4. Mini-project: Coordinate descent

In [1]:

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import time
import pandas as pd
from pandas import read_csv
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import KFold
from sklearn.metrics import accuracy_score as acc
from sklearn.model_selection import LeaveOneOut
from sklearn.neighbors import KNeighborsClassifier
import matplotlib
from sklearn.preprocessing import MinMaxScaler
from sklearn.linear_model import LogisticRegression
from mlxtend.feature_selection import SequentialFeatureSelector as sfs
from sklearn.model_selection import cross_val_score
from sklearn.metrics import log_loss
import sys

if not sys.warnoptions:
    import warnings
    warnings.simplefilter("ignore")
```

In [2]:

```
dataframe = read_csv('heart.csv')
dataframe
```

Out[2]:

	age	sex	cp	trestbps	chol	fbs	restecg	thalach	exang	oldpeak	slope	ca	thal	target
0	63	1	3	145	233	1	0	150	0	2.3	0	0	1	1
1	37	1	2	130	250	0	1	187	0	3.5	0	0	2	1
2	41	0	1	130	204	0	0	172	0	1.4	2	0	2	1
3	56	1	1	120	236	0	1	178	0	0.8	2	0	2	1
4	57	0	0	120	354	0	1	163	1	0.6	2	0	2	1
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
298	57	0	0	140	241	0	1	123	1	0.2	1	0	3	0
299	45	1	3	110	264	0	1	132	0	1.2	1	0	3	0
300	68	1	0	144	193	1	1	141	0	3.4	1	2	3	0
301	57	1	0	130	131	0	1	115	1	1.2	1	1	3	0
302	57	0	1	130	236	0	0	174	0	0.0	1	1	2	0

303 rows × 14 columns

In [3]:

```
# Separate features from labels
data = dataframe.values
X, y = data[:, :-1], data[:, -1]
```

(a) A short, high-level description of your coordinate descent method.

In particular, you should give a concise description of how you solve problems (i) and (ii) above. Do you need the function L(·) to be differentiable, or does it work with any loss function?

(i) Which coordinate to choose?

Answer: We will choose the coordinate that minimize the wi by running all coordinates in cyclic order at each iteration.

(ii) How to set the new value of wi?

Answer: For setting the new value of wi, we will use the current value of wi subtract value of the stepsize times gradient.

Do you need the function L(·) to be differentiable, or does it work with any loss function?

Answer: Yes, the function L(·) needs to be differentiable and convex.

Summary of the coordinate descent method

Answer: For the loss function, we will minimize it by minimizing each of the individual dimensions of it in a cyclic fashion, while holding the values of it in the other dimensions fixed.

(b) Convergence.

Under what conditions do you think your method converges to the optimal loss? There's no need to prove anything: just give a few sentences of brief explanation.

Answer: After minimizing the cost function with respect to each coordinate, the cost function has bounded level sets and is in some sense strictly convex.

(c) Experimental results.

Begin by running a standard logistic regression solver (e.g., from scikit-learn) on the training set. It should not be regularized: if the solver forces you to do this, just set the regularization constant suitably to make it irrelevant. Make note of the final loss L∗.

In [4]:

```
# Build logistic regression classifier
clf = LogisticRegression(random_state = 42).fit(X, y)
proba = clf.predict_proba(X)
log_loss_value = log_loss(y, proba)
print('Final loss\n L* =', log_loss_value)
```

```
Final loss
 L* = 0.3534163611333882
```

In [5]:

```
# normalize data
X_normalize = (X - np.mean(X,axis = 0))/(np.max(X, axis = 0) - np.min(X, axis = 0))
intercept = np.ones((X.shape[0], 1))
intercept = intercept.reshape(X.shape[0], 1)
X_norm = np.hstack((X_normalize, intercept))
X_norm.shape
```

Out[5]:

```
(303, 14)
```

Then, implement your coordinate descent method and run it on this data.

In [6]:

```
# predict y
def predict_function(w, x):
    return 1/(1 + (np.exp(-(np.dot(w.T, x.T)))))

# calculate the loss
def loss_function(x, y, y_predict):
    return -(1/x.shape[0]) * np.sum(y * np.log(y_predict) + (1 - y) * np.log(1 - y_predict))

# calculate gradient
def gradient_function(x, y, y_predict):
    return list(np.dot((y_predict-y), x)[0])
```

In [7]:

```
# calculate the loss by using the coordinate descent method
def coordinate_descent(x, y, iteration, etha):
    w = np.zeros((x.shape[1],1))
    index = 0
    loss = []
    n = (x.shape[1] - 1)

    for i in range(iteration):
        y_predict = predict_function(w, x)
        gradient = gradient_function(x, y, y_predict)
        w[index%n] = w[index%n] - etha*gradient[index%n]
        index += 1
        loss.append(loss_function(x, y, y_predict))
    return loss
```

In [8]:

```
loss_coordinate_descent = coordinate_descent(X_norm, y, 1000, 0.01)
print('The minimum loss about the coordinate descent is:\n', min(loss_coordinate_descent))
```

```
The minimum loss about the coordinate descent is:
 0.35857565006240405
```

Finally, compare to a method that chooses coordinates i uniformly at random and then updates wi using your method (we'll call this "random-feature coordinate descent").

In [9]:

```
def random_coordinate_descent(x, y, number_iteration, etha):
    w = np.zeros((x.shape[1],1))
    loss = []
    n = (x.shape[1] - 1)

    for i in range(number_iteration):
        y_predict = predict_function(w, x)
        gradient = gradient_function(x, y, y_predict)
        current_index = np.random.randint(0,n)
        w[current_index] = w[current_index] - etha*gradient[current_index]
        loss.append(loss_function(x, y, y_predict))
    return loss
```

In [10]:

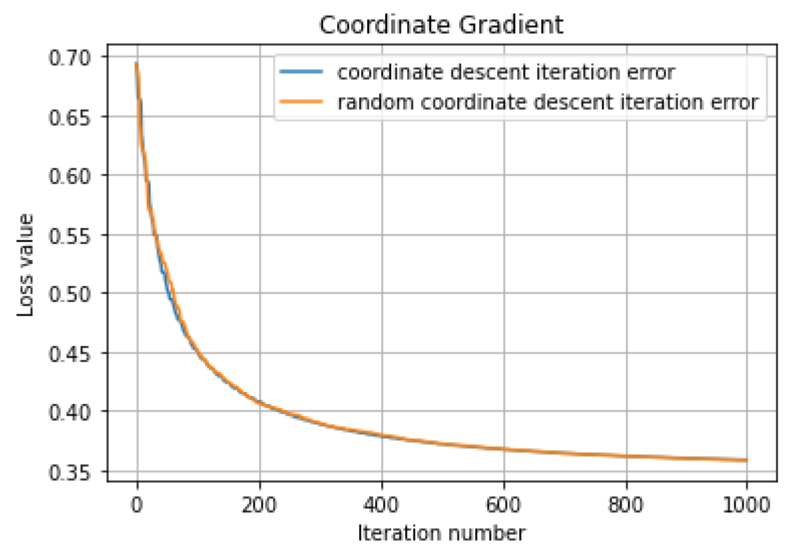
```
loss_random_coordinate_descent = random_coordinate_descent(X_norm, y, 1000, 0.01)
print('The minimum loss about the coordinate descent is:\n', min(loss_random_coordinate_descent))
```

```
The minimum loss about the coordinate descent is:
 0.35817122582262023
```

Produce a clearly-labeled graph that shows how the loss of your algorithm's current iterate— that is, L(wt)—decreases with t; it should asymptote to L∗. On the same graph, show the corresponding curve for random-feature coordinate descent.

In [11]:

```
iteration = np.arange(0,1000)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.title('Coordinate Gradient')
plt.plot(iteration, loss_coordinate_descent, label = 'coordinate descent iteration error')
plt.plot(iteration, loss_random_coordinate_descent, label = 'random coordinate descent iteration error')
plt.legend()
plt.grid()
plt.show()
```


Coordinate Gradient
coordinate descent iteration error
random coordinate descent iteration error
Loss value
Iteration number
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0
200
400
600
800
1000