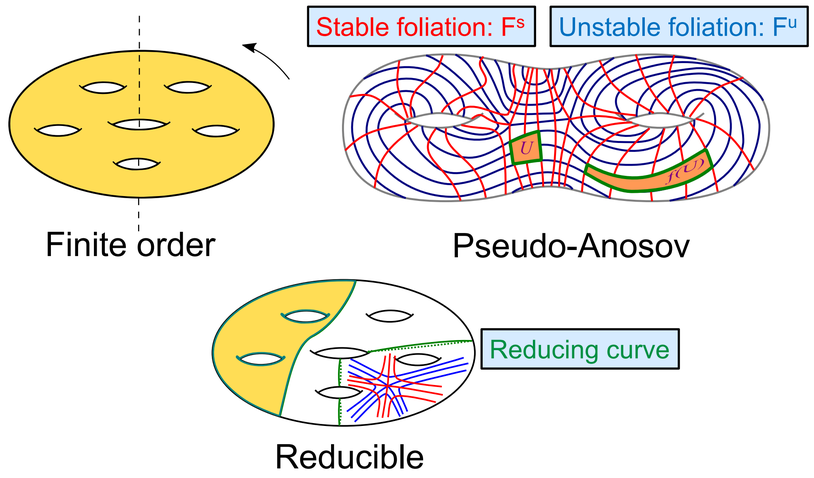
Fast Nielsen-Thurston classification

Balázs Strenner
Georgia Institute of Technology
(joint with Dan Margalit and Öykü Yurttaş)

Nielsen-Thurston Classification

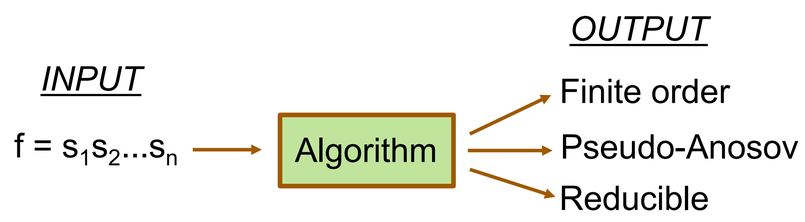
Mod(S) = mapping class group of a surface S

Every element of the mapping class group is either



Nielsen-Thurston Classification Problem

Fix a finite generating set for Mod(S).



Running time:
function of n.

+ order, reducing curves, foliations and stretch factors on all pieces

Main theorem

Theorem (Margalit-Strenner-Yurttaş)

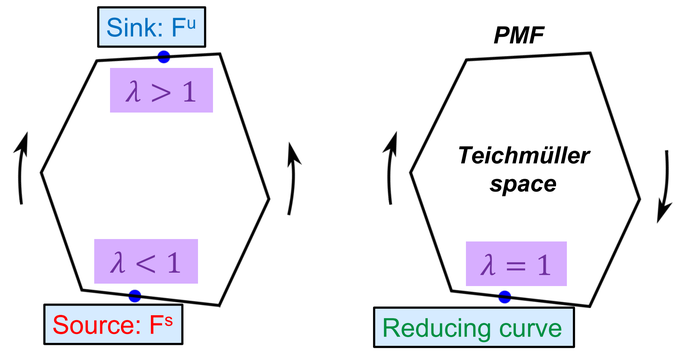
Quadratic time algorithm for the Nielsen-Thurston Classification Problem.

Work in progress (also in quadratic time)
veering triangulation, flat structure, permutation of components, etc. ⟹ conjugacy problem.

Previous algorithms and implementations are either exponential (Bestvina-Handel algorithm as XTrain, Trains), are not provably fast and do not handle the reducible case (flipper, Dynn), or do not work for closed surfaces (all four).

Classification by fixed points on PMF

Finite order elements fix all of PMF (up to power).

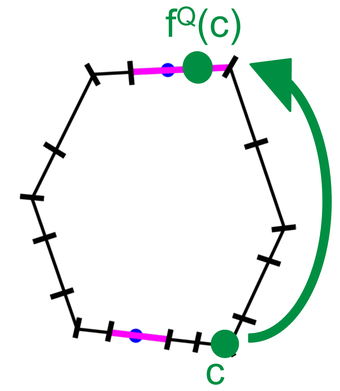


Pseudo-Anosov

Reducible
(up to power)

Outline of the algorithm

1. Fix some c in PMF.
2. Compute $f^Q(c)$ for some Q = O(1).
3. Compute the acting matrix at $f^Q(c)$.
4. Eigendata gives Nielsen-Thurston type.



Theorem (MSY)

O(1) iterations are sufficient.

Macaw (implementation in progress)

Features already implemented:

- Solves the word problem
- Recognizes finite order mapping classes
- Computes the order
- Approximates factors
- Works for closed surfaces*

*For closed surfaces the charts on PMF are much more complicated than for punctured surfaces. Details were worked out by Ian Katz, Yandi Wu, Yihan Zhou and the author as part of a summer REU project at Georgia Tech in 2017.