Due at 1700, Fri. Feb. 13 in homework box under stairs, first floor Cory .

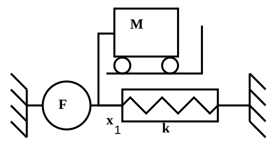
Note: up to 2 students may turn in a single writeup. Reading Nise 4.

1. (20 pts) Linearization

A magnetic actuator has force given by $F = \frac{\mu}{(d_o - x_1)^2} i^2$ where d_o is the nominal magnetic gap, and i is solenoid current. The magnetic actuator has mass M and has a return spring with non-linear stiffness $F_k = kx_1^2$ and can be modelled as shown below.

a. Write the dynamic equations in state space form $\dot{\mathbf{x}} = f(\mathbf{x}, u)$, with x_1 and $\dot{x}_1$ the states, and i the input.

b. Write the dynamic equations in state space form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$ for the system linearized about a non-zero operating point $i = i_o$, $x_1 = x_o$, and $x_2 = \dot{x}_1 = 0$.



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2. (20 pts) 2nd order step response (Nise 4.6)

A memory system can be made using a mechanical head positioning system to read data stored on a surface, e.g. the IBM millipede. The head positioning system can be approximately modelled by the transfer function from applied force $F(s)$ to output position $X(s)$:

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Assume that $m = 10^{-11} kg$, $b = 2 \times 10^{-6} N \cdot sec \cdot m^{-1}$ and $k = 2N \cdot m^{-1}$.

a) Find the pole locations and sketch in the s-plane, and find ζ, ω_n, and ω_d.

b) For a 1 μN step, determine peak overshoot (μm), time to peak T_p, and time for settling to within 1 nm of final value.

c) Repeat b) for 100 nN step.

d) Sketch $x(t)$ noting peak value, time to peak, 2% settling time, and location of relative maxima/minima.

3. (15 pts) Second order poles (Nise 4.6)

For each pair of second-order system step response specifications, find the location of the second order pair poles.

a. %OS = 20%; $T_s = 4$ seconds.

b. %OS = 40%; $T_s = 4$ seconds.

b. $T_p = 2$ sec; $T_s = 4$ seconds.

4. (25 pts) Time Domain Solution (Nise 4.10)

Given the following state-space representation find $y(t)$ using the Laplace transform method. Here $u(t)$ is the unit step.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) , \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{and } \ y = [1 \quad 3]\mathbf{x}$$

5. (20 pts) Time Domain Solution (Nise 4.11)

Find $\mathbf{x}(t)$ and $y(t)$ using convolution (4.109) for the following system with unit step input $u(t)$:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) , \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{and } \ y = [1 \quad 3]\mathbf{x}$$