

IIT CS536: Science of Programming

Homework 2: State, Expression and Types

My Dinh

1 State

Task 1.1 (Written, 9 points)

- a) $\{x = 8, y = 3, b = [4; 5; 6; 7]\}$
- b) 1
- c) $\{x = 2, y = 3, b = [4; 5; 6; 7], z = 4\}$

Task 1.2 (Written, 12 points)

a) Yes

b) No

Explain: Variable x in the proposition is bounded by $\forall x$, so the state does not affect it. If $x > 16$ then proposition is False. Hence, $\{x = 2, y = 4\} \not\models \forall x \in \mathbb{Z}. x \leq y^2$.

c) Yes

d) Yes

Task 1.3 (Written, 5 points)

State σ so that $\sigma \models p$ is

$$\sigma = \{x = 30, y = 12, z = 6, b = [0, 5, 6]\}$$

Task 1.4 (Written, 7 points)

- a) $\sigma \models (\exists x \in \mathbb{Z}. x < 0)$ if for **all** states σ , it is true that $\sigma[x \mapsto n] \models x < 0$, for **some** $n \in \mathbb{Z}$.
- b) $\sigma \models (\forall x \in \mathbb{Z}. x < 0)$ if for **all** states σ , it is true that $\sigma[x \mapsto n] \models x < 0$, for **all** $n \in \mathbb{Z}$.
- c) $\sigma \models (\exists x \in \mathbb{Z}. \forall y \in \mathbb{Z}. x < y^2)$ if for **all** states σ , it is true that $\sigma[x \mapsto n][y \mapsto m] \models x < y^2$, for **some** $n \in \mathbb{Z}$ and for **all** $m \in \mathbb{Z}$.

Task 1.5 (Written, 7 points)

The definition of a predicate function $P(a, b, m, n)$ is

$$P(a, b, m, n) = \forall i \in \mathbb{Z}. \exists j \in \mathbb{Z}. (0 \leq i \wedge i < m) \wedge (0 \leq j \wedge j < n) \rightarrow a[i] > b[j]$$

2 Expression and Types

Task 2.1 (Written, 10 points)

- a) Legal
- b) Legal
- c) Illegal
- d) Legal
- e) Legal

(It might be illegal if each element in a 2-dimensional array is a 1-dimensional array. But since the definition of multi-dimensional array in our programming language is 1-dimensional, then each indexing (e.g. $a[i]$, $a[i][j]$, etc.) results an expression)

Task 2.2 (Written, 15 points)

- a) 7
- b) 30
- c) 6
- d) 5
- e) 5

3 One more wrap-up questions

Task 3.1 (Written, 0 points)

I spent about 6 hours on this homework, 4 hours actual working on the homework and 2 hour going back to the lecture recording and notes to understand the materials.