Nearest neighbor for handwritten digit recognition

In this notebook we will build a classifier that takes an image of a handwritten digit and outputs a label 0-9. We will look at a particularly simple strategy for this problem known as the **nearest neighbor classifier**.

To run this notebook you should have the following Python packages installed:

- `numpy`
- `matplotlib`
- `sklearn`

1. The MNIST dataset

`MNIST` is a classic dataset in machine learning, consisting of 28x28 gray-scale images handwritten digits. The original training set contains 60,000 examples and the test set contains 10,000 examples. In this notebook we will be working with a subset of this data: a training set of 7,500 examples and a test set of 1,000 examples.

In [1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import time

## Load the training set
train_data = np.load('MNIST/train_data.npy')
train_labels = np.load('MNIST/train_labels.npy')

## Load the testing set
test_data = np.load('MNIST/test_data.npy')
test_labels = np.load('MNIST/test_labels.npy')
```

In [2]:
```
## Print out their dimensions
print("Training dataset dimensions: ", np.shape(train_data))
print("Number of training labels: ", len(train_labels))
print("Testing dataset dimensions: ", np.shape(test_data))
print("Number of testing labels: ", len(test_labels))
```

```
Training dataset dimensions:  (7500, 784)
Number of training labels:  7500
Testing dataset dimensions:  (1000, 784)
Number of testing labels:  1000
```

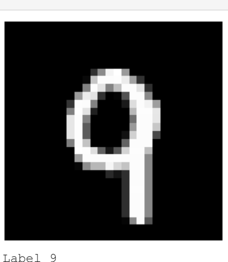
In [3]:
```
## Compute the number of examples of each digit
train_digits, train_counts = np.unique(train_labels, return_counts=True)
print("Training set distribution:")
print(dict(zip(train_digits, train_counts)))

test_digits, test_counts = np.unique(test_labels, return_counts=True)
print("Test set distribution:")
print(dict(zip(test_digits, test_counts)))
```

```
Training set distribution:
{0: 750, 1: 750, 2: 750, 3: 750, 4: 750, 5: 750, 6: 750, 7: 750, 8: 750, 9: 750}
Test set distribution:
{0: 100, 1: 100, 2: 100, 3: 100, 4: 100, 5: 100, 6: 100, 7: 100, 8: 100, 9: 100}
```

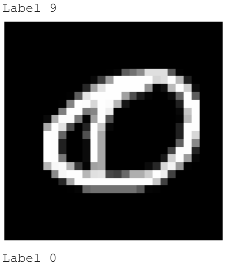
2. Visualizing the data

Each data point is stored as 784-dimensional vector. To visualize a data point, we first reshape it to a 28x28 image.

In [4]:
```
## Define a function that displays a digit given its vector representation
def show_digit(x):
    plt.axis('off')
    plt.imshow(x.reshape((28,28)), cmap=plt.cm.gray)
    plt.show()
    return

## Define a function that takes an index into a particular data set ("train" or "test") and displays that im
def vis_image(index, dataset="train"):
    if(dataset=="train"):
        show_digit(train_data[index,])
        label = train_labels[index]
    else:
        show_digit(test_data[index,])
        label = test_labels[index]
    print("Label " + str(label))
    return

## View the first data point in the training set
vis_image(0, "train")

## Now view the first data point in the test set
vis_image(0, "test")
```



```
Label 9
```



```
Label 0
```

3. Squared Euclidean distance

To compute nearest neighbors in our data set, we need to first be able to compute distances between data points. A natural distance function is *Euclidean distance*: for two vectors $x, y \in \mathbb{R}^d$, their Euclidean distance is defined as

$$\|x - y\| = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}.$$

Often we omit the square root, and simply compute *squared Euclidean distance*:

$$\|x - y\|^2 = \sum_{i=1}^{d} (x_i - y_i)^2.$$

For the purposes of nearest neighbor computations, the two are equivalent: for three vectors $x, y, z \in \mathbb{R}^d$, we have $\|x - y\| \leq \|x - z\|$ if and only if $\|x - y\|^2 \leq \|x - z\|^2$.

Now we just need to be able to compute squared Euclidean distance. The following function does so.

In [5]:
```
## Computes squared Euclidean distance between two vectors.
def squared_dist(x,y):
    return np.sum(np.square(x-y))

## Compute distance between a seven and a one in our training set.
print("Distance from 7 to 1: ", squared_dist(train_data[4,],train_data[5,]))

## Compute distance between a seven and a two in our training set.
print("Distance from 7 to 2: ", squared_dist(train_data[4,],train_data[1,]))

## Compute distance between two seven's in our training set.
print("Distance from 7 to 7: ", squared_dist(train_data[4,],train_data[7,]))
```

```
Distance from 7 to 1:  5357193.0
Distance from 7 to 2:  12451684.0
Distance from 7 to 7:  5223403.0
```

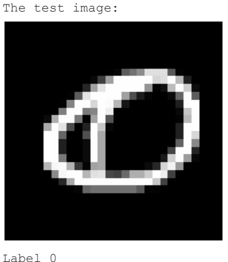
4. Computing nearest neighbors

Now that we have a distance function defined, we can now turn to nearest neighbor classification.

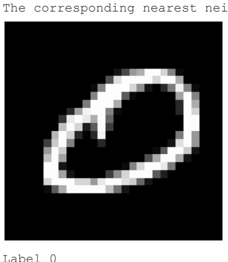
In [6]:
```
## Takes a vector x and returns the index of its nearest neighbor in train_data
def find_NN(x):
    # Compute distances from x to every row in train_data
    distances = [squared_dist(x,train_data[i,]) for i in range(len(train_labels))]
    # Get the index of the smallest distance
    return np.argmin(distances)

## Takes a vector x and returns the class of its nearest neighbor in train_data
def NN_classifier(x):
    # Get the index of the the nearest neighbor
    index = find_NN(x)
    # Return its class
    return train_labels[index]
```

In [7]:
```
## A success case:
print("A success case:")
print("NN classification: ", NN_classifier(test_data[0,]))
print("True label: ", test_labels[0])
print("The test image:")
vis_image(0, "test")
print("The corresponding nearest neighbor image:")
vis_image(find_NN(test_data[0,]), "train")
```

```
A success case:
NN classification:  0
True label:  0
The test image:
```



```
Label 0
The corresponding nearest neighbor image:
```



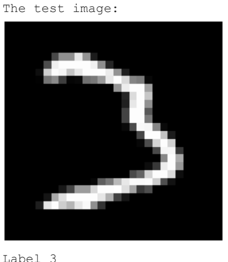
```
Label 0
```

In [8]:
```
test_data.shape
```

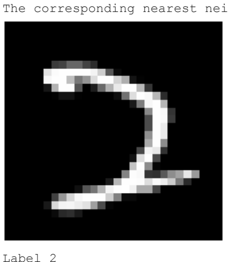
Out[8]:
```
(1000, 784)
```

In [9]:
```
## A failure case:
print("A failure case:")
print("NN classification: ", NN_classifier(test_data[39,]))
print("True label: ", test_labels[39])
print("The test image:")
vis_image(39, "test")
print("The corresponding nearest neighbor image:")
vis_image(find_NN(test_data[39,]), "train")
```

```
A failure case:
NN classification:  2
True label:  3
The test image:
```



```
Label 3
The corresponding nearest neighbor image:
```



```
Label 2
```

5. Processing the full test set

Now let's apply our nearest neighbor classifier over the full data set.

Note that to classify each test point, our code takes a full pass over each of the 7500 training examples. Thus we should not expect testing to be very fast. The following code takes about 100-150 seconds on 2.6 GHz Intel Core i5.

In [10]:
```
## Predict on each test data point (and time it!)
t_before = time.time()
test_predictions = [NN_classifier(test_data[i,]) for i in range(len(test_labels))]
t_after = time.time()

## Compute the error
err_positions = np.not_equal(test_predictions, test_labels)
error = float(np.sum(err_positions))/len(test_labels)

print("Error of nearest neighbor classifier: ", error)
print("Classification time (seconds): ", t_after - t_before)
```

```
Error of nearest neighbor classifier:  0.046
Classification time (seconds):  49.61181592941284
```

6. Faster nearest neighbor methods

Performing nearest neighbor classification in the way we have presented requires a full pass through the training set in order to classify a single point. If there are N training points in $\mathbb{R}^d$, this takes $O(Nd)$ time.

Fortunately, there are faster methods to perform nearest neighbor look up if we are willing to spend some time preprocessing the training set. `scikit-learn` has fast implementations of two useful nearest neighbor data structures: the *ball tree* and the *k-d tree*.

In [11]:
```
from sklearn.neighbors import BallTree

## Build nearest neighbor structure on training data
t_before = time.time()
ball_tree = BallTree(train_data)
t_after = time.time()

## Compute training time
t_training = t_after - t_before
print("Time to build data structure (seconds): ", t_training)

## Get nearest neighbor predictions on testing data
t_before = time.time()
test_neighbors = np.squeeze(ball_tree.query(test_data, k=1, return_distance=False))
ball_tree_predictions = train_labels[test_neighbors]
t_after = time.time()

## Compute testing time
t_testing = t_after - t_before
print("Time to classify test set (seconds): ", t_testing)

## Verify that the predictions are the same
print("Ball tree produces same predictions as above? ", np.array_equal(test_predictions, ball_tree_predictio
```

```
Time to build data structure (seconds):  0.40031003952026367
Time to classify test set (seconds):  7.583951950073242
Ball tree produces same predictions as above?  True
```

In [12]:
```
from sklearn.neighbors import KDTree

## Build nearest neighbor structure on training data
t_before = time.time()
kd_tree = KDTree(train_data)
t_after = time.time()

## Compute training time
t_training = t_after - t_before
print("Time to build data structure (seconds): ", t_training)

## Get nearest neighbor predictions on testing data
t_before = time.time()
test_neighbors = np.squeeze(kd_tree.query(test_data, k=1, return_distance=False))
kd_tree_predictions = train_labels[test_neighbors]
t_after = time.time()

## Compute testing time
t_testing = t_after - t_before
print("Time to classify test set (seconds): ", t_testing)

## Verify that the predictions are the same
print("KD tree produces same predictions as above? ", np.array_equal(test_predictions, kd_tree_predictions))
```

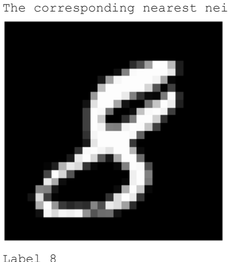
```
Time to build data structure (seconds):  0.3857090473175049
Time to classify test set (seconds):  8.6266348361969
KD tree produces same predictions as above?  True
```

Worksheet 1 Question 7

(a) For test point 100, print its image as well as the image of its nearest neighbor in the training set. Put these images in your writeup. Is this test point classified correctly?

In [13]:
```
## Now view the 100th data point in the test set
vis_image(99, "test")
print("The corresponding nearest neighbor image:")
vis_image(find_NN(test_data[99,]), "train")
```

```
Label 8
The corresponding nearest neighbor image:
```



```
Label 8
```

Conclusion: Yes, this test point classified correctly.

(b) The confusion matrix for the classifier is a 10 × 10 matrix Nij with 0 ≤ i, j ≤ 9, where Nij is the number of test points whose true label is i but which are classified as j. Thus, if all test points are correctly classified, the off-diagonal entries of the matrix will be zero.

• Compute the matrix N for the 1-NN classifier and print it out.

• Which digit is misclassified most often? Least often?

In [14]:
```
from sklearn.metrics import confusion_matrix
cm = confusion_matrix(test_predictions, test_labels)
cm
print('Confusion matrix of the classifier:\n', cm)
```

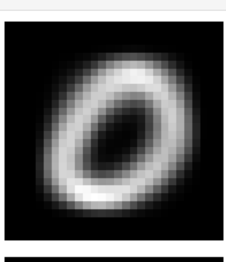
```
Confusion matrix of the classifier:
 [[ 99   0   0   0   0   1   0   0   2   1]
 [  0 100   1   0   0   0   0   4   0   1]
 [  0   0  94   2   0   0   0   0   1   1]
 [  0   0   1  91   0   0   0   0   1   1]
 [  0   0   0   2  97   0   0   1   1   2]
 [  1   0   0   4   0  98   1   0   0   1]
 [  0   0   0   0   0   0  99   0   1   0]
 [  0   0   3   0   0   0   0  94   1   3]
 [  0   0   1   1   0   0   0   0  92   0]
 [  0   0   0   0   3   1   0   1   1  90]]
```

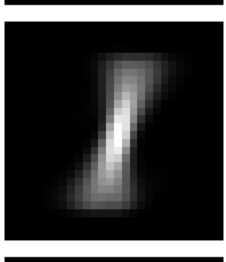
In [15]:
```
x = cm[0][0]
for i in range(0, 10):
    for j in range (0, 10):
        if i == j:
            if cm[i][j] <= x:
                x = cm[i][j]
        most_often = np.where(cm == x)
most_often
print('Digits that misclassified most often:\n', most_often[0])

least_often = np.where(cm == cm.max())
least_often[0]
print('\nDigits that misclassified least often:\n', least_often[0])
```

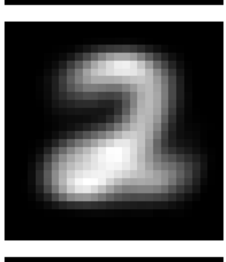
```
Digits that misclassified most often:
 [9]

Digits that misclassified least often:
 [1]
```

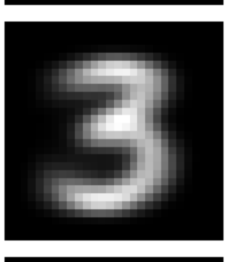
(c) For each digit 0 ≤ i ≤ 9: look at all training instances of image i, and compute their mean. This average is a 784-dimensional vector. Use the show digit routine to print out these 10 average-digits.

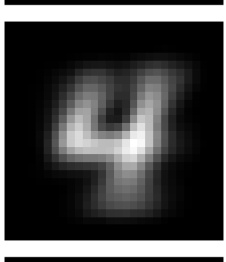
In [16]:
```
zero = np.mean(train_data[train_labels == 0,], axis = 0)
one = np.mean(train_data[train_labels == 1,], axis = 0)
two = np.mean(train_data[train_labels == 2,], axis = 0)
three = np.mean(train_data[train_labels == 3,], axis = 0)
four = np.mean(train_data[train_labels == 4,], axis = 0)
five = np.mean(train_data[train_labels == 5,], axis = 0)
six = np.mean(train_data[train_labels == 6,], axis = 0)
seven = np.mean(train_data[train_labels == 7,], axis = 0)
eight = np.mean(train_data[train_labels == 8,], axis = 0)
nine = np.mean(train_data[train_labels == 9,], axis = 0)

show_digit(zero)
show_digit(one)
show_digit(two)
show_digit(three)
show_digit(four)
show_digit(five)
show_digit(six)
show_digit(seven)
show_digit(eight)
show_digit(nine)
```

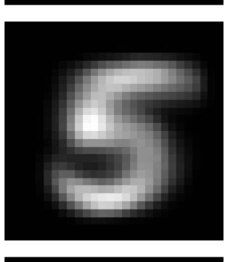


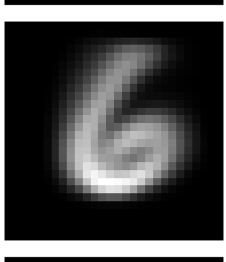


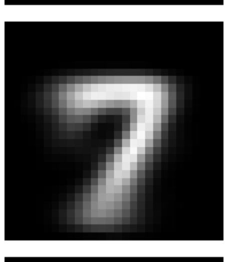


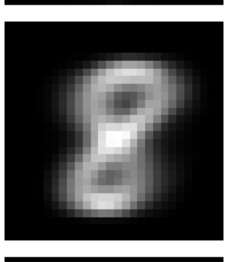


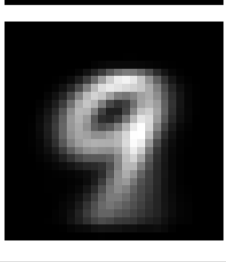












In [ ]: