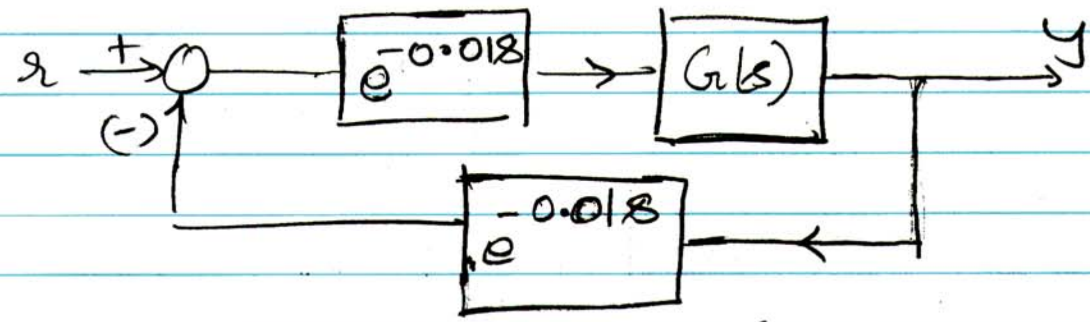
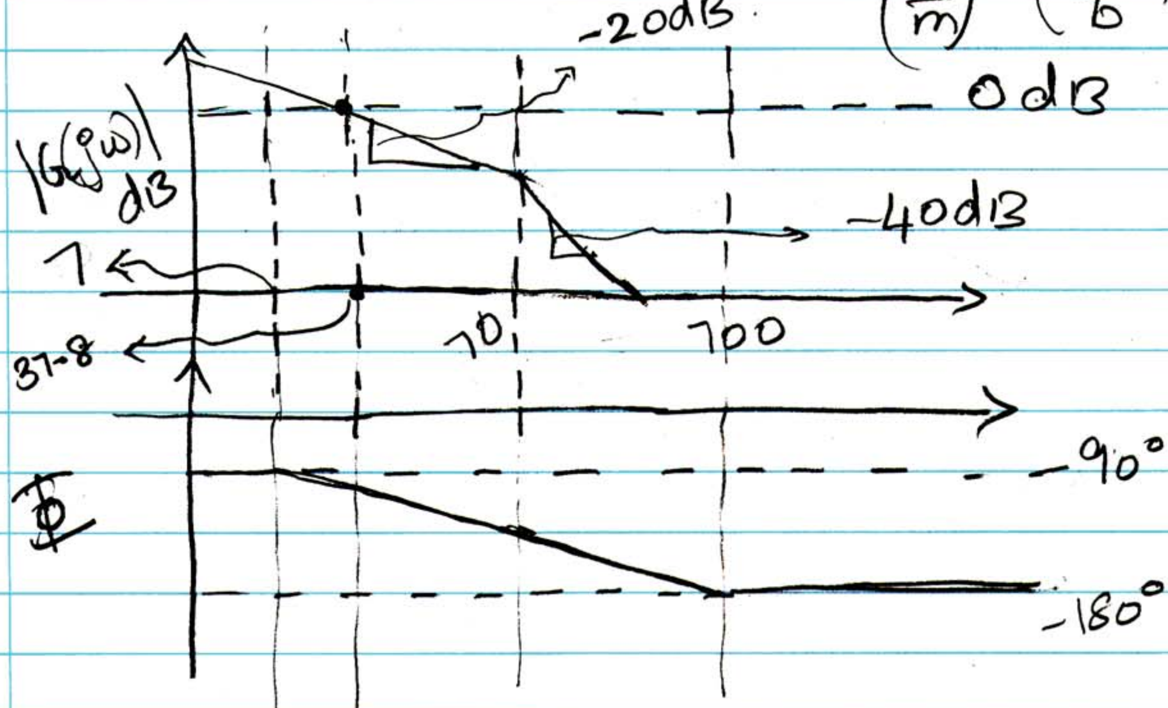
EE128/ME134 PS8 SOLUTIONS

1 (a) $G(s) = \dfrac{3000}{s(s+b/m)}$



OLTF: $e^{-0.028}\, G(s)$.

(b) Without Delay: $G_0 = G(s) = \dfrac{3000}{\left(\frac{b}{m}\right) s \left(\frac{sm}{b}+1\right)}$



$\Rightarrow$ $\boxed{G_M = \infty \; ; \; \phi_M = 61.68°}$

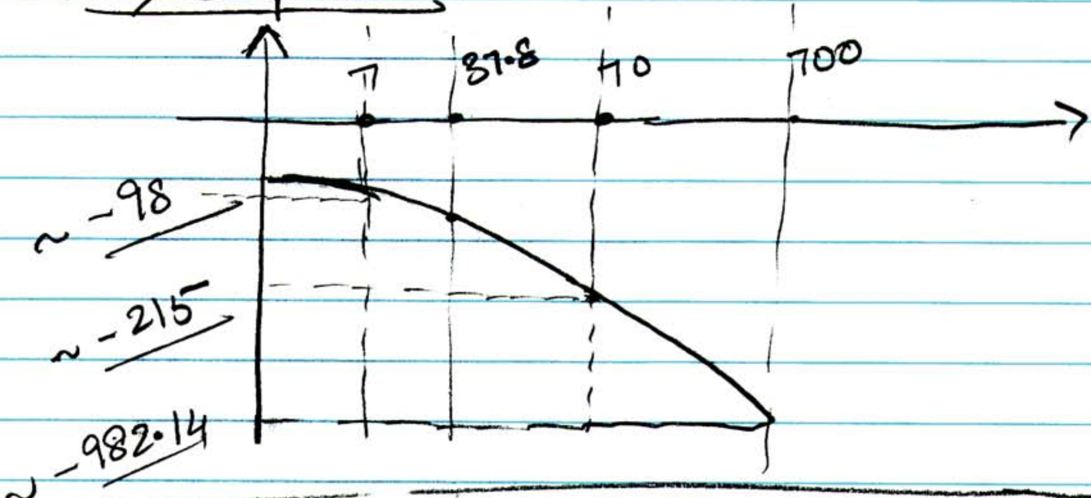
EE128 PS8 SOLUTIONS

With Delay: $G_o(s) = e^{-0.02s} G(s)$

$$= \frac{e^{-0.02s} \cdot 3000}{\left(s \frac{b}{m}\right)\left(\frac{s}{b/m} + 1\right)}$$

$|G_o(s)| = |G(s)| \Rightarrow$ magnitude plot remains the same.

Phase plot:



$$\boxed{G_M = 2.73 \text{ dB}, \quad \Phi_m = 18.45^\circ}$$

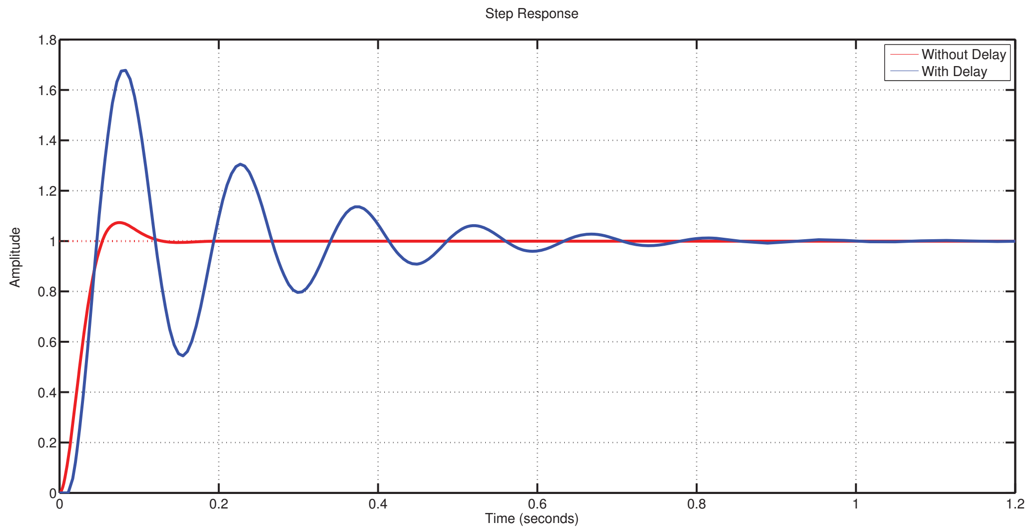


Figure 1: Step response of the closed loop system with and without delays

2. a. Bode plot:

$$G(s) = \frac{K(s+20)(s+25)}{s(s+6)(s+9)(s+14)} = \frac{K(20)(25)(\frac{s}{20}+1)(\frac{s}{25}+1)}{(6)(9)(14)\, s(\frac{s}{6}+1)(\frac{s}{9}+1)(\frac{s}{14}+1)}$$

	Magnitude	Phase
DC:	$20\log\left(\frac{K(20)(25)}{(6)(9)(14)}\right)$ assume $K=1$ $= 20\log\left(\frac{20(25)}{6(9)(14)}\right) = -3.6\text{ dB}$	$0°$
$\frac{1}{s}$:	-20 dB/dec with 0dB at $\omega=1$	$-90°$
$\frac{1}{(\frac{s}{6}+1)}$:	-20 dB/dec at $\omega=6$	$-45°$/dec from $\omega=0.6$ to 60
$\frac{1}{(\frac{s}{9}+1)}$:	-20 dB/dec at $\omega=9$	$-45°$/dec from $\omega=0.9$ to 90
$\frac{1}{(\frac{s}{14}+1)}$:	-20 dB/dec at $\omega=14$	$-45°$/dec from $\omega=1.4$ to 140
$(\frac{s}{20}+1)$:	$+20$ dB/dec at $\omega=20$	$+45°$/dec from $\omega=2$ to 200
$(\frac{s}{25}+1)$:	$+20$ dB/dec at $\omega=25$	$+45°$/dec from $\omega=2.5$ to 250

Bode plot attached

b. Calculating K for $\Phi_M = 30°$:

Need K such that for ω_D where $\phi = -180+30 = -150$, the magnitude is 0dB

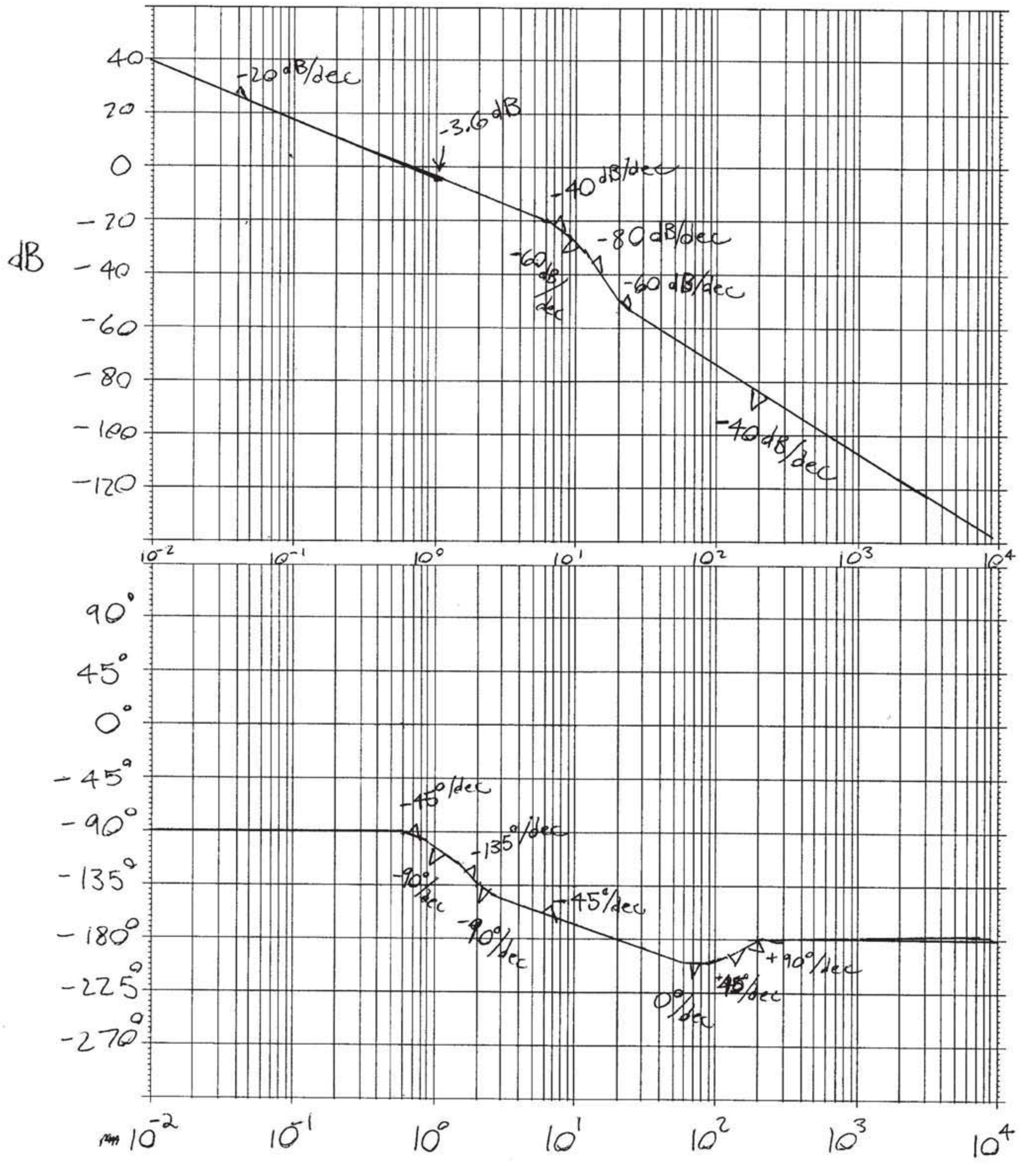
From Bode plot this is approximately $\omega_D \approx 4$ rad/s

Can also be directly calculated:

$$G(j\omega) = \frac{(j\omega+20)(j\omega+25)}{(j\omega)(j\omega+6)(j\omega+9)(j\omega+14)}$$

$$\measuredangle G(j\omega_D) = \tan(-150°)$$

$$\boxed{\omega_D = 4.67 \text{ rad/s}}$$



dB
40
20
0
-20
-40
-60
-80
-100
-120
-20 dB/dec
-3.6 dB
-40 dB/dec
-80 dB/dec
-60 dB/dec
-60 dB/dec
-40 dB/dec
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
90°
45°
0°
-45°
-90°
-135°
-180°
-225°
-270°
-45°/dec
-135°/dec
-90°/dec
-90°/dec
-45°/dec
0°/dec
+45°/dec
+90°/dec
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

Now need to shift magnitude such that the magnitude of $G(j\omega)$ is 0 dB (gain of 1) at $\omega_B = 4.67$ rad/s:

Curret magnitude at $\omega_B = 4.67$ rad/s:

$$|G(j(4.67))| = 0.0985 \Rightarrow -20.1\text{ dB}$$

So need to multiply by K such that $|G(j(4.67))| = 1$ (gain ~~shifts~~ up by 20.1 dB)

$$K = \frac{1}{|G(j(4.67))|} = \frac{1}{0.0985} = 10.15 \Rightarrow 20.1\text{ dB}$$

$$\boxed{K = 10.15}$$

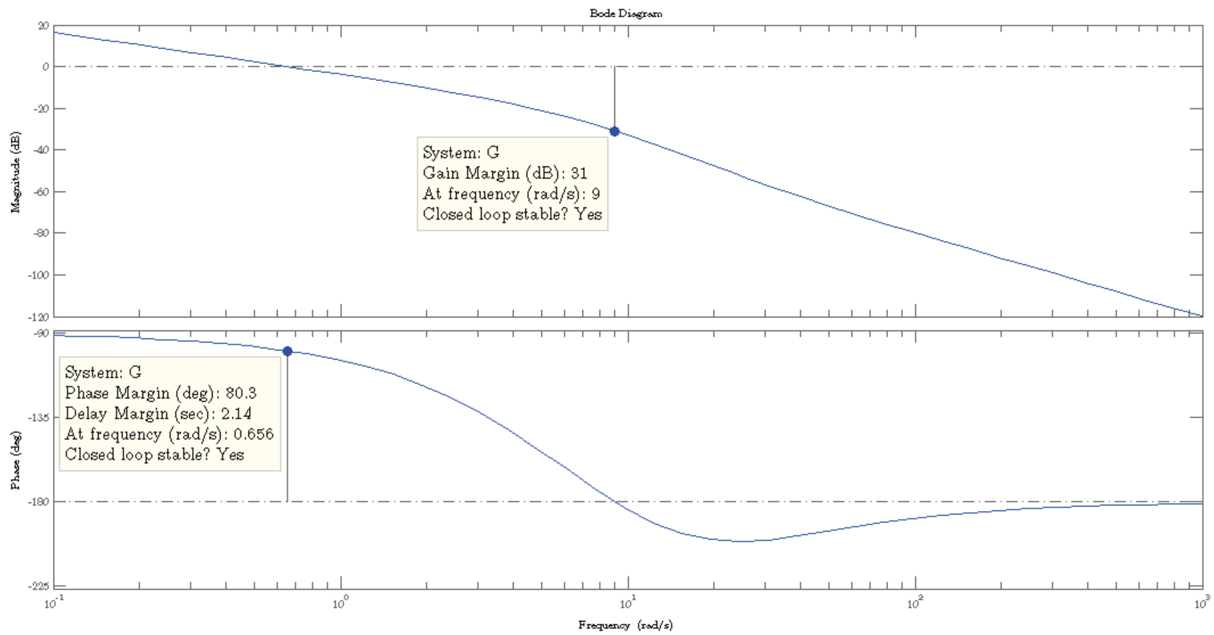
Bode plots before and after attached. Note the magnitude shift and that the phase remains unchanged.

C. $\Phi_M = 30^\circ = \tan^{-1} \dfrac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$

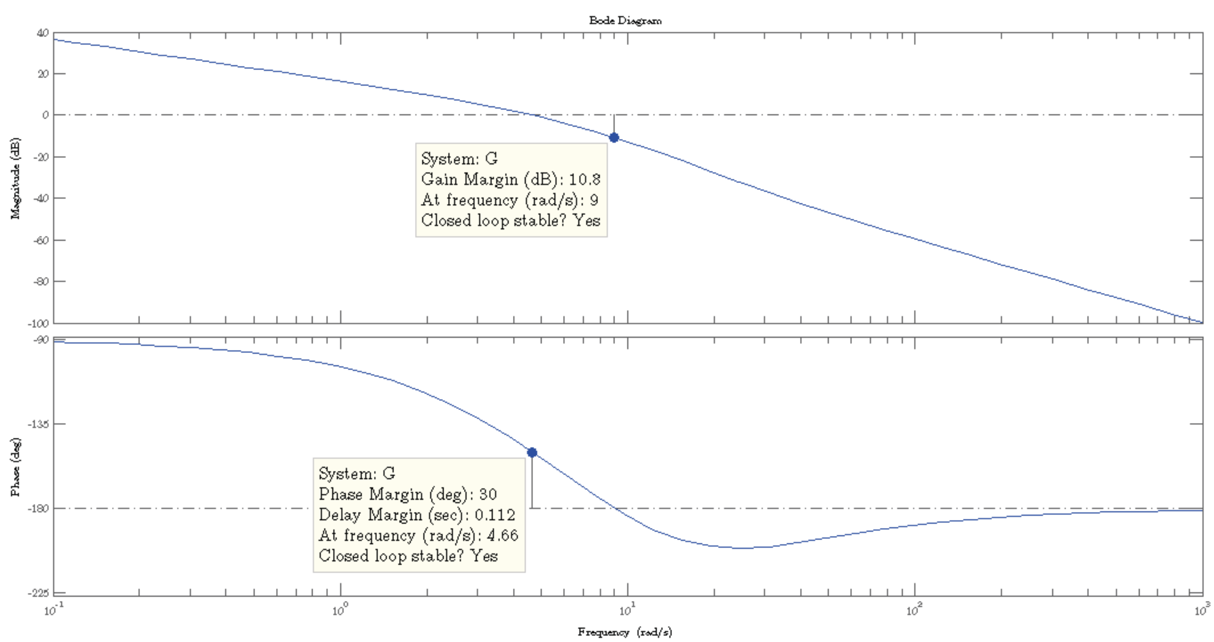
$$\boxed{\zeta = 0.269}$$

Step response attached

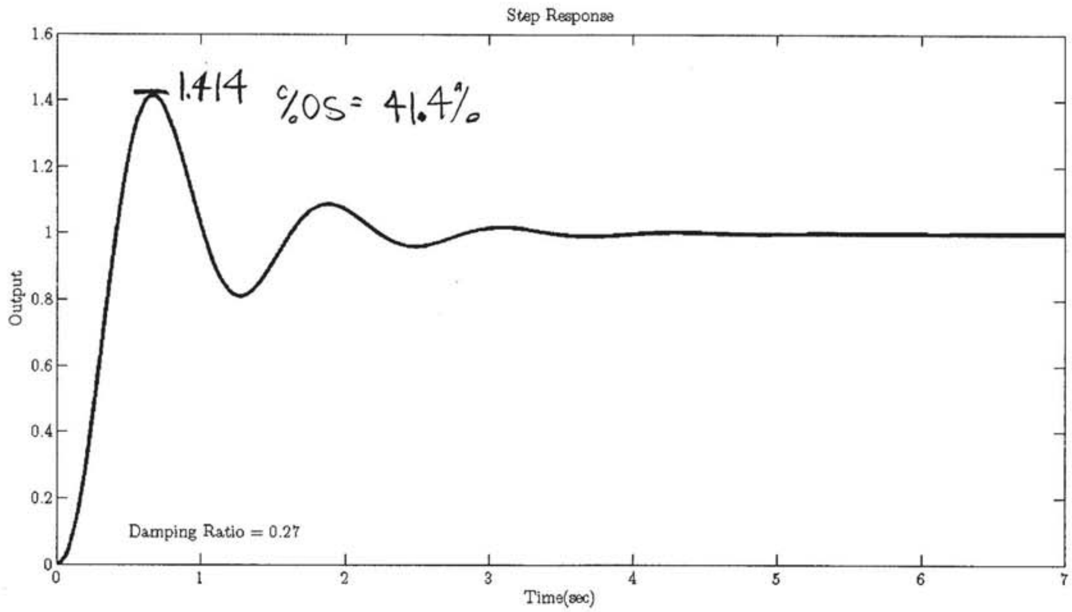
Before Gain



After Gain



Step Response



$$\zeta = \frac{-\ln\left(\frac{OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{OS}{100}\right)}} = 0.27$$

Prob 3 starts.

(3) <u>Lag</u> <u>Compensation</u>:

$$G = \frac{k(s+4)}{(s+2)(s+6)(s+8)}.$$

- choosing gain k.

we want $k_p = G(0) = \frac{4k}{12 \times 8} = \frac{k}{24} = 100$

$\Rightarrow \boxed{k = 2400.}$

- <u>Designing</u> <u>Lag</u> <u>Compensation</u>:

$$G_c(s) = \left(\frac{s/\omega_u + 1}{s/\omega_l + 1}\right)$$

where ω_u is upper cut off frequency

and ω_l is lower cutoff frequency.

Required $\phi_M = 40°$

set $\phi_M = 40° + 12° = 52° = 0.91$ rad.

$\Rightarrow \angle G(j\omega_p) = \underbrace{0.91}_{\phi_M} - \pi = -2.23 \text{ rad} = -128°$

$\Rightarrow \omega_p \approx 11.8$ rad/sec

$$\Rightarrow |G(j\omega_p)| = \left|\frac{2400\,(j11.8+4)}{(j11.8+2)(j11.8+6)(j11.8+8)}\right|$$

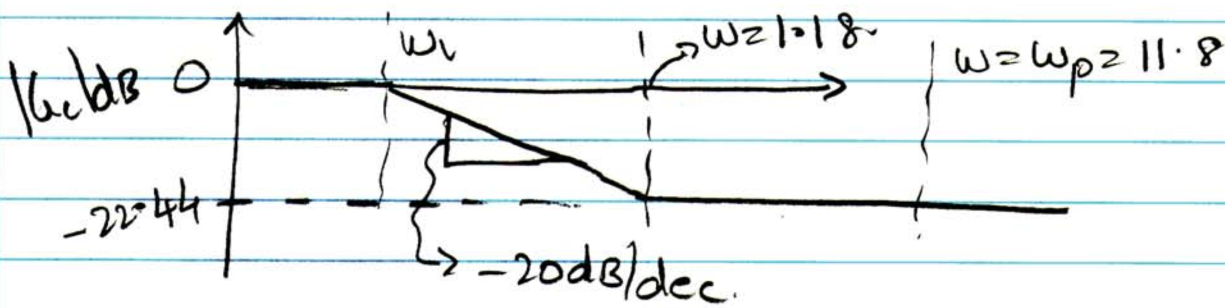
$\Rightarrow |G(j\omega_p)| = 13.24 = 22.44$ dB.

$\Rightarrow$ we want $|G_c|_{dB}$ to be -22.44 dB

at $\omega = \omega_p = 11.8$

choose $\omega_u = \frac{11.8}{10} = 1.18$ rad/sec. (standard rule)

$\Rightarrow |G_c|_{dB}$ looks like the following:



Choosing ω_l:

With a -20dB/dec from $\omega_l \rightarrow \omega_u$

we have, $22.44 = 20\log_{10}(\omega_u - \omega_l) - 20\log_{10}\omega_l$

$$22.44 = 20\log_{10}\left(\frac{1.18-\omega_l}{\omega_l}\right)$$

$$\Rightarrow \left(\frac{22.44}{20}\right) = \log_{10}\left(\frac{1.18-\omega_l}{\omega_l}\right)$$

$$10^{(22.44/20)} = \frac{1.18-\omega_l}{\omega_l} \Rightarrow \boxed{\omega_l \approx 0.083}$$

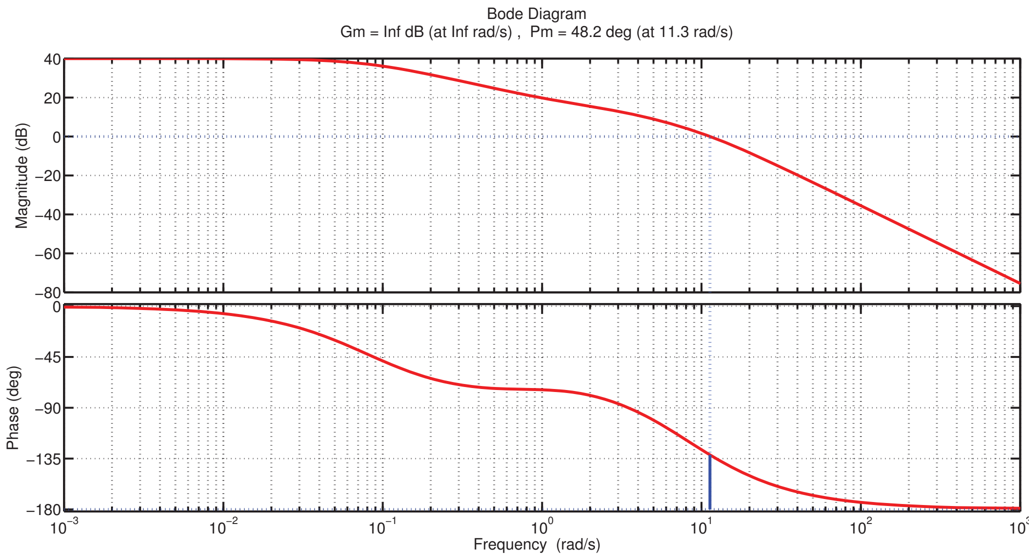


Figure 2: Bode Plot of the compensated system

4. Design a lead compensator for $K_v = 4$ and a phase margin of $40°$.

$$G(s) = \frac{K}{s(s+3)(s+15)(s+20)}$$

Satisfy steady state error with gain K:

$$K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} \frac{s\,K}{s(s+3)(s+15)(s+20)} = \frac{K}{3(15)(20)} = 4$$

So $K = 3600$

$$G(s) = \frac{3600}{s(s+3)(s+15)(s+20)}$$

Bode plots for $K = 3600$ attached.

Gain Margin $= 7.99$ dB at $\omega = 4.87$ rad/s

Phase Margin $= 27.9°$ at $\omega = 2.83$ rad/s

Want phase margin $\phi_D = 40°$ with compensator of form: $G_c(s) = \frac{1}{\beta} \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$

Accounting for shift in phase margin frequency by compensator we want to add $\phi_c = 40° - 27.9° + \underbrace{16}° = 28.1°$ of phase with compensator

this number was selected after designing multiple times

Finding β:

$$\phi_{max} = 28.1° = \sin^{-1} \frac{1-\beta}{1+\beta}$$

$$\beta = 0.3596$$

$$|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}} = \frac{1}{\sqrt{0.272}} = 1.67$$ ← gain at freq for max phase angle

$$= 4.44\,\text{dB}$$

So want to select ω_{max} of compensator such that the phase margin of the compensated system is at ω_{max}. So gain of uncompensated system must be -4.44 dB at ω_{max}. So select ω_{max} = frequency at which the magnitude of uncomp system is -5.65 dB.

$$|G(j\omega_{max})| = \left| \frac{4500}{(j\omega_{max})(j\omega_{max}+3)(j\omega_{max}+15)(j\omega_{max}+20)} \right| = 10^{\frac{-4.44dB}{20}}$$

$\omega_{max} = 3.88$ rad/s

$$G_c = \frac{1}{\beta} \frac{(s+\frac{1}{T})}{(s+\frac{1}{\beta T})}$$

Finding pole and zero for compensator:

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

$$3.88 = \frac{1}{T\sqrt{0.3596}}$$

$$T = 0.43$$

$$\beta T = (0.3596)(0.43) = 0.155$$

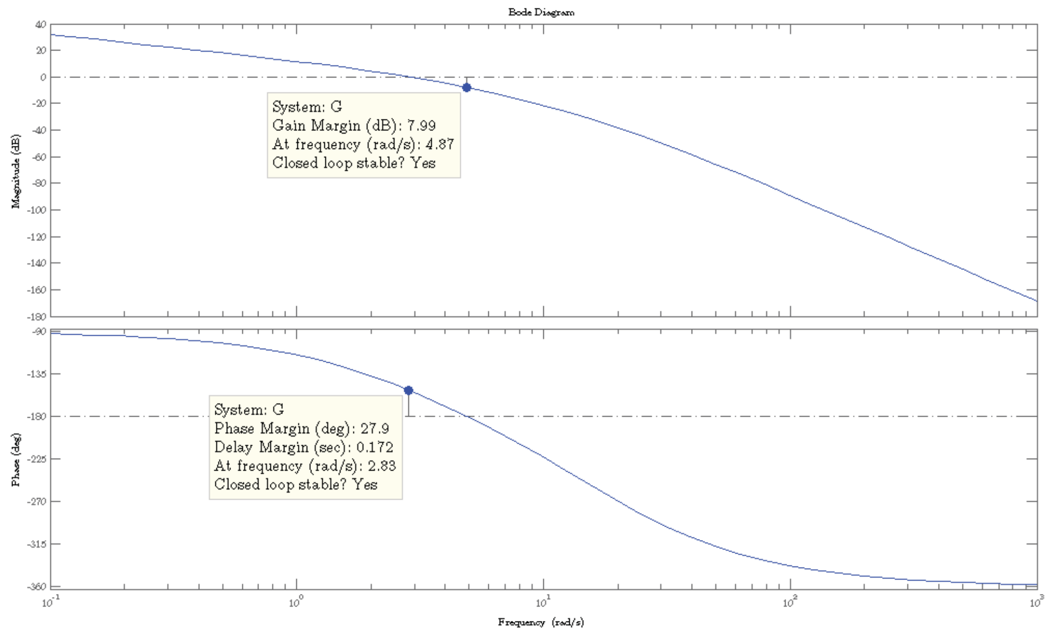
So: $$G_c = \frac{1}{\beta} \frac{(s+\frac{1}{T})}{(s+\frac{1}{\beta T})} = \frac{1}{0.3956} \frac{(s+\frac{1}{0.43})}{(s+\frac{1}{0.155})}$$

$$\boxed{G_c = 2.78 \frac{(s+2.33)}{(s+6.47)}}$$

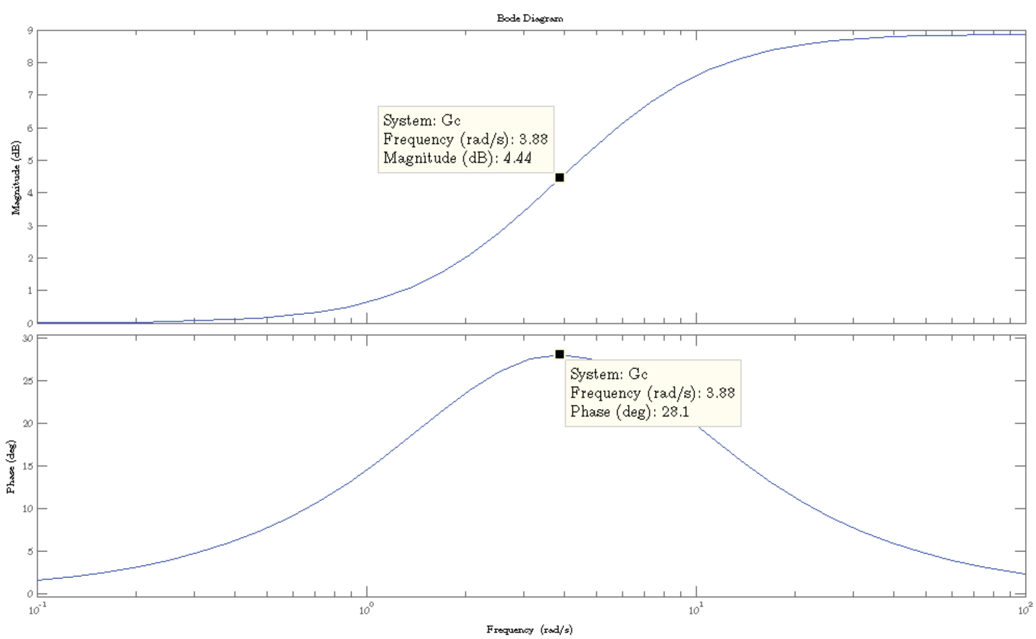
Bode plot for compensator alone and compensated system are attached.

$$G_c(s)G(s) = \frac{10008(s+2.33)}{s(s+6.47)(s+3)(s+15)(s+20)}$$

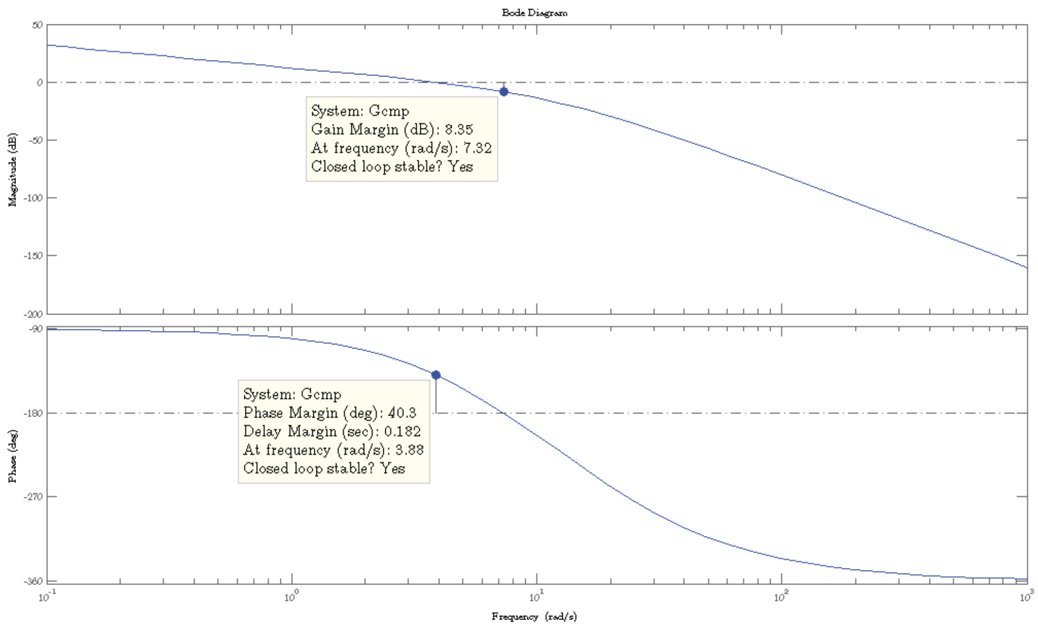
Uncompensated System



Compensated System



Lead Compensator Alone



Prob 5 starts

(5) Let $G_c(s) = \frac{s+c}{s}$.

$\Rightarrow G_{OL}(s) = G_c(s)\, G(s)$.

$$= \frac{(s+c)\, 100k}{s^2 (s+36)(s+100)}.$$

$$\Rightarrow K_v = \lim_{s \to 0} \frac{s\,(s+c)\,(100k)}{s^2 (s+36)(s+100)}$$

$= \infty$.

$\Rightarrow$ '0' steady state error to ramp input.

Desired %OS = 9.5 %

$\Rightarrow 0.095 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$$\Rightarrow \xi^2 = \frac{\ln(0.095)^2}{[\pi^2 + \ln(0.095)^2]}$$

$\Rightarrow \xi = 0.6$.

$$\Rightarrow \Phi_M = \tan^{-1}\left[\frac{2\xi}{\sqrt{-2\xi^2 + \sqrt{1+4\xi^4}}}\right]$$

$= 1.032$ rad.

$\Rightarrow \phi_M = 59.13^\circ$

Set $\Phi_M = 59.13^\circ + 12^\circ = 71.13^\circ$

'ω' at which $\angle G(j\omega) = 71.13^\circ - 180^\circ = -108.87^\circ$

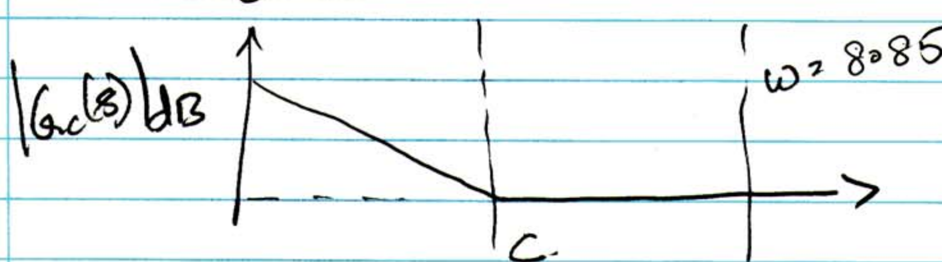
$\Rightarrow \angle G(j\omega) = -\angle j\omega - \angle j\omega+36 - \angle j\omega+100$

$= -108.87^\circ$

$\Rightarrow \omega = 8.85$ rad/sec (using matlab)

$$|G(j\omega)| = \left| \frac{100k}{j\omega(j\omega+36)(j\omega+100)} \right| = 0.003k.$$

Choosing c:



By the usual choose 'c' to be 'one decade' below $\omega = 8.85 \Rightarrow c = 0.885$

$$\Rightarrow |G_c(j\omega)| = |G_c(j8.85)|$$

$$= \left| \frac{j8.85 + 0.885}{j8.85} \right| \approx 1.$$

to set $\omega_{\Phi_m} = \omega = 8.85$ we need

$$|G_c(j\omega)|\,|G(j\omega)| = 1.$$

$$\Rightarrow \boxed{k = \frac{1}{0.003}}$$

- Attached plot verifies that the designed controller achieves $OS\% = 9.5\%$

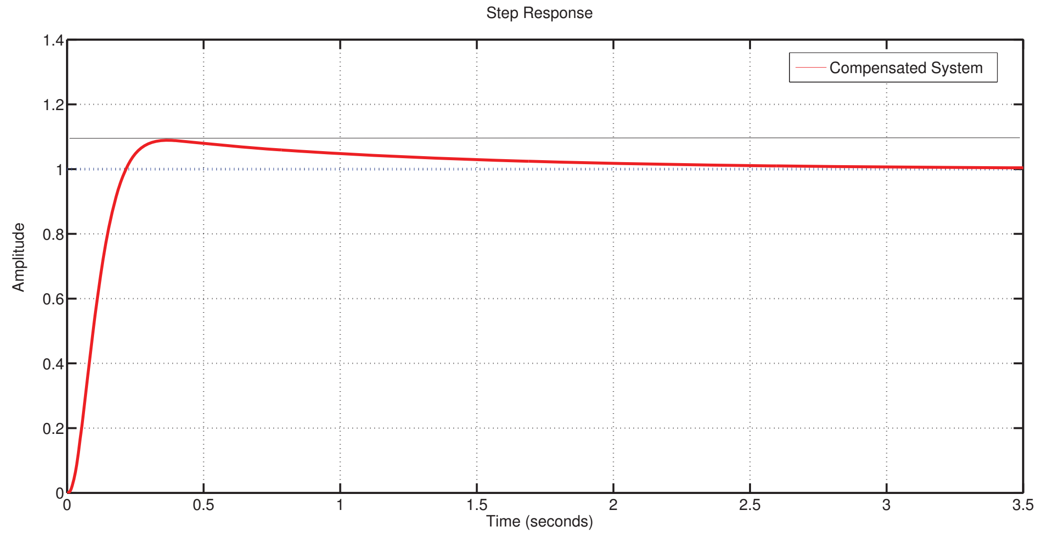


Figure 3: Step response of the closed loop system

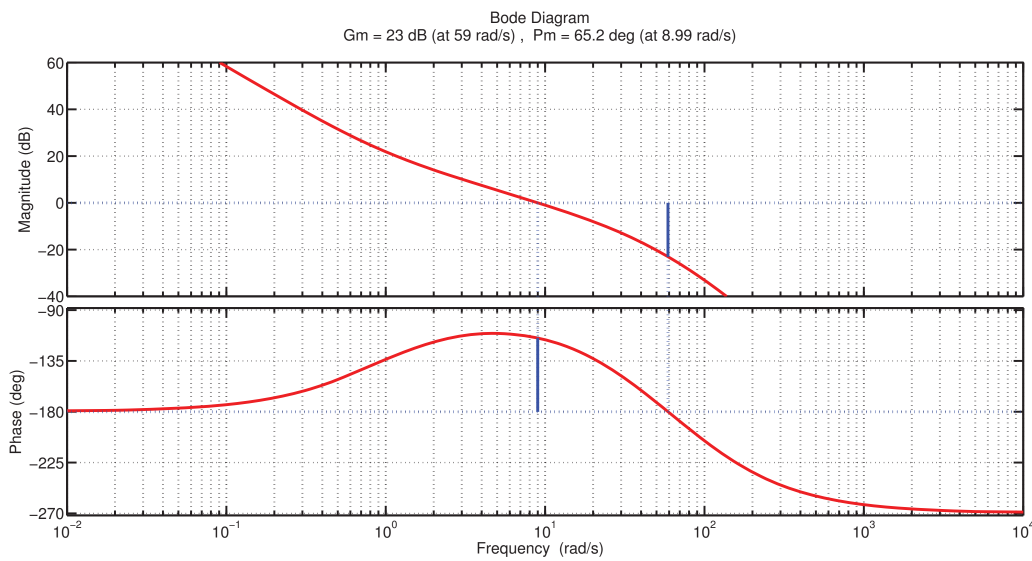


Figure 4: Bode plot of OLTF with compensator