Worksheet 5

1

(a)

$mean = (1 + 3 + 4 + 6)/4 = 3.5$

$MSE = ((1 - 3.5)^2 + (3 - 3.5)^2 + (4 - 3.5)^2 + (6 - 3.5)^2))/4 = 3.25$

(b)

$MSE = ((1 - 1)^2 + (3 - 1)^2 + (4 - 4)^2 + (6 - 4)^2)/4 = 2$

(c)

$y = x + 1$

$MSE = ((2 - 1)^2 + (3 - 2)^2 + (5 - 4)^2 + (6 - 5)^2)/4 = 1$

2

(a)

$b = 0$

$$MSE(a, b) = \frac{1}{n}\sum_{i=1}^{n} (y^{(i)} - (ax^{(i)} + b))^2 = \frac{1}{n}\sum_{i=1}^{n} (y^{(i)} - ax^{(i)})^2$$

$$L(a, b) = \frac{1}{n}\sum_{i=1}^{n} (y^{(i)} - ax^{(i)})^2$$

(b)

$$\frac{dL}{da} = \frac{2}{n}\sum_{i=1}^{n} (y^{(i)} - ax^{(i)})(- x^{(i)}) = 0$$

$$a = \frac{\sum_{i=1}^{n} y^{(i)}x^{(i)}}{\sum_{i=1}^{n} (x^{(i)})^2}$$

3

(a)

$$\frac{dL}{ds} = \frac{2}{n}\sum_{i=1}^{n} (x_i - s)(- 1) = \frac{-2}{n}\sum_{i=1}^{n} (x_i - s)$$

(b)

$$\frac{dL}{ds} = \frac{2}{n}\sum_{i=1}^{n} (x_i - s)(- 1) = 0$$

$$s = \frac{1}{n}\sum_{i=1}^{n} x_i = \overline{x}$$

4

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right|$

$\frac{dL}{ds} = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| \times 1 = 0$

$\sum_{i=1}^{n}\left|x_i - s\right| = 0$

$s\ should\ be\ the\ median\ of\ the\ numbers\ x_1, ..., x_n$

(a)

$mean = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 90)/9 = 14$

(b)

$s = 14$

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| = 16.89$

(c)

$s = 5$

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| = 11.22$

(d)

5

5

$L = \sum_{i=1}^{n}|y^{(i)} - \widehat{y}^{(i)}| = \frac{1}{n}\sum_{i=1}^{n}|y^{(i)} - \tilde{\omega} \cdot \tilde{x}^{(i)}| = ||y - X\tilde{\omega}||$

$and\ it\ is\ minimized\ at\ \omega = (X^TX)^{-1}(X^Ty).$

6

(a)

$\frac{1}{n}(1^T y)$

(b)

XX^T

(c)

$\frac{1}{n}(1^T X)$

(d)

$\frac{1}{n}(X^T X)$