IIT CS536: Science of Programming

Homework 6: Loop Bounds, Nondeterminism, Prallelism

My Dinh

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1 Loop Bounds and Proof Outline

Task 1.1 (Written, 10 points).

$$\{n > 0\}$$
$$i := n;$$
$$\{\textbf{inv}\ i \geq 1 \wedge n > 0\}$$
$$\{\textbf{dec}\ i/2\}$$
$$\textsf{while}\ (i > 1)\ \{$$
$$\quad i := i\ /\ \overline{2}$$
$$\}\qquad \{n > 0 \wedge i = 1\}$$

Task 1.2 (Written, 8 points).

$$\{0 \leq i < |a| \wedge a[i] \geq 1\}$$
$$x := \overline{1};$$
$$k := \overline{0};$$
$$\{\textbf{inv}\ x = 2^k \wedge x \leq a[i]\}$$
$$\{\textbf{dec}\ a[i] - x\}$$
$$\textsf{while}\ (x * 2 \leq a[i])\ \{$$
$$\quad k := k + \overline{1};$$
$$\quad x := x * \overline{2}$$
$$\}\qquad \{x = 2^k \wedge x \leq a[i] \wedge a[i] < 2^{k+1}\}$$

2 Nondeterminism

Task 2.1 (Written, 8 points).

states σ	P_1	P_2
$\sigma \vDash e_1 \wedge e_2$	Executes s_1	Nondeterministically executes s_1 or s_2
$\sigma \vDash e_1 \wedge \neg e_2$	Executes s_1	Executes s_1
$\sigma \vDash \neg e_1 \wedge e_2$	Executes s_2	Executes s_2
$\sigma \vDash \neg e_1 \wedge \neg e_2$	Executes s_2	Aborts with Runtime Error

Task 2.2 (Written, 12 points).

a) $$\begin{aligned} M(\textsf{havoc}\ i; a[i] := \overline{1}, \sigma) &= U_{\sigma' \in M(\textsf{havoc}\ i, \sigma)} M(a[i] := \overline{1}, \sigma') \\ &= U_{\sigma' \in \{\sigma[i \mapsto n | n \in \mathbb{Z}]\}} M(a[i] := \overline{1}, \sigma') \\ &= M(a[i] := \overline{1}, \sigma[i \mapsto n | n \in \mathbb{Z}]) \\ &= \{\sigma[i \mapsto n | n \in [0, |a|)][a[i] \mapsto 1], \{\bot_e\}\} \end{aligned}$$

b) $\{\{x = -10\}, \{x = 10\}, \{\bot_d\}\}$

Because $x := x - \overline{1}$ might be chosen to execute until $x = -10$ and terminate the loop. Or $x := x + \overline{1}$ might be chosen to execute until $x = 10$ and terminate the loop. Or $x := x - \overline{1}$ and $x := x + \overline{1}$ are chosen to execute so that x value always stays between -10 and 10 and an infinite loop (divergence) would occur.

c) $M((\mathsf{branch}\ \{x \geq y \to x := x - \overline{2} \ \square\ y \geq x \to y := y + \overline{1}\}); \mathsf{if}\ x < y\ \mathsf{then}\ \{z := y\}\ \mathsf{else}\ \{z := x\}, \{x = 3, y = 3\})$
$= U_{\sigma' \in M(\mathsf{branch}\ \{x \geq y \to x := x - \overline{2} \ \square\ y \geq x \to y := y + \overline{1}\}, \{x=3, y=3\})} M(\mathsf{if}\ x < y\ \mathsf{then}\ \{z := y\}\ \mathsf{else}\ \{z := x\}, \sigma')$
$= U_{\sigma' \in \{\{x=1, y=3\}, \{x=3, y=4\}\}} M(\mathsf{if}\ x < y\ \mathsf{then}\ \{z := y\}\ \mathsf{else}\ \{z := x\}, \sigma')$
$= M(\mathsf{if}\ x < y\ \mathsf{then}\ \{z := y\}\ \mathsf{else}\ \{z := x\}, \{x = 1, y = 3\}) \cup M(\mathsf{if}\ x < y\ \mathsf{then}\ \{z := y\}\ \mathsf{else}\ \{z := x\}, \{x = 3, y = 4\})$
$= M(z := y, \{x = 1, y = 3\}) \cup M(z := y, \{x = 3, y = 4\})$
$= \{\{x = 1, y = 3, z = 3\}, \{x = 3, y = 4, y = 4\}\}$

Task 2.3 (Written, 12 points).

a) $wlp(\mathsf{branch}\ \{x \geq y \to max := x \ \square\ y \geq x \to max := y\}, max \geq 0)$
$= (x \geq y \to wlp(max := x, max \geq 0)) \wedge (y \geq x \to wlp(max := y, max \geq 0))$
$= (x \geq y \to x \geq 0) \wedge (y \geq x \to y \geq 0)$

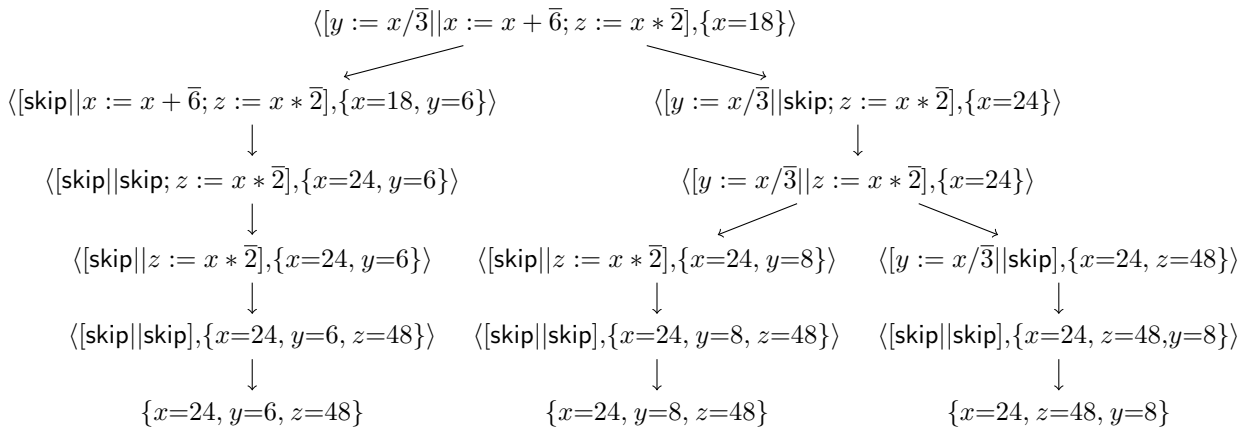
b) $sp(x \geq y, \mathsf{branch}\ \{x \geq y \to y := y + \overline{1} \ \square\ y \geq x \to x := x + \overline{1}\})$
$= sp(x \geq y \wedge x \geq y, y := y + \overline{1}) \vee sp(x \geq y \wedge y \geq x, x := x + \overline{1})$
$= sp(x \geq y, y := y + \overline{1}) \vee sp(x = y, x := x + \overline{1})$
$= (x \geq y_0 \wedge y = y_0 + 1) \vee (y = x_0 \wedge x = x_0 + 1)$

c) $wlp(\mathsf{havoc}\ x; y := y + \overline{1}, y \geq \dfrac{x}{|x|}) = wlp(\mathsf{havoc}\ x, wlp(y := y + \overline{1}, y \geq \dfrac{x}{|x|}))$
$= wlp(\mathsf{havoc}\ x, y + 1 \geq \dfrac{x}{|x|})$
$= \forall x_0 \in \mathbb{Z}. y + 1 \geq \dfrac{x_0}{|x_0|}$

3 Parallel Programs

Task 3.1 (Written, 10 points).

a) Evaluation graph for $\langle [y := x/\overline{3} || x := x + \overline{6}; z := x * \overline{2}], \{x = 18\}\rangle$:



b) $\{\{x = 24, y = 6, z = 48\}, \{x = 24, y = 8, z = 48\}\}$

Because from the evaluation graph in part (a), the different states that the program can be evaluated to are $\{x = 24, y = 6, z = 48\}$ and $\{x = 24, y = 8, z = 48\}$.

4 One more wrap-up question

Task 4.1 (Written, 8 points).
I spent about 3 hours on this homework, in total 1.5 hours of actual working time.