

**Due at 1700, Fri. Feb. 27 in homework box under stairs, first floor Cory .**

Note: up to 2 students may turn in a single writeup. Reading Nise 8.

1. (30 pts) Root locus sketching (Nise 8.6)

For each part below with open loop transfer function  $G(s)$  in unity gain feedback (Fig.1):

- Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.
- Find  $j\omega$  axis intercepts if any.
- Hand sketch root locus.
- Specify range of  $k$  for stability.
- Verify your root locus using MATLAB.

a)  $G(s) = \frac{k}{s(s^2 + 25s + 100)}$

b)  $G(s) = \frac{k(s+2)}{s(s^2 + 4s + 8)}$

c)  $G(s) = \frac{k(s+1)^2}{s^2(s+4)^2}$

2. (15 pts) Root locus (Nise 8.7)

Given the unity gain feedback system in Fig. 1, where

$$G(s) = \frac{K}{(s+1)(s+3)(s+6)(s+8)}$$

- Find and hand sketch the root locus.
- Using the second order approximation:
- Find the value of  $K$  that will yield a settling time of 4 seconds.
  - Find the value of  $K$  that will yield a critically damped system.
  - Use MATLAB to plot the step response for b) and c) and compare to approximation estimate.

3. (15 pts) Root locus (Nise 8.6, 8.9)

Consider the unity gain feedback system in Fig. 1 with  $G(s) = \frac{k2(s-1)}{s^2+2s+2}$ . Here  $-\infty < k < \infty$

- Apply root locus rules: specify real axis segments, break-away and break-in locations on real axis, and angle of departure from complex poles.
- Find the  $j\omega$  crossing using Routh-Hurwitz.
- Hand sketch the closed-loop root locus for positive and negative  $k$ .
- Find the range of  $k$  for stability.

4. (20 pts) Root Locus (Nise 8.6, 8.8)

A mammalian regulation system (perhaps cholesterol, etc.) is postulated to be of the form of a proportional-plus-integral controller, where  $G_c(s) = K_p + \frac{K_I}{s}$ . The plant might be modelled as a simple accumulator (integrator)  $P(s) = \frac{1}{V_o s}$  where  $V_o$  is a volume. The open loop transfer function for a system in unity feedback (Fig.1) is given by:  $G(s) = G_c(s)P(s)$ .

- Sketch the root locus as a function of  $K_p$  assuming  $K_I > 0$  is a constant.
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5. (20 pts) Root locus (Nise 8.8)

The open loop transfer function for a system in unity feedback (Fig.1) is given by:

$$G(s) = \frac{s+2}{(s+3)(s^2+cs+9)}$$

- Determine the characteristic equation for the closed loop system.
- Sketch the root locus with respect to positive values of  $c$ , showing direction in which  $c$  increases on the locus.

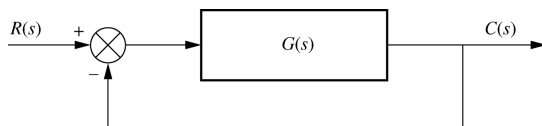


Fig. 1. Control System Block Diagram.