$DSE210 - Final$

1

(a)

$Answer:$

$\frac{13}{52} = \frac{1}{4}$

(b)

$Answer:$

$\frac{52-13}{52} = \frac{3}{4}$

(c)

$Answer:$

$\frac{12+13}{51} = \frac{25}{51}$

(d)

$Answer:$

$\frac{1}{2}$

(e)

$P_r(x = 1st\ ace|y = 1st\ heart) = \frac{P_r(x=1st\ ace \cap y=1st\ heart)}{P_r(y=1st\ heart)}$

$= \frac{2/52}{26/52}$

$= \frac{1}{13}$

$Answer:$

$\frac{1}{13}$

(f)

$P_r(x = 2nd\ ace|y = 1st\ ace) = \frac{P_r(x=2nd\ ace \cap y=1st\ ace)}{P_r(y=1st\ ace)}$

$= \frac{4/52 \times 3/51}{4/52}$

$= 3/51$

$Answer:$

$\frac{3}{51}$

2

- $A:$ $first\ card\ is\ a\ ten,\ B:\ tenth\ card\ is\ a\ jack$

$Answer:$

$dependent$

- $A:$ $first\ card\ is\ a\ ten,\ B:\ second\ card\ is\ a\ heart$

$Answer:$

$independent$

- $A:$ $second\ card\ is\ a\ heart,\ B:\ fifth\ card\ is\ a\ club$

$Answer:$

$dependent$

3

(a)

$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1 = \frac{1}{120}$

$Answer:$

$\frac{1}{120}$

(b)

$6 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$Answer:$

$\frac{1}{36}$

(c)

$x = \begin{cases} 1, \text{ run into enemy} \\ 0, \text{ not run into enemy} \end{cases} \qquad p = 20\%$

$E(x) = 1 \times 20\% + 0 \times 80\% = 0.2$

$Let\ T\ be\ the\ number\ of\ trips$

$E(T) = 1 \times P_r(T = 1) + 2 \times P_r(T = 2) + \ldots$

$= 1 \times p + 2 \times (1 - p)p + \ldots$

$= \frac{1}{p}$

$= 5$

$Answer:$

5

(d)

$P_r(man) = 40\%$

$P_r(women) = 60\%$

$P_r(left\_handed|man) = 20\%$

$P_r(left\_handed|women) = 10\%$

$$P_r(left\_handed) = P_r(left\_handed|men)P_r(men) + P_r(left\_handed|women)P_r(women)$$

$= 20\% \times 40\% + 10\% \times 60\%$

$= 0.14$

$$P_r(women|left\_handed) = P_r(women \cap left\_handed)/P_r(left\_handed)$$

$= P_r(left\_handed|women)P_r(women)/P_r(left\_handed)$

$= 10\% \times 60\%/0.14$

$= \frac{3}{7}$

Answer :

$\frac{3}{7}$

4

$P_r(rash|insect\ bite) = 1$

$P_r(rash|no\ insect\ bite) = \frac{1}{8}$

$P_r(insect\ bite) = \frac{1}{9}$

(a)

$$P_r(rash) = P_r(rash|insect\ bite)P_r(insect\ bite)$$
$$+ P_r(rash|no\ insect\ bite)P_r(no\ insect\ bite)$$

$= 1 \times \frac{1}{9} + \frac{1}{8} \times (1 - \frac{1}{9})$

$= \frac{2}{9}$

$Answer:$

$\frac{2}{9}$

(b)

$P_r(insect\ bite|rash) = P_r(insect\ bite\ \cap\ rash)/P_r(rash)$

$= P_r(rash|insect\ bite)P_r(insect\ bite)/P_r(rash)$

$= 1 \times \frac{1}{9}/\frac{2}{9}$

$= \frac{1}{2}$

$Answer:$

$\frac{1}{2}$

5

(a)

$E(X) = (1 + 2 + 3) \times \frac{1}{12} + (4 + 5 + 6) \times \frac{1}{4} = \frac{17}{4}$

$Answer:$

$\frac{17}{4}$

(b)

$E(X^2) = (1^2 + 2^2 + 3^2) \times \frac{1}{12} + (4^2 + 5^2 + 6^2) \times \frac{1}{4} = \frac{245}{12}$

$var(X) = E(X^2) - (E(X))^2 = \frac{245}{12} - (\frac{17}{4})^2 = 2.3542$

$Answer:$

2.3542

(c)

$X_1, ..., X_i : independent$

$E(Z) = E(X_1 + X_1 + ... + X_{100}) = E(X_1) + E(X_1) + ... + E(X_{100})$

$= 100 \times \frac{17}{4} = 425$

$Answer:$

425

(d)

$X_1, ..., X_i$: *independent*

$var(Z) = var(X_1 + X_2 + ... + X_{100}) = var(X_1) + var(X_1) + ... + var(X_{100})$

$= 100 \times 2.3542 = 235.42$

Answer :

235.42

6

(a)

X, Y, Z : *independent*

$E(W) = E(X - Y + Z) = E(X) - E(Y) + E(Z)$

$= 1 - 0 + 2 = 3$

Answer :

3

(b)

$var(W) = var(X - Y + Z) = var(X) + var(-Y) + var(Z)$

$= 16 + 4 + 9 = 29$

Answer :

29

7

$$corr(X_1, X_2) = \frac{cov(X_1, X_2)}{std(X_1)std(X_2)} = \frac{E(X_1X_2) - E(X_1)E(X_2)}{std(X_1)std(X_2)} = 0.25$$

$$E(X_1) = 0, \quad p_1 = 0.5, \quad E(X_1^2) = 1, \; var(X_1) = E(X_1^2) - (E(X_1))^2 = 1$$

$$E(X_2) = 0.5, \; p_2 = 0.25, \; E(X_2^2) = 1, \; var(X_2) = E(X_2^2) - (E(X_2))^2 = 0.75$$

$$cov(X_{1,}X_2) = corr(X_1, X_2)std(X_1)std(X_2) = 0.25 \times 1 \times \sqrt{0.75} = \frac{\sqrt{3}}{8} = 0.22$$

Answer :

Model the data by bivariate Gaussian, parameterized by :

$$mean\ \mu = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \ and\ covariance\ matrix\ \sum = \begin{pmatrix} 1 & 0.22 \\ 0.22 & 0.75 \end{pmatrix}$$

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(a)

Answer :

$$\lambda = \frac{5+3+1+0+0+1+2+4+3+4}{10} = 2.3$$

(b)

Answer :

$$P_r(X_2 \geq 4) = 0.5$$

(c)

$$|V| = 4$$

$$V = (1, 3, 1, 2)$$

Answer :

$$p_1 = \frac{1+1}{7+4} = \frac{2}{11}$$

$$p_2 = \frac{3+1}{7+4} = \frac{4}{11}$$

$$p_3 = \frac{1+1}{7+4} = \frac{2}{11}$$

$$p_4 = \frac{2+1}{7+4} = \frac{3}{11}$$

9

$\mu = 12.2$

$\sigma = 5.4/\sqrt{100} = 0.54$

$\mu \pm 2\sigma = 12.2 \pm 2 \times 0.54 = 12.2 \pm 1.08$

Answer :

$[11.12, 13.28]$

10

(a)

$Answer:$

Null hypothesis: the two distributions(scores in Genius Academy students and scores in other local high school students) have the same mean.

(b)

Assume the null is true. Let be the common mean.

The sample average in Genius Academy students, call it X_2, is roughly normal with mean μ and standard deviation $\sigma_2 = \frac{150}{\sqrt{100}}$

The sample average in local high school students, call it X_1, is roughly normal with mean μ and standard deviation $\sigma_1 = \frac{200}{\sqrt{100}}$

The difference $X_2 - X_1$ is therefore normally distributed with mean 0 and standard deviation $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 25$

$$z - statistic = \frac{observed - expected}{std} = \frac{(1930-1860)-0}{25} = 2.8$$

$Answer:$

2.8

(c)

$Answer:$

$p\ value = P_r(being\ more\ than\ 2.8\ standard\ deviations\ above\ the\ mean)$
$= 0.002555$

Conclusion: strong evidence that the difference between these observed averages is significant.

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(a)

p : actual fraction of computer science freshman had prior programming experience

$\widehat{p} = 0.4$

$std(\widehat{p}) \approx \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{0.24}{100}} = 0.049$

X_i *is approximated by* $N(0.4, 0.049)$ *normal distribution.*

95% *confidence interval for* X :

$0.4 \pm 2 \times 0.049$

Answer :

$[0.302, 0.498]$

(b)

$\widehat{p} = 0.4$

$2std(\widehat{p}) = 0.01$

$std(\widehat{p}) = 0.005 = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$

Answer :

$n = 9600$