Due at 1700, Fri. Apr. 17 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 5.6-5.8, 12-12.8.

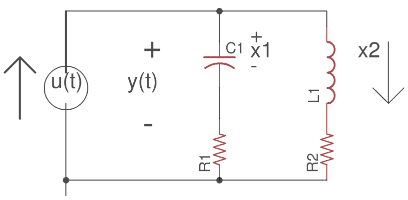
1. (15 pts) Controllability and Observability (Nise 12.3, 12.6)

For the circuit below, input is current $u(t)$, output is voltage $y(t)$, states $x1$ = capacitor voltage, and $x2$ = inductor current.

a) Write state and output equations for the circuit.

b) Find conditions for $R_1, R_2, L1, C1$ that make the system controllable and observable.

c) Interpret the conditions that make the system lose controllability or observability in terms of the time constants of the system.



2. (15 pts) Control Form transformation (Nise 12.4)

Given the following

$$\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -3 & 1 \\ -1 & 2 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{z}$$

Find the transformation P such that $(\bar{A}, \bar{B})$ is in phase variable form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$. Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{x}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.

3. (30 pts) Steady State Error/Integral Control (Nise 12.8)

Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1 u = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [10\ \ 0]\ \mathbf{x} \tag{1}$$

a) Given error $e(t) = r(t) - y(t)$ where $r(t)$ is a scalar, evaluate the steady state error $\lim_{t\to\infty} e(t)$ for input $r(t)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where K_1 is chosen so that the closed loop poles are at $s_i = -4, -5$.

b) Add an integrator to the plant, using a new state vector $\mathbf{x} = [x_1 x_2 x_N]^T$, write the new state and output equations, and find gains such that the closed-loop poles are at $s_i = -4, -5, -25$. Evaluate the steady-state error for a step input.

c) Plot the step response for both systems in Matlab, and compare.

4. (30 pts) State Feedback versus Observer Feedback

Given the following system (e.g. inverted pendulum with position sensor)

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ 6 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}$$

a) Design a state feedback controller $u = [-k_1 - k_2]\mathbf{x}$ such that the closed loop system has $\zeta = 0.5$ and $\omega_n = 6$.

b) Use Matlab to plot the states $\mathbf{x}(t)$ for $t > 0$ for the system with state feedback.

c) Design a critically damped observer $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ with both observer poles at $s = -10$.

d) Write the state space equations for the controller with $u = -K\hat{\mathbf{x}}$. (This should have 4 state variables.) From the separation principle, what are the eigenvalues of the combined system with observer and feedback control?

e) Use Matlab to plot the states $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ for $t > 0$ for the closed loop system using $u = [-k_1 - k_2]\hat{\mathbf{x}}$. (Suggestion, use `sys = ss(A0,B0,C0,D0)` and `lsimplot(sys)`, where `A0, B0, C0, D0` are the matrices for the system with observer).

f) Compare the responses in part b) and e). What differences are there?

5. (10 pts) Cayley-Hamilton (handout)

Every square matrix satisfies its own characteristic equation; that is, $\Delta(A) = [0]$. A 2×2 matrix A has characteristic polynomial $\Delta(\lambda) = \lambda^2 + 2\lambda + 3$. Reduce polynomial $P(A) = A^4 + 3A^3 + 2A^2 + A + 2I$ to the lowest order equivalent polynomial. (Hint: $P(A) = Q(A)\Delta(A) + R(A)$, where $Q(A)$ is the quotient and $R(A)$ is the remainder for $P(A)/\Delta(A)$.