

DSE_HW3_Day3
Worksheet 6

1

(a)

Answer :

positively correlated

(b)

Answer :

negatively correlated

(c)

Answer :

uncorrelated

2

(a)

$$P_r(X = 0) = \frac{1}{2}$$

$$P_r(X = 1) = \frac{1}{4}$$

$$P_r(X = -1) = \frac{1}{4}$$

$$P_r(Y = 0) = \frac{1}{2}$$

$$P_r(Y = 1) = \frac{1}{4}$$

$$P_r(Y = -1) = \frac{1}{4}$$

$$P_r(X = 0, Y = 0) = \frac{1}{2}$$

$$P_r(X = 0)P_r(Y = 0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Answer :

Dependent

(b)

$$\begin{aligned} E(X) &= (-1) \times \left(\frac{1}{8} + 0 + \frac{1}{8}\right) + 0 + 1 \times \left(\frac{1}{8} + 0 + \frac{1}{8}\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} E(Y) &= (-1) \times \left(\frac{1}{8} + 0 + \frac{1}{8}\right) + 0 + 1 \times \left(\frac{1}{8} + 0 + \frac{1}{8}\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} E(XY) &= (-1) \times \frac{1}{8} \times 2 + 1 \times \frac{1}{8} \times 2 \\ &= 0 \end{aligned}$$

Answer :

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\text{std}(X)\text{std}(Y)} = \frac{E(XY) - E(X)E(Y)}{\text{std}(X)\text{std}(Y)} = 0$$

3

$$Y = 2X$$

$$E(X) = 0$$

$$std(X) = 10$$

$$var(X) = 10^2 = E(X^2) - (E(X))^2$$

$$E(Y) = E(2X) = 0$$

$$std(Y) = std(2X) = 2std(X) = 20$$

(a)

Answer :

$$cov(X, Y) = E(XY) - E(X)E(Y)$$

$$= 2(E(X^2) - (E(X))^2)$$

$$= 2var(X) = 200$$

(b)

Answer :

$$corr(X, Y) = \frac{cov(X, Y)}{std(X)std(Y)} = \frac{200}{10 \times 20} = 1$$

4

(a)

$$E(x) = \mu_1 = 2$$

$$std(x) = 1$$

$$E(y) = \mu_2 = 4$$

$$std(y) = 0.5$$

$$corr(x, y) = -0.5 = \frac{E(xy) - E(x)E(y)}{std(x)std(y)} = \frac{cov(x, y)}{1 \times 0.5}$$

$$cov(x, y) = -0.25$$

Answer :

parameters

$$mean \mu = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$covariance \ matrix \ \Sigma = \begin{pmatrix} 1 & -0.25 \\ -0.25 & 0.25 \end{pmatrix}$$

(b)

$$x = y$$

$$E(x) = 1$$

$$std(x) = 1$$

$$cov(x, y) = E(xy) - E(x)E(y) = E(x^2) - (E(x))^2 = 1$$

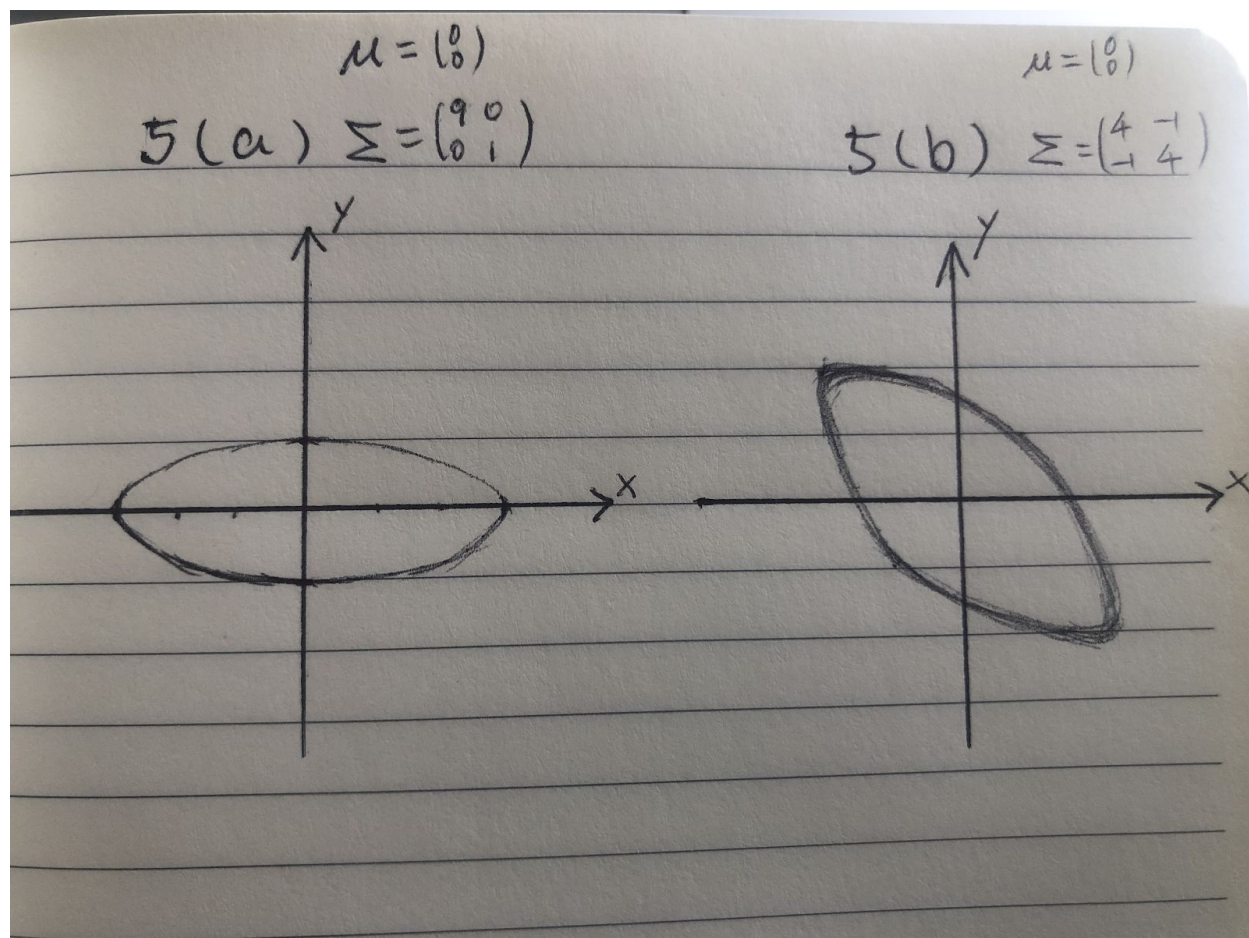
Answer :

parameters

$$mean \ \mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$covariance \ matrix \ \Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

5



6

```
[1]: import matplotlib.pyplot as plt
from scipy.stats import multivariate_normal
import numpy as np

mean1 = [0, 0]
cov1 = [[9, 0], [0, 1]]

mean2 = [0, 0]
cov2 = [[4, -1], [-1, 4]]

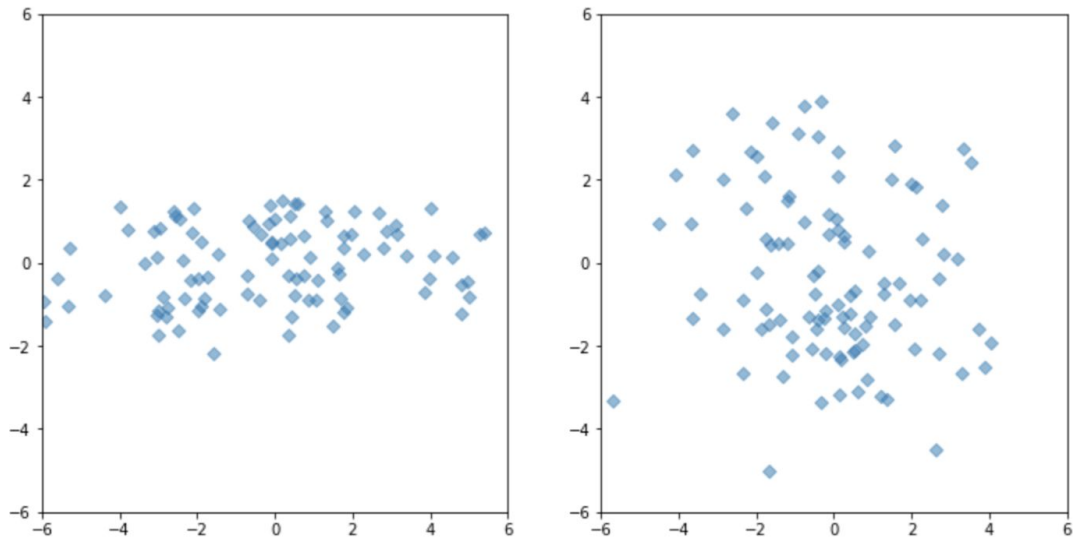
x1, y1 = np.random.multivariate_normal(mean1, cov1, 100).T
x2, y2 = np.random.multivariate_normal(mean2, cov2, 100).T

f, (ax1, ax2) = plt.subplots(1, 2, figsize = (12, 6))

ax1.scatter(x1, y1, marker = 'D', alpha = 0.5)
ax2.scatter(x2, y2, marker = 'D', alpha = 0.5)

ax1.set(xlim=(-6, 6), ylim=(-6, 6))
ax2.set(xlim=(-6, 6), ylim=(-6, 6))

plt.show()
```



Worksheet 7

1

$$\sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

Answer :

$$\frac{1}{\sqrt{14}}(1, 2, 3) = \frac{\sqrt{14}}{14}(1, 2, 3)$$

Or it can be written as

$$\frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

2

$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

Answer :

$$\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

3

Answer :

$$\{x \in R^d : \|x\| = 5\}$$

4

Answer :

$$\omega = (2, -1, 6)$$

Or it can be written as

$$\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

5

Answer :

$$A : 10 \times 30$$

$$B : 30 \times 20$$

6

(a)

Answer :

$$X : n \times d$$

(b)

Answer :

$$XX^T : n \times n$$

(c)

Answer :

$$(XX^T)_{ij} = \sum_{l=1}^d X_{il} X_{lj}^T = \sum_{l=1}^d X_{il} X_{jl} \quad \times$$

$$x_i x_j$$

7

Answer :

$$x^T x x^T x x^T x = \|x\|^6 = 10^6$$

8

If $x = (1, 3, 5)$ is a 3×1 vector.

Answer :

$$x^T x = \|x\|^2 = 1^2 + 3^2 + 5^2 = 35$$

$$xx^T = (1, 3, 5)^T (1, 3, 5) =$$

$$\begin{pmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{pmatrix}$$

If $x = (1, 3, 5)$ is a 1×3 vector.

Answer :

$$xx^T = \|x\|^2 = 1^2 + 3^2 + 5^2 = 35$$

$$x^T x = (1, 3, 5)^T (1, 3, 5) =$$

$$\begin{pmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{pmatrix}$$

9

$$\cos\theta = \frac{x \cdot y}{\|x\| \|y\|} = \frac{2}{2 \times 2} = \frac{1}{2}$$

Answer :

$$\theta = 60^\circ$$

10

Answer :

$$\begin{pmatrix} 3 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 6 \end{pmatrix}$$

11

Answer :

(a) (b) (c)

12

(a)

Answer :

$$|A| = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$$

(b)

Answer :

$$A^{-1} = \text{diag}(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8})$$

13

(a)

Answer :

$$UU^T = I$$

(b)

Answer :

$$U^{-1} = U^T$$

14

$$1 \times z - 2 \times 3 = 0$$

Answer :

$$z = 6$$