

1. (10 pts) Partial fraction expansion (Nise 2.2)
 Find the inverse Laplace transform of the following function using partial fraction expansion:

$$\frac{s-2}{s(s+1)(s+4)^2}$$

2. (15 pts) Laplace transform review (Nise 2.2)
 For each transfer function below determine $h(t)$.

i) $H_1(s) = \frac{1}{s^2+4s+53}$ ii) $H_2(s) = \frac{s}{s^2+4s+53}$ iii) $H_3(s) = \frac{s+3}{s^2+4s+53}$
 iv) $H_4(s) = \frac{s^2}{s^2+4s+53}$ v) $H_5(s) = \frac{s^2+4}{s^2+4s+53}$

3. (10 pts) Initial value, final value (Nise 2.2)
 For each of the following Laplace transforms $Y_i(s)$ determine $y_i(t=0^+)$ and if the limit exists, $\lim_{t \rightarrow \infty} y_i(t)$:

i) $Y_1(s) = \frac{1}{s(s+3)}$ ii) $Y_2(s) = \frac{1}{s^2(s+3)}$ iii) $Y_3(s) = \frac{s+1}{s(s+3)}$
 iv) $Y_4(s) = \frac{s-3}{s(s+3)}$ v) $Y_6(s) = \frac{1}{(s+1)(s+2)s}$

4. (15 pts) Electrical circuit example (Nise 2.4)
 For the circuit in Fig. 1. below, using ideal op-amp assumptions, determine $H(s) = \frac{V_o(s)}{V_i(s)}$.

5. (15 pts) Equivalent models (Nise 2.5)
 For the translational mechanical system in Fig. 2, write the transfer function relating input force $f(t)$ to output velocity $\dot{x}_2(t)$.

6. (20 pts) Equivalent electrical circuit (Nise 2.9)
 Draw the equivalent electrical circuit for the system in Fig. 2, (with voltage corresponding to force, and current corresponding to velocity $\dot{x}_2(t)$), and re-derive the transfer function from voltage input to current output for the circuit to verify that it is equivalent to the transfer function found in problem 5 above.

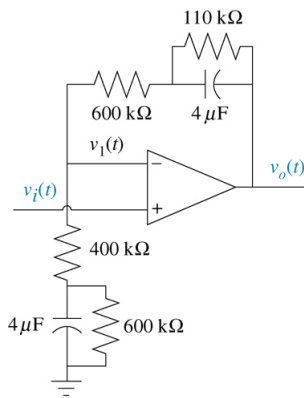


Fig. 1

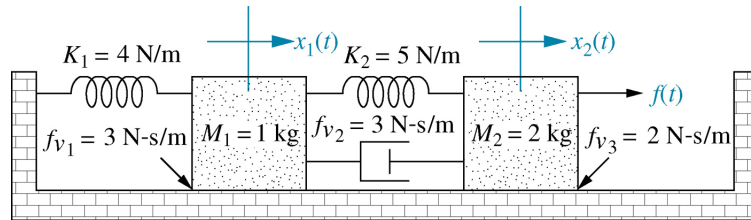


Fig. 2

7. (15 pts) Linearization (Nise 2.11)
 A system is described by $f(t) = m\ddot{x}(t) + b\dot{x}(t) + f_s(x, t)$, where f_s is the force from a non-linear spring. The spring is defined by $x_s(t) = 1 - e^{-f_s(t)}$ where $x_s(t)$ is the spring displacement. Find the transfer function $X(s)/F(s)$ for small excursions around $f(t) = 1$.