Worksheet 4 — Solutions

1. $(1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$

2. $(-1/\sqrt{2}, 1/\sqrt{2})$ and $(1/\sqrt{2}, -1/\sqrt{2})$

3. $x \cdot x = 25 \Leftrightarrow \|x\| = 5$. All points of length 5: a sphere, centered at the origin, of radius 5.

4. $f(x) = 2x_1 - x_2 + 6x_3 = w \cdot x$ for $w = (2, -1, 6)$.

5. A is 10×30 and B is 30×20

6. (a) X is $n \times d$
   (b) XX^T is $n \times n$
   (c) $(XX^T)_{ij} = x^{(i)} \cdot x^{(j)}$

7. $((x^T x)(x^T x)(x^T x)) = (\|x\|^2)^3 = 10^6$

8. $x^T x = \|x\|^2 = 35$ and
$$x^T x = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{bmatrix}$$

9. The angle θ between x and y satisfies $\cos\theta = x^T y / \|x\|\|y\| = 1/2$, so θ is 60 degrees.

10.
$$M = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 6 \end{bmatrix}$$

11. *Symmetric Matrices*

    (a) $(AA^T)^T = (A^T)^T A^T = AA^T$, Thus AA^T is symmetric.
    (b) $(A^T A)^T = A^T (A^T)^T = A^T A$, Thus $A^T A$ is symmetric.
    (c) $(A + A^T)^T = (A^T + A) = (A + A^T)$, Thus $(A + A^T)$ is symmetric
    (d) $(A - A^T)^T = (A^T - A) \neq (A - A^T)$, Thus $(A - A^T)$ need not be symmetric

12. (a) $|A| = 8! = 40320$
    (b) $A^{-1} = \mathrm{diag}(1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8)$

13. *Orthonormal matrices*

    (a) UU^T is the identity matrix
    (b) $U^{-1} = U^T$

14. Since A is singular matrix, $|A| = 0 \implies z - 6 = 0 \implies z = 6$