IIT CS536: Science of Programming

Homework 1: Course Bascis, Logic

My Dinh

1 Course Basics

Task 1.1 (Written, 10 points).

a) Both

b) Neither

c) Verification

d) Verification

e) Test

Task 1.2 (Written, 10 points).

a) $p \wedge q \vee r \neq r \vee q \wedge p$

b) $p \wedge q \vee r \neq p \wedge r \vee q$

c) $\neg p \vee q \rightarrow \neg(p \wedge \neg q) \equiv ((\neg p) \wedge q) \rightarrow \neg(p \wedge (\neg q))$

d) $p \rightarrow q \neq \neg p \rightarrow \neg q$

e) $p \wedge \neg p = F$

2 Propositional Logic

Task 2.1 (Written, 9 points).

a) Contigency:
$\{P = T; Q = T; R = T\} \vDash (P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \rightarrow Q) \rightarrow R)$
$\{P = F; Q = F; R = F\} \nvDash (P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \rightarrow Q) \rightarrow R)$

b) Tautology: $\sigma \vDash (P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ for all σ

c) Contigency:
$\{P = T; Q = T\} \vDash \neg P \wedge Q \leftrightarrow \neg Q \wedge P$
$\{P = T; Q = F\} \nvDash \neg P \wedge Q \leftrightarrow \neg Q \wedge P$

Task 2.2 (Programming, 5 points).

The proof for this problem is included in `tauto.log` file.

Task 2.3 (Written, 3 points).

We didn't prove that statement by proving $((P \rightarrow Q) \wedge \neg P) \vee P \Rightarrow T$ because then we will have to assume that $((P \rightarrow Q) \wedge \neg P) \vee P$ is true, which is the statement we have to prove.

Task 2.4 (Programming, 5 points).

The proof for this problem is included in `uncurry.log` file.

3 Predicate Logic

Task 3.1 (Written, 4 points).

$\exists l \in \mathbb{A}.\exists u \in \mathbb{A}.\forall x \in \mathbb{A}.(l \leq x \leq u)$

Task 3.2 (Written, 9 points).

a) False. $\not\models \forall x \in \mathbb{Z}.\exists y \in \mathbb{Z}.x = y*2$ because if $x = 2n+1$ for $n \in \mathbb{Z}$ then $x = 2y \Leftrightarrow 2n+1 = 2y \Leftrightarrow y = n+0.5$ is not an integer, which contradicts with our assumption that $y \in \mathbb{Z}$.

b) True. If x is prime then either a or b has to be 1, which means either that $a > 1 = F$ or $b > 1 = F \Rightarrow (\exists a \in \mathbb{Z}.\exists b \in \mathbb{Z}.a > 1 \wedge b > 1 \wedge a * b = x) = F \Rightarrow \neg(\exists a \in \mathbb{Z}.\exists b \in \mathbb{Z}.a > 1 \wedge b > 1 \wedge a * b = x) = T$.

c) True. $\models \forall x \in \mathbb{Z}.\exists y \in \mathbb{Z}.y = x * 2$ because or every integer x, we can find an even integer y that satisfies $y = 2x$ by multiplying x with 2.

Task 3.3 (Programming, 10 points).

The proof for this problem is included in `pred.log` file.

4 One more wrap-up question

Task 4.1 (Written, 0 points)

I spent 4 hours on this homework, in totla hours of actual working time.