Ps 1	PS 1	EE128

1. Find inv. laplace using partial fraction expansion

$$F(s)=\frac{s-2}{s(s+1)(s+4)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+4)^2} + \frac{D}{s+4}$$

$$A = sF(s)\Big|_{s=0} = \frac{s-2}{(s+1)(s+4)^2}\Big|_{s=0} = \frac{-2}{1(4^2)} = -\frac{1}{8}$$

$$B = (s+1)F(s)\Big|_{s=-1} = \frac{s-2}{s(s+4)^2}\Big|_{s=-1} = \frac{-3}{(-1)(3^2)} = \frac{1}{3}$$

$$C = (s+4)^2F(s)\Big|_{s=-4} = \frac{s-2}{s(s+1)}\Big|_{s=-4} = \frac{-6}{-4(-3)} = \frac{-1}{2}$$

$$D = \left[\frac{d}{ds}(s+4)^2F(s)\right]\Big|_{s=-4} = \left[\frac{d}{ds}\frac{s-2}{s(s+1)}\right]_{s=-4} = \frac{s(s+1)-(s-2)(2s+1)}{(s(s+1))^2}\Big|_{s=-4}$$

$$= \frac{-5}{24}$$

$$F(s)=\frac{s-2}{s(s+1)(s+4)^2} = \frac{-1}{8}\left(\frac{1}{s}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{s+1}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{(s+4)^2}\right) + \left(\frac{-5}{24}\right)\left(\frac{1}{s+4}\right)$$

$$\boxed{f(t) = -\frac{1}{8} + \frac{1}{3}e^{-t} - \frac{1}{2}te^{-4t} - \frac{5}{24}e^{-4t}}$$

2. Determine $h(t)$

a. $$H_1(s) = \frac{1}{s^2+4s+53} = \frac{7}{7}\left(\frac{1}{s^2+4s+4+49}\right) = \frac{1}{7}\left(\frac{7}{(s+2)^2+7^2}\right)$$

$$\boxed{h_1(t) = \frac{1}{7}e^{-2t}\sin(7t)u(t)}$$

b. $$H_2(s) = \frac{s}{s^2+4s+53} = sH_1(s)$$

$$h_2(t) = \frac{d}{dt}h_1(t) = \frac{d}{dt}\left[\frac{1}{7}e^{-2t}\sin(7t)u(t)\right] = \frac{-2}{7}e^{-2t}\sin(7t)u(t) + \frac{1}{7}e^{-2t}7\cos(7t)u(t)$$

$$+\frac{1}{7}e^{-2t}\underbrace{\sin(7t)\delta(t)}_{0 \text{ at } t=0}$$

$$\boxed{h_2(t) = e^{-2t}\cos(7t)u(t) - \frac{2}{7}e^{-2t}\sin(7t)u(t)}$$

c. $$H_3(s) = \frac{s+3}{s^2+4s+53} = \frac{s}{s^2+4s+53} + \frac{3}{s^2+4s+53} = H_2(s) + \frac{3}{7}\left(\frac{7}{(s+2)^2+7^2}\right)$$

$$h_3(t) = e^{-2t}\cos(7t)u(t) - \frac{2}{7}e^{-2t}\sin(7t)u(t) + \frac{3}{7}e^{-2t}\sin(7t)u(t)$$

$$\boxed{h_3(t) = \left[e^{-2t}\cos(7t) + \frac{1}{7}e^{-2t}\sin(7t)\right]u(t)}$$

d. $H_4(s) = \dfrac{s^2}{s^2+4s+53} = s\,H_2(s)$

$h_4(t) = \dfrac{d}{dt} h_2(t) = \dfrac{d}{dt}\left[\left(e^{-2t}\cos(7t) - \frac{2}{7}e^{-2t}\sin(7t)\right)u(t)\right]$

$= \left[-2e^{-2t}\cos(7t) + e^{-2t}7(-\sin(7t)) + \frac{4}{7}e^{-2t}\sin(7t) - \frac{2}{7}e^{-2t}7\cos(7t)\right]u(t)$

$+\left(e^{-2t}\overset{1}{\cancel{\cos}}(7t) - \frac{2}{7}e^{-2t}\overset{0}{\cancel{\sin}}(7t)\right)\delta(t)$

$$\boxed{h_4(t) = -4e^{-2t}\cos(7t) - \frac{45}{7}e^{-2t}\sin(7t) + \delta(t)}$$

e. $H_5(s) = \dfrac{s^2+4}{s^2+4s+53} = \dfrac{s^2}{s^2+4s+53} + \dfrac{4}{s^2+4s+53} = H_4(s) + \dfrac{4}{7}\left(\dfrac{7}{(s+2)^2+7^2}\right)$

$h_5(t) = -4e^{-2t}\cos(7t)u(t) - \frac{45}{7}e^{-2t}\sin(7t)u(t) + \delta(t) + \frac{4}{7}e^{-2t}\sin(7t)u(t)$

$$\boxed{h_5(t) = \left[-4e^{-2t}\cos(7t) - \frac{41}{7}e^{-2t}\sin(7t)\right]u(t) + \delta(t)}$$

3. Determine $y_i(t=0^+)$ and $\lim_{t\to\infty} y_i(t)$ if exists.

a. $Y_1(s) = \dfrac{1}{s(s+3)}$

$y(0) = \lim_{s\to\infty} sY_1(s) = \lim_{s\to\infty} \dfrac{1}{s+3} = 0$

$\boxed{y_1(0) = 0}$

$y_1(\infty) = \lim_{s\to 0} sY_1(s) = \lim_{s\to 0}\dfrac{1}{s+3} = \dfrac{1}{3}$

$\boxed{y_1(\infty) = \frac{1}{3}}$

b. $Y_2(s) = \dfrac{1}{s^2(s+3)}$

$y_2(0) = \lim_{s\to\infty} sY_2(s) = \lim_{s\to\infty}\dfrac{1}{s(s+3)} = 0$

$\boxed{y_2(0) = 0}$

$\boxed{y_2(\infty) \text{ d.n.e, double pole at origin.}}$

c. $Y_3(s) = \dfrac{s+1}{s(s+3)}$

$y_3(0) = \lim_{s\to\infty} sY_3(s) = \lim_{s\to\infty}\dfrac{s+1}{s+3} = 1$

$\boxed{y_3(0) = 1}$

$y_3(\infty) = \lim_{s\to 0} sY_3(s) = \lim_{s\to 0}\dfrac{s+1}{s+3} = \dfrac{1}{3}$

$\boxed{y_3(\infty) = \frac{1}{3}}$

d. $Y_4(s) = \dfrac{s-3}{s(s+3)}$

$y_4(0) = \lim_{s\to\infty} sY_4(s) = \lim_{s\to\infty}\dfrac{s-3}{s+3} = 1$

$\boxed{y_4(0) = 1}$

$y_4(\infty) = \lim_{s\to 0} sY_4(s) = \lim_{s\to 0}\dfrac{s-3}{s+3} = -1$

$\boxed{y_4(\infty) = -1}$

e. $Y_6(s) = \frac{1}{(s+1)(s+2)s}$

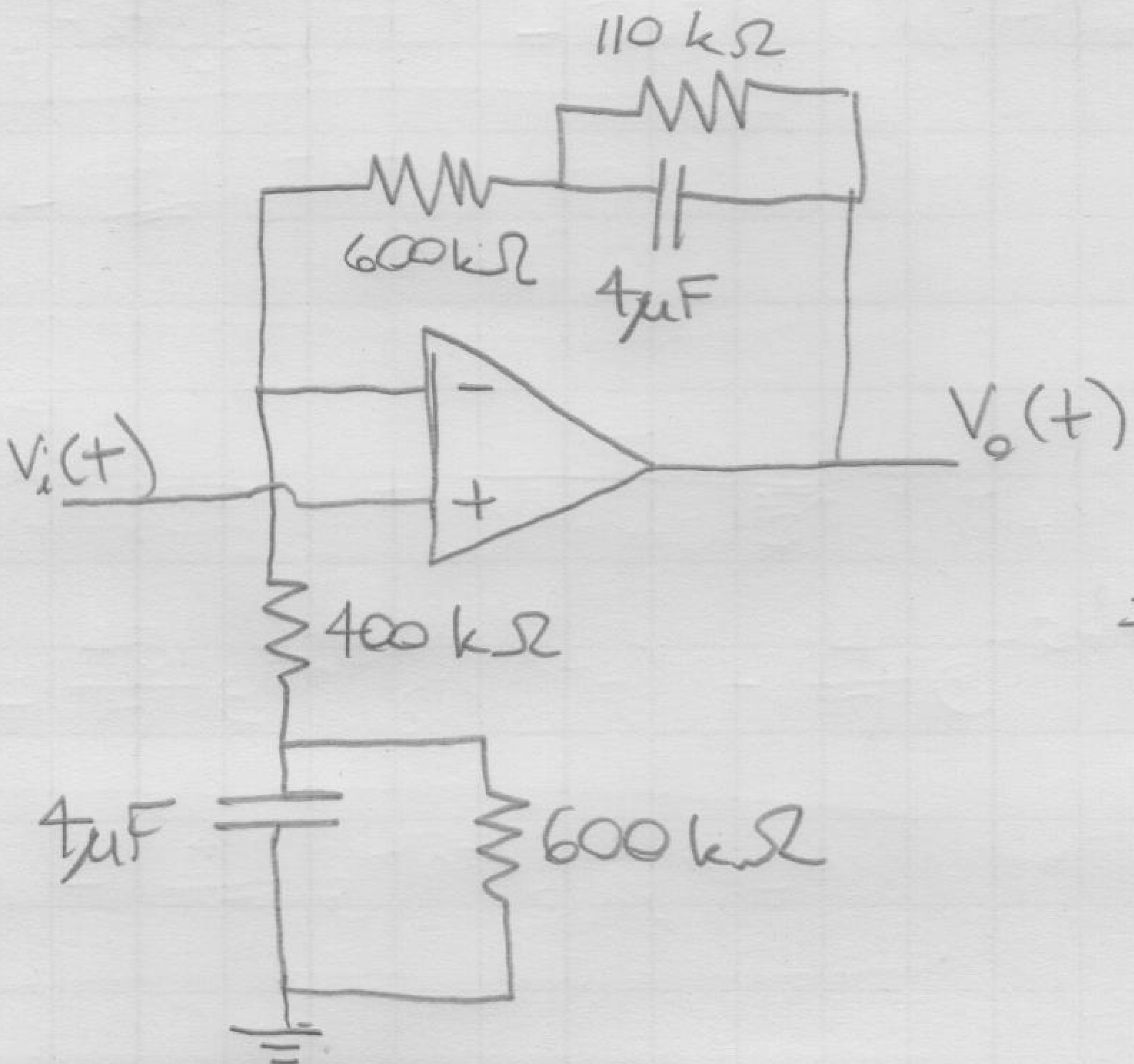
$y_6(0) = \lim_{s\to\infty} sY_6(s) = \lim_{s\to\infty} \frac{1}{(s+1)(s+2)} = 0$

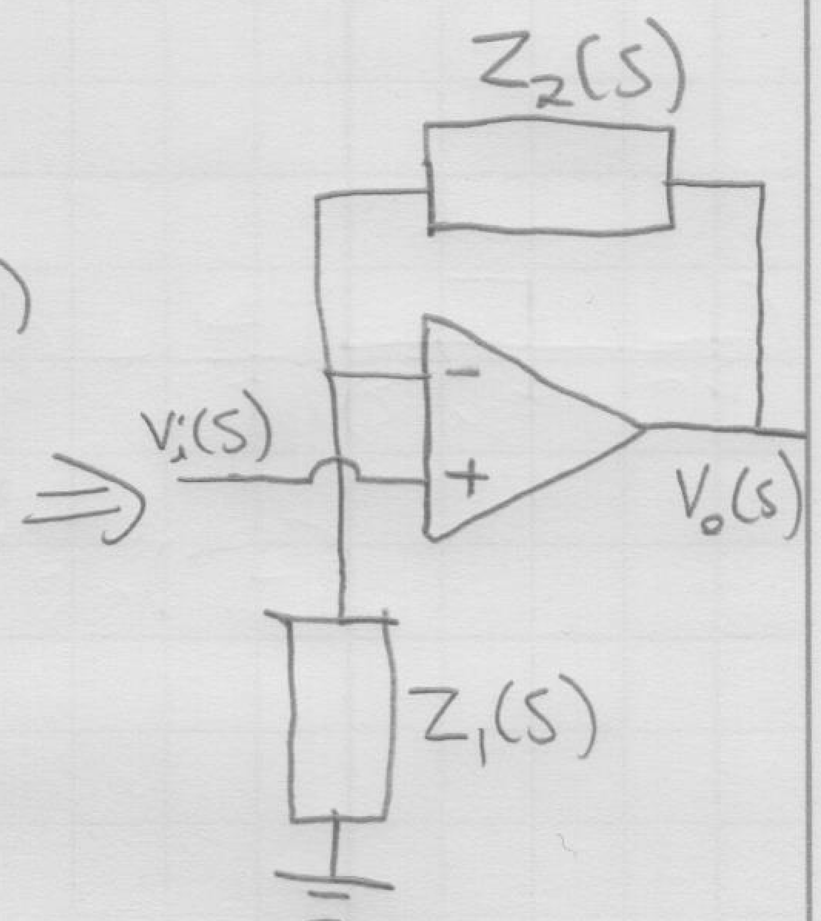
$\boxed{y_6(0) = 0}$

$y_6(\infty) = \lim_{s\to 0} sY_6(s) = \lim_{s\to 0} \frac{1}{(s+1)(s+2)} = \frac{1}{2}$

$\boxed{y_6(\infty) = \frac{1}{2}}$

4. Find $H(s) = \frac{V_o(s)}{V_i(s)}$





$$\frac{V_o(s) - V_i(s)}{Z_2(s)} = \frac{V_i(s) - 0}{Z_1(s)}$$

$$V_o(s) - V_i(s) = \frac{Z_2(s)}{Z_1(s)} V_i(s)$$

$$V_o(s) = \frac{Z_2(s) + Z_1(s)}{Z_1(s)} V_i(s)$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

$$= \frac{400\times10^3 + \frac{600\times10^3}{2.4s+1} + 600\times10^3 + \frac{110\times10^3}{0.44s+1}}{400\times10^3 + \frac{600\times10^3}{2.4s+1}}$$

$$\boxed{H(s) = \frac{1056s^2 + 3368s + 1710}{422.4s^2 + 1400s + 1000}}$$

$$Z_1(s) = 400\times10^3 + \frac{1}{\frac{1}{600\times10^3} + 4\times10^{-6}s} = 400\times10^3 + \frac{600\times10^3}{2.4s+1}$$

$$= \frac{960\times10^3 s + 1000\times10^3}{2.4s+1}\ \Omega$$

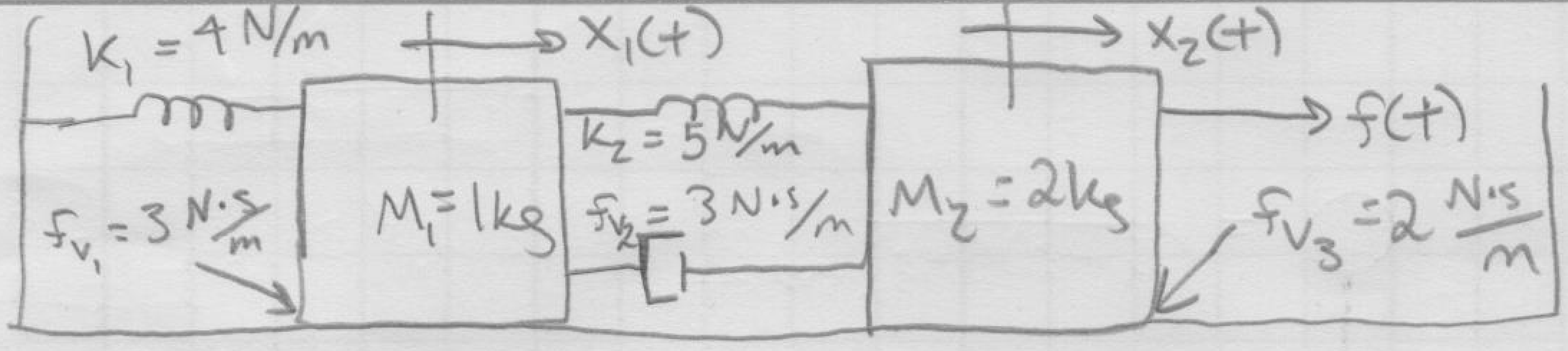
$$= \frac{960s + 1000}{2.4s+1}\ k\Omega$$

$$Z_2(s) = 600\times10^3 + \frac{1}{\frac{1}{110\times10^3} + 4\times10^{-6}s}$$

$$= \frac{264\times10^3 s + 710\times10^3}{0.44s+1}\ \Omega$$

$$= \frac{260s + 710}{0.44s+1}\ k\Omega$$

5.



Write TF relating input force $f(t)$ to output velocity $\dot{x}_2(t)$:

So $H(s) = \dfrac{sX_2(s)}{F(s)} = sG(s)$ where $G(s) = \dfrac{X_2(s)}{F(s)}$

Equations of motion:

$$(1)\quad [M_1s^2 + (f_{v_1}+f_{v_2})s + (K_1+K_2)]X_1(s) - [f_{v_2}s + K_2]X_2(s) = 0$$

$$(2)\quad [M_2s^2 + (f_{v_2}+f_{v_3})s + K_2]X_2(s) - [f_{v_2}s + K_2]X_1(s) = F(s)$$

Plug $X_1(s)$ from (1) in for $X_1(s)$ of (2)

$$[M_2s^2 + (f_{v_2}+f_{v_3})s + K_2]X_2(s) - \frac{[f_{v_2}s+K_2][f_{v_2}s+K_2]}{[M_1s^2 + (f_{v_1}+f_{v_2})s + (K_1+K_2)]}X_2(s) = F(s)$$

$$X_2(s)\frac{[M_2s^2 + (f_{v_2}+f_{v_3})s + K_2][M_1s^2 + (f_{v_1}+f_{v_2})s + (K_1+K_2)] - [f_{v_2}s+K_2][f_{v_2}s+K_2]}{[M_1s^2 + (f_{v_1}+f_{v_2})s + (K_1+K_2)]}$$

$= F(s)$

$$\Rightarrow \frac{[2s^2+5s+5][s^2+6s+9] - [3s+5][3s+5]}{[s^2+6s+9]}X_2(s) = F(s)$$

$$\frac{2s^4+17s^3+44s^2+45s+20}{s^2+6s+9}X_2(s) = F(s)$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{s^2+6s+9}{2s^4+17s^3+44s^2+45s+20}$$

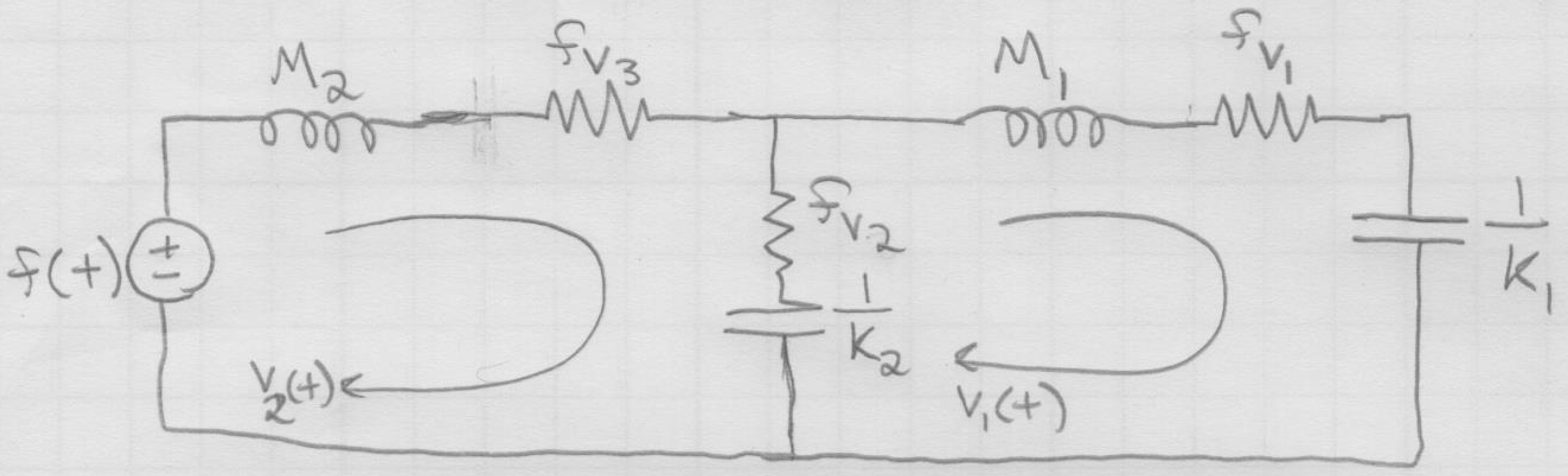
$$\boxed{H(s) = sG(s) = \frac{sX_2(s)}{F(s)} = \frac{s^3+6s^2+9s}{2s^4+17s^3+44s^2+45s+20}}$$

6. Draw electrical equivalent circuit for mechanical system in problem 5.

Convert equations of motion to velocity:

$$\left[M_1 s + (f_{v_1} + f_{v_2}) + \frac{K_1 + K_2}{s}\right]V_1(s) - \left[f_{v_2} + \frac{K_2}{s}\right]V_2(s) = 0$$

$$\left[M_2 s + f_{v_2} + f_{v_3}) + \frac{K_2}{s}\right]V_2(s) - \left[f_{v_2} + \frac{K_2}{s}\right]V_1(s) = F(s)$$



where V_i is current around mesh i and $f(t)$ is an applied voltage

Using Kirchhoff's voltage law:

$$\left[M_2 s + (f_{v_3} + f_{v_2}) + \frac{K_2}{s}\right]V_2(s) - \left[f_{v_2} + \frac{K_2}{s}\right]V_1(s) = F(s)$$

$$\left[M_1 s + (f_{v_1} + f_{v_2}) + \left(\frac{K_2}{s} + \frac{K_1}{s}\right)\right]V_1(s) - \left[f_{v_2} + \frac{K_2}{s}\right]V_2(s) = 0$$

Since $V_1(s) = sX_1(s)$ and $V_2(s) = sX_2(s)$, it is evident that the equations of motion, and therefore, the transfer function from $F(s)$ to $V_2(s)$ are the same as problem 5.

7. $f(t) = m\ddot{x}(t) + b\dot{x}(t) + f_s(x,t)$

$x_s(t) = 1 - e^{-f_s(t)}$, $x_s(t)$ is spring displacement, f_s is nonlinear spring force

Linearize around $f(t) = 1 = f_o$

$$f_s(t) = -\ln(1 - x_s(t))$$

$$f(t) = m\frac{\delta^2 x}{dt^2} + b\frac{\delta x}{\delta t} - \ln(1 - x(t))$$

To linearize, replace $f(t)$ with $\delta f + f_o$ and $x(t)$ with $\delta x + x_o$

Linearizing $\ln(1 - x(t)) = f_s(x,t)$

$$f_s(x) = f_s(x_o) + \left.\frac{\delta f_s}{\delta x}\right|_{x = x_o} \delta x$$

$$\ln(1-x) = \ln(1-x_o) + \left.\frac{\delta \ln(1-x)}{\delta x}\right|_{x = x_o} \delta x$$

$$\ln(1-x) = \ln(1-x_o) - \left.\frac{1}{1-x}\right|_{x = x_o} \delta x$$

$$\ln(1-x) = \ln(1-x_o) - \frac{1}{1-x_o}\delta x$$

$$\text{So: } \delta f + f_o = m\frac{d^2 \delta x}{dt} + b\frac{d\delta x}{dt} - \ln(1 - x_o) + \frac{1}{1-x_o}\delta x$$

Need x_o:

At $f(t) = 1$, $\delta x = 0$ and $\delta f = 0$, so

$$f_o = 1 = -\ln(1 - x_o)$$

$$x_o = 1 - e^{-1} = 0.6321$$

So:

$$\delta f + 1 = m\frac{d^2\delta x}{dt} + b\frac{d\delta x}{dt} - \ln(1 - 0.6321) + \frac{1}{1-0.6321}\delta x$$

$$\delta f + 1 = m\frac{d^2\delta x}{dt} + b\frac{d\delta x}{dt} + 1 + e\,\delta x$$

$$\delta f = m\frac{d^2\delta x}{dt} + b\frac{d\delta x}{dt} + e\,\delta x$$

$$F(s) = ms^2X(s) + bsX(s) + eX(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + e}}$$