Due at 1700, Fri. Apr. 10 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 5.6-5.8, 12-12.6.

1. (20 pts) (Nise 5.6,5.7)
Consider the plant, where $G(s) = Y(s)/U(s)$:

$$G(s) = \frac{20s(s+7)}{(s+3)(s+7)(s+9)}.$$

a. Draw the signal graph in phase variable form and write the corresponding state equations.
b. Draw the signal graph in parallel form and write the corresponding state equations.
c. Draw the signal graph in cascade form and write the corresponding state equations.
d. Draw the signal graph in observer canonical form and write the corresponding state equations.

2. (25 pts) State Feedback/Pole placement (Nise 12.2)
Consider the plant, where $G(s) = Y(s)/U(s)$:

$$G(s) = \frac{20(s+2)}{s(s+5)(s+7)}.$$

a. Draw the signal graph in phase variable form and write the corresponding state equations.
b. Find $K = [k_1\ k_2\ k_3]$ such that feedback $u = -Kx$ yields 10% overshoot and $T_s = 2$ seconds for a step response with a pair of dominant poles. (Place third pole 10 times further from $j\omega$ axis as the dominant pole pair.

3. (20 pts) Diagonalization (Nise 5.8)
Consider the homogenous system:

$$\dot{\mathbf{x}} = A\mathbf{x} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \text{with init. cond. } \mathbf{x_o}(t=0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Consider invertible square matrix P and $x = P\bar{x}$. (Matlab use suggested).
a) Find P and $\bar{A}$ such that $\bar{A} = P^{-1}AP$ is diagonal .
b) Using $e^{\bar{A}t}$, and P, P^{-1} as needed, find $\mathbf{x}(t)$ for $t > 0$.

4. (25 pts) Observer (Nise 12.5)
Given the plant:

$$G(s) = \frac{1}{s(s+3)(s+7)}$$

where state variables are not available.
a. Express $G(s)$ in observer canonical form, $\dot{x} = Ax + Bu$.
b. Design an observer: $\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$ for the observer canonical variables to yield a 2nd order transient response with $\zeta = 0.4$ and $\omega_n = 75$. (The third pole should be placed 10 times further from the imaginary axis than the dominant poles.)
c. Using Matlab, compare the state variables in G for a step input with the observer estimate.(That is, plot $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$.)

5. (10 pts) Controllability (Nise 12.3)

Consider the system $\dot{\mathbf{x}} = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} u(t)$

For what values of k_1 and k_2 is the system completely controllable?