EECS 117 Homework Assignment 1 Spring 2004

2-2. Resistive loads. Since $R_s = 0$, $\Gamma_s = -1$.

(a) For $R_L = 25\Omega$, we have

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

At $t = 0$, an incident voltage of amplitude given by

$$\mathcal{V}_1^+ = \frac{Z_0}{R_s + Z_0} V_0 = \frac{50}{0 + 50}(3\text{ V}) = 3\text{ V}$$

is launched at the source end of the line. Using these values, the bounce diagram and the load voltage $\mathcal{V}_L(t)$ versus t are as shown.

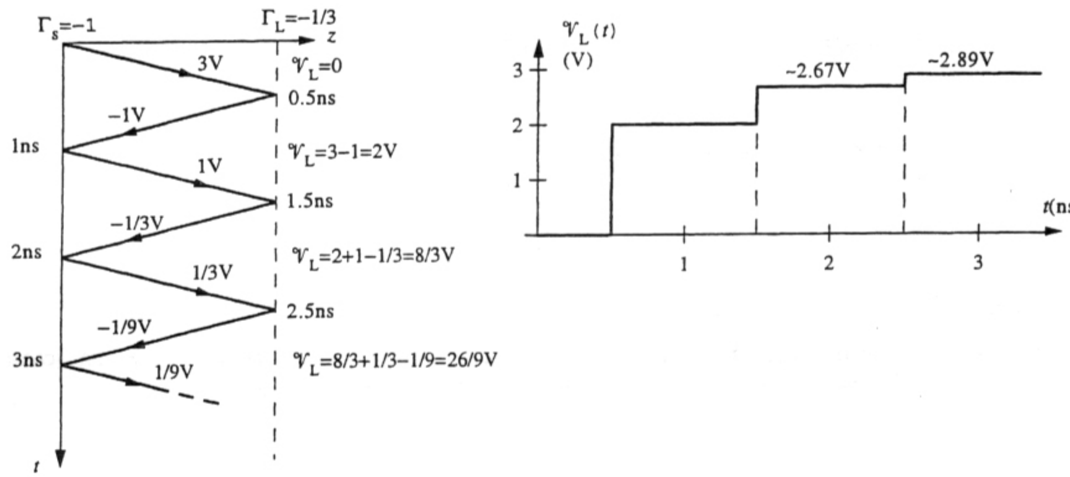

Fig. 2.2. Figure for Problem 2-2a. $R_L = 25\Omega$.

(b) For $R_L = 50\Omega$, $\Gamma_L = 0$. The bounce diagram and the sketch of the load voltage for this case are as shown.

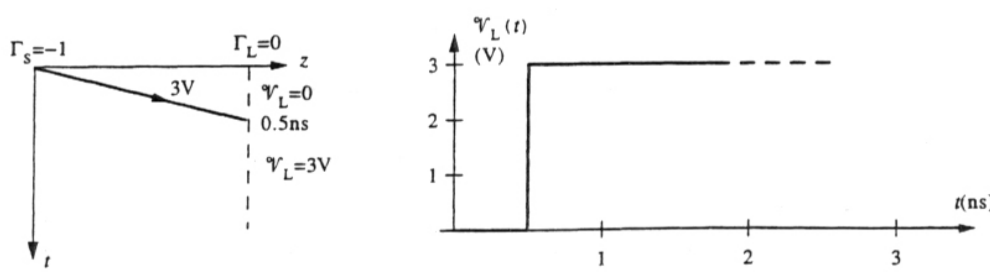

Fig. 2.3. Figure for Problem 2-2a. $R_L = 50\Omega$.

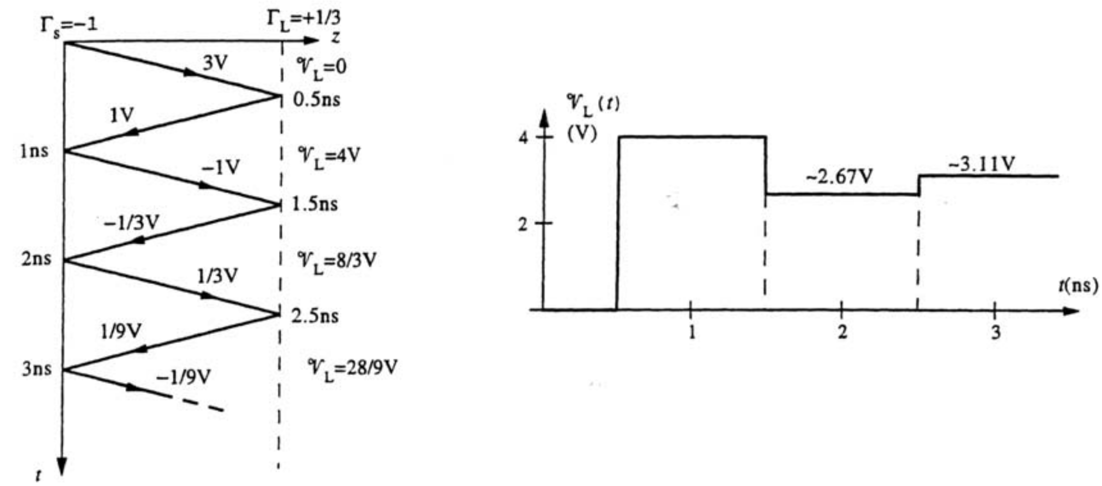

Fig. 2.4. Figure for Problem 2-2c. $R_L = 100\Omega$.

(c) For $R_L = 100\Omega$, the load reflection coefficient is

$$\Gamma_L = \frac{100-50}{100+50} = +\frac{1}{3}$$

Again, the bounce diagram and $\mathcal{V}_L(t)$ versus t are as shown.

2-3. Ringing. Using the line parameters $L = 4.5$ nH-cm^{-1} and $C = 0.8$ pF-cm^{-1}, the characteristic impedance Z_0 and the phase velocity v_p of the line can be calculated as

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4.5\times10^{-9}}{0.8\times10^{-12}}} = 75\Omega$$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.5\times10^{-9})(0.8\times10^{-12})}} \simeq 16.7 \text{ cm-ns}^{-1}$$

The one-way time delay t_d from one to the other end of the line is $t_d = l/v_p \simeq$(30 cm)/(16.7 cm-ns^{-1})=1.8 ns. The reflection coefficients at the two ends of the line are

$$\Gamma_s = \frac{R_s - Z_0}{R_s + Z_0} = \frac{15-75}{15+75} = -\frac{2}{3}$$

and $\Gamma_L = +1$ (since $R_L = \infty$) respectively. At $t = 0$, an incident voltage of

$$\mathcal{V}_1^+ = \frac{Z_0}{R_s + Z_0}V_0 = \frac{75}{15+75}(3.6 \text{ V}) = 3 \text{ V}$$

is launched from the source end of the line. Using these values, we draw the bounce diagram and sketch the load-end voltage $\mathcal{V}_L(t)$ versus t as shown.

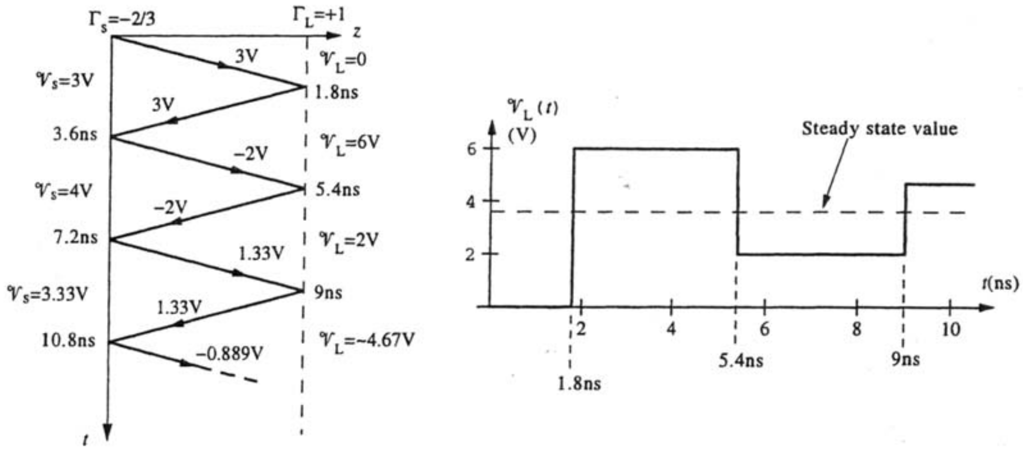

Fig. 2.5. Figure for Problem 2-3. Bounce diagram and $\mathcal{V}_L(t)$ versus t.

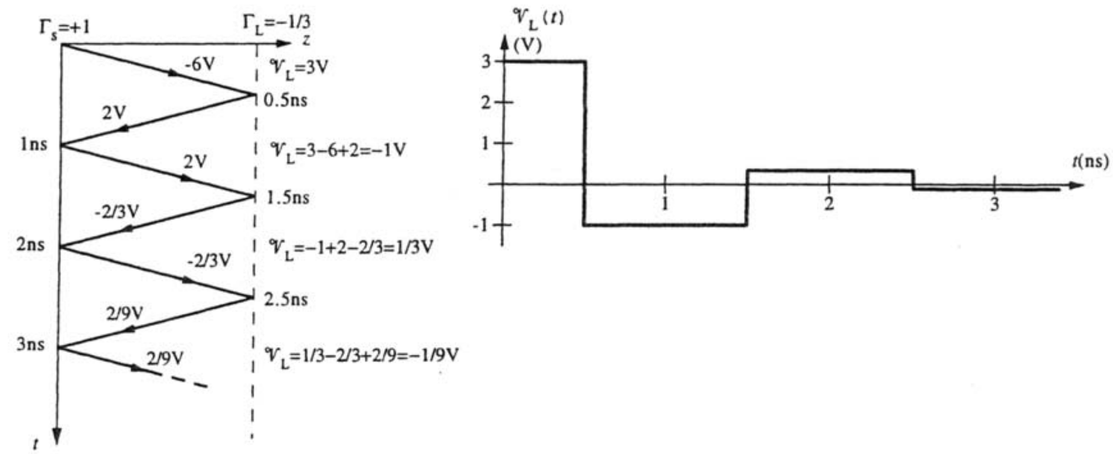

Fig. 2.6. Figure for Problem 2-4a. Bounce diagram and $\mathcal{V}_L(t)$ versus t for $R_L = 25\Omega$.

2-4. Discharging of a charged line. At $t = 0^-$ (i.e., immediately before the switch opens when steady-state conditions are in effect), the source end current $\mathcal{I}_s$ is given by $\mathcal{I}_s(0^-) = V_0/R_L = 3/R_L$. At $t = 0^+$ (i.e., immediately after the switch opens), the source end current is suddenly forced to go to zero, i.e., $\mathcal{I}_s(0^+) = 0$. We can represent this change in $\mathcal{I}_s$ as a new disturbance launched from the source end of the line starting at $t = 0$. The amplitude of this new current and its accompanying voltage are given respectively by $\mathcal{I}_1^+ = \mathcal{I}_s(0^+) - \mathcal{I}_s(0^-) = -3/R_L$ and $\mathcal{V}_1^+ = Z_0\mathcal{I}_1^+ = -3Z_0/R_L$. This disturbance will not affect the value of the load end voltage until $t = t_d = 0.5$ ns. So, for $t < t_d$, $\mathcal{V}_L(t) = 3$ V, regardless of the value of R_L. Using the reflection coefficient values calculated in Problem 2-2, we draw the bounce diagram and sketch $\mathcal{V}_L(t)$ as a function of t for each value of R_L, as shown.

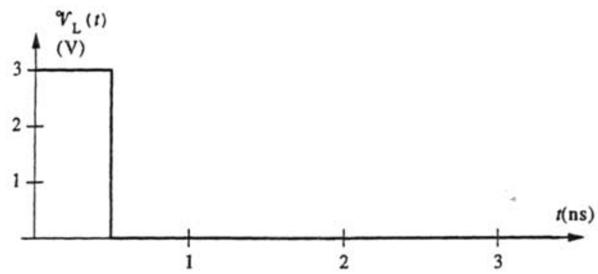

Fig. 2.7. Figure for Problem 2-4b. Bounce diagram and $\mathcal{V}_{\mathrm{L}}(t)$ versus t for $R_{\mathrm{L}} = 50\Omega$.

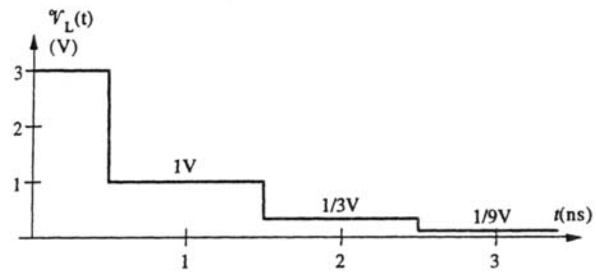

Fig. 2.8. Figure for Problem 2-4c. Bounce diagram and $\mathcal{V}_{\mathrm{L}}(t)$ versus t for $R_{\mathrm{L}} = 100\Omega$.

2-7. Observer on the line. (a) Using the sketch of $\mathcal{V}_{\mathrm{ctr}}$ versus t provided, the initial voltage launched on the line is given by

$$\mathcal{V}_1^+ = \frac{Z_0}{R_s + Z_0} V_0 = \frac{Z_0}{100 + Z_0}(1\ \mathrm{V}) = 0.5\ \mathrm{V} \quad \rightarrow \quad Z_0 = 100\Omega$$

The reflected voltage is given by

$$\mathcal{V}_1^- = \Gamma_{\mathrm{L}} \mathcal{V}_1^+ = \frac{R_{\mathrm{L}} - Z_0}{R_{\mathrm{L}} + Z_0} \mathcal{V}_1^+ = \frac{R_{\mathrm{L}} - 100}{R_{\mathrm{L}} + 100}(0.5\ \mathrm{V})$$

which reaches the center of the line at $t = 1.5t_d$. From the $\mathcal{V}_{\mathrm{ctr}}$ sketch, we have $\mathcal{V}_1^- = 0.3$ V. Substituting above, we find $R_{\mathrm{L}} = 400\Omega$.

(b) For $t_w = 1.5t_d$, the variation of the voltage $\mathcal{V}_{\mathrm{ctr}}$ versus t is as shown.

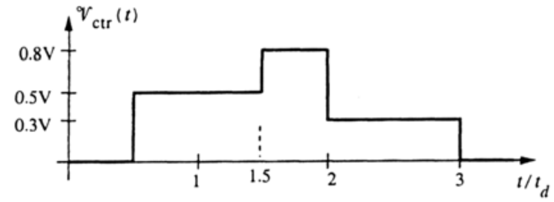

Fig. 1.1. Figure for Problem 2-7. $\mathcal{V}_{\mathrm{ctr}}$ versus t for $t_w = 1.5t_d$.

2-12. Multiple lines. The source and the load reflection coefficients are $\Gamma_s = 0$, $\Gamma_{L2} = 0$ and

$$\Gamma_{L3} = \frac{120 - 60}{120 + 60} = +\frac{1}{3}$$

respectively. At the junction where the three lines meet, the junction reflection coefficient is given by

$$\Gamma_{1\to 23} = \Gamma_{2\to 13} = \frac{(120 \parallel 60) - 120}{(120 \parallel 60) + 120} = -0.5$$

if the signal arrives at the junction from line 1 or 2 and

$$\Gamma_{3\to 12} = \frac{(120 \parallel 120) - 60}{(120 \parallel 120) + 60} = 0$$

if the signal arrives at the junction from line 3. The bounce diagram and the voltages $\mathcal{V}_1(t)$, $\mathcal{V}_2(t)$ and $\mathcal{V}_3(t)$ are sketched as shown.

2-23. Terminated IC interconnects. (a) In the series termination network, $R_T = Z_0 - R_s = 50 - 14 = 36\Omega$ so that $\Gamma_s = 0$. In the parallel termination network, $R_T = Z_0 = 50\Omega$ so that $\Gamma_L = 0$. In both circuits, the load gate at the end of the interconnect reaches steady state at $t = t_d = 1.5$ ns. However, the steady-state voltages are different from one another. In the series termination network, the steady-state load voltage at HIGH state is $V_{ss} = V_{HIGH} = 5$ V whereas in the parallel termination network, it is

$$V_{ss} = \frac{R_T}{R_s + R_T} V_{HIGH} = \frac{50}{14 + 50}(5 \text{ V}) \simeq 3.91 \text{ V}$$

This difference makes the parallel network more vulnerable to noise at HIGH state. Series termination network is the natural choice for low-power dissipation at steady-state since the steady-state current $I_{ss} = 0$ and as a result, no power is dissipated. In the parallel termination network however, a steady-state current flows through the circuit at HIGH state given by

$$I_{ss} = \frac{V_{HIGH}}{R_s + R_T} = \frac{5 \text{ V}}{14\Omega + 50\Omega} \simeq 78.1 \text{ mA}$$

resulting in a power dissipation of $P = I_{ss}^2(R_s + R_T) \simeq 0.39$ W.

2-28. Capacitive load. At $t = 0$, an incident voltage of amplitude

$$\mathcal{V}_1^+ = \frac{100}{100 + 100}(5 \text{ V}) = 2.5 \text{ V}$$

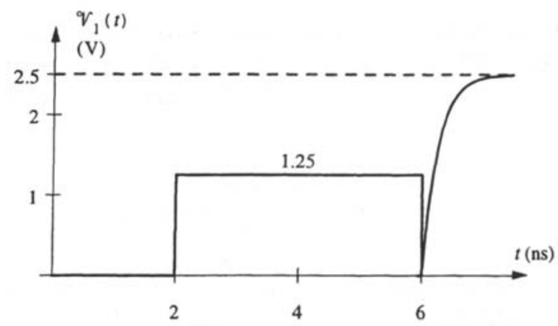

Fig. 2.34. Figure for Problem 2-28. $\mathcal{V}_1(t)$ versus t.

is launched at the source end of the 100Ω line. This voltage reaches the resistance $R_1 = 100\Omega$ at $t = t_d = 2$ ns. The reflection coefficient at this junction for a voltage arriving from the 100Ω onto the 50Ω line is

$$\Gamma_{1\to 2} = \frac{(R_1 \parallel 50) - 100}{(R_1 \parallel 50) + 100} = -0.5$$

Therefore, a voltage of $\mathcal{V}_1^- = \Gamma_{1\to 2}\mathcal{V}_1^+ = (-0.5)(2.5\text{V}) = -1.25$ V reflects back to the 100Ω line and $\mathcal{V}_1^{+*} = \mathcal{V}_1^+ + \mathcal{V}_1^- = 1.25$ V is transmitted onto the 50Ω line. Hence, $\mathcal{V}_1(t) = \mathcal{V}_1^{+*}$, or using step function notation, $\mathcal{V}_1(t) = 1.25u(t - 2\text{ ns})$ V until a new voltage disturbance arrives this position. The reflected voltage $\mathcal{V}_1^-$ reaches the source end at $t = 2t_d = 4$ ns and is completely absorbed since $\Gamma_s = 0$. The transmitted voltage reaches the 5 pF capacitor at $t = 2t_d = 4$ ns. Since this is a first-order circuit, the voltage reflected from the capacitor is of the form

$$\mathcal{V}_1^{-*}(t) = u(t - 4\text{ ns})[K_1 + K_2 e^{-(t-2t_d)/\tau}]$$

where τ is the time constant of the circuit given by $\tau = (50\Omega)(5\text{ pF}) = 0.25$ ns and $2t_d$ is the total transmission line delay for the step voltage to travel from the source to the 5 pF capacitor. Initially the capacitor is uncharged and appears like a short circuit, i.e., the capacitor voltage at $t = 4$ ns is $\mathcal{V}_c(t = 4\text{ ns}) = \mathcal{V}_1^{+*} + \mathcal{V}_1^{-*}(t = 4\text{ ns}) = 0$ from which $\mathcal{V}_1^{-*}(t = 4\text{ ns}) = -\mathcal{V}_1^{+*} = -1.25$ V, therefore $K_1 + K_2 = -1.25$ V. At $t = \infty$, $\mathcal{V}_1^{-*}(\infty) = K_1 = \mathcal{V}_1^{+*}(\infty) = +1.25$ V since the capacitor is fully charged and appears as an open circuit. Thus, $K_1 = 1.25$ V. Hence $K_2 = -2.5$ V and so the reflected voltage is given by

$$\mathcal{V}_1^{-*}(t) = 1.25u(t - 4\text{ ns})[1 - 2e^{-(t-4\text{ ns})/(0.25\text{ ns})}]\text{ V}$$

This voltage reaches R_1 at $t = 6$ ns where no reflection back to the 50Ω line occurs since the reflection coefficient for a voltage arriving from the 50Ω line onto the 100Ω line is

$$\Gamma_{2\to 1} = \frac{(100 \parallel 100) - 50}{(100 \parallel 100) + 50} = 0$$

Therefore, the voltage $\mathcal{V}_1^{-*}(t)$ continues to travel on the 100Ω line towards the source. Hence, the voltage $\mathcal{V}_1(t)$ across the R_1 resistor is now given by $\mathcal{V}_1(t) = \mathcal{V}_1^{+*} + \mathcal{V}_1^{-*}(t)$ which can also be written as

$$\mathcal{V}_1(t) = 1.25u(t - 2\text{ ns}) + 1.25u(t - 6\text{ ns})[1 - 2e^{-(t-6\text{ ns})/(0.25\text{ ns})}]\text{ V}$$

When $\mathcal{V}_1^{-*}(t)$ arrives the source end, it is completely absorbed since $\Gamma_s = 0$. The sketch of $\mathcal{V}_1(t)$ versus t is as shown.

2-31. Unknown lumped element. (a) From the sketch, the amplitude of the incident voltage is

$$\mathcal{V}_1^+ = \frac{Z_{01}}{R_s + Z_{01}} V_0 = \frac{V_0}{2}$$

from which $Z_{01} = R_s = 50\Omega$. The initial amplitude of the reflected voltage is $\mathcal{V}_1^-(t = t_1^+/2) = \mathcal{V}_s(t = t_1^+) - \mathcal{V}_1^+ = V_0/2$ which means that the unknown element at first appears to be an open circuit (i.e., $\Gamma_{1\to2}(t = t_1) = +1$). Therefore, this element must be an inductor which is initially uncharged and which initially behaves as an open circuit to maintain the zero value of its initial current. The value of the inductor can be expressed in terms of the time constant as

$$\tau = \frac{L}{Z_{01} + Z_{02}} = 0.1 \text{ ns}$$

where to solve for L, we need to know the value of Z_{02}. As $t \to \infty$ (i.e., steady state), the inductor turns into a short circuit when the source-end voltage becomes

$$\mathcal{V}_s(t \to \infty) = \frac{Z_{02}}{R_s + Z_{02}} V_0 = \frac{V_0}{4} \quad \to \quad Z_{02} = \frac{R_s}{3} \simeq 16.7\Omega$$

Using $Z_{02} \simeq 16.7\Omega$ in the time constant expression above yields $L \simeq 6.67$ nH.

(b) Since this is a first-order circuit, the voltage across the inductor can be expressed as

$$\mathcal{V}_{\text{ind}}(t) = u(t - 0.5t_1)[K_1 + K_2 e^{-(t-0.5t_1)/(0.1 \text{ ns})}]$$

Using the initial and final conditions, we have

$$\mathcal{V}_{\text{ind}}(0.5t_1) = K_1 + K_2 = \mathcal{V}_1^+ + \mathcal{V}_1^-(t = 0.5t_1) = V_0 \quad \text{and} \quad \mathcal{V}_{\text{ind}}(\infty) = K_1 = 0$$

from which $K_2 = V_0 - K_1 = V_0$. Therefore, the inductor voltage is given by

$$\mathcal{V}_{\text{ind}}(t) = V_0 u(t - 0.5t_1) e^{-(t-0.5t_1)/(0.1 \text{ ns})}$$

This expression is sketched with respect to time as shown.