EECS C128/ ME C134

Final

Wed. Dec. 14, 2011

0810-1100 am

Name:____________________________

SID:_____________

- Closed book. One page, 2 sides of formula sheets. No calculators.
- There are 8 problems worth 100 points total.

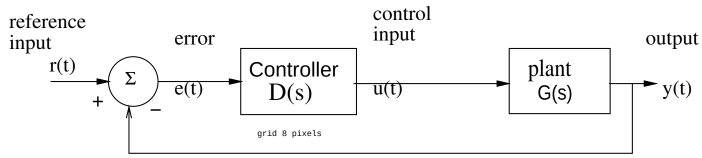
| Problem | Points | Score |
|---|---|---|
| 1 | 16 | |
| 2 | 12 | |
| 3 | 8 | |
| 4 | 16 | |
| 5 | 11 | |
| 6 | 16 | |
| 7 | 13 | |
| 8 | 8 | |
| Total | 100 | |

| | |
|---|---|
| $\tan^{-1}\frac{1}{2} = 26.6°$ | $\tan^{-1} 1 = 45°$ |
| $\tan^{-1}\frac{1}{3} = 18.4°$ | $\tan^{-1}\frac{1}{4} = 14°$ |
| $\tan^{-1}\sqrt{3} = 60°$ | $\tan^{-1}\frac{1}{\sqrt{3}} = 30°$ |
| $\sin 30° = \frac{1}{2}$ | $\cos 60° = \frac{\sqrt{3}}{2}$ |

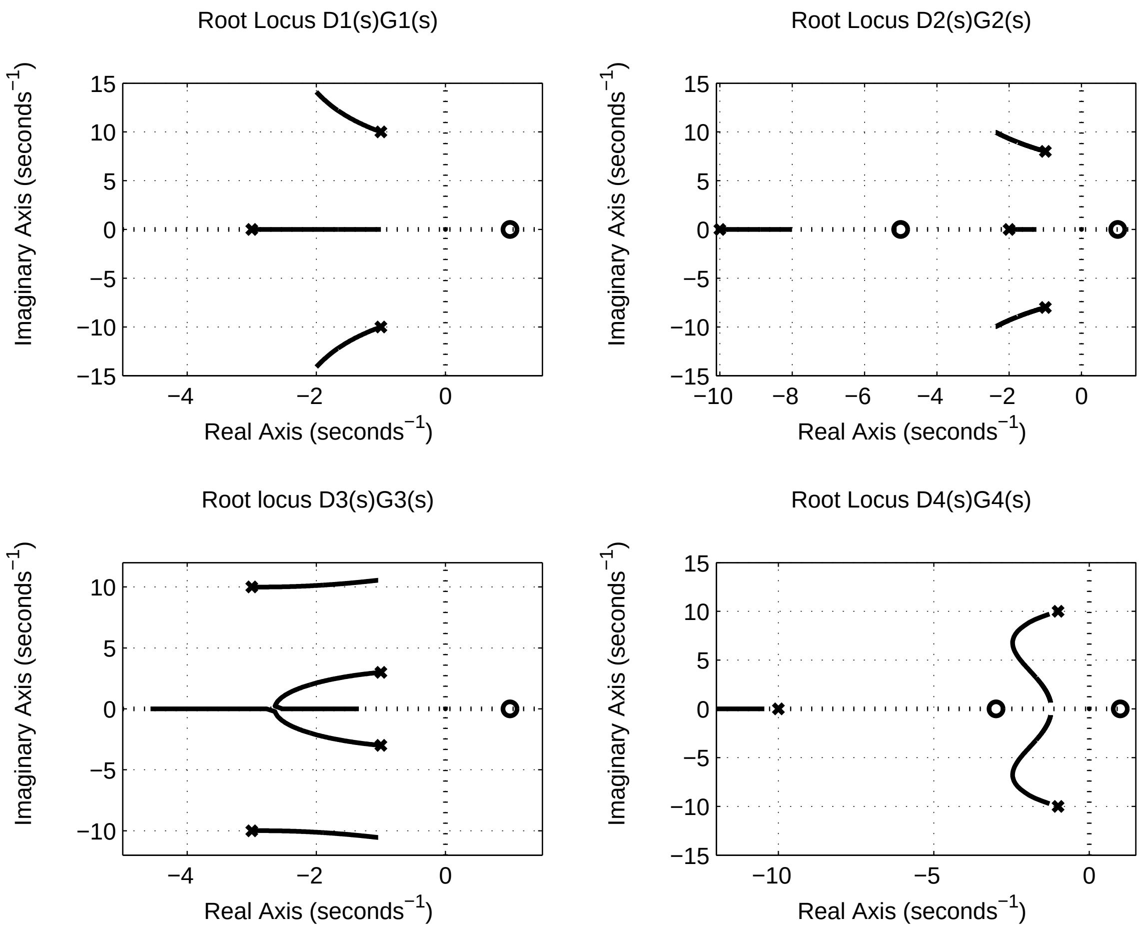
| | |
|---|---|
| $20\log_{10} 1 = 0dB$ | $20\log_{10} 2 = 6dB$ |
| $20\log_{10}\sqrt{2} = 3dB$ | $20\log_{10}\frac{1}{2} = -6dB$ |
| $20\log_{10} 5 = 20db - 6dB = 14dB$ | $20\log_{10}\sqrt{10} = 10$ dB |
| $1/e \approx 0.37$ | $1/e^2 \approx 0.14$ |
| $1/e^3 \approx 0.05$ | $\sqrt{10} \approx 3.16$ |

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of 'F' and a letter will be written for your file and to the Office of Student Conduct.

**Problem 1 (16 pts)**




For the above system, the root locus is shown for 4 different controller/plant combinations, $D_1(s)G_1(s), ..., D_4(s)G_4(s)$. (Note: the root locus shows open-loop pole locations for $D(s)G(s)$, and closed-loop poles for $\frac{DG}{1+DG}$).




[4 pts] a) For each set of open-loop poles and zeros given above, choose the best corresponding open-loop Bode plot W,X,Y, or Z from the next page:

(i) $D_1(s)G_1(s)$: Bode Plot ____

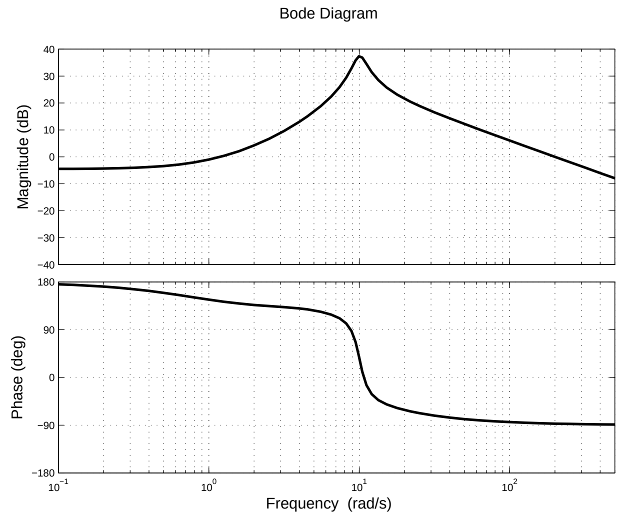
(ii) $D_2(s)G_2(s)$: Bode plot ____

(iii) $D_3(s)G_3(s)$: Bode plot ____

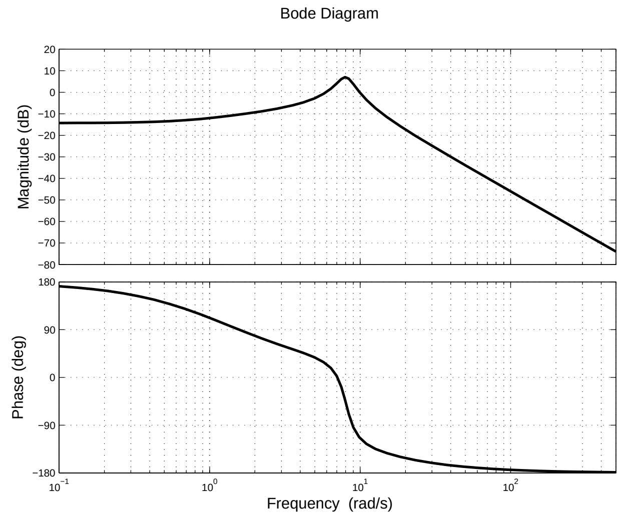
(iv) $D_4(s)G_4(s)$: Bode Plot ____

**Problem 1, cont.**

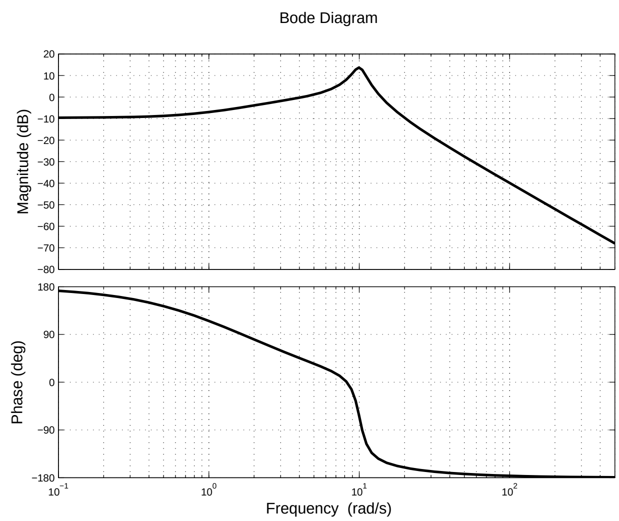
The open-loop Bode plots for 4 different controller/plant combinations, $D_1(s)G_1(s), ..., D_4(s)G_4(s)$ are shown below.



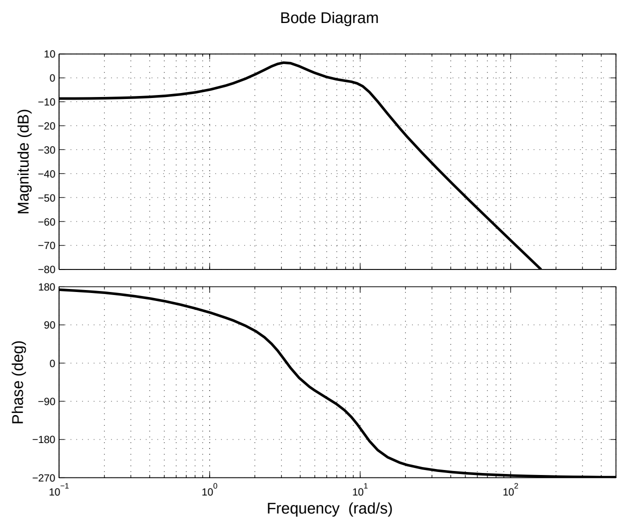

W




X




Y




Z

[8 pts] b) For each Bode plot, estimate the phase and gain margin:

(i) Bode plot W: phase margin ____ (degrees) at $\omega =$ ____
Bode plot W: gain margin ___ dB at $\omega =$ ____

(ii) Bode plot X: phase margin ____ (degrees) at $\omega =$ ____
Bode plot X: gain margin ___ dB at $\omega =$ ____

(iii) Bode plot Y: phase margin ____ (degrees) at $\omega =$ ____
Bode plot Y: gain margin ___ dB at $\omega =$ ____

(iv) Bode plot Z: phase margin ____ (degrees) at $\omega =$ ____
Bode plot Z: gain margin ___ dB at $\omega =$ ____

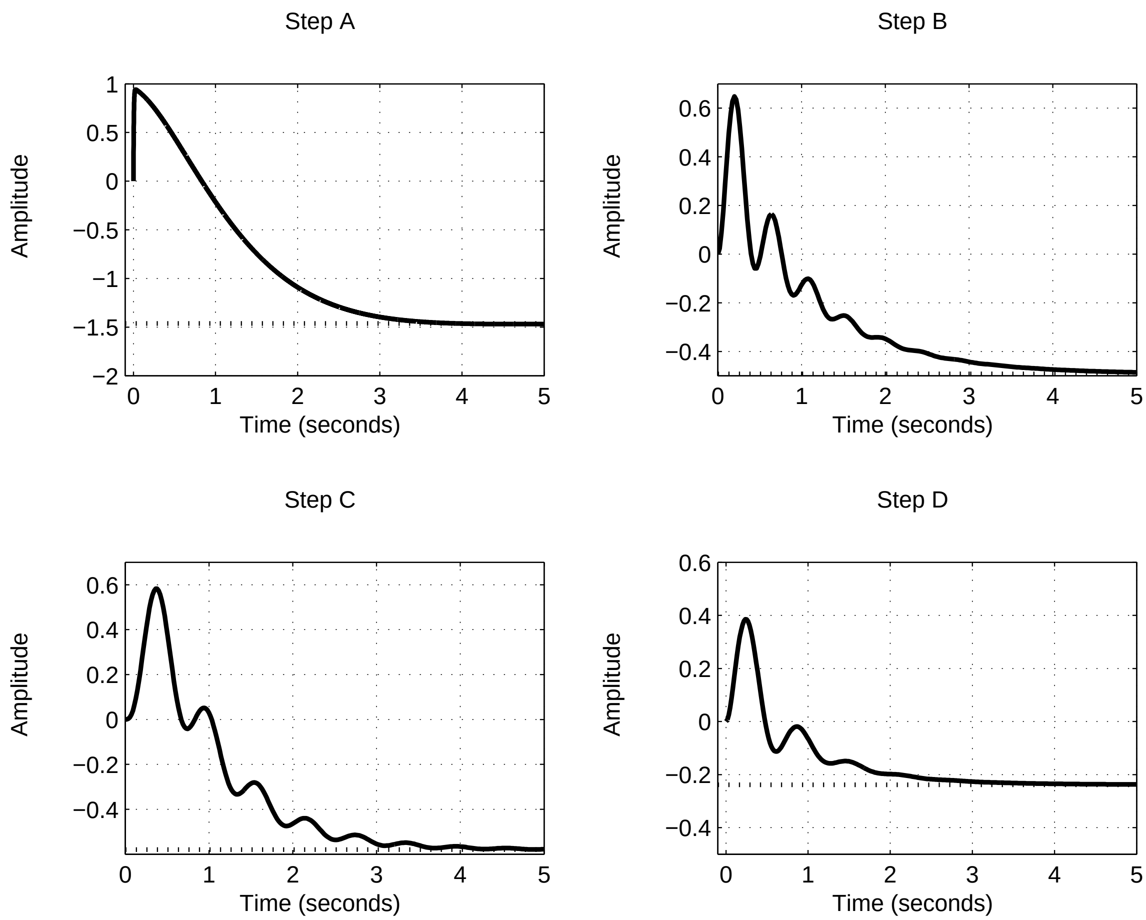
**Problem 1, cont.**
s
c) For each closed loop controller/plant with root locus as given in part a), choose the best corresponding closed-loop step response (A-D)

(i) $D_1(s)G_1(s)$: step response ____

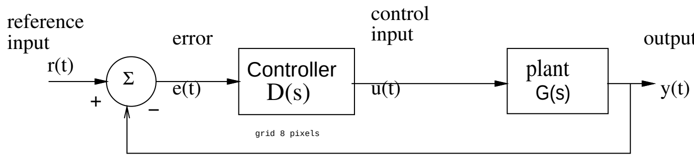
(ii) $D_2(s)G_2(s)$: step response ____

(iii) $D_3(s)G_3(s)$: step response ____

(iv) $D_4(s)G_4(s)$: step response ____



**Problem 2 (12 pts)**

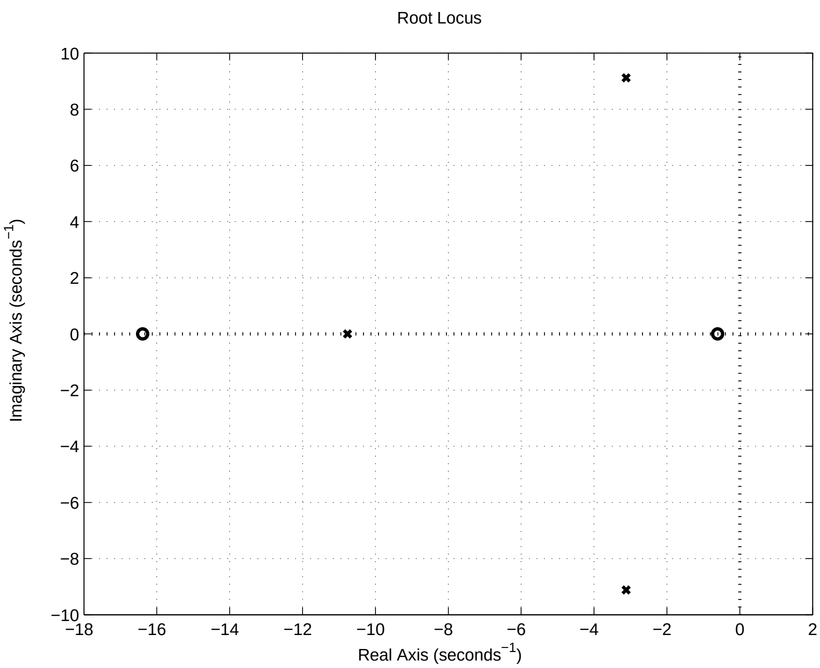



You are given the open loop plant $G(s) = \frac{100}{s^2+17s+60}$. The system is to be controlled using a lag controller. with $D(s) = \frac{s+10}{s+\alpha}$.

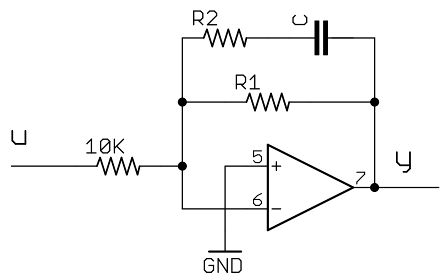
Given: the roots of $s^3 + 17s^2 + 160s + 1000 \approx (s + 10.7)(s + 3.11 + 9.1j)(s + 3.11 - 9.1jj)$

[8 pts] a) Sketch the positive root locus as $\alpha$ varies, noting asymptote intersection point and angle of departure.

[4 pts] b)
(i) approximate asymptote intersection point $s =$ _______
(ii) approximate angle of departure for the poles: ___ ___ ___



**Problem 3 (8 pts)**
Consider the following circuit for a *lag* compensator:




[4 pts] a) Find the transfer function $\frac{Y(s)}{U(s)}$

[4 pts]b) Suppose the desired behaviour of this circuit is that the (asymptotic) phase response is $-90°$ between $100 rad/s$ and $1000 rad/s$. At every other frequency the phase response should be greater than $-90°$. If $C = 1\mu F$, what are the resistor values, $R_1$ and $R_2$?

**Problem 4 (16 pts)**

You are given the following plant

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}(t), \quad y = [4\ 1]\ \mathbf{x} \qquad \mathbf{x}(t=0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[2 pts] a) Determine if the system is controllable and observable.

[4 pts] b) Find feedback gains $K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$ such that with control $\mathbf{u} = K(\mathbf{r} - \mathbf{x})$, the controller has closed loop poles at -2 and -4.

$k_1 =$ _______

$k_2 =$ _______

[2 pts] c) Draw a block diagram of the controlled system using integrators, summing junctions, and scaling functions. (Every signal should be a scalar, no vectors.)

**Problem 4, cont**

You are given the following plant

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1\mathbf{u} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}(t), \quad y = [4\ 1]\ \mathbf{x} \qquad \mathbf{x}(t=0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[2 pts] d) Determine if the system $\{A_1, B_1, C_1\}$ is controllable and observable.

[4 pts] e) Find feedback gains $K = [k_1 k_2]$ such that with control $u = K(\mathbf{r} - \mathbf{x})$, the controller has closed loop poles at -2 and -4.

$k_1 =$ _______

$k_2 =$ _______

[2 pts] f) Draw a block diagram of the controlled system using integrators, summing junctions, and scaling functions. (Every signal should be a scalar, no vectors.)

**Problem 5 (11 pts)**

[3 pts] a) Given the following system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \qquad\qquad y = C\mathbf{x}$$

The state is transformed by a non-singular $P$ such that $\bar{\mathbf{x}} = P\mathbf{x}$. Thus $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{\mathbf{x}}$.

Find $\bar{A}$ $\bar{B}$ $\bar{C}$ in terms of $A, B, C, P$:

$\bar{A}$ =: _______

$\bar{B}$ =:_______

$\bar{C}$ =:_______

[4 pts] b) You are given the following system:

$$\begin{bmatrix} -2 & -1 \\ -9 & 6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [3 \ \ -1] \ \mathbf{x}$$

Find the transformation $P$ and $\bar{A}$ such that $\bar{A} = P^{-1}AP$ is in **modal canonical** (diagonal) form.

$$P = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right] \qquad \bar{A} = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

[4 pts] d) Find $e^{\bar{A}t}$ and $e^{At}$:

$$e^{\bar{A}t} = \left[\begin{array}{c|c} \qquad\qquad & \qquad\qquad \\ \hline & \end{array}\right] \qquad e^{At} = \left[\begin{array}{c|c} \qquad\qquad\qquad\qquad & \qquad\qquad\qquad\qquad \\ \hline & \end{array}\right]$$

**Problem 6 (16 pts)**

The simplified dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2}$$

$y$ is the position of the ball; $u$ is the current through the coil (in amps); $c$ is a constant that describes the magnetic force between the coil and the ball. The system is linearized at equilibrium position $y_0$ with equilibrium input $u_e$:

$$y = y_0 + \delta y \qquad\qquad u = u_e + \delta u$$

$$u_e = y_0\sqrt{\frac{mg}{c}}$$

The linearized state space equations are:

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}0 & 1\\ \frac{2g}{y_0} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{-2}{y_0}\sqrt{\frac{cg}{m}}\end{bmatrix}\delta u\\ &= \begin{bmatrix}0 & 1\\ 200 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -20\end{bmatrix}\delta u\\ \delta y &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\end{aligned}$$

[1 pts] (a) What are the units of $c$? Assume that all other quantities are SI standard (kilograms, meters, amps, etc).

[4 pts] (b) We want to build a regulator to keep the ball at $y_0$. We will design a state feedback scheme, $\delta u = -Kx$, so that the poles of the linearized system are at $s = -20, -12$. Find $K$.

$K = [\qquad\qquad\qquad\qquad]$

[3 pts] (c) Assume that you can directly access $x_1$ and $x_2$. You build your regulator as described above, and it successfully levitates the ball. You decide to try levitating four steel balls at the same time. Now $m$ is four times bigger; everything else stays the same. Is your linearized system still stable? Will the steel balls be stable at $y_0$? Why or why not?

[4 pts] (d) Return to the one-ball problem. Assuming that the only accessible output of the plant is $y$, you will need an observer in order to implement state feedback. Draw a block diagram of your regulator system. Use one block labelled "Plant", with input $u$ and output $y$; one block labelled "Observer", with output $\hat{x}_1$ and $\hat{x}_2$ (you decide what the input should be); static gains; and addition junctions. Every signal should be scalar (no vectors). Label as many signals as you can. (Note that you're not being asked to design the observer gain).

[2 pts] (e) What are some sensible values for the poles of the observer?

[2 pts] (f) Does using an observer introduce any new problems if you try to levitate four balls, as in (c)?

**Problem 7. (13 pts)**

You are given a continuous time plant described by the following state equation.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu$$

The system is driven with a D/A converter such that $u(t) = u[n]$ for $nT < t < nT + T$. (That is, the input is held constant, by a zero-order hold equivalent.) Every $T$ seconds the state of the system is measured with an A/D converter, that is $\mathbf{x}[n] = \mathbf{x}(nT)$.

Recall that the solution for the continuous time system is given by:

$$\mathbf{x}(t) = e^{A(t-t_o)}\mathbf{x}(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}Bu(\tau)d\tau. \tag{1}$$

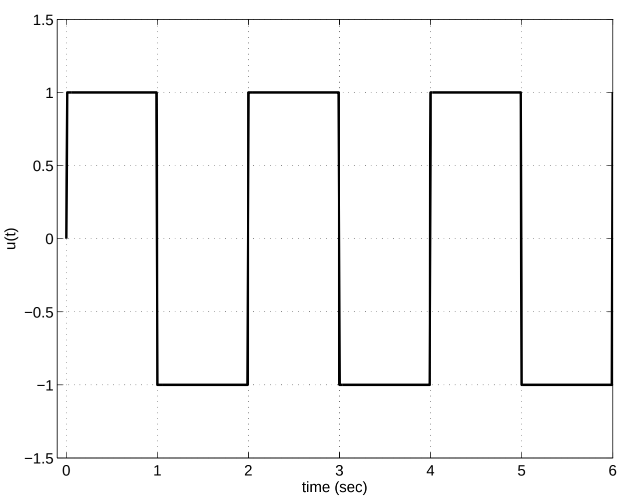
[3 pts] a) For the zero input response, $(\mathbf{x}(t=0) = \mathbf{x}_o, u(t) = 0)$
Find:(in terms of $A$ and $\mathbf{x_o}$)

$\mathbf{x}[0] =:$ _______
$\mathbf{x}[1] =:$ _______
$\mathbf{x}[n] =:$ _______

[3 pts] b) For the zero state response, $(\mathbf{x}(t=0) = \mathbf{0})$
Find:(in terms of $A, B, u$)

$\mathbf{x}[0] =:$ _______
$\mathbf{x}[1] =:$ _______
$\mathbf{x}[n] =:$ _______

[3 pts] c) Consider the CT system $\dot{x} = -x + u$. With $u(t)$ as shown, sketch $x(t)$ for $0 < t < 6$sec with initial condition $x(0) = 0$.




s

d) Let $T = 1sec$. For the CT system $\dot{x} = -x + u$, $x_o = 0$, with zero-order hold on input, determine the value of $x$ at following steps (Answers may be left in terms of $e$.) Consider $u[0] = 1, u[1] = -1$ etc.

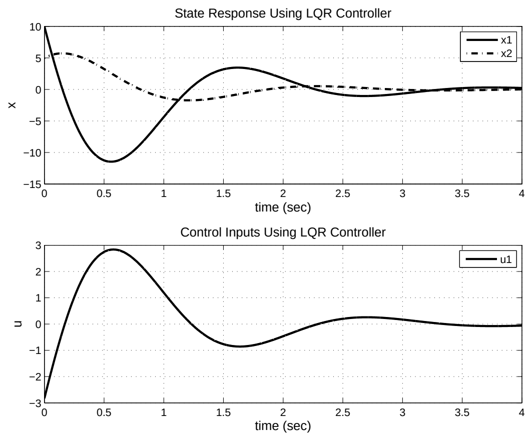
$\mathbf{x}[0] =:$ _______
$\mathbf{x}[1] =:$ _______
$\mathbf{x}[2] =:$ _______
$\mathbf{x}[3] =:$ _______

## Problem 8 (8 pts)

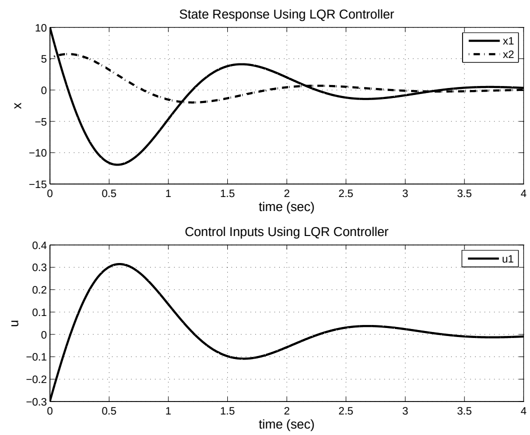
You are given the following plant

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} -2 & -10 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \quad y = [1\ 4]\ \mathbf{x} \qquad \text{and} \quad \mathbf{x}(t=0) = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

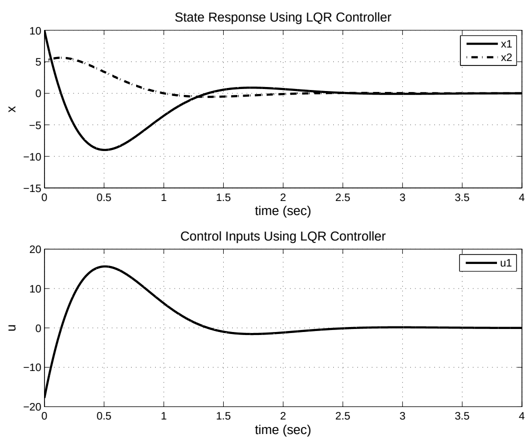
The LQR method is used to find the linear control $u = -Kx$ which minimizes the cost $J = \int_0^\infty (x^T Qx + u^T Ru)dt$, where $Q$ and $R$ are positive semi-definite. Four responses of the closed-loop system $A$,$B$,$C$,$D$ are shown below for different choices of $Q, R$. Match the plots with $Q$ $R$ weights below.



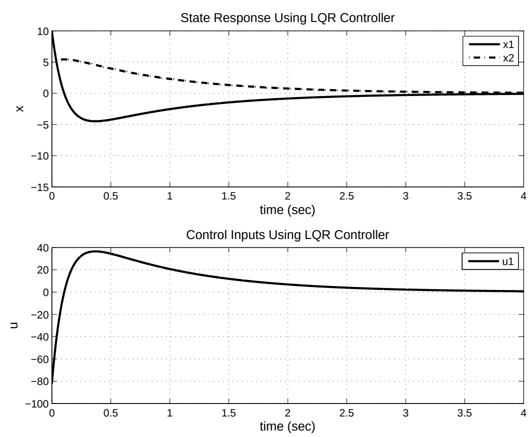

A




B




C




D

[4 pts] For each plot of $\mathbf{x}(t)$ and $u(t)$ choose the appropriate $Q, R$ pair, writing in the appropriate letter.

| | |
|---|---|
| $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R = [1]$ <br> Plot: ____ | $Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} R = [1]$ <br> Plot: ____ |
| $Q = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \quad R = [1]$ <br> Plot: ____ | $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R = [10]$ <br> Plot: ____ |