Worksheet 6

1

(a)

no

(b)

yes, significant amount of inherent uncertainty.

(c)

no

(d)

yes, significant amount of inherent uncertainty.

2

(a)

$P_r(y = 1|x) = \frac{1}{1+e^{-(w\cdot x+b)}} = 1/2$

$e^{-(w\cdot x+b)} = 1$

$w \cdot x + b = 0$

$x = \frac{-b}{w}$

(b)

$P_r(y = 1|x) = \frac{1}{1+e^{-(w\cdot x+b)}} = 3/4$

$e^{-(w\cdot x+b)} = \frac{1}{3}$

$w \cdot x + b =- ln\frac{1}{3} = ln3 - ln1$

$x = (ln3 - ln1 - b)/w$

(c)

$P_r(y = 1|x) = \frac{1}{1+e^{-(w\cdot x+b)}} = 1/4$

$e^{-(w\cdot x+b)} = 3$

$- (w \cdot x + b) = ln3$

$x =- (ln3 + b)/w$

Worksheet 7

3

(a)

$\frac{dL(w)}{dw_1} = 2w_1 + 2$

$\frac{dL(w)}{dw_2} = 4w_2 - 4$

$\frac{dL(w)}{dw_3} = 2w_3 - 2w_4$

$\frac{dL(w)}{dw_4} =- 2w_3 + 2w_4$

$$\nabla L(w) = \begin{pmatrix} 2w_1 + 2 \\ 4w_2 - 4 \\ 2w_3 - 2w_4 \\ 2w_4 - 2w_3 \end{pmatrix}$$

(b)

$w = (0, 0, 0, 0)$

$\frac{dL(w)}{dw_1} = 2w_1 + 2 = 2$

$\frac{dL(w)}{dw_2} = 4w_2 - 4 =- 4$

$\frac{dL(w)}{dw_3} = 2w_3 - 2w_4 = 0$

$\frac{dL(w)}{dw_4} =- 2w_3 + 2w_4 = 0$

$w_{t+1} = w_t - \eta \nabla L(w_t) = (0, 0, 0, 0) - \eta(2, - 4, 0, 0) = \eta(- 2, 4, 0, 0)$

(c)

$\frac{dL(w)}{dw_1} = 2w_1 + 2 = 0$

$\frac{dL(w)}{dw_2} = 4w_2 - 4 = 0$

$\frac{dL(w)}{dw_3} = 2w_3 - 2w_4 = 0$

$\frac{dL(w)}{dw_4} =- 2w_3 + 2w_4 = 0$

$w = (- 1, 1, w_3, w_4),\ w_3 = w_4$

$L(w) = 1$

(d)

$No,\ there\ is\ no\ unique\ solution\ w\ at\ which\ this\ minimum\ is\ realized.$

4

(a)

$$\nabla L(w) = -2\sum_{i=1}^{n}(y^{(i)} - w \cdot x^{(i)})x^{(i)} + 2\lambda w$$

(b)

$$w_{t+1} = w_t - \eta_t \sum_{i=1}^{n} \nabla L(w_t; x^{(i)}, y^{(i)})$$

$$= w_t - \eta_t(2\sum_{i=1}^{n}(y^{(i)} - w_t \cdot x^{(i)})(-x^{(i)}) + 2\lambda w_t)$$

(c)

$$w_{t+1} = w_t - \eta_t \nabla L(w_t; x, y)$$

$$= w_t - \eta_t(2(y^{(i)} - w_t \cdot x^{(i)})(-x^{(i)}) + 2\lambda w_t)$$