Exploring different prior choices for the expectation-maximization algorithm

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Abstract

The expectation-maximization (EM) algorithm is one of the most effective ways to solve for the maximum a posteriori (MAP) estimate of a mixture of Gaussians (MOG). In this project we explore how prior choice affects the performance of this algorithm, both in terms of computational speed and accuracy. Using real-world datasets, we find that the MOGs are solved most efficiently by choosing priors from visual inspection. Choosing priors based on statistics of bins on the data is a close second, and is the superior method which does not require human input. We also find that numerous safeguards need to be added to the EM algorithm to ensure convergence on the correct MAP estimate. Without these safeguards, the algorithm might converge on sub-optimal local maxima, and this is more likely the further away the initial priors are from the true data-generating MOG distributions.

**Index Terms**: mixture of Gaussians, expectation maximization,

# Introduction

The MOG is a useful model in many industry and research settings. It has been used to assist with spam detection by clustering heterogeneous documents to make the feature space easier to understand. It has also been used to classify emotions in speech audio, thus paving the way for more responsive human-computer interactions.

The MOG model is often solved with the EM algorithm. This is a numerical procedure which begins with a set of possible distributions, i.e. priors, and adjusts their parameters iteratively to yield their MAP estimates. In this project, we explore how the choice of initial priors affects the computational efficiency (in terms of number of iterations necessary for convergence at a predetermined tolerance level) and accuracy (in terms of returning the correct distributions when they are known, or returning distributions with the highest log-likelihood when they are unknown) of the EM algorithm. We experiment with three different Normal priors: random priors which do not depend on the data, priors which depend purely on statistics of the data, and priors which are chosen by human inspection. We tested these methods on univariate synthetic and real-world datasets to compare their performances. We attempted to use our own EM algorithm, but we switched to a pre-existing implementation after seeing it fail to converge for certain priors. Our results are relevant to anyone interested in solving MOGs more efficiently and accurately, or interested in gaining a deeper understanding of the EM algorithm.

# Methods

We carried out this project entirely in R and RStudio. Our scripts and experiment log are available at this Github repo: [github.com/damondpham/EM](https://github.com/damondpham/EM)

## Priors

We tried three different methods of choosing the priors:

Table 1: *Summary of Prior Choice Methods*

|  |  |  |  |
| --- | --- | --- | --- |
| Method | Mu | Sigma | Pr(Class) |
| Random | ~Unif(0, 1) | ~Unif(0, 1) | Equal priors |
| Quantile | Spaced evenly on range of data | Std. Dev. of data | Equal priors |
| Inspection | Visually chosen to maximize likelihood | | |

(The number of classes is predetermined.) Below are more detailed descriptions and rationales for each prior choice method.

### Random Method

This is the same method used for in-class assignment   
#3. All initializing values do not depend on the data at all. The means and standard deviations are drawn from a uniform distribution on zero and one. Prior probabilities are equal. We chose this formulation so that we could compare our results to those obtained in in-class assignment #3.

### Quantile Method

This new method uses statistics of the data to choose the prior parameters. The means are chosen to be spaced evenly on the range of the data. Specifically, they are the

(1)

(2)

(3)

where is the number of priors and is the sequence of integers from to . (If , the median of the data, i.e. the 50th percentile, will be used instead.). The standard deviations of the distributions were set to the sample standard deviation. Prior probabilities are equal.

We suspected that the magnitude and spacing of these priors will be much more reasonable than the Random Method, allowing for quicker convergence. Also, these priors can be calculated automatically from the data, requiring no human input.

### Inspection Method

Our other new method relies entirely on qualitative human decisions. Kushal looked at the data histograms and chose the means, standard deviations, and priors, such that the visually estimated likelihood of the data is maximized.

## Data

### Synthetic

### Real-world

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## EM Algorithm

Talk about how it works.

## Controlling the other hyperparameters

An example of a table is shown in Table 1. The caption text must be above the table.

Table 1: *This is an example of a table.*

|  |  |
| --- | --- |
| Ratio | Decibels |
| 1/1 | 0 |
| 2/1 | ≈ 6 |
| 3.16 | 10 |
| 1/10 | 20 |
| 10/1 | -20 |
| 100/1 | 40 |
| 1000/1 | 60 |

Figure 1: *Schematic diagram of speech production.*

## Equations

Equations should be placed on separate lines and numbered. Examples of equations are given below. Particularly,

 (1)

where  is a special warping function

 (2)

A residue theorem states that

 (3)

Applying (3) to (1), it is quite straightforward to see that

 (4)

Finally we have proven the secret theorem of all speech sciences. No more math is needed to show how useful the result is!

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# Discussion

## Convergence speed

## Model accuracy

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# Conclusions

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