Worksheet 5

1

(a)

$mean = (1 + 3 + 4 + 6)/4 = 3.5$

$MSE = ((1 - 3.5)^2 + (3 - 3.5)^2 + (4 - 3.5)^2 + (6 - 3.5)^2))/4 = 3.25$

(b)

$MSE = ((1 - 1)^2 + (3 - 1)^2 + (4 - 4)^2 + (6 - 4)^2)/4 = 2$

(c)

$y = x + 1$

$MSE = ((2 - 1)^2 + (3 - 2)^2 + (5 - 4)^2 + (6 - 5)^2)/4 = 1$

2

(a)

$b = 0$

$$MSE(a, b) = \frac{1}{n}\sum_{i=1}^{n}(y^{(i)} - (ax^{(i)} + b))^2 = \frac{1}{n}\sum_{i=1}^{n}(y^{(i)} - ax^{(i)})^2$$

$$L(a, b) = \frac{1}{n}\sum_{i=1}^{n}(y^{(i)} - ax^{(i)})^2$$

(b)

$$\frac{dL}{da} = \frac{2}{n}\sum_{i=1}^{n}(y^{(i)} - ax^{(i)})(-x^{(i)}) = 0$$

$$a = \frac{\sum_{i=1}^{n} y^{(i)}x^{(i)}}{\sum_{i=1}^{n}(x^{(i)})^2}$$

3

(a)

$$\frac{dL}{ds} = \frac{2}{n}\sum_{i=1}^{n}(x_i - s)(-1) = \frac{-2}{n}\sum_{i=1}^{n}(x_i - s)$$

(b)

$$\frac{dL}{ds} = \frac{2}{n}\sum_{i=1}^{n}(x_i - s)(-1) = 0$$

$$s = \frac{1}{n}\sum_{i=1}^{n}x_i = \overline{x}$$

4

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right|$

$\frac{dL}{ds} = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| \times 1 = 0$

$\sum_{i=1}^{n}\left|x_i - s\right| = 0$

s should be the median of the numbers $x_1, \ldots, x_n$

(*a*)

$mean = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 90)/9 = 14$

(*b*)

$s = 14$

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| = 16.89$

(*c*)

$s = 5$

$L(s) = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - s\right| = 11.22$

(*d*)

5

5

$L = \sum_{i=1}^{n} |y^{(i)} - \hat{y}^{(i)}| = \frac{1}{n}\sum_{i=1}^{n} |y^{(i)} - \tilde{\omega} \cdot \tilde{x}^{(i)}| = ||y - X\tilde{\omega}||$

and it is minimized at $\omega = (X^T X)^{-1}(X^T y)$.

6

(a)

$\frac{1}{n}(1^T y)$

(b)

XX^T

(c)

$\frac{1}{n}(1^T X)$

(d)

$\frac{1}{n}(X^T X)$

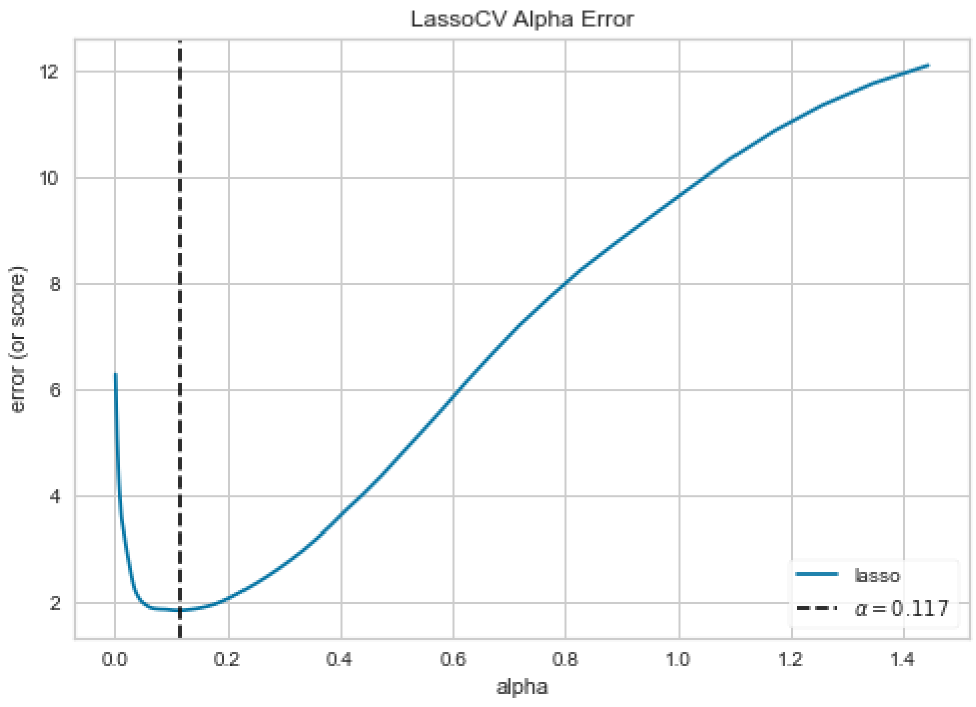
In [1]:
```
from pandas import read_csv
import numpy as np
from sklearn.linear_model import Lasso
from sklearn.linear_model import LassoCV
from sklearn.model_selection import train_test_split
from yellowbrick.regressor.alphas import alphas
```

In [2]:
```
# Load dataset
dataframe = read_csv('mystery.dat', header = None, sep = ',')

print('Shape of the mystery data:\n', dataframe.shape)
print('\nHead of the mystery data:\n', dataframe.head())
```

```
Shape of the mystery data:
 (101, 101)

Head of the mystery data:
         0        1        2        3        4        5        6        7    \
0  0.63311 -1.71313 -0.48056 -0.32540 -0.05102  0.05634 -1.63462 -0.58081
1  0.82710 -0.45099  0.62209 -0.24694  0.53069  0.84492  0.37463 -0.61650
2 -0.25135 -0.22821 -0.65147  0.52365 -0.58971  0.02787  0.27812  0.21289
3  0.46192  0.16546  2.87388 -0.65411  0.76601  1.54346 -1.08101 -1.00728
4 -1.50107  2.05339  0.03820  0.27116 -0.07920 -0.53648  0.32249 -0.57844

       8        9    ...      91       92       93       94       95   \
0  0.70627 -2.06938  ...  0.69346  0.49371 -0.15578  1.02650  0.48640
1 -0.00887  0.51328  ...  1.12702  0.53821  1.69800  0.65812  0.18004
2  1.08754 -0.28801  ... -0.08484  0.00259  1.98580  0.39629 -1.37305
3 -0.01883  0.41995  ...  0.72389  1.27442 -0.69487  0.47128  0.54426
4 -0.66211 -0.73749  ...  0.07916 -0.34523  1.09813  1.78102 -1.06170

       96       97       98       99       100
0  0.32758 -2.28887 -0.00430 -0.39673 -6.07560
1  0.31097  0.42096  0.43610 -0.09575  4.03525
2  1.66343 -1.25645 -0.41212  0.78800 -3.57768
3  1.18577  0.00389  0.90909  1.44143 -1.01789
4 -1.74101  1.96249 -0.86213 -1.88139  4.54025

[5 rows x 101 columns]
```

In [3]:
```
# Separate features from labels
data = dataframe.values
X, y = data[:, :-1], data[:, -1]

# Divide into training and test set
x_train,x_test,y_train,y_test = train_test_split(X,y, random_state = 3, test_size = 0.2)
```

In [4]:
```
# Find the best alpha
alphas(LassoCV(random_state = 0), X, y)
```



Out[4]:
```
AlphaSelection(ax=<AxesSubplot:title={'center':'LassoCV Alpha Error'}, xlabel='alpha', ylabel='error (or sco
re)'>,
               estimator=LassoCV(random_state=0))
```

In [5]:
```
# Evaluate an lasso regression model on the dataset
# When I apply the best alpha value 0.117 above, it only select 7 features.
# So here, I modified the alpha value to fit the requirement of this question for 10 features.
lasso = Lasso(alpha = 0.07, normalize = True)
lasso.fit(x_train,y_train)
predict = lasso.predict(x_test)

print('Lasso score:\n',lasso.score(x_test,y_test))
print('\nLasso coefficients:\n',lasso.coef_)
```

```
Lasso score:
 0.43029417215164845

Lasso coefficients:
 [-0.          0.62526249  0.44797675 -0.          0.69402542  0.
  0.77809284  0.         -0.          0.          0.35679046  0.
  0.01922474 -0.         -0.          0.          0.02210121  0.
  0.3265981  -0.          0.          0.          0.73002793 -0.
  0.         -0.          0.3514545   0.          0.          0.
  0.         -0.         -0.         -0.          0.          0.
 -0.         -0.         -0.         -0.         -0.         -0.
  0.         -0.         -0.         -0.          0.          0.
  0.         -0.         -0.          0.          0.         -0.
 -0.          0.          0.          0.          0.         -0.
  0.          0.         -0.         -0.         -0.         -0.
 -0.          0.          0.          0.         -0.          0.
 -0.          0.         -0.          0.         -0.         -0.
 -0.         -0.          0.          0.          0.         -0.
  0.          0.          0.          0.         -0.          0.
 -0.         -0.          0.          0.          0.          0.
 -0.          0.         -0.          0.        ]
```

(a) Explain your strategy in one or two sentences. Hint: you will find it helpful to look over the routines in sklearn.linear model.

For this question, I will use Lasso regression model to select features, which is a regression method that involves penalizing the absolute size of the regression coefficients. By penalizing, we end up in a situation where some of the parameter estimates may be exactly zero, so the larger the penalty applied, the further estimates are shrunk towards zero.

(b) Which ten features did you identify? You need only give their coordinate numbers, from 1 to 100.

In [6]:
```
# Find the coordinate numbers
num = np.where(lasso.coef_ != 0)

# Since the coordinate numbers are from 1 to 100, not the index 0 to 99,
# so I add one to the num[0].
print('Coordinate numbers for ten features:\n', num[0] + 1)
```

```
Coordinate numbers for ten features:
 [ 2  3  5  7 11 13 17 19 23 27]
```

In [ ]:

Loading [MathJax]/jax/output/CommonHTML/fonts/TeX/fontdata.js