DSE210_HW2

Worksheet 4

1.

X_1 *and* X_2 *are the outcomes in a 2 throws*

X *is the minimum of* X_1 *and* X_2

$X \in \{1, 2, 3, 4, 5, 6\}$

$Total\ outcomes\ = 36$

Answer :

$$P_r(X = 1) = \frac{1}{6} \times \frac{6}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{11}{36}$$

$$P_r(X = 2) = \frac{1}{6} \times \frac{5}{6} + \frac{4}{6} \times \frac{1}{6} = \frac{9}{36} = \frac{1}{4}$$

$$P_r(X = 3) = \frac{1}{6} \times \frac{4}{6} + \frac{3}{6} \times \frac{1}{6} = \frac{7}{36}$$

$$P_r(X = 4) = \frac{1}{6} \times \frac{3}{6} + \frac{2}{6} \times \frac{1}{6} = \frac{5}{36}$$

$$P_r(X = 5) = \frac{1}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{3}{36} = \frac{1}{12}$$

$$P_r(X = 6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

2.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$
$$T = \# \; of \; rolls$$
$$p = \tfrac{1}{6}$$

$$E(T) = 1 \cdot P_r(X = 1) + 2 \cdot P_r(X = 2) + ...$$
$$= 1 \times \tfrac{1}{6} + 2 \times \tfrac{5}{6} \times \tfrac{1}{6} + 3 \times (\tfrac{5}{6})^2 \times \tfrac{1}{6} + ...$$
$$= \tfrac{1}{p} = 6$$

Answer :

$$T = 6$$

3.

$$P_r(1) = P_r(2) = P_r(3) = P_r(4) = \tfrac{1}{8}$$
$$P_r(5) = P_r(6) = \tfrac{1}{4}$$

(a)

$$E(Z) = 1 \cdot P_r(1) + 2 \cdot P_r(2) + ... + 6 \cdot P_r(6)$$
$$= (1 + 2 + 3 + 4) \times \tfrac{1}{8} + (5 + 6) \times \tfrac{1}{4}$$
$$= 4$$

$Answer\ :\ 4$

(b)

$$var(Z) = E[(Z - E(Z))^2] = E(Z^2) - (E(Z))^2$$
$$= 1^2 \cdot P_r(1) + 2^2 \cdot P_r(2) + ... + 6^2 \cdot P_r(6) - 4^2$$
$$= (1 + 4 + 9 + 16) \times \tfrac{1}{8} + (25 + 36) \times \tfrac{1}{4} - 16$$
$$= 19 - 16$$
$$= 3$$

$Answer\ :\ 3$

(c)

$$P_r(5\ rolls\ are\ 6) = \binom{10}{5} \cdot \left(\tfrac{3}{4}\right)^5 \cdot \left(\tfrac{1}{4}\right)^5 = 0.058399$$

$Answer\ :\ 0.058399$

(d)

$T = \#\ of\ rolls \qquad p = \frac{1}{4}$

$$E(T) = 1 \cdot P_r(X = 1) + 2 \cdot P_r(X = 2) + \ldots$$

$$= 1 \times \tfrac{1}{4} + 2 \times \tfrac{3}{4} \times \tfrac{1}{4} + 3 \times \left(\tfrac{3}{4}\right)^2 \times \tfrac{1}{4} + \ldots$$

$$= \frac{1}{p}$$

$$= 4$$

$$T = 4$$

$Answer\ :\ 4$

(e)

$$E(T) = 4 + 4 = 8$$

$Answer\ :\ 8$

8.

(a)

$var(X) = (std(X))^2 = 4$

$Answer\ :\ 4$

(b)

$Z = 10X$

$E(Z) = E(10X) = 10E(X) = 10 \times 5 = 50$

$Answer\ :\ 50$

(c)

$std(Z) = std(10X) = \sqrt{var(10X)} = \sqrt{10^2 var(X)}$

$= 10 \times 2 = 20$

$Answer\ :\ 20$

(d)

$var(Z) = var(10X) = 100var(X) = 400$

$Answer\ :\ 400$

9.

(a)

$P_r(\textit{exactly one person chooses the number } i)$

$= \binom{n}{1}\left(\frac{9}{10}\right)^{n-1}\frac{1}{10}$

$\textit{Answer}: \left(\frac{9}{10}\right)^{n-1}\frac{n}{10}$

(b)

$X_i = \begin{cases} 1 \textit{ if exactly one person chooses floor } i \\ 0 \textit{ otherwise} \end{cases}$

$E(X_i) = \binom{n}{1}\left(\frac{9}{10}\right)^{n-1}\frac{1}{10}$

$E(X) = E(X_1) + E(X_2) + \ldots + E(X_{10})$

$= 10 \times \binom{n}{1}\left(\frac{9}{10}\right)^{n-1}\frac{1}{10}$

$= n\left(\frac{9}{10}\right)^{n-1}$

$\textit{Answer}: n\left(\frac{9}{10}\right)^{n-1}$

11.

$Answer:$

$(a)\ dependent$

$(b)\ dependent$

$(c)\ independent$

$(d)\ dependent$

12.

(a)

$E(X) = np = 200 \times 5\% = 10$

$var(X) = np(1-p)$

$= 200 \times 5\% \times (1 - 5\%)$

$= 9.5$

$Answer: E(X) = 10 \quad var(X) = 9.5$

(b)

$p = 0.05$

$\binom{200}{10} p^{10}(1-p)^{200-10} \approx 0.128$

$Answer: 0.128$

18.

$$P_r(X=1) = \frac{1}{12} + \frac{1}{24} + \frac{1}{8} = \frac{1}{4}$$
$$P_r(X=3) = \frac{1}{12} + \frac{1}{24} + \frac{1}{8} = \frac{1}{4}$$
$$Then\ P_r(X=2) = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$$
$$P_r(X=1, Y=1) = P_r(X=1) \cdot P_r(Y=1)$$
$$\frac{1}{12} = \left(\frac{1}{12} + \frac{1}{24} + \frac{1}{8}\right) \cdot \left(\frac{1}{12} + P_r(X=2, Y=1) + \frac{1}{12}\right)$$
$$Then\ P_r(X=2, Y=1) = \frac{1}{6}$$
$$Then\ P_r(Y=1) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12} = \frac{1}{3}$$

$$P_r(X=2, Y=2) = P_r(X=2) \cdot P_r(Y=2)$$
$$P_r(X=2, Y=2) = \frac{1}{2} \cdot \left(\frac{1}{24} + P_r(X=2, Y=2) + \frac{1}{24}\right)$$
$$Then\ P_r(X=2, Y=2) = \frac{1}{12}$$

$$P_r(Y=1) = \frac{1}{12} + \frac{1}{6} + \frac{1}{12} = \frac{1}{3}$$
$$P_r(Y=2) = \frac{1}{24} + \frac{1}{12} + \frac{1}{24} = \frac{1}{6}$$
$$Then\ P_r(X=2, Y=3) = 1 - \frac{1}{3} - \frac{1}{6} - \frac{1}{8} - \frac{1}{8} = \frac{1}{4}$$

$Answer: \frac{1}{6}\quad \frac{1}{12}\quad \frac{1}{4}$

20.

(a)

$$X_i = \{ {}^{-1\ left\ \ \frac{1}{3}}_{1\ \ right\ \frac{2}{3}} \qquad X = \sum_{1}^{n} X_i$$

$$E(X) = E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n}((-1) \times \frac{1}{3} + 1 \times \frac{2}{3})$$

$$= \frac{n}{3}$$

$Answer : \frac{n}{3}$

(b)

$$E(X_i) = (-1) \times \frac{1}{3} + 1 \times \frac{2}{3} = \frac{1}{3}$$

$$E(X_i^2) = (-1)^2 \times \frac{1}{3} + 1^2 \times \frac{2}{3} = 1$$

$$var(X_i) = E(X_i^2) - (E(X_i))^2$$

$$= 1 - (\frac{1}{3})^2$$

$$= \frac{8}{9}$$

$$var(X) = \sum_{i=1}^{n} var(X_i) = \frac{8n}{9}$$

$Answer : \frac{8n}{9}$

(c)

$$std(X) = \sqrt{var(X)} = \sqrt{\tfrac{8n}{9}} = \tfrac{2}{3}\sqrt{2n}$$

$$E(X) \pm 2std(X) = \tfrac{n}{3} \pm \tfrac{4}{3}\sqrt{2n}$$

$$Answer : \tfrac{n}{3} \pm \tfrac{4}{3}\sqrt{2n}$$

Worksheet 5

1.

(a)

$\mu = 10$

$\sigma^2 = 16$

$P_r(X \geq 10) = P_r(Z \geq \frac{10-10}{4}) = P_r(Z \geq 0)$

$= 0.5$

$Answer : 0.5$

(b)

$P_r(X = 10) = P_r(Z = \frac{10-10}{4}) = P_r(Z = 0)$

$= 0$

$Answer : 0$

(c)

$P_r(X \geq 14) = P_r(Z \geq \frac{14-10}{4}) = P_r(Z \geq 1)$

$= 0.1587$

$Answer : 0.1587$

(d)

$P_r(X \leq 2) = P_r(Z \leq \frac{2-10}{4}) = P_r(Z \leq -2)$

$= 0.0228$

$Answer : 0.0228$

2.

$(a)\ E(X) = 0 \times \frac{22}{500} + 1 \times \frac{66}{500} + 2 \times \frac{106}{500} + ... + 8 \times \frac{10}{500}$

$= 3.154$

$Answer : 3.154$

(b)

Answer :

$\lambda = 3.154 \qquad N = 500$

$$N_0 = N \cdot P_r(k=0) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 21$$
$$N_1 = N \cdot P_r(k=1) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 67$$
$$N_2 = N \cdot P_r(k=2) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 106$$
$$N_3 = N \cdot P_r(k=3) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 112$$
$$N_4 = N \cdot P_r(k=4) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 88$$
$$N_5 = N \cdot P_r(k=5) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 56$$
$$N_6 = N \cdot P_r(k=6) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 29$$
$$N_7 = N \cdot P_r(k=7) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 13$$
$$N_8 = N \cdot P_r(k=8) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 5$$
$$N_9 = N \cdot P_r(k=9) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 2$$
$$N_{\geq 10} = N \cdot P_r(k \geq 10) = 500e^{-\lambda}\frac{\lambda^k}{k!} \approx 1$$

4.

(a) $Answer:$

$Max-likelihood\ estimate\ of\ bias:\ p = 1$

(b)

$p = \frac{k+1}{n+2} = \frac{20+1}{20+2} = \frac{21}{22}$

$Answer: \frac{21}{22}$

(c)

$pp(1-p)(1-p)pp$

$= (\frac{21}{22})^4 \times (\frac{1}{22})^2$

$= 0.0017$

$Answer: 0.0017$

5.

(a)

Answer :

$(3,3,2,0)$

(b)

Answer :

$p_1 = \frac{3}{8}$

$p_2 = \frac{3}{8}$

$p_3 = \frac{2}{8} = \frac{1}{4}$

$p_4 = \frac{0}{8} = 0$

(c)

$|V| = 4$

$Answer:$

$$p_1 = \frac{3+1}{8+4} = \frac{1}{3}$$

$$p_2 = \frac{3+1}{8+4} = \frac{1}{3}$$

$$p_3 = \frac{2+1}{8+4} = \frac{1}{4}$$

$$p_4 = \frac{0+1}{8+4} = \frac{1}{12}$$

8.

(*a*)

Answer :

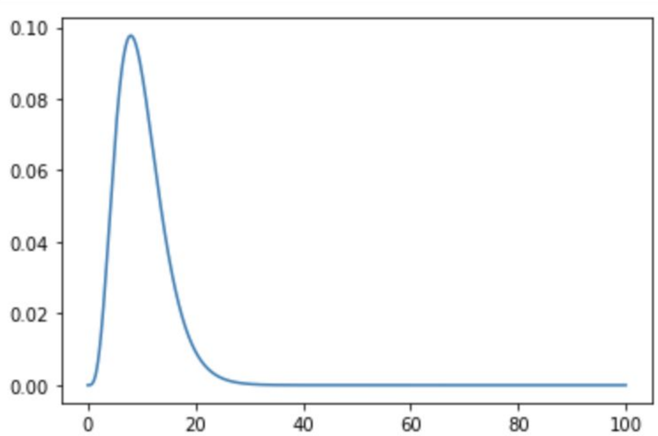
```
[1]: from scipy.stats import chi2
     import matplotlib.pyplot as plt
     import numpy as np

[2]: df = 10

[3]: x = np.linspace(0,100,1000)

[4]: density = chi2.pdf(x, df, loc=0, scale=1)

[5]: plt.plot(x, density)
     plt.show()
```




```
[6]: rv = chi2.rvs(df, size=1000000)

[7]: print(np.median(rv))
     9.348860396016848
```

(b)

$$median \approx k(1 - \tfrac{2}{9k})^3 = 10(1 - \tfrac{2}{90})^3 \approx 9.35$$

$Answer : 9.35$