DSE 220: Machine learning

Worksheet 2 — Useful distance functions

1. Consider the two points $x = (-1, 1, -1, 1)$ and $x' = (1, 1, 1, 1)$.
   (a) What is the L_2 distance between them?
   (b) What is the L_1 distance between them?
   (c) What is the L_∞ distance between them?
2. For the point $x = (1, 2, 3, 4)$ in $\mathbb{R}^4$, compute the following.
   (a) $\|x\|_1$
   (b) $\|x\|_2$
   (c) $\|x\|_\infty$
3. For each of the following norms, consider the set of points with length ≤ 1. In each case, state whether this set is shaped like a *ball*, a *diamond*, or a *box*.
   (a) ℓ_2
   (b) ℓ_1
   (c) ℓ_∞
4. List all points in $\mathbb{R}^2$ with $\|x\|_1 = \|x\|_2 = 1$.
5. Which of these distance functions is a *metric*? If it is not a metric, state which of the four metric properties it violates.
   (a) Let $\mathcal{X} = \mathbb{R}$ and define $d(x, y) = x - y$.
   (b) Let Σ be a finite set and $\mathcal{X} = \Sigma^m$. The *Hamming distance* on $\mathcal{X}$ is $d(x, y) = \#$ of positions on which x and y differ.
   (c) Squared Euclidean distance on $\mathbb{R}^m$, that is, $d(x, y) = \sum_{i=1}^m (x_i - y_i)^2$. (It might be easiest to consider the case $m = 1$.)
6. Suppose d_1 and d_2 are two metrics on a space $\mathcal{X}$. Define d to be their sum: $d(x, y) = d_1(x, y) + d_2(x, y)$. Is d necessarily a metric? Either show that it is or give a counterexample.