Problem (1)

2 wire topology
$l \triangleq$ length of line $= 4,260$ km
$a \triangleq$ radius of each line $=$ 1cm
$d \triangleq$ spacing between the two lines $=$ 5 cm

a) From Inan & Inan, table 2.2, page 85:

$$L' = 0.4 \ln\left[\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right] \ \mu H/m$$

$d = 5$ cm
$a = 1$ cm

$\Rightarrow L' = 0.627 \ \mu H/m$

$L = L' . l$

$= 0.627 \mu H/m \ . \ 4,260$ km

$\Rightarrow \boxed{L = 2.67 \ H}$

b) Let $R \triangleq$ resistance of each wire at DC

$$R = \frac{\rho . l}{A}$$

$\rho_{copper} = 1.7 \times 10^{-8} \ \Omega . m$
$A = \pi a^2$, $a = 1$ cm $\Rightarrow A = \pi . 10^{-4}$ m
$l = 4,260$ Km

$\Rightarrow \boxed{R = 230 \Omega}$

c) From Inan & Inan, table 2.2, page 85:

$$C' = \frac{27.8}{\ln\left[\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]}$$

$d = 5\,cm$

$a = 1\,cm$

$\Rightarrow \quad C' = 17.74\ pF/m$

$C = C' \cdot l$

$\quad = 17.74\ pF/m \times 4,260\ km$

$\Rightarrow \boxed{C = 75.6\ \mu F}$

d) let $t_p \triangleq$ propagation delay along the line.

$$t_p = \frac{\ell}{v}$$

if the line is in air,

$$v = 3 \times 10^8 \text{ m/s}$$

$$\Rightarrow t_p = \frac{4,260 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}}$$

$$\Rightarrow \boxed{t_p = 14.2 \text{ ms}}$$

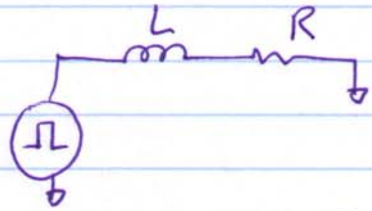
e) Increasing the voltage between the two lines increases the E field in the dielectric. If the E field increases beyond a critical value E_{crit}, the dielectric breaks down resulting in large dielectric current and the transmission line becomes disfunctional.

Assuming the dielectric is Teflon,
$$E_{crit,\,Teflon} = 60 \ \frac{\text{MV}}{\text{m}}$$

$$V_{max} = E_{crit} \times d$$
$$= 60 \frac{\text{MV}}{\text{m}} \times 5 \text{ cm}$$

$$\Rightarrow \boxed{V_{max} = 3.\ \text{MV}}$$

g) simple LR circuit



from (a) : $L = 2.67\ H$

from (b) : $R = 2 \times 230\,\Omega = 460\,\Omega$ (each wire has resistance of $230\,\Omega$)

let $\tau \triangleq$ circuit time constant

$\Rightarrow \tau = \frac{L}{R}$

$= \frac{2.67\,H}{460\,\Omega}$

$\Rightarrow \tau = 5.72\,ms$

let $t_r \triangleq$ rise time

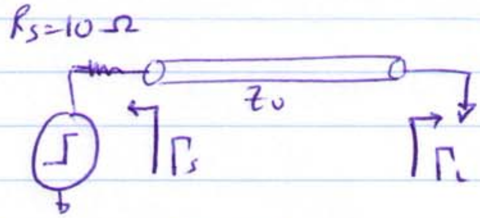
$t_r \simeq 5\tau$

$\Rightarrow \boxed{t_r \simeq 28.6\ ms}$

The pulse contains many high frequency components. Since $\lambda = \frac{c}{f}$, $\ell \gg \lambda$ for the high frequency components.

In this case, the line exhibits distributed effects and should be modeled as a transmission line. LR circuits do not model reflection OR wave phenomena.

h)

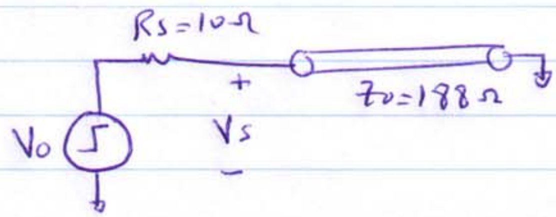


$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{0.627\ \mu H/m}{17.74\ pF/m}}$$

$$\Rightarrow Z_0 = 188\,\Omega$$

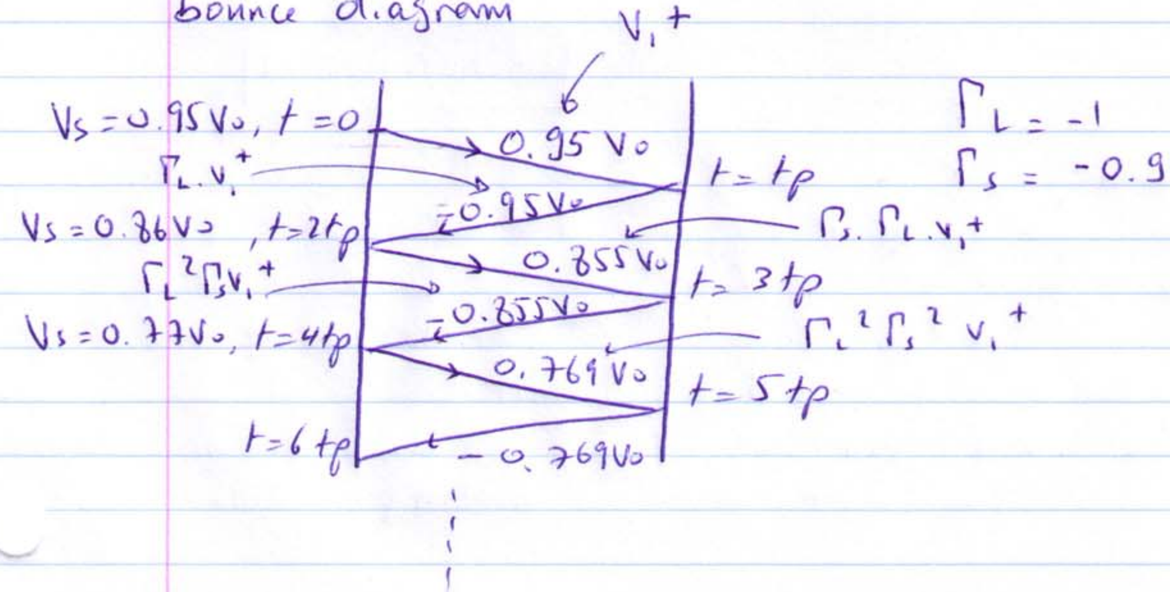
$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} \Rightarrow \Gamma_L = -1$$

$$\Gamma_s = \frac{R_s - Z_0}{R_s + Z_0} = \frac{10 - 188}{10 + 188} \Rightarrow \Gamma_s = -0.9$$



$$@\ t = 0:\ V_s = V_1^+ = \frac{Z_0}{R_s + Z_0} \cdot V_0 = \frac{188}{10+188} V_0 = 0.95\,V_0$$

bounce diagram



$V_s\big|_{t=0} = 0.95\,V_0$

$V_s\big|_{t=\infty} = 0$

We need to find when V_s becomes $0.1 \times V_s\big|_{t=0}$

From the bounce diagram, we can deduce that:

$$V_s\big|_{t=2nt_p} = (\Gamma_L . \Gamma_s)^n . V_s\big|_{t=0} \quad , n>1$$

we need $V_s = 0.1\, V_s\big|_{t=0}$

$\Rightarrow$ solve for n, replacing $V_s\big|_{t=2nt_p}$ by $0.1\, V_s\big|_{t=0}$

$$\Rightarrow \quad 0.1\, V_s\big|_{t=0} = (\Gamma_L . \Gamma_s)^n\, V_s\big|_{t=0}$$

$$\Rightarrow \quad 0.1 = (0.9)^n$$

$$\Rightarrow \quad \log_{0.9}(0.1) = \log_{0.9}(0.9^n) = n . \log_{0.9}(0.9) = n$$

$$\Rightarrow \quad n = \log_{0.9}(0.1)$$

$$\Rightarrow \quad n = 22$$

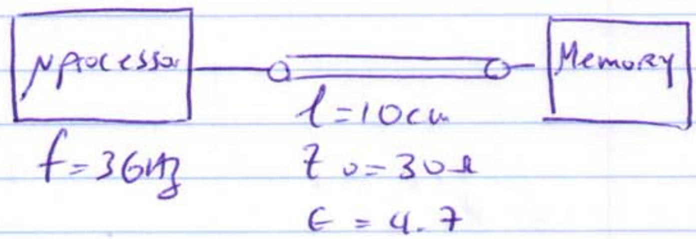
V_s of $0.1\, V_s\big|_{t=0}$ occurs @ $t = 2n\, t_p$

$t_p = 14.2\,ms$ (from (d))

$$\Rightarrow \quad \boxed{t = 624.8\ ms}$$

Problem (6)

$f = 3\text{ GHz}$
$l = 10\text{ cm}$
$\epsilon = 4.7$
$Z_0 = 30\,\Omega$



$$v = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{3\times 10^8\text{ m/s}}{\sqrt{4.7 \times 1}}$$

$\Rightarrow \quad v = 1.38 \times 10^8\text{ m/s}$

To read data from the memory, the signal has to propagate from the μprocessor to the memory and back from the memory to the μprocessor

$\Rightarrow$ it take $2 \times \frac{l}{v} = 2 \times \frac{10\text{ cm}}{1.38\times10^8\text{ m/s}} = 1.45\text{ ns}$

clock cycle $= \frac{1}{f} = \frac{1}{3\times10^9 s} = \frac{1}{3}\text{ ns}$

$\Rightarrow$ number of cycles to read data $= \frac{1.45\text{ ns}}{\frac{1}{3}\text{ ns}} = \boxed{4.35\text{ cycles.}}$

It is recommended to terminate the line at both ends with an impedance equal to $Z_0 = 30\,\Omega$. This stops reflections at both ends resulting in faster steady state convergence.