

IIT CS536: Science of Programming

Homework 3: Statements, Semantics, Errors and Hoare Triples

My Dinh

1 Statement Syntax

Task 1.1 (Written, 8 points)

```
x:=0;
i:=0;
while(i<size(a)){
    x:=x+a[i];
    i:=i+1
}
```

2 Semantics

Task 2.1 (Written, 10 points)

a) Steps of evaluation

$$\begin{aligned} & \langle s, \{n = 5\} \rangle \\ \rightarrow & \langle b; s, \{n = 5\} \rangle \\ \rightarrow & \langle n := 3 * n + 1; s, \{n = 5\} \rangle \\ \rightarrow^2 & \langle s, \{n = 16\} \rangle \\ \rightarrow^2 & \langle n := n/2; s, \{n = 16\} \rangle \\ \rightarrow^2 & \langle s, \{n = 8\} \rangle \\ \rightarrow^2 & \langle n := n/2; s, \{n = 8\} \rangle \\ \rightarrow^2 & \langle s, \{n = 4\} \rangle \\ \rightarrow^2 & \langle n := n/2; s, \{n = 4\} \rangle \\ \rightarrow^2 & \langle s, \{n = 2\} \rangle \\ \rightarrow^2 & \langle n := n/2; s, \{n = 2\} \rangle \\ \rightarrow^2 & \langle s, \{n = 1\} \rangle \\ \rightarrow & \langle skip, \{n = 1\} \rangle \end{aligned}$$

b) $M(s, \sigma)$ is $\Sigma_5 = \{\{n = 1\}\}$.

Task 2.2 (Written, 12 points)

- a) $\{\perp_d\}$
- b) $\Sigma_3 = \{\{n = 0\}\}$
- c) $\{n = -1\}$
- d) $\{x = 1, a = [0, 3, 2, 1], y = 3\}$
- e) $\{x = -1, a = [0, 3, 2, 1], z = 0\}$
- f) $\{\perp_e\}$

3 Hoare triples

Task 3.1 (Written, 16 points)

- a) Unsatisfied.
Because the the final state $\{i = 0, x = 6\}$ does not satisfy $i > 0$.
- b) Satisfied.
Because the starting state $\{i = 3, x = 1\}$ satisfies $i > 0$ and the program terminates then the final state $\{i = 0, x = 6\}$ satisfies $i \geq 0 \wedge x \geq 1$.
- c) Satisfied.
Because the starting state $\{i = 3, x = 1\}$ satisfies $i > 0$, then the program terminates and the final state $\{i = 0, x = 6\}$ satisfies $i \geq 0 \wedge x \geq 1$.
- d) Satisfied.
Because the starting state $\{i = -1, x = 1\}$ does not satisfy $i > 0$ (a contradiction).
- e) Satisfied.
Because the starting state $\{i = -1, x = 1\}$ does not satisfy $i > 0$ (a contradiction).
- f) Unsatisfied.
Because the final state $\{i = 0, x = 0\}$ does not satisfy $i \geq 0 \wedge x \geq 1$.
- g) Unsatisfied.
Because the final state $\{i = 0, x = 6\}$ does not satisfy $x = i!$.
- h) Satisfied.
Because the starting state $\{i = 3, x = 1, k = 3\}$ satisfies $i > 0 \wedge i = k$ and the program terminates then the final state $\{i = 0, x = 6, k = 3\}$ satisfies $x = k!$.

Task 3.2 (Written, 12 points)

- a) Valid
- b) Invalid. Fix precondition: $[x \geq 0 \wedge y > 0] z := x/y [z \geq 0]$.
- c) Valid
- d) Invalid. Fix precondition: $\{i \geq 0 \wedge |a| > i \wedge (\forall i. (0 \leq i \wedge i < |a|) \rightarrow a[i] \geq 0)\} x := a[i] \{x \geq 0\}$.

Task 3.3 (Written, 7 points)

The triple is

$[n \geq 0 \wedge ((n = 2k \wedge r = 1) \vee (n = 2k + 1 \wedge r = -1))] m := n; \text{while } n \neq 0 \{r := r * -2; n := n - 1\} [r = 2^m]$

4 One more wrap-up question

Task 4.1 (Written, 0 points)

I spent about 2 to 3 hours on this homework, in total of 1 hour of actual working time.