Due at 1700, Fri. May. 1 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise Ch. 13, DT handout (Lec. 24).

1. (10 pts) Phase Variable form (Nise 3.5)
Consider the transfer function (with $T = 1$)

$$\frac{Y(z)}{U(z)} = \frac{z^{-2} + 4z^{-3}}{1 + 6z^{-1} + 11z^{-2} + 6z^{-3}}$$

a) Draw a block diagram for the system in phase variable form using a cascaded section of delay blocks z^{-1}.
b) Write the the system in phase variable form: $x(k+1) = Gx(k) + Hu(k)$ and $y(k) = Cx(k) + Du(k)$.

2. (10 pts) SS to TF (Nise 3.6, 13.3, DT handout)
Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}(k+1) = G\mathbf{x}(k) + Hu(k) = \begin{bmatrix} 1 & 0.2 \\ 0.5 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.4 \\ -0.5 \end{bmatrix} u \tag{1}$$

a. Find the transfer function $\frac{X(z)}{U(z)}$. b. Is the system BIBO stable?

3. (15 pts) Laplace to Z conversion (Nise 13.3)
Given $H(s) = \frac{1}{(s+\alpha)^2}$ and sample rate T. find $H(z)$ using the definition of Z transform, i.e. $H(z) = \Sigma_k h(kT) z^{-k}$.

4. (15 pts) Sampling Rate (DT Handout)
A continuous time plant has transfer function $F(s) = \frac{K_p}{s+10}$.
a. In CT with unity gain feedback, K_p is chosen so that the steady state error for a step input $r(t)$ is less than 0.1. Find K_p and the closed loop pole location.
b. Find the discrete time equivalent system for $F(s)$ in state space such that $x((k+1)T) = Gx(kT) + Hu(kT)$ where $u(kT) = K_p(r(kT) - x(kT))$.
c. Algebraically find the maximum T for which the closed loop system is stable with steady state error less than 0.1.

5. (20 pts) Transient performance using gain compensation (Nise 13.9)
Given a CT plant $G(s) = \frac{K}{s(s+1)}$.
a. With sample period $T = 0.2$, find $G(z)$, the Z transform of $G(s)$.
b. Sketch the root locus for $G(z)$ in unity gain feedback, and find the range of K for stability.
c. With unity gain feedback, find the value of K for 20% overshoot, and note the K in root locus.
d. Plot step response for the closed-loop DT system in Matlab.

6. (30 pts) Steady State Error/DT Integrator (Nise 12.8, 13.7, 13.8)
Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}(k+1) = G_1\mathbf{x}(k) + H_1 u(k) = \begin{bmatrix} 0 & 1 \\ -.64 & 1.6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k), \quad y = [1 \;\; 0]\, \mathbf{x} \tag{2}$$

a) Given error $e(k) = r(k) - y(k)$ where $r(k)$ is a scalar, evaluate the steady state error $\lim_{k\to\infty} e(k)$ for input $r(k)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where K_1 is chosen so that the closed loop poles are at $z_i = 0.3 \pm 0.3j$.
b) Add a DT integrator to the plant, with $X_N(k+1) = X_n(k) + e(k)$, where the error $e(k) = r(k) - C\mathbf{x}$. Using a new state vector $\mathbf{x} = [x_1 \; x_2 \; x_N]^T$, write the new state and output equations for DT, equivalent to Nise eq. (12.115ab).
c) Find gains such that the 3 closed-loop poles with the DT integrator are at $z_i = 0.3 \pm 0.3j, 0.1$. Evaluate the steady-state error for a step input.
d) Plot the step response for both systems in Matlab, (hint `tf(num,den,-1)`) and compare.