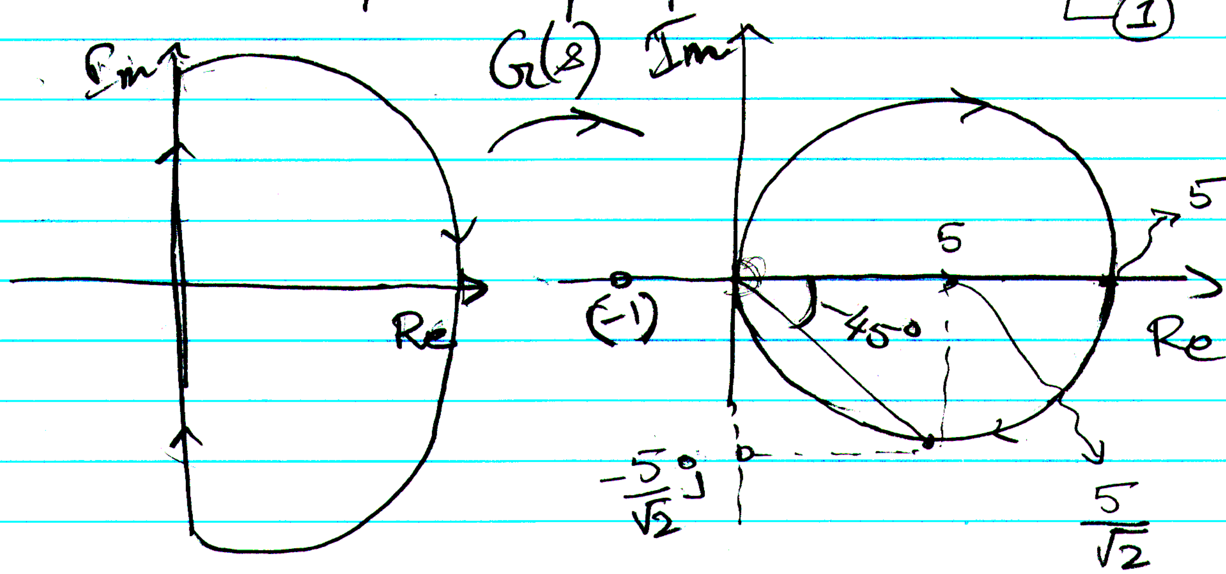
EE128 P87 SOLUTIONS

1

(a) $G(s) = \left(\frac{5}{s+1}\right)$

# of open loop poles in RHP = 0. — (1)



ccw of -1 = 0 — (2)

(1) ⇒ $P = 0$, (2) ⇒ $N = 0$

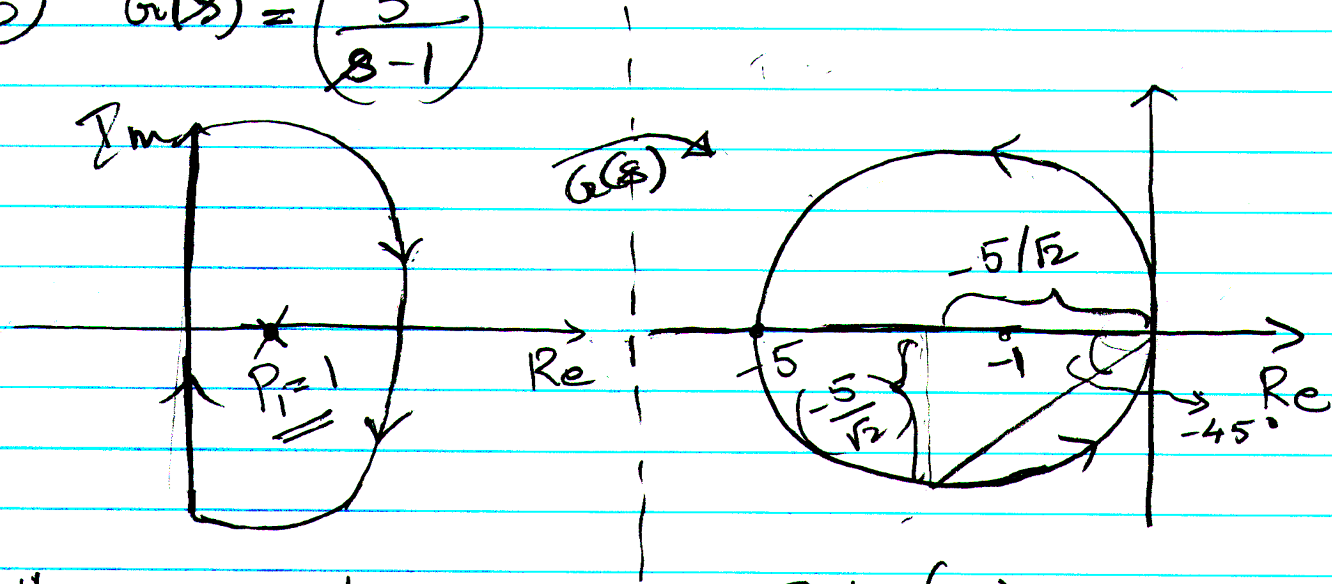
⇒ $\boxed{Z = 0}$

closed loop pole location:

roots of $1 + \frac{5}{s+1} = 0$ ⇒ $\boxed{s = -6}$

which does lie in the LHP.

(b) $G(s) = \left(\dfrac{5}{s-1}\right)$



# of open loop poles in RHP (P) = 1

ccw about '-1' (N) = 1.

$\Rightarrow \boxed{Z = P - N = 0}$

Closed loop poles are roots of :

$1 + G(s) = 0 \Rightarrow 1 + \dfrac{5}{s-1} = 0 \Rightarrow \boxed{s = -4}$ (3)

(3) also $\Rightarrow$ $Z = 0$

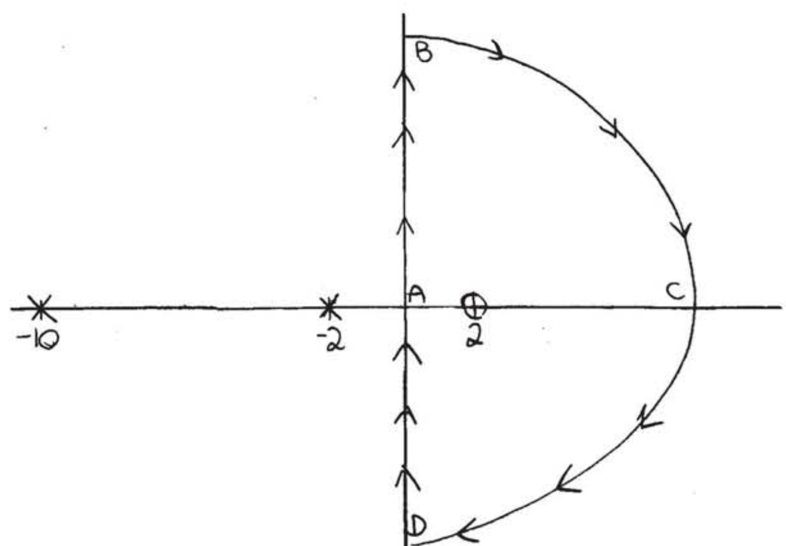
2. $G(s) = \frac{k(s-2)}{(s+10)(s+2)}$

a. $\frac{k(-2)(\frac{s}{-2}+1)}{10(2)(\frac{s}{10}+1)(\frac{s}{2}+1)}$

	Mag	Phase
DC:	$20\log\left(\left\lvert\frac{-2}{10(2)}\right\rvert\right) = 20\log(0.1) = -20\text{dB}$	$0°$
$\left(\frac{s}{-2}+1\right)$:	$+20$ dB/dec from $\omega = 2$	Starts at $180°$ (RHP zero) $-45°$/dec from $\omega = 0.2$ to 20
$\frac{1}{(\frac{s}{10}+1)}$:	-20 dB/dec from $\omega = 10$	$-45°$/dec from $\omega = 1$ to 100
$\frac{1}{(\frac{s}{2}+1)}$:	-20 dB/dec from $\omega = 2$	$-45°$/dec from $\omega = 0.2$ to 20

Bode plot attached

b.



for $K=1$

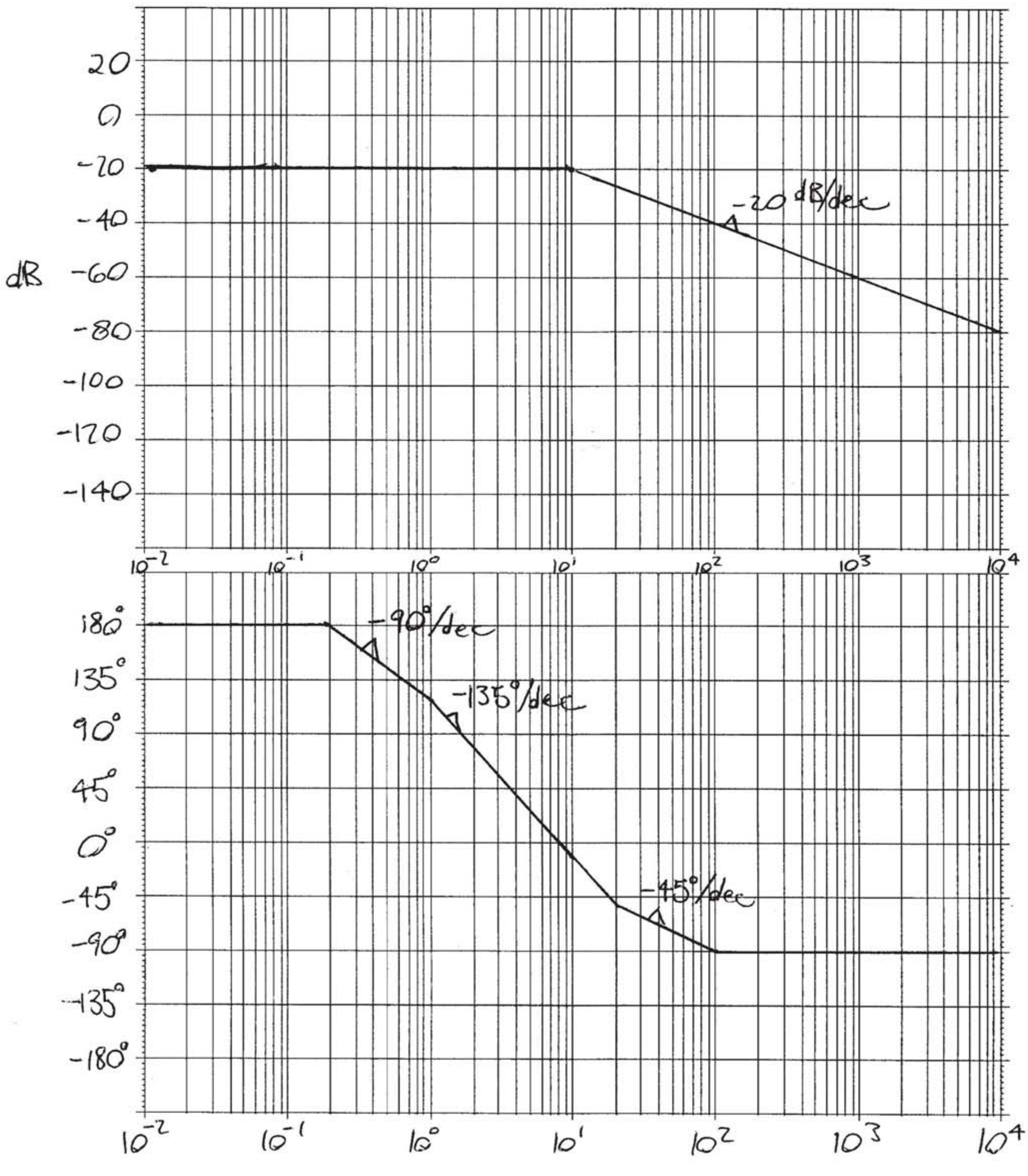
$$G(j\omega) = \frac{(j\omega-2)}{(j\omega+10)(j\omega+2)} = \frac{(j\omega-2)}{(20-\omega^2)+j12\omega} \cdot \frac{((20-\omega^2)-j12\omega)}{((20-\omega^2)-j12\omega)}$$

$$= \frac{(14\omega^2-40) - j(\omega^3-44\omega)}{(20-\omega^2)^2+(12\omega)^2}$$

Real-axis crossings:
Imaginary part $= 0 \Rightarrow \omega^3 - 44\omega = 0$ at $\omega = 0, \pm 2\sqrt{11}$

For these points, real part:

$\omega = 0$	$\omega = 2\sqrt{11}$	$\omega = -2\sqrt{11}$
$\sigma = -0.1$	$\sigma = 0.08$	$\sigma = 0.08$



As k varies, add $20\log(k)$ to magnitude

Imaginary axis crossings:

Real part $= 0 \Rightarrow 14\omega^2 - 40 = 0$ at $\omega = \pm\sqrt{\frac{20}{7}}$

For these points, imaginary part:

$\omega = \sqrt{\frac{20}{7}}$	$\omega = -\sqrt{\frac{20}{7}}$
$j0.1$	$-j0.1$

As $\omega \to \infty$:

Can approximate $G(j\omega) \approx \frac{j\omega^3}{\omega^4} = \frac{j}{\omega}$

So $\lim_{\omega \to \infty} G(j\omega) = \lim_{\omega \to \infty} \frac{j}{\omega} = 0$

As $\omega \to \infty$, nyquist plot approaches 0 from $-90°$

$$\underbrace{-2*90}_{\text{2 poles}} + \underbrace{90}_{\text{1 zero}} = -90$$

Summary:

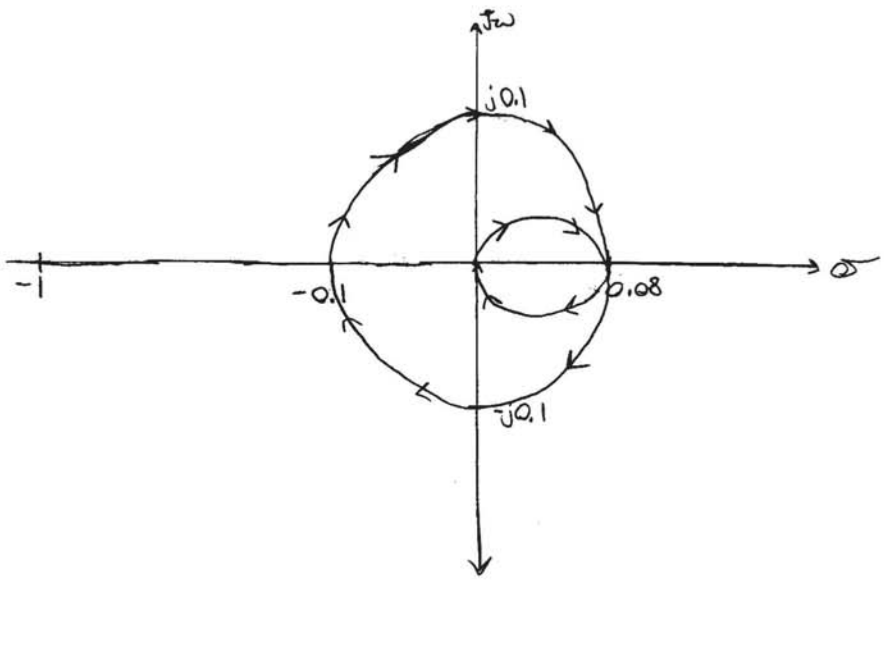
From $A \to B$:

$\omega: 0 \to \infty$

Intersects real axis at -0.1, 0.08

Intersects $j\omega$ axis at $j0.1$

Starts at -0.1, ends at 0, approaches 0 from $-90°$



C. Nyquist intersects real axis in LHP at -0.1.

So if $K = 10$, it will intersect at -1

$K < 10$ stable $(N = 0,\ P = 0,\ Z = P - N = 0)$

$K = 10$ marginally stable

$K > 10$ unstable $(N = 1,\ P = 0,\ Z = P - N = 1$ so 1 unstable CL pole$)$

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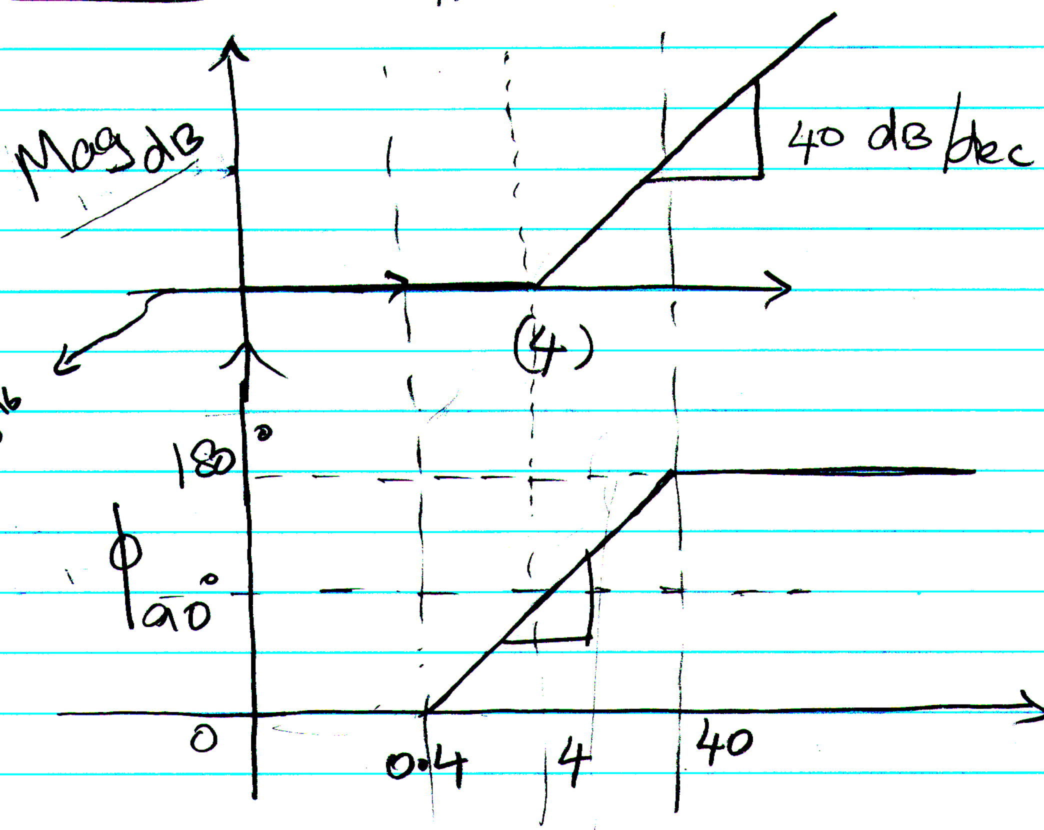
3) a) $G(s) = \dfrac{k(s+4)^2}{(s+1)^4}$.

(i) Factor : $(s+4)^2$



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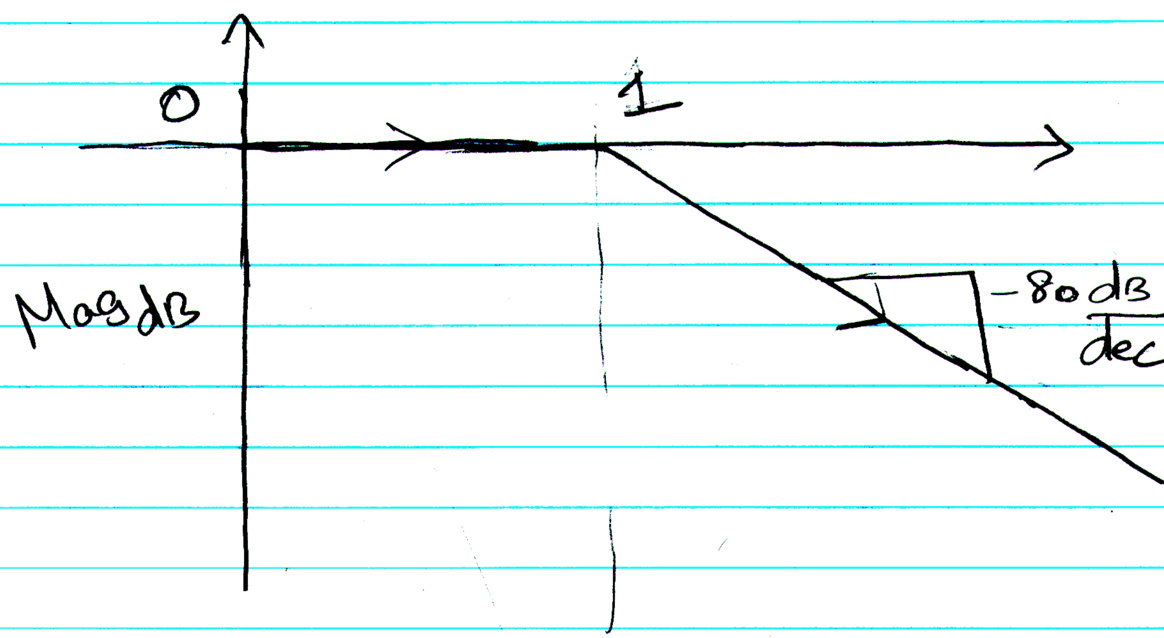
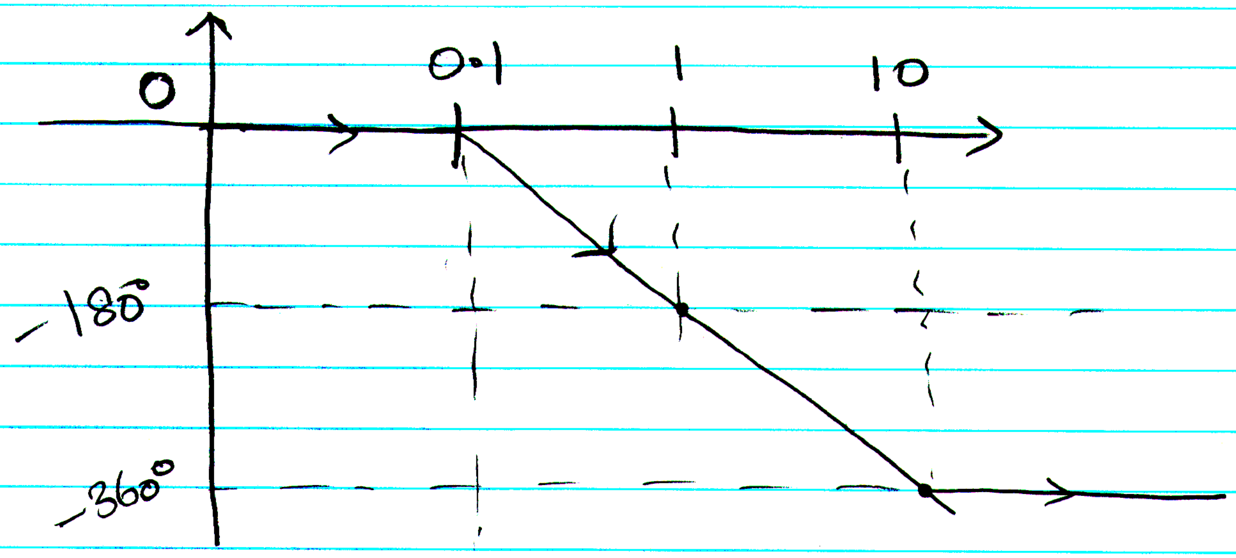
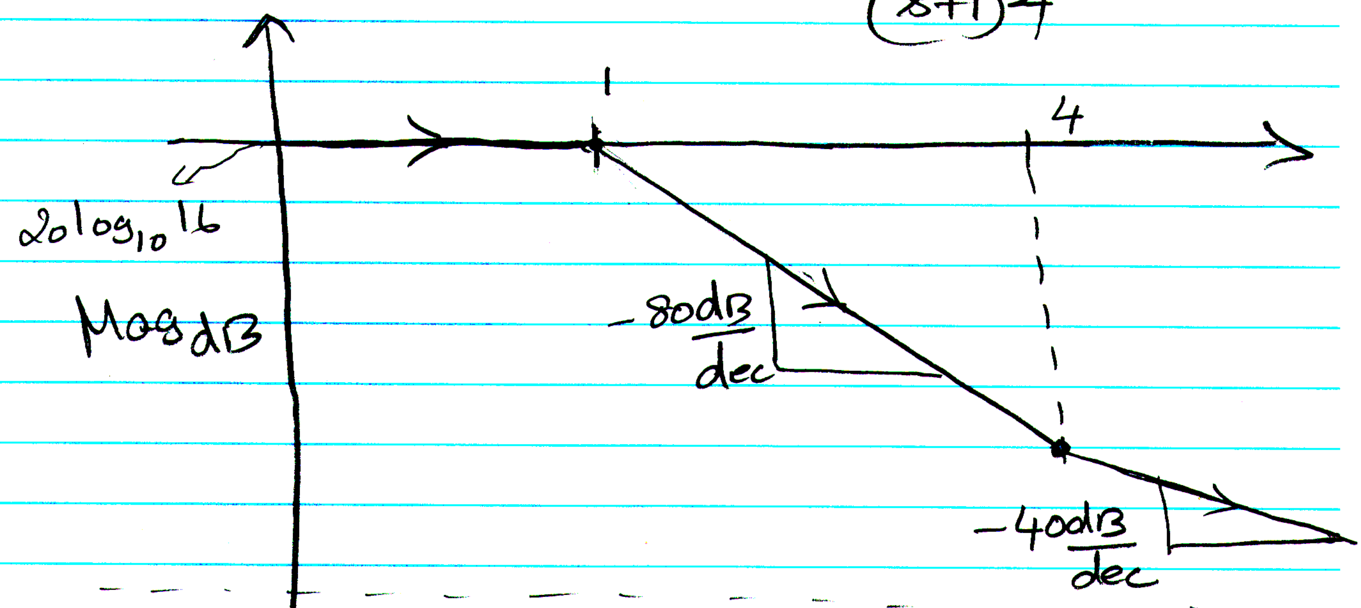
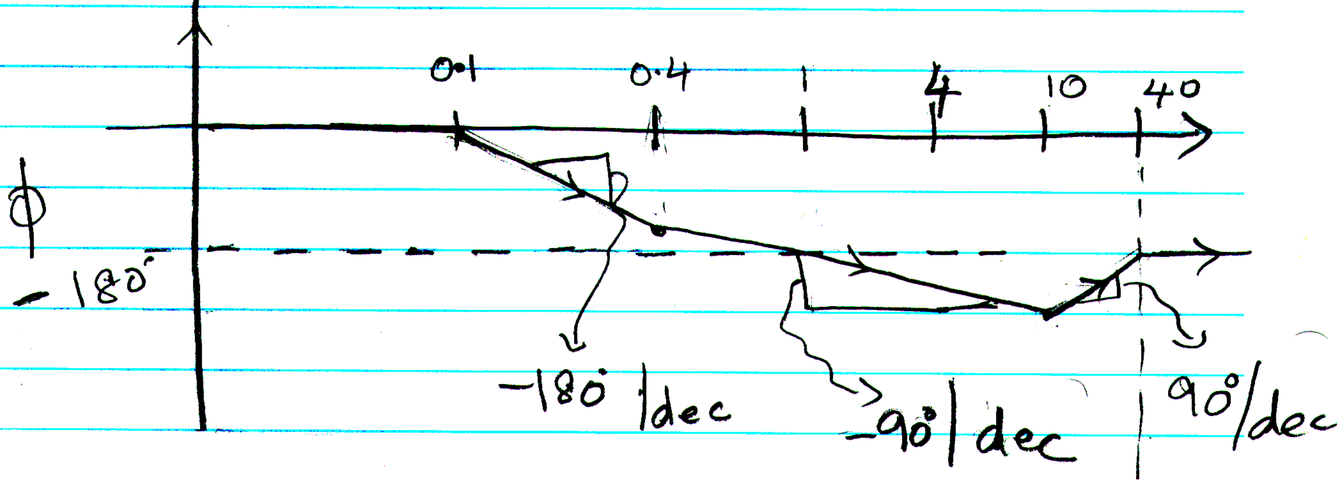
(ii) Factor : $1/(s+1)^4$



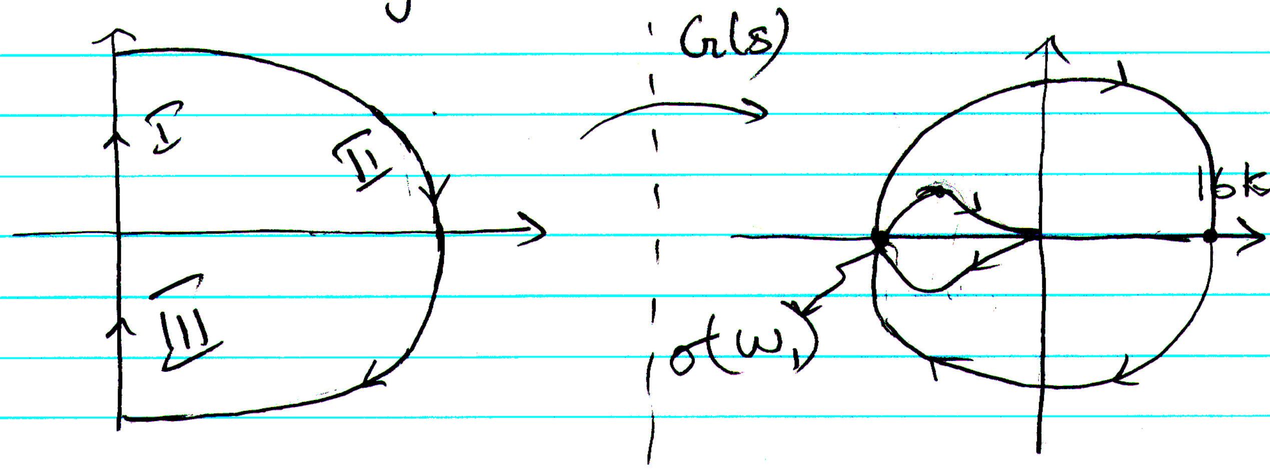


Bode Plot for $G(s) = \frac{K(s+4)^2}{(s+1)^4}$





(b) Nyquist diagram follows from the Bode diagram



At ω_1:

$$\mathrm{Im}\left(\frac{(j\omega_1+4)^2}{(j\omega_1+1)^4}\right) = 0$$

$$\Rightarrow \quad 4\omega_1\left[\omega_1^4+5\omega_1^2-14\right] = 0$$

$$\Rightarrow \quad \boxed{\omega_1 = 0 \quad \text{or} \quad \omega_1 = \sqrt{2}}$$

at $\omega_1 = \sqrt{2}$: $\quad \left|G(j\sqrt{2})\right| = \dfrac{k(j\sqrt{2}+4)^2}{(j\sqrt{2}+1)^4}$

$$= (2)k$$

$$\Rightarrow \quad \sigma(\omega_1) = -2k.$$

Now: $\quad P = 0$

$$\Rightarrow \quad Z = 0 \iff N = 0.$$

$\Rightarrow \boxed{K < 1/2}$

For K > 1/2, N = 2 => Z = 2 => There are two right half planes.

4. $G(s) = \frac{2(s+5)}{s(s^2+2s+8)}$

a. Bode plots attached

b. Gain Margin:

By calculation

Finding $\omega > 0$ such that phase is -180°

$$G(j\omega) = \frac{2(j\omega+5)}{j\omega((j\omega)^2+2(j\omega)+8)} = \frac{2(j\omega+5)}{-2\omega^2 - j(\omega^3-8\omega)} \cdot \frac{(-2\omega^2 + j(\omega^3-8\omega))}{-2\omega^2 + j(\omega^3-8\omega)}$$

$$= \frac{2(-(\omega^4+2\omega^2) + j(3\omega^3-40\omega))}{(2\omega^2)^2 + (\omega^3-8\omega)^2}$$

$$\frac{(3\omega^3-40\omega)}{-(\omega^4+2\omega^2)} = \tan(\pi) = 0$$

$$3\omega^3 - 40\omega = 0$$

$$\omega = 0, \pm 2\sqrt{\frac{10}{3}}$$

So phase is -180° at $\omega = 2\sqrt{\frac{10}{3}}$

Magnitude at $\omega = 2\sqrt{\frac{10}{3}}$:

$$\left|G\left(j2\sqrt{\tfrac{10}{3}}\right)\right| = 0.375$$

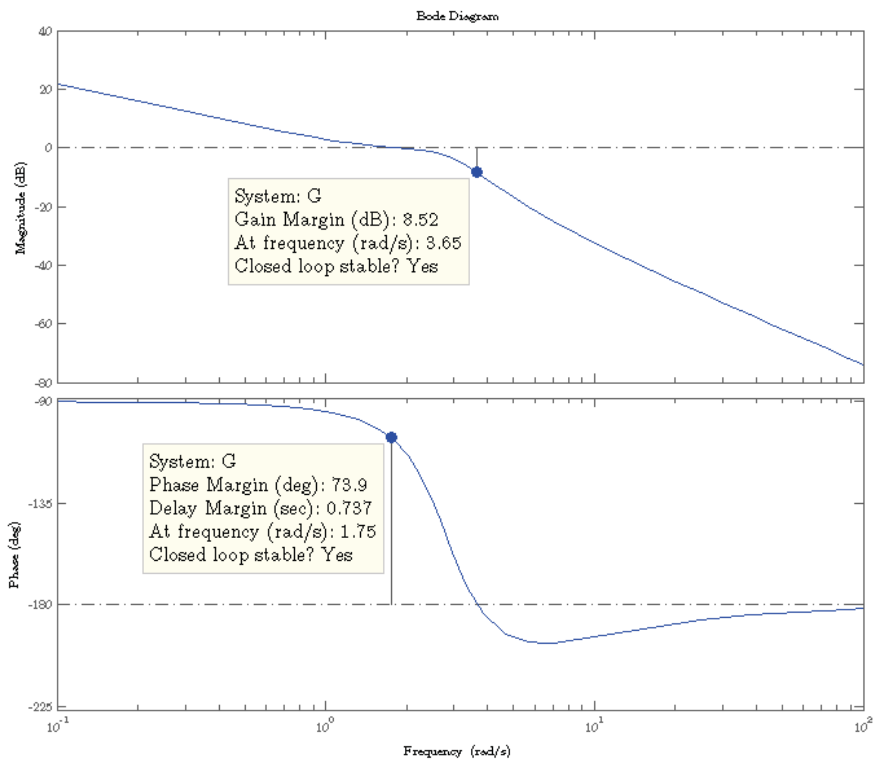
$$\text{Gain Margin} = 0 - 20\log(0.375)$$

$$\boxed{\text{Gain Margin} = 8.52\,\text{dB}}$$

By inspection:

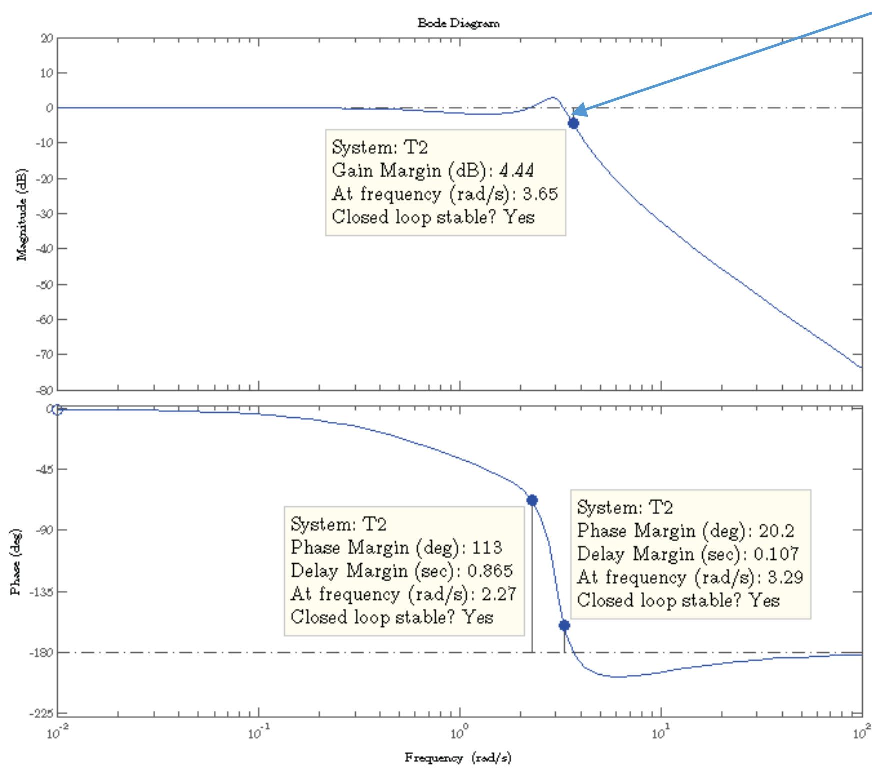
This can also be estimated from the bode plot. It is the gain necessary to cause the magnitude to be 0 dB when the phase is -180°, as indicated on the plot.

Open loop Bode Plot



Closed loop Bode plot

Closed loop bandwidth is frequency at -3dB is approximately 3.5 rad/s



Phase Margin:

By calculation:

Finding $\omega > 0$ such that gain is 0 dB.

So, $|G(j\omega)| = 1$

$$G(j\omega) = \frac{-2(\omega^4 + 2\omega^2) + j2(3\omega^3 - 40\omega)}{(2\omega^2)^2 + (\omega^3 - 8\omega)^2}$$

$$|G(j\omega)| = \sqrt{\left(\frac{-2(\omega^4+2\omega^2)}{(2\omega^2)^2+(\omega^3-8\omega)^2}\right)^2 + \left(\frac{2(3\omega^3-40\omega)}{(2\omega^2)^2+(\omega^3-8\omega)^2}\right)^2} = 1$$

$\omega = 1.7514$

Phase at $\omega = 1.7514$:

$$\tan^{-1}\left(\frac{(3\omega^3 - 40\omega)}{-(\omega^4 + 2\omega^2)}\right) = -106.1^\circ$$

Phase margin $= -106.1^\circ - (-180^\circ)$

$\boxed{\text{Phase margin} = 73.9^\circ}$

By inspection

Can be estimated from the Bode plot. The phase change necessary to cause the 0 dB magnitude point to be at a phase of -180°, as indicated on the plot.

C. Find closed loop bandwidth, ω_{BW}, where open loop magnitude is between -6 and -7.5 dB if phase is between -135° and -225°.

For $|G(j\omega)| = -6$ dB $= 0.5012$

$\omega = 3.31 \Rightarrow$ Phase $= -170.5^\circ$

For $|G(j\omega)| = -7.5$ dB $= 0.4217$

$\omega = 3.51 \Rightarrow$ Phase $= -176.5^\circ$

Both are within -135° and -225°

So using $\omega_{BW} = 3.5$ rad/s as the closed loop bandwidth

(Note: this is the closed loop bandwidth, NOT open loop bandwidth. ω_{BW} can also be estimated from the closed loop Bode plot, attached)

$$\phi_M = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

$$73.9° = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

$$\boxed{\text{Damping ratio} = \zeta = 0.912}$$

$$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = 0.092\%$$

$$\boxed{\%OS = 0.092\%}$$

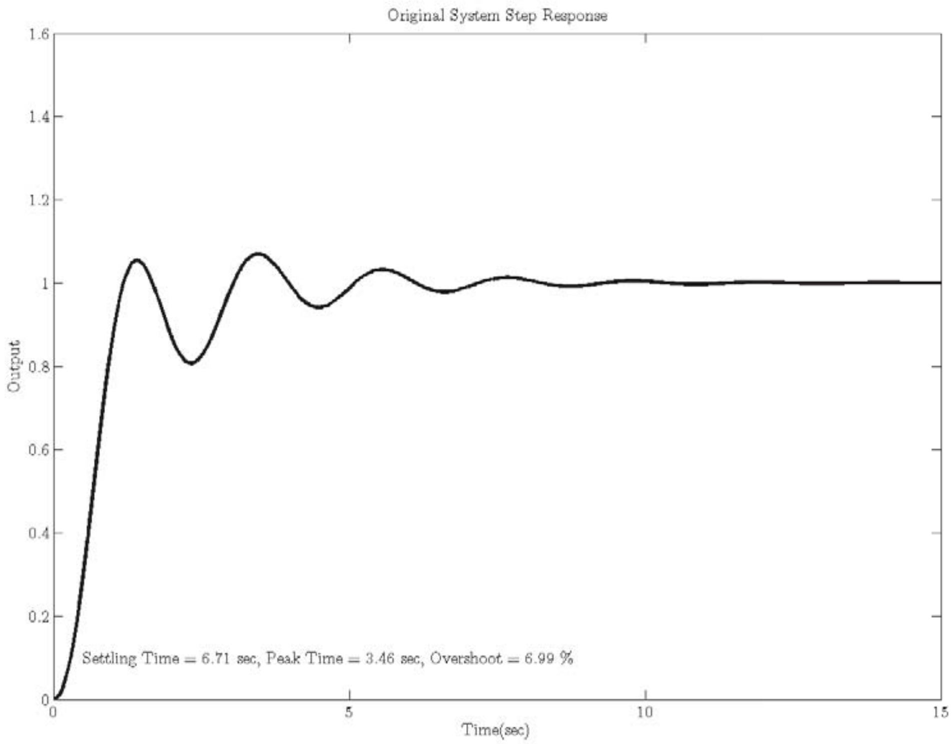
$$T_s = \frac{4}{\omega_{BW}\zeta} \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 0.92\,s$$

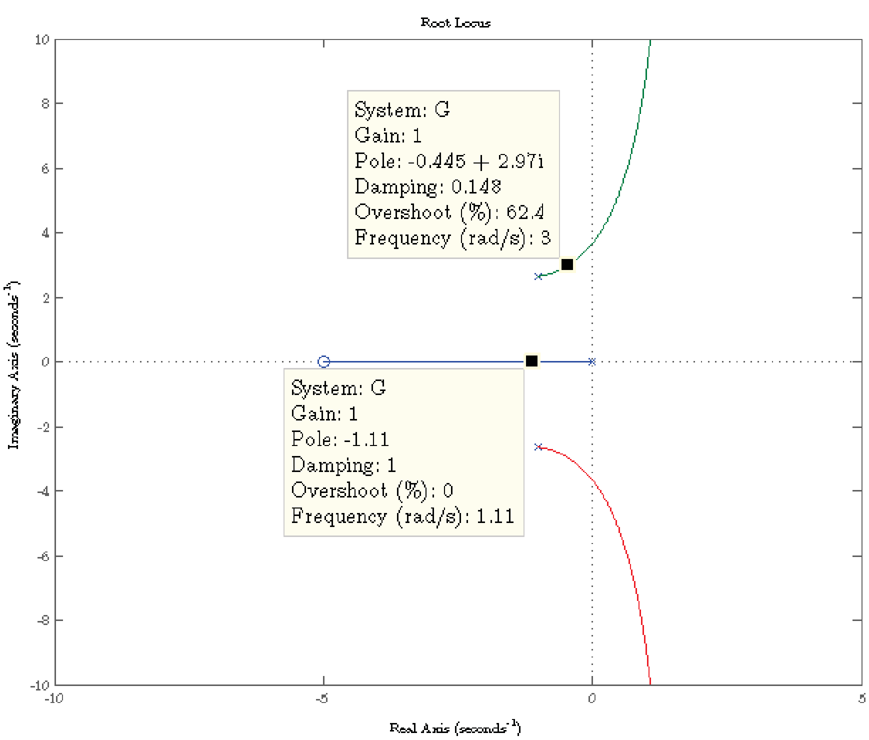
$$\boxed{T_s = 0.92\,s}$$

$$T_p = \frac{\pi}{\omega_{BW}\sqrt{1-\zeta^2}} \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 1.65\,s$$

$$\boxed{T_p = 1.60\,s}$$

d. It can be seen from the attached plot that the step response properties are not close to our estimates. This is because our second order approximation is not valid in this case. The attached root locus shows that the CL poles are at -1.11, $-0.45 \pm j2.97$. The poles are not far enough apart for a dominant pair for the approximation.


Original System Step Response
Output
Time(sec)
Settling Time = 6.71 sec, Peak Time = 3.46 sec, Overshoot = 6.99 %


Root Locus
System: G
Gain: 1
Pole: -0.445 + 2.97i
Damping: 0.148
Overshoot (%): 62.4
Frequency (rad/s): 3
System: G
Gain: 1
Pole: -1.11
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 1.11
Imaginary Axis (seconds⁻¹)
Real Axis (seconds⁻¹)