EE128 PS2: Comments on Question 4

Let $x'(t) = x(t) + \delta x(t)$ be the output to the input $u'(t) = u(t) + \delta u(t)$ (a perturbation $\delta u(t)$) s.t. $\dot{x}(t) = f(x(t), u(t)$. Then its clear that,

$$\begin{aligned} \dot{x}'(t) = \dot{x}(t) + \delta \dot{x}(t) &= f(x(t) + \delta x(t), u(t) + \delta u(t)) \qquad (1) \\ &\approx f(x(t), u(t)) + \frac{\partial f(x(t), u(t))}{\partial x} \delta x(t) + \frac{\partial f(x(t), u(t))}{\partial u} \delta u(t) \end{aligned}$$

This then reduces to,

$$\delta \dot{x}(t) = \frac{\partial f(x(t), u(t))}{\partial x} \delta x(t) + \frac{\partial f(x(t), u(t))}{\partial u} \delta u(t) \qquad (2)$$

In this particular problem you are only given the operating point $((x(t), u(t))$ at a particular instant of time and not its explicit time dependence. That is the operating point given in the problem is the value of $x(t)$ and $u(t)$ at a particular time instant t_0. Denote $x(t_0)$ by x_0 and $u(t_0)$ by u_0. Then Eq. (2) can be expressed as the following which gives us the linearization at (x_0, u_0).

$$\delta \dot{x}(t) = \frac{\partial f(x, u)}{\partial x}_{@(x=x_0, u=u_0)} \delta x(t) + \frac{\partial f(x, u)}{\partial u}_{@(x=x_0, u=u_0)} \delta u(t) \qquad (3)$$