Professor Fearing EECS120/Problem Set 8 v 1.0 Fall 2016

Due at 4 pm, Fri. Oct. 21 in HW box under stairs (1st floor Cory)MT Wed Oct 26 410-6 pm.

Closed book, closed notes. Two sides 8.5x11 inch formula sheet. Coverage PS1-PS8, Lectures 1-14.

Note: $\Pi(t) \;=\; u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

Problem 1 LTI Properties (20 pts)

[7.5 pts] a. Classify the following systems, with input $x(t)$ and output $y(t)$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+0.5 for correct, 0 for blank, -0.5 for incorrect).

System	Causal	Linear	Time-invariant
a. $y(t) = x(t)\cos(2\pi t)$			
b. $y(t) = x(t) * u(t-2)$			
c. $y(t) = 3x(t+1) + 1$			
d. $y(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$			
e. $y(t) = x(t) - \frac{1}{2}\frac{dy(t)}{dt}$			

[6 pts] b. Two of the systems above (a,b,c,d,e) are not BIBO stable. Note below which systems are not BIBO stable, and then find a bounded input $x(t)$ which gives rise to an unbounded output $y(t)$ for each of these systems.

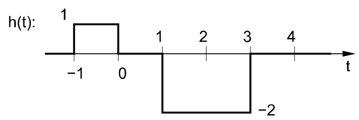
System 1:

Bounded input $x(t) =$ _________

System 2:

Bounded input $x(t) =$ _________

[6.5 pts] c. An LTI system has impulse response $h(t)$ as shown below:



Given input $x(t) = u(t+1)$. Sketch the output $y(t)$ on the grid below, noting key times and amplitudes.

2. (20 pts) Consider an LTI system (with input $x[n]$ and output $y[n]$) defined by the difference equation:

$$y[n] = -0.25x[n] + 0.5x[n-1] - 0.25x[n-2]$$

a. Determine if this system is causal and/or stable.

b. Determine the frequency response $H(e^{j\omega})$ and sketch its magnitude $|H(e^{j\omega})|$ as a function of ω. Determine the type of filter (low pass, highpass, bandpass, or bandstop) realized by this system.

c. Determine whether this system is linear phase.

d. Draw a block diagram implementing this system with delay, summation, and multiplication blocks.

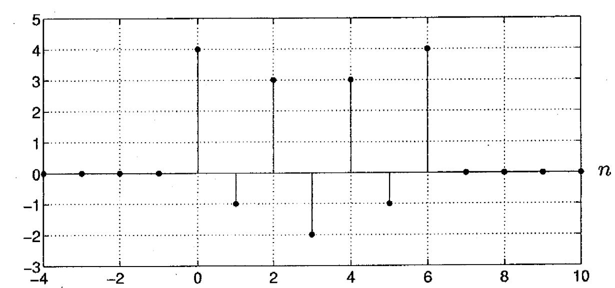
Given the sequence x[n] depicted below, determine the following:

e. $X(e^{j0})$

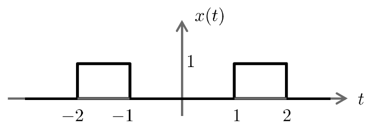
f. $X(e^{j\pi})$

g. $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$

h. $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$



3. a) (~~15~~ 7 points) Determine the Fourier transform of the continuous-time signal $x(t)$ depicted below:

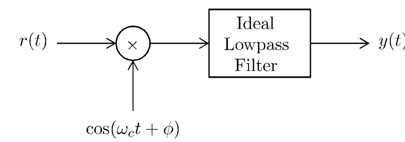


b) (~~5~~ 3 points) Sketch the phase plot for $X(j\omega)$.

~~3.~~ 4. (~~20~~ 10 points) Let $s(t)$ be a real-valued signal for which $S(j\omega) = 0$ when $|\omega| > \omega_c$. Amplitude modulation is performed to produce the signal:

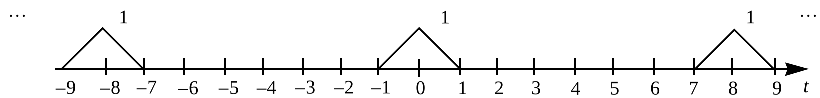
$$r(t) = s(t)\cos(\omega_c t)$$

and the demodulation scheme below is applied to $r(t)$ at the receiver. The constant ϕ represents a phase error that arises when the modulator and demodulator are not synchronized. Determine $y(t)$ assuming that the ideal lowpass filter has a cutoff frequency of ω_c and a passband gain of 2. Your answer should depend only on $s(t)$ and ϕ.



Problem ~~3~~ 5 (~~35~~ 20 points) Fourier Series

[~~10~~ 6 pts.] a) $x(t)$ is a periodic function as shown:



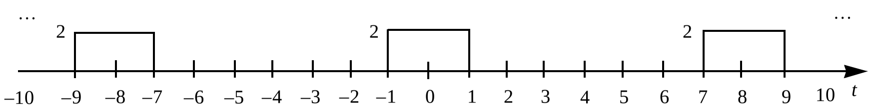
$x(t)$ can be represented as a Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$.

Find ω_0 and X_k.

$\omega_0 =$

$X_k =$

[~~10~~ 4 pts.] b) $a(t)$ is a periodic function as shown:



$a(t)$ can be represented as a Fourier Series $(t) = \sum_{k=-\infty}^{\infty} \frac{2\sin(k\pi/4)}{k\pi} e^{j\frac{k\pi}{4}t}$.

What is the time average power in $a(t)$? ________________________

What is the time average power at the fundamental frequency in $a(t)$? _____________________

Problem ~~3~~ 5 (cont.)

[~~15~~ 10 pts.] c) Consider a system whose behavior is specified by the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d^2x(t)}{dt^2} + \frac{\pi^2}{16}x(t)$$

with input $x(t)$ and output $y(t)$.

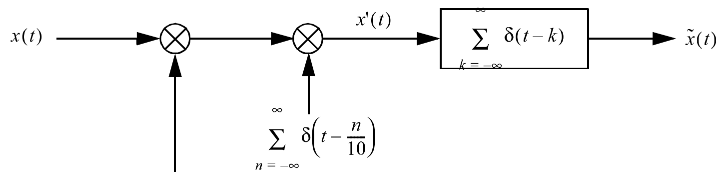
If the input to the system is the periodic function $a(t)$ from part ~~3c~~ 5b above, express the output as a Fourier Series $b(t) = \sum_{k=-\infty}^{\infty} b_k e^{j\frac{k\pi}{4}t}$. Find b_k.

$b_k =$

Bonus (2 pts.) (only applicable if you got b_k right): What is the time average power at the fundamental frequency in $b(t)$? __________________________

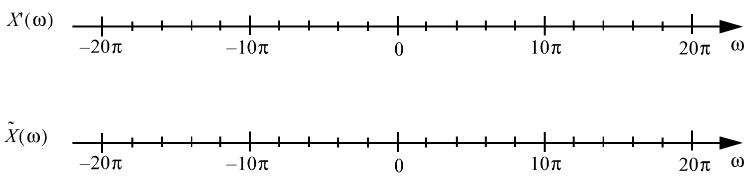
Problem ~~7~~ 6 (~~35~~ 20 points)

A system is described by the following block diagram:



$$w(t) = \begin{cases} 1 \text{ if} & -.5 \le t < .5 \\ 0 & \text{otherwise} \end{cases}$$

[~~30~~ 16 pts.] a) Let $x(t) = \cos 4\pi t$. Sketch $X'(\omega)$ and $\tilde{X}(\omega)$, labelling peak magnitude, zero crossing(s), and spacing. (Hint: $X'(\omega)$ and $\tilde{X}(\omega)$ should be real.)



[~~5~~ 4 pts.] b) What is the relationship between $\tilde{X}(\omega)$ and the 10 point DFT of $x[n] = X[k]$ (where $x[n] = x(0)\ldots x(.9)$)? Explain why. (What is the effect of not shifting the window $w(t)$ by $T/2$?)