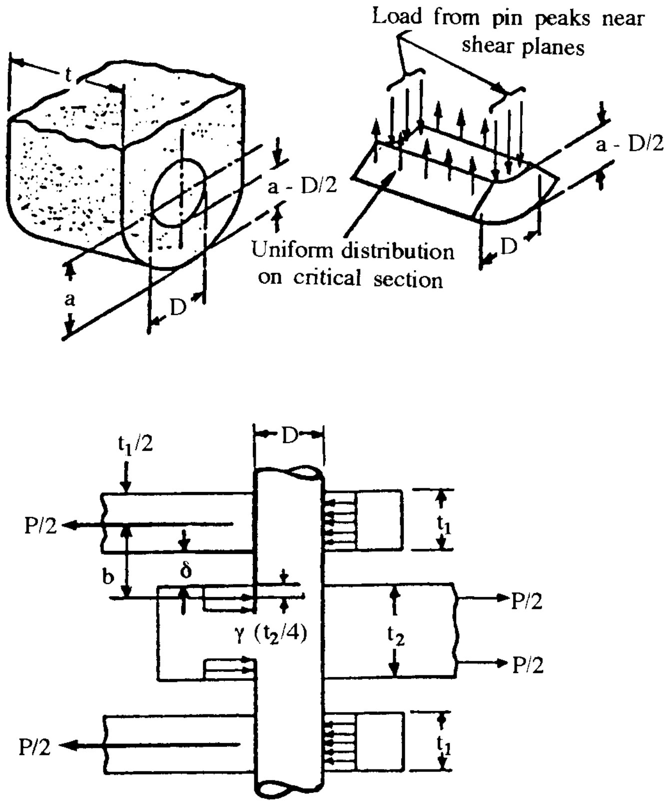


Fig. 9.8.8 Pin Moment Arm for Determination of Bending Moment

Since a weak or smaller pin can cause an inner lug (t_2) to fail at a smaller load, larger pins (ample MS) are always recommended. The moment arm is given by:

$$b = \frac{t_1}{2} + \delta + \gamma(\frac{t_2}{4}) \qquad \text{Eq. 9.8.8}$$

Compute the following two values:

$$\frac{(P_u)_{min}}{A_{br} F_{tux}}$$

$$r = \frac{a - \frac{D}{2}}{t_2}$$

where:
- t_1 – Outer lug thickness
- t_2 – Inner lug thickness
- $(P_u)_{min}$ – The smaller of P_{bux} (Eq. 9.8.1) and P_{tu} (Eq. 9.8.2) for the inner lug
- P_{tux} – Lug material across grain "F" (see sketch in Fig. 9.8.5)
- D – Pin diameter or D_b (if bushing used)
- δ – Gap (lug chamfer or use flange bushings) as shown in Fig. 9.8.9
- γ – Reduction factor (only applies to the inner lug) as given in Fig. 9.8.10

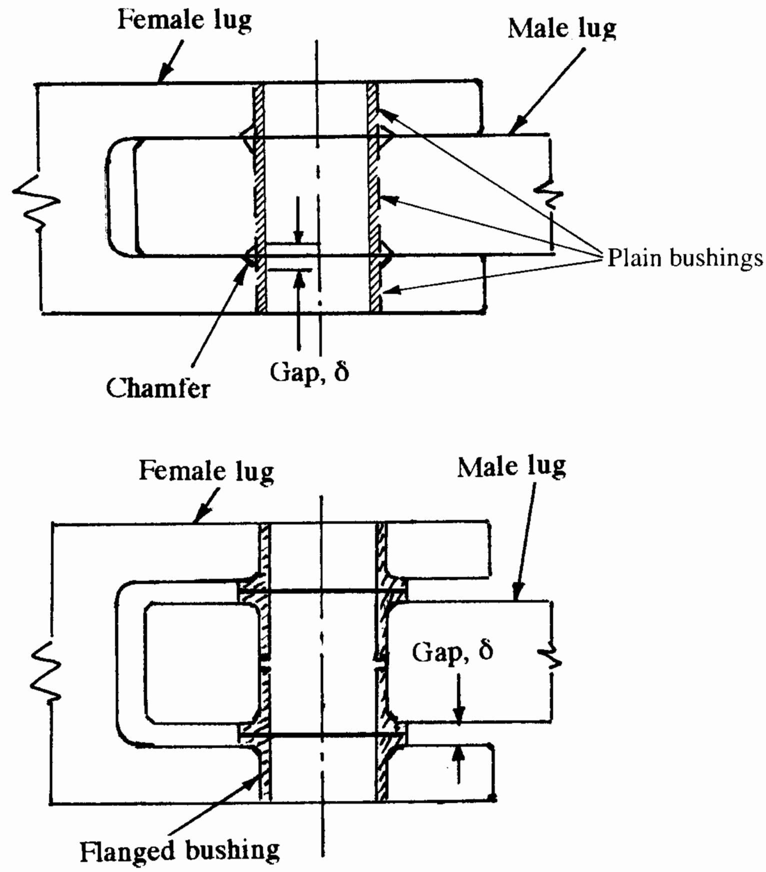
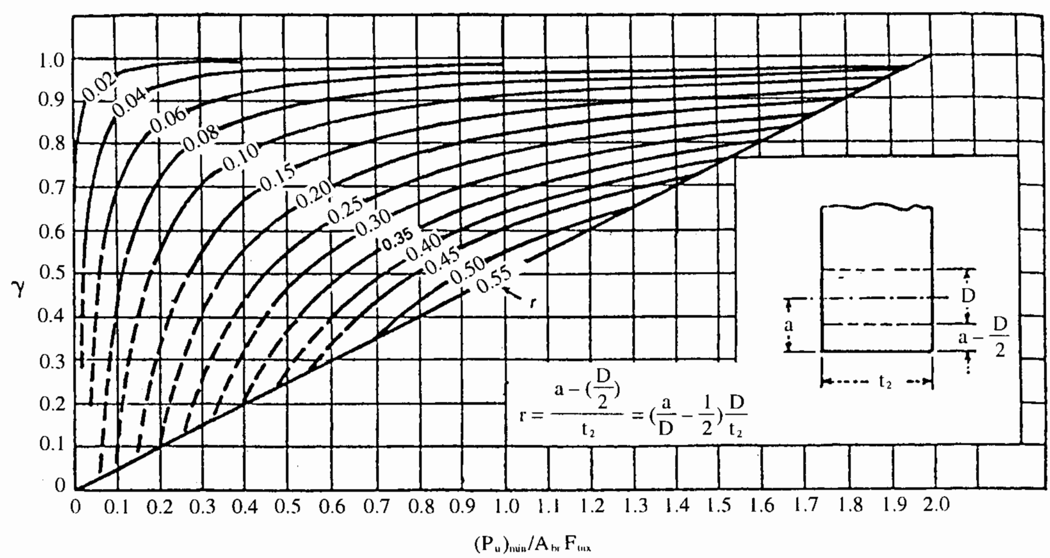


Fig. 9.8.9 Gap Definition, δ



(Dashed lines indicate region where these theoretical curves are not substantiated by test data)

Fig. 9.8.10 Inner Lug Peaking Factors for Pin Bending, γ

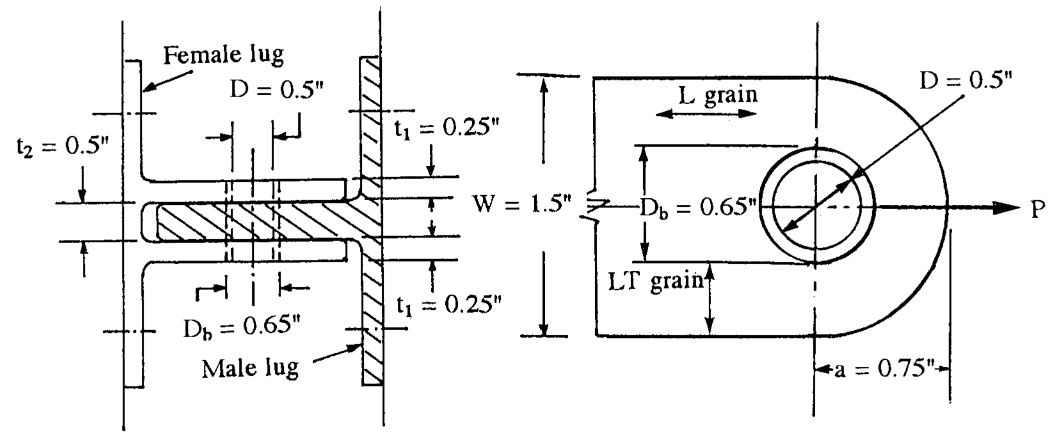
The pin bending moment, $M = \frac{P}{2} b$ Eq. 9.8.9

$$MS = \frac{\text{Ultimate bending moment}}{\lambda M} - 1 \quad \text{where } \lambda - \text{Fitting factor}$$

The ultimate bending moment for F_{tu} = 125 ksi AN steel pin can be obtained from Fig. 9.2.7; use the plastic bending method to obtain the ultimate bending moment for other H.T. steel pin materials from Chapter 6.4.

Example 1:

Find the MS of a lug under an axial concentrated load of P = 15 kips (case I) with the following given data:



Lug: Aluminum 7075-T6 Extrusion (for material properties, see Fig. 4.3.5)

W = 1.5" a = 0.75" t_1= 0.25" t_2 = 0.5"

F_{tu} = 81 ksi; F_{ty} = 72 ksi; F_{cy} = 72 ksi ('A' values of L-grain direction)

F_{tu} = 74 ksi; F_{ty} = 65 ksi; F_{cy} = 71 ksi ('A' values of LT-grain direction)

Pin: (H.T. = 125 ksi, material properties, see Fig. 4.3.6)

F_{tu} = 125 ksi; F_{su} = 75 ksi; F_{cy} = 109 ksi ('S' values)

D = 0.5" (for pin shear allowable, see Fig. 9.2.5)

Bushing: (H.T. = 125 ksi; for material properties, see Fig. 4.3.6)

D_b = 0.65" (outside diameter)

Compute:

$$\frac{a}{D_b} = \frac{0.75}{0.65} = 1.15; \quad \frac{W}{D_b} = \frac{1.5}{0.65} = 2.3$$

$$\frac{D_b}{t} = \frac{0.65}{0.5} = 1.3; \quad A_{br} = Dt = 0.65 \times 0.5 = 0.325$$

$$A_t = (W - D_b)t = (1.5 - 0.65) \times 0.5 = 0.425$$

Design requirements:

- Use fitting factor λ = 1.15
- Minimum MS = 0.2

(a) Shear-bearing failure (from Eq. 9.8.1):

$$P_{bru} = k_{br} F_{tux} A_{br}$$
$$= 0.97 \times 81 \times 0.325 = 25.5 \text{ kips}$$

$$MS = \frac{25.5}{1.15 \times 15} - 1 = 0.48 > 0.2 \qquad \text{O.K.}$$

The above 1.15 is fitting factor (λ)

(b) Tension failure (from Eq. 9.8.2):

$$P_{tu} = k_t F_{tu} A_t$$
$$= 0.95 \times 82 \times 0.425 = 33.1 \text{ kips}$$

$$MS = \frac{33.1}{1.15 \times 15} - 1 = 0.92 > 0.2 \text{ (min. MS requirement)} \qquad \text{O.K.}$$

(c) Yield failure – lug (from Eq. 9.8.3):

$$P_y = C\left(\frac{F_{tyx}}{F_{tux}}\right)(P_u)_{min}$$
$$= 1.1 \times \frac{72}{81} \times 22.7 = 22.2 \text{ kips}$$

$$MS = \frac{22.2}{1.15 \times \frac{15}{1.5}} - 1 = 0.93 > 0.2 \qquad \text{O.K.}$$

(d) Yield failure – bushing (from Eq. 9.8.4):

$$P_{bry} = 1.85 F_{cy} A_{brb}$$
$$= 1.85 \times 75 \times 0.25 = 34.7 \text{ kips}$$

$$MS = \frac{34.7}{1.15 \times \frac{15}{1.5}} - 1 = \text{high} > 0.2 \qquad \text{O.K.}$$

(e) Pin shear-off failure (from Eq. 9.8.7):

$$P_{p,s} = 2 P_{s,all}$$
$$= 2 \times 14.7 = 29.4 \text{ kips}$$

where: $P_{s,all} = 14.7$ kips from Fig. 9.2.7.

$$MS = \frac{29.4}{1.15 \times 15} - 1 = 0.7 > 0.2 \qquad \text{O.K.}$$

(f) Pin bending failure (from Eq. 9.8.8):

$$b = \frac{t_1}{2} + \delta + \gamma\left(\frac{t_2}{4}\right) \qquad \text{Eq. 9.8.8}$$

$$r = \frac{a - \frac{D_b}{2}}{t_2} = \frac{0.75 - \frac{0.65}{2}}{0.5} = 0.85$$

$$\frac{(P_u)_{min}}{A_{br} F_{tux}} = \frac{22.7}{0.325 \times 81} = 0.86$$

$\gamma = 0.43$ (from Fig. 9.8.10) and $\delta = 0.02''$ (gap from chamfers)

$$b = \frac{0.25}{2} + 0.02 + 0.43 \frac{0.5}{4} = 0.2$$

$$M = \frac{P}{2} b$$

$$= \frac{15}{2} 0.2 = 1.5 \text{ in.-kips}$$

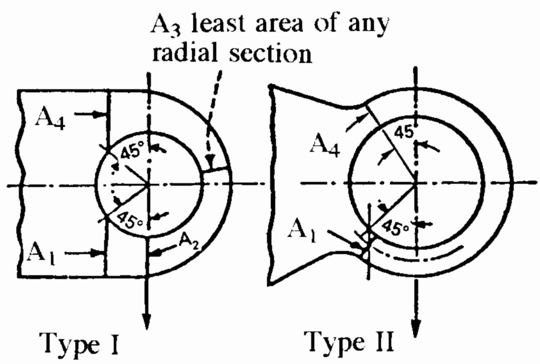
The allowable pin bending moment is 2.21 in.-kips (H.T. = 125 ksi) from Fig. 9.2.7.

$$MS = \frac{2.21}{1.15 \times 1.5} - 1 = \underline{0.28} > 0.2 \qquad \text{O.K.}$$

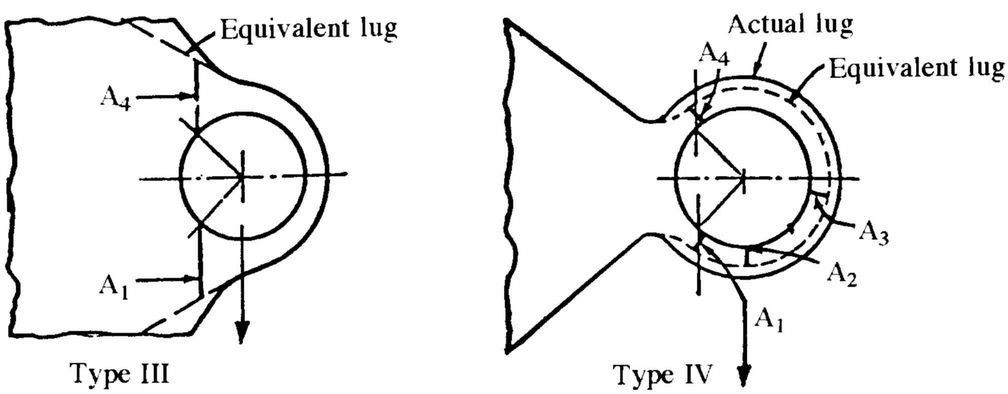
(g) The final minimum MS = **0.28** [pin bending is critical from Step (f)].

(C) CASE II – TRANSVERSE LOAD ($\alpha = 90°$)

The lug failure modes for this transverse load case is shown in Fig. 9.8.11.



(a) Common shapes



(b) Unusual Shapes

(Locations of Cross-Section Areas A_1, A_2, A_3, and A_4 are shown)

Fig. 9.8.11 Lugs Subjected to Transverse Load

(a) Compute:

$$A_{br} = D\,t$$

$$A_{av} = \frac{6}{\dfrac{3}{A_1} + \dfrac{1}{A_2} + \dfrac{1}{A_3} + \dfrac{1}{A_4}} \qquad \text{Eq. 9.8.10}$$

$$\frac{A_{av}}{A_{br}}$$

(b) The ultimate load is obtained using:

$$P_{tru} = k_{tru} A_{br} F_{tuy} \qquad \text{Eq. 9.8.11}$$

where: P_{tru} – Ultimate transverse load
k_{tru} – Efficiency factor for transverse ultimate load from Fig. 9.8.12
A_{br} – Projected bearing area
F_{tuy} – Ultimate tensile stress of lug material in y-direction

The load that can be carried by cantilever beam action is indicated very approximately by curve A in Fig. 9.8.12; if the efficiency factor falls below curve A, a separate calculation as a cantilever beam is warranted, as shown in Fig. 9.8.13.

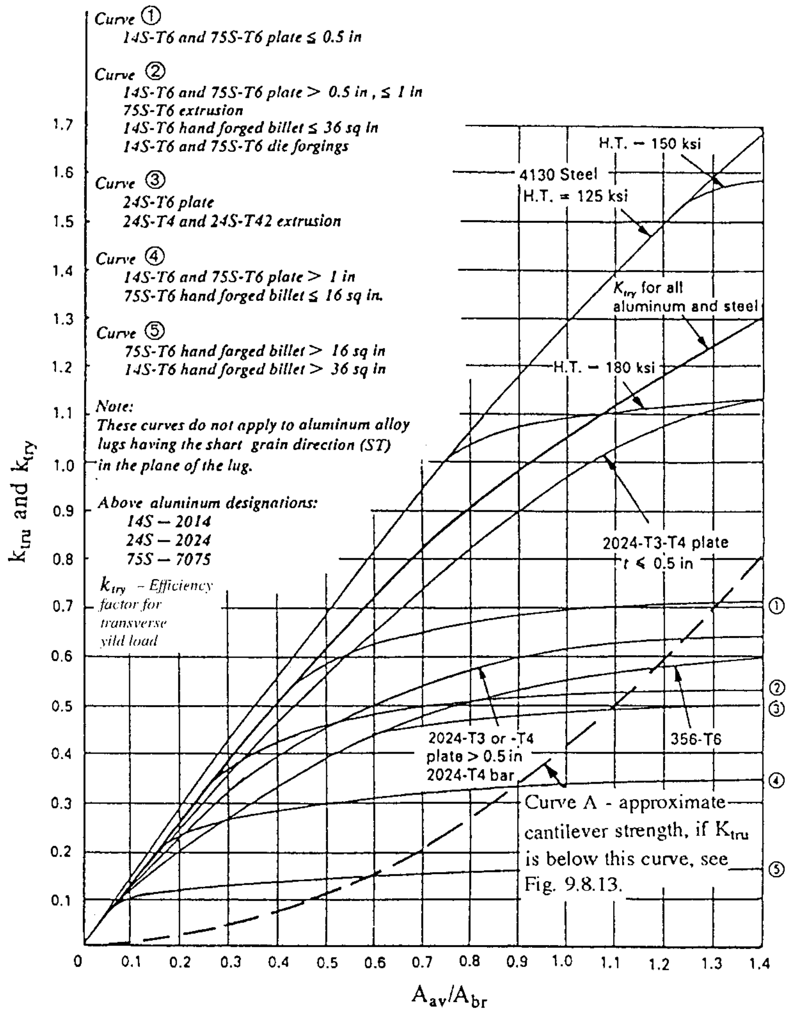


Fig. 9.8.12 Efficiency Factor for Transverse Load, k_{tru} and k_{try}

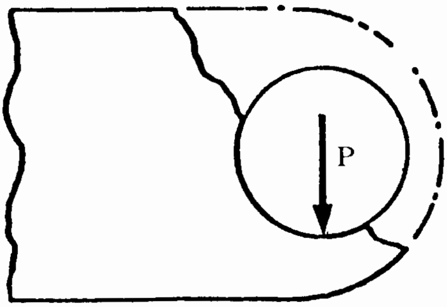


Fig. 9.8.13 Cantilever Beam Action of the Portion of the Lug Under Load

(c) The yield load is given by:

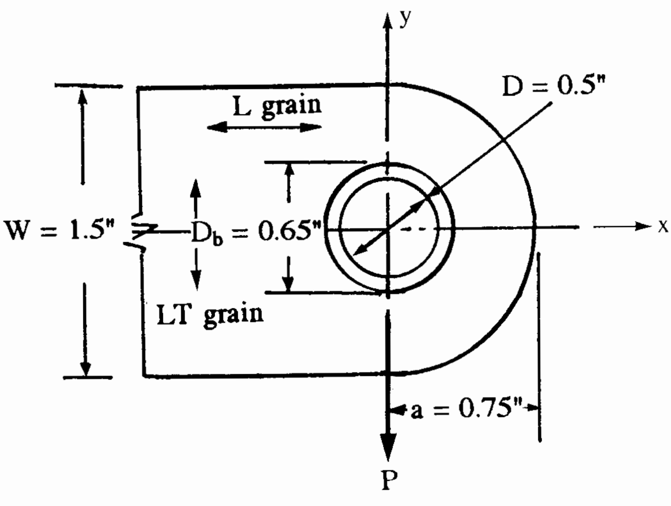
$P_y = k_{try} A_{br} F_{tyy}$ Eq. 9.8.12

where: P_y – Yield transverse load
k_{try} – Efficiency factor for transverse yield load from Fig. 9.8.12
A_{br} – Projected bearing area
F_{tyy} – Tensile yield stress of lug material in y-direction

(d) Determine the yield load using Eq. 9.8.4, namely $P_{bry} = 1.85\, F_{cy} A_{brb}$

Example 2:

Assume use of the same lug given in Example 1 except use the Case II transverse load (P = 15 kips) application as shown below:



(a) Compute:

$$A_1 = A_4 = (0.75 - \frac{0.65}{2}\sin 45°) \times 0.5 = (0.75 - 0.23) \times 0.5 = 0.26$$

$$A_2 = A_3 = (0.75 - \frac{0.65}{2}) \times 0.5 = 0.21$$

$$A_{br} = 0.65 \times 0.5 = 0.325$$

From Eq. 9.8.10:

$$A_{av} = \frac{6}{\frac{3}{0.26} + \frac{1}{0.21} + \frac{1}{0.21} + \frac{1}{0.26}} = 0.24$$

$$\frac{A_{av}}{A_{br}} = \frac{0.24}{0.325} = 0.74$$

(b) The ultimate load is obtained using Eq. 9.8.11:

From curve ② of Fig. 9.8.12, obtain $k_{tru} = 0.51$:

$$P_{tru} = k_{tru} A_{br} F_{tuy}$$
$$= 0.51 \times 0.325 \times 74 = 12.3 \text{ kips}$$

(c) The yield load is obtained using Eq. 9.8.12:

From curve k_{try} of Fig. 9.8.12, obtain $k_{tru} = 0.86$

$$P_y = k_{try} A_{br} F_{tyy}$$
$$= 0.86 \times 0.325 \times 65 = 18.2 \text{ kips}$$

(d) The min. MS $= \frac{12.3}{1.15 \times 15} - 1 = -0.29$ (ultimate load) NO GOOD

(e) Therefore the allowable ultimate load (considering fitting factor of 1.15 and 20% margin) is:

$P = \frac{12.3}{1.15 \times 1.2} = 8.9$ kips instead of the given load of 15 kips.

(D) CASE III – OBLIQUE LOAD ($\alpha = 0°$ BETWEEN $90°$)

Use the following interaction equation to size the oblique load case as shown in Fig. 9.8.14 below:

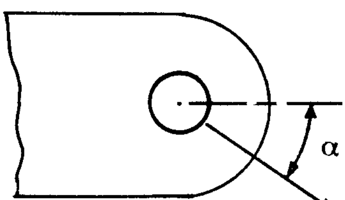


Fig. 9.8.14 Oblique Load Case

(a) Ultimate load:

$$MS = \frac{1}{(R_{a,u}^{1.6} + R_{tr,u}^{1.6})^{0.625}} - 1 \qquad \text{Eq. 9.8.13}$$

where: $R_{a,u}$ – Axial component ($\alpha = 0°$) of applied ultimate load divided by smaller of P_{bru} (from Eq. 9.8.1) or P_u (from Eq. 9.8.2)

$R_{tr,u}$ – Transverse component ($\alpha = 90°$) of applied ultimate load divided by P_{tru} (from Eq. 9.8.11)

(b) Yield load:

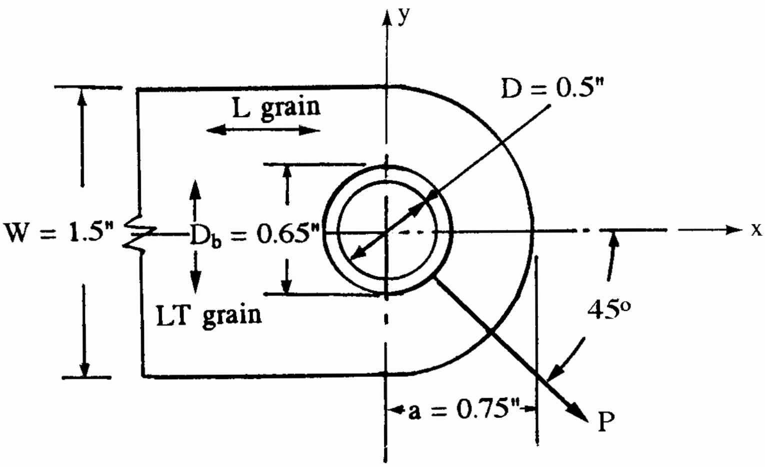
$$MS = \frac{1}{(R_{a,y}^{1.6} + R_{tr,y}^{1.6})^{0.625}} - 1 \qquad \text{Eq. 9.8.14}$$

where: $R_{a,y}$ – Axial component ($\alpha = 0°$) of applied limit load divided by $P_{y,o}$ ($P_{y,o}$ = P_y from Eq. 9.8.3)

$R_{tr,y}$ – Transverse component ($\alpha = 90°$) of applied limit load divided by $P_{y,90}$ ($P_{y,90} = P_y$ from Eq. 9.8.12)

Example 3:

Assume use of the same lug given in Example 1 except use the Case III oblique load (P = 15 kips) application as shown below:



(a) Ultimate load:

P_{bru} = 22.7 kips [see Step (a) of Example 1]

P_{tu} = 33.1 kips [see Step (b) of Example 1]

P_{tru} = 12.3 kips [see Step (b) of Example 2]

$$R_{a,u} = \frac{P\cos 45°}{\text{Smaller of } P_{bru} \text{ or } P_{tu}}$$

$$= \frac{15 \times 0.707}{22.7} = 0.47$$

$$R_{tr,u} = \frac{P\sin 45°}{P_{tru}}$$

$$= \frac{15 \times 0.707}{12.3} = 0.86$$

From Eq. 9.8.13:

$$MS = \frac{1}{1.15(0.47^{1.6} + 0.86^{1.6})^{0.625}} - 1 = -0.17 < 0.2 \qquad \text{N.G.}$$

Note: See discussion in Step (c).

(b) Yield load:

$P_{y,o}$ = 22.2 kips [see Step (c) of Example 1]

$P_{y,90}$ = 18.2 kips [see Step (c) of Example 2]

$$R_{a,y} = \frac{\frac{P}{1.5}\cos 45°}{P_{y,o}}$$

$$= \frac{\frac{15}{1.5} \times 0.707}{22.2} = 0.32$$

$$R_{tr,y} = \frac{\frac{P}{1.5}\sin 45°}{P_{y,90}}$$

$$= \frac{\frac{15}{1.5} \times 0.707}{18.2} = 0.39$$

From Eq. 9.8.14:

$$MS = \frac{1}{1.15(0.32^{1.6} + 0.39^{1.6})^{0.625}} - 1 = 0.59 > 0.2 \qquad \text{O.K.}$$

(c) Discussion

- Ample MS for yield load condition
- Not enough MS for ultimate load condition
- Allowable ultimate load can be calculated following the same procedures or re-size this lug

(E) TUBULAR PIN IN LUG

Under conditions of very high concentrated loads, the use of a tubular pin to encase a large diameter pin in the lug can provide greater bearing area and less pin bending deformation due to its greater moment of inertia. A nearly even distribution of bearing or compression stress on lug thickness can be thus obtained in addition to a reduction of wear and an improvement of the fatigue life of the lug. There are basically two types of design approaches:

- A tubular pin can be used as the shear fuse (breakaway design conditions for landing gear trunnion and engine pylon mount fittings)
- Double pin design (another smaller pin is used inside the tubular pin) provides fail-safe capability for vital hinge applications (e.g., horizontal tail pivot and certain hinges on control surfaces)

Design considerations:

- $\frac{D}{t}$ ratio $5 \leftrightarrow 10$
- $0.5 > \frac{t_1}{D} < 1.0$

where: D – Tubular pin outside diameter
t – Tubular wall thickness
t_1 – Female lug wall thickness (see Fig. 9.8.8)

Use the following interaction equation to determine MS:

(a) Ultimate load:

$$MS = \frac{1}{R_{s,u}^{\ 2} + R_{b,u}} - 1 \qquad \text{Eq. 9.8.15}$$

where: $R_{s,u}$ – Ultimate shear stress divided by allowable shear stress (F_s from Fig. 9.8.15)
$R_{b,u}$ – Ultimate bending stress divided by modulus of rupture bending stress (F_b from Fig. 9.8.16) for the tubular pin