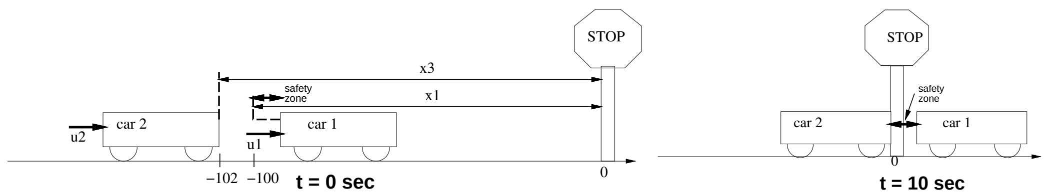
Due at 1700, Fri. Apr. 24 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 5.6-5.8, 12-12.8.

1. (20 pts) Gain and phase margin (review) (Nise 10.5-10.7)
For each $G_i(s)$ below, i) draw Bode plot, ii) estimate gain and phase margin from Bode plot, iii) draw Nyquist plot, iv) evaluate stability, gain, and phase margin from Nyquist plot.
a) $G_1(s) = \frac{100}{(s+1)(s+10)}$
b) $G_2(s) = \frac{100}{(s+1)(s-10)}$
c) $G_2(s) = \frac{100}{(s-1)(s+10)}$



2. (35 pts) Linear Quadratic Regulator (handout)
Consider a pair of cars in a "platoon" regulated to a stopping position, as illustrated above. The dynamics of car 1 are ($\dot{x_2} = \ddot{x_1} = 2u_1$ and car 2 has a plant model $\dot{x_4} = \ddot{x_3} = 0.5u_2$ where u_1 and u_2 are the car's thrust due to engine and braking. (The offset of x_1 from car 1's rear bumper prevents the cars from colliding when $x_3 = x_1$.) The outputs of the system are $y_1 = x_1$ and $y_2 = x_3 - x_1$. Note that if $y_2 > 0$ then car 2 has intruded on the safety zone of car 1.
Initial conditions are car 1 at -100 m, 25 m/s, and car 2 at -102 m, 28 m/s.
a) Write the system equations in state space form.
b) Use the LQR method (from Matlab function `lqr(sys,Q,R)`, with `Q=diag([2.5,10,2.5,10])` and `R=diag([1,1])` to find an optimal K for the state feedback control $\mathbf{u} = -K_b\mathbf{x}$. Plot $\mathbf{x}(t)$ and $\mathbf{u}(t)$ for the given initial condition (Matlab `initial`) and state feedback with gain K_b. How long does it take to get to with 1 m of the stop sign? What is the velocity at 10 sec? Are there any problems with the control?
c) Find a new cost function $Q = \mathtt{diag}([\mathtt{q}_1\ \mathtt{q}_2\ \mathtt{q}_3\ \mathtt{q}_4])$ which keeps car 2 within 2 m of car 1 for the whole trajectory, while maintaining $y_2 < 0$ to prevent a collision, and distance from stop sign less than 1 m at 10 sec with velocity less than 1 m/s. (Suggestion: let $q_1 = 2.5$ and $q_2 = 10$ and search for new q_3 and q_4.) Plot $\mathbf{x}(t)$ and $\mathbf{u}(t)$ for the given initial condition (Matlab `initial`) and state feedback with new gain K_c.
Suggestion: you may change R (but keep diagonal).
d) Find the solution to the Riccati equation P using Matlab function `care(A,B,Q,R)` and estimate the cost $J = (x^T P x)(0)$ for each of b) and c).
e) Briefly compare the tradeoffs between control effort and time response between the two cases.

3. (35 pts) Discrete Time Control (Handout)
Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -320 & -152 & -22 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \quad y = [2\ 1\ 0]\ \mathbf{x}$$

the corresponding discrete time (DT) system is
$x[n+1] = Gx[n] + Hu[n]$, $y[n] = Cx[n]$

which can be found using the Matlab function `c2d(sys,T,'zoh')`.

a) With initial condition $x_0 = [1 \ \ 0 \ \ 0]'$, plot the ZIR using Matlab function `initial()` for the CT system and the DT system (with $T = 0.4$ sec).
b) For output feedback control $u = k(r - y)$, sketch the root locus for the equivalent transfer function for the continuous time (CT) system.
c) Determine the closed loop pole locations for the CT system for $k = 20$ and plot the closed-loop step response using Matlab.
d) The closed loop DT system has state equation
$x[n+1] = (G - kHC)x[n] + kHr[n]\ , \quad y[n] = Cx[n]$
(which can be found using the Matlab `feedback` function). Using Matlab, determine the closed loop pole locations for the DT system for $k = 20$ and sampling period $T = 0.4$ sec and plot the step response.
e) Use Matlab (iteratively if necessary) to find a sampling period T which gives a closed-loop step response that is "reasonably close" to the CT closed-loop step response. Determine closed-loop pole locations, and plot the DT step response.
f) Briefly explain why the CT and DT ZIR responses from a) above are reasonably close, but the closed loop responses from c) and d) (with $T = 0.4$ sec) do not agree at all. (Hint, consider e^{AT}.)

4. (10 pts) Z transform (Nise 13.3)
For each $F(z)$, find $f(kT)$ using partial fraction expansion.

a) $F(z) = \frac{(z+4)(z+1)}{(z-0.3)(z-0.6)}$

b) $F(z) = \frac{(z+0.2)(z+1)}{z(z-0.1)(z-0.2)(z-0.3)}$