# Penman-Monteith (hourly) Reference Evapotranspiration Equations for Estimating ETos and ETrs with Hourly Weather Data



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# Penman-Monteith (hourly) Reference Evapotranspiration Equations for Estimating $ET_{os}$ and $ET_{rs}$ with Hourly Weather Data


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## Overview

The following text is a description of the steps needed to estimate reference evapotranspiration ($ET_{ref}$) for a 0.12 m tall reference surface ($ET_{os}$) and for a 0.50 m tall reference surface ($ET_{rs}$) using hourly weather data as adopted by the Environmental Water Resources Institute - American Society of Civil Engineers (ASCE-EWRI, 2004). Note that the steps are in the same sequence as one would use when write computer code. The steps to calculate the Penman equation estimate of $ET_p$ for a short canopy with no canopy resistance is also provided.

## Data Requirements

Site characteristics including the latitude (+ for north and – for south), longitude (+ for west and – for east) and elevation (m) above sea level must be input. The required weather data includes hourly solar radiation (MJ $m^{-2}h^{-1}$), mean air temperature ($^{o}$C), mean wind speed (m $s^{-1}$) and mean dew point temperature ($^{o}$C). The air and dew point temperatures should be measured at between 1.5 and 2.0 m height and the wind speed should be measured at 2.0 m height. For wind speeds measured at some height other than 2.0 m, the wind speed at 2 m height ($u_2$) can be estimated as:

$$u_2 = u_z\left(\frac{4.87}{ln(67.8z_w - 5.42)}\right)$$

where $u_z$ = wind speed (m $s^{-1}$) at height $z_w$ (m) above the ground.

**STEP 1:** Extraterrestrial radiation ($R_a$) is calculated for each hour using the following equations from Duffie and Beckman (1980).

**$G_{SC}$ = solar constant in MJ $m^{-2}$ $min^{-1}$**

$G_{SC} = 0.082$

**$\sigma$ = Steffan-Boltzman constant in MJ m$^{-2}$ h$^{-1}$ K$^{-4}$**

$\sigma = 2.04 \times 10^{-10}$

**$\phi$ = Latitude in radians converted from latitude (*L*) in degrees**

$$\phi = \frac{\pi L}{180}$$

*J* = day of the year (1-366)

**$d_r$ = correction for eccentricity of Earth's orbit around the sun**

$$d_r = 1 + 0.033 cos\left(\frac{2\pi}{365} J\right) \tag{1}$$

**$\delta$ = Declination of the sun above the celestial equator in radians**

$$\delta = 0.409 sin\left(\frac{2\pi}{365} J - 1.39\right) \tag{2}$$

$L_m$ = station longitude in degrees

$L_z$ = longitude of the local time meridian

$L_z$ = 120$^o$ for Pacific Standard Time

**$S_c$ = solar time correction for wobble in Earth's rotation**

$$b = \frac{2\pi(J - 81)}{364} \tag{3}$$

$$S_c = 0.1645 sin(2b) - .1255 cos(b) - 0.025 sin(b) \tag{4}$$

*t* = local standard time (h)

**$\omega$ = hour angle in radians**

$$\omega = \frac{\pi}{12}\left[(t - 0.5) + \frac{L_z - L_m}{15} - 12 + S_c\right] \tag{5}$$

**$\omega_1$ = hour angle ½ hour before $\omega$ in radians**

$$\omega_1 = \omega - \left(\frac{1}{2}\right)\left(\frac{\pi}{12}\right) \tag{6}$$

***ω₂* = hour angle ½ hour after *ω* in radians**

$$\omega_2 = \omega + \left(\frac{1}{2}\right)\left(\frac{\pi}{12}\right) \quad (7)$$

***θ* = solar altitude angle in radians**

$$sin\,\theta = (\omega_2 - \omega_1)\, sin\,\phi\, sin\,\delta + cos\,\phi\, cos\,\delta (sin\,\omega_2 - sin\,\omega_1) \quad (8)$$

**$R_a$ = extraterrestrial radiation (MJ $m^{-2}$ $h^{-1}$)**

$$R_a = \frac{12}{\pi}(60 G_{SC}) d_r\, sin\,\theta \quad (9)$$

***β* = solar altitude in degrees**

$$\beta = \frac{180}{\pi} sin^{-1}[sin\,\phi\, sin\,\delta + cos\,\phi\, cos\,\delta\, cos\,\omega] \quad (10)$$

**STEP 2:** Calculate the hourly net radiation ($R_n$) expected over grass in MJ $m^{-2}$ $h^{-1}$ using equations from Allen et al. (1994).

**$R_{so}$ = clear sky total global solar radiation at the Earth's surface in MJ $m^{-2}$ $h^{-1}$**

$$R_{so} = R_a \left(0.75 + 2.0 \times 10^{-5} E_l\right) \quad (11)$$

where $E_l$ = elevation above mean sea level (m)

**$e_s$ = saturation vapor pressure (kPa) at the mean hourly air temperature (*T*) in $^o$C**

$$e_s = 0.6108\, exp\left[\frac{17.27T}{T + 237.3}\right] \quad (12)$$

**$e_a$ = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature**

$$e_a = 0.6108\, exp\left[\frac{17.27T_d}{T_d + 237.3}\right] \quad (13)$$

***ε′* = apparent 'net' clear sky emissivity**

$$\varepsilon' = 0.34 - 0.14\sqrt{e_a} \quad (14)$$

Note that $\varepsilon' = \varepsilon_{vs} - \varepsilon_a$, where $\varepsilon_{vs}$ is the emissivity of the grass and $\varepsilon_a$ is the emissivity from the atmosphere. It is called 'apparent' because the temperature from a standard shelter rather than the surface temperature and atmosphere temperature are used to calculate the 'net' long–wave radiation balance. Equation 11 is called the 'Brunt form' equation for net emittance because the form of the equation is similar to Brunt's equation for apparent long-wave emissivity from a clear sky.

***f* = a cloudiness function of *$R_S$* and *$R_{SO}$***

$$f = 1.35\frac{R_S}{R_{SO}} - 0.35 \tag{15}$$

with the restriction that $0.3 < R_s/R_{so} \leq 1.0$ and $R_s/R_{so} = 0$ whenever $\beta < 17.2^o$ (=0.300 radians) above the horizon. When using a spreadsheet program, put the value $f = 0.6$ in the cell before the first data cell in the column for $f$. For each sequential hour interval, whenever $\beta < 17.2^o$, let the value for $f$ equal the previous $f$ value. When the corresponding $\beta \geq 17.2^o$, use the $R_s/R_{so}$ and Equation 15 to calculate the $f$ values. The values for $f$ will fall between 0.05 and 1.00. If this procedure is followed, the nighttime values for $f$ will equal the $f$ value at the end of the previous daylight period until the next daylight period. The nighttime $f$ values are used to estimate the effect of cloud cover on $R_n$ during the night. This method is used in the PMhr.xls program.

***$R_{ns}$* = net short wave radiation as a function of measured solar radiation (*$R_s$*) in MJ $m^{-2}$ $h^{-1}$**

$$R_{ns} = (1 - 0.23)R_s \tag{16}$$

To convert $R_s$ from W $m^{-2}$ to MJ $m^{-2}$ $h^{-1}$, multiply by 0.0036.

***$R_{nl}$* = net long wave radiation in MJ $m^{-2}$ $h^{-1}$**

$$R_{nl} = -f\varepsilon'\sigma(T + 273.15)^4 \tag{17}$$

***$R_n$* = net radiation over grass in MJ $m^{-2}$ $h^{-1}$**

$$R_n = R_{ns} + R_{nl} \tag{18}$$

**STEP 3:** Calculate $ET_o$ using the Penman-Monteith equation as presented by Allen et al. (1994)

***$B_p$* = barometric pressure in kPa as a function of elevation (*$E_l$*) in meters**

$$B_p = 101.3\left(\frac{293 - 0.0065E_l}{293}\right)^{5.26} \tag{19}$$

***$\lambda$* = latent heat of vaporization in (MJ $kg^{-1}$)**

$$\lambda = 2.45 \tag{20}$$

**$\gamma$ = psychrometric constant in kPa $^{o}C^{-1}$**

$$\gamma = 0.00163\frac{B_p}{\lambda} \quad (21)$$

**$r_a$ = aerodynamic resistance in s $m^{-1}$ is estimated for a 0.12 m tall crop as a function of wind speed ($u_2$) in m $s^{-1}$ as:**

$$r_a = \frac{208}{u_2} \quad (22)$$

**Modified psychrometric constant ($\gamma$*)**

For the short 0.12 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of $r_s = 50$ s $m^{-1}$ and an aerodynamic resistance of $r_a = 208/u_2$ are used to calculate modified psychrometric constant as:

$$\gamma^* = \gamma\left(1+\frac{r_s}{r_a}\right) \approx \gamma(1+0.24u_2) \quad (23)$$

During the night (when $R_n \le 0$), a canopy resistance of $r_s = 200$ s $m^{-1}$ and an aerodynamic resistance of $r_a = 208/u_2$ $\gamma$* are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1+\frac{r_s}{r_a}\right) \approx \gamma(1+0.96u_2) \quad (24)$$

For wind speeds less than 0.5 m $s^{-1}$, the wind speed is set equal to 0.5 m $s^{-1}$ for both Eqs. 23 and 24. For the 0.50 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of $r_s = 30$ s $m^{-1}$ and an aerodynamic resistance of $r_a = 118/u_2$ s $m^{-1}$ are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1+\frac{r_s}{r_a}\right) \approx \gamma(1+0.25u_2) \quad (25)$$

During the night (when $R_n \le 0$), a canopy resistance of $r_s = 200$ s $m^{-1}$ and an aerodynamic resistance of $r_a = 118/u_2$ s $m^{-1}$ are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1+\frac{r_s}{r_a}\right) \approx \gamma(1+1.7u_2) \quad (26)$$

For wind speeds less than 0.5 m $s^{-1}$, the wind speed is set equal to 0.5 m $s^{-1}$ for both Eqs. 25 and 26.

**$\Delta$ = slope of the saturation vapor pressure curve (kPa $^{o}C^{-1}$) at mean air temperature ($T$)**

$$\Delta = \frac{4099e_s}{(T+237.3)^2} \quad (27)$$

**$G$ = soil heat flux density (MJ m$^{-2}$ h$^{-1}$)**

For $ET_{os}$, let $G = 0.1\ R_n$ when $R_n > 0$ and let $G = 0.5\ R_n$ for $R_n < 0$. For $ET_{rs}$, let $G = 0.04\ R_n$ when $R_n > 0$ and $G = 0.2\ R_n$ when $R_n \leq 0$.

***R* is the radiation term of the Penman-Monteith and Penman equations in mm d$^{-1}$ .**

When $R_n > 0$, for $ET_{os}$, the radiation term contribution to $ET$ is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.24U_2)} \tag{28}$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.96U_2)} \tag{29}$$

When $R_n > 0$, for $ET_{rs}$, the radiation term contribution to $ET$ is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.25U_2)} \tag{30}$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 1.7U_2)} \tag{31}$$

For the $ET_p$ (Penman equation), the radiation term contribution to $ET$ is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma} \tag{32}$$

for both day and night calculations.

**A = aerodynamic term of the Penman-Monteith equation in mm d$^{-1}$ with $u_2$ the wind speed at 2 m height**

When $R_n > 0$, for $ET_{os}$, the aerodynamic contribution to $ET$ is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.24u_2)} \tag{33}$$

And during the night, it is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.96 u_2)} \qquad (34)$$

When $R_n > 0$, for $ET_{rs}$, the aerodynamic contribution to *ET* is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.25 u_2)} \qquad (35)$$

And during the night, it is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 1.7 u_2)} \qquad (36)$$

For $ET_p$, the aerodynamic contribution to *ET* during daytime and nighttime is calculated as:

$$A_p = \frac{\left(\frac{37\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma} \qquad (37)$$

**Reference evapotranspiration**

For a short (0.12 m) canopy, the Penman-Monteith reference evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \qquad (38)$$

Similarly, for a tall (0.5 m) canopy, the Penman-Montieth reference evapotranspiration is calculated as:

$$ET_{rs} = R_r + A_r \qquad (39)$$

For a short (0.12 m) tall canopy, the Penman evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \qquad (40)$$

In equations 38-40, the units are mm $h^{-1}$.

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