

Example:

a) non-perturbate ETP

→ a') perturbate ETP

	$s_1$	$s_2$	...	$s_n$	
$w_1$	$c_{11}$	$c_{12}$	...	$c_{1n}$	$a_1$
$w_2$	$c_{21}$	$c_{22}$	...	$c_{2n}$	$a_2$
...	...	...	...	...	...
$w_m$	$c_{m1}$	$c_{m2}$	...	$c_{mn}$	$a_m$
	$b_1$	$b_2$	...	$b_n$	

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

	$s_1$	$s_2$	...	$s_n$	
$w_1$	$c_{11}$	$c_{12}$	...	$c_{1n}$	$a_1 + \epsilon$
$w_2$	$c_{21}$	$c_{22}$	...	$c_{2n}$	$a_2 + \epsilon$
...	...	...	...	...	...
$w_m$	$c_{m1}$	$c_{m2}$	...	$c_{mn}$	$a_m + \epsilon$
	$b_1$	$b_2$	...	$b_n + m \cdot \epsilon$	

$$\sum_{i=1}^m (a_i + \epsilon) = \sum_{i=1}^m a_i + m \cdot \epsilon = \sum_{j=1}^n b_j + m \cdot \epsilon$$

Examples:

Solve the next TP. if the second <sup>B.A.</sup> solution which we <sup>will</sup> obtain it isn't <sup>the</sup> optimal solution don't continue the algorithm:

a)

	$s_1$	$s_2$	$s_3$	
$w_1$	3	2	1	30
$w_2$	1	4	2	20
$w_3$	2	1	3	20
	20	15	20	

Because,  $\sum_{i=1}^3 a_i = 70 > \sum_{j=1}^3 b_j = 55$

we have a No TP, so we must to equilibrate.

①

	$s_1$	$s_2$	$s_3$	$s_4^f$	
$w_1$	3	2	1	0	30, 10, 0
$w_2$	1	4	2	0	20, 15, 0
$w_3$	2	1	3	15	20, 15, 0
	20	15	20	15	

For ETP we applied the diagonal method to find the iBAS:  $\bar{x}_0$   $\left\{ \begin{array}{l} \bar{x}_{11}=20; \bar{x}_{12}=10; \bar{x}_{22}=5; \\ \bar{x}_{23}=15; \bar{x}_{33}=5; \bar{x}_{34}=15 \end{array} \right.$   
the other  $\bar{x}_{ij} = * = 0$   
 $f(\bar{x}_0) = 145$

② We check if  $\bar{x}_0$  it's optimal solution:

$$\delta_{13} = -1 + 2 - 4 + 2 = -1$$

$$\delta_{14} = -0 + 0 - 3 + 2 - 4 + 2 = -3$$

$$\delta_{21} = -1 + 3 - 2 + 4 = 4 > 0$$

$$\delta_{24} = -0 + 0 - 3 + 2 = -1$$

$$\delta_{31} = -2 + 3 - 2 + 4 - 2 + 3 = 4 > 0$$

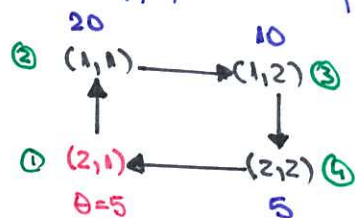
$$\delta_{32} = -1 + 4 - 2 + 3 = 4 > 0$$

$\Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_0$  it isn't optimal solution!!

I choose  $\delta_{21,11}$

③ We apply the input criteria:  $\delta_{ij} = \max \{ \delta_{ke} > 0 \} = \max \{ \delta_{21}, \delta_{31}, \delta_{32} \} \stackrel{(!)}{=} \delta_{21} \Rightarrow x_{21} \downarrow$

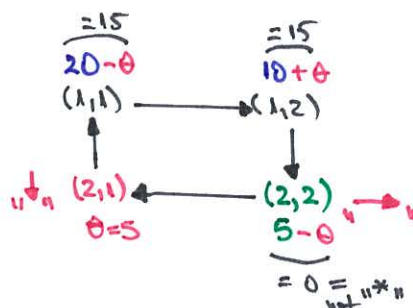
④ We apply the "output criteria":



$$\theta = \min \{ x_{ij} / x_{ij} \text{ been into even no. cells} \} = \min \{ \underbrace{x_{11}}_{=20}, \underbrace{x_{22}}_{=5} \} = 5 \Rightarrow x_{22} \rightarrow$$

⑤ We determine the new solution:  $\bar{X}_1$

	$s_1$	$s_2$	$s_3$	$s_4$	
$w_1$	15	15	*	*	30
$w_2$	5	*	5	*	20
$w_3$	*	*	5	15	20
	20	15	20	15	



$$\bar{X}_1: \begin{cases} \bar{x}_{11}=15, \bar{x}_{12}=15, \bar{x}_{21}=5, \bar{x}_{23}=5, \bar{x}_{33}=5, \bar{x}_{34}=15 \rightarrow \text{basic components} \\ \bar{x}_{13}=\bar{x}_{14}=\bar{x}_{22}=\bar{x}_{24}=\bar{x}_{31}=\bar{x}_{32}=0 (= " * ") - \text{non-basic (free) components} \end{cases}$$

$$f(\bar{X}_1) = 115 (< f(\bar{X}_0))$$

② We check if  $\bar{X}_1$  it is a optimal solution:

$$\delta_{13} = -1 + 2 - 1 + 3 = 3 > 0 \Rightarrow (\exists) \delta_{ij} > 0 \Leftrightarrow \bar{X}_1 \text{ - it isn't optimal solution} \Rightarrow \text{the algorithm must to continue ... !!}$$

b)

	$s_1$	$s_2$	$s_3$	
$w_1$	20	0	*	20, 0
$w_2$	*	15	*	15, 0
$w_3$	*	5	5	10, 5, 0
	20	20	5	

$$\bar{X}_0: \begin{cases} \bar{x}_{11}=20; \bar{x}_{12}=0; \bar{x}_{22}=15; \bar{x}_{32}=5; \bar{x}_{34}=5 \rightarrow \text{basic component} \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0 (= " * ") \rightarrow \text{non-basic components} \end{cases}$$

$$f(\bar{X}_0) = 125$$

it's degenerate BAS(!!!)

(no, it's possible to appear the cycle phenom.)

we apply the perturbation method

	$s_1$	$s_2$	$s_3$	
$w_1$	20	$\epsilon$	*	$20+\epsilon, \epsilon, 0$
$w_2$	*	$15+\epsilon$	*	$15+\epsilon, 0$
$w_3$	*	$5-2\epsilon$	$5+3\epsilon$	$10+\epsilon, 5+3\epsilon, 0$
	20	$20-\epsilon$	$5+3\epsilon$	

$$\Rightarrow \bar{X}_0 = \begin{cases} \bar{x}_{11}=20; \bar{x}_{12}=\epsilon; \bar{x}_{22}=15+\epsilon; \bar{x}_{32}=5-2\epsilon; \bar{x}_{34}=5+3\epsilon \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0 (= " * ") - \text{non-basic components} \end{cases}$$

$$f(\bar{X}_0) = 125 + 6\epsilon$$

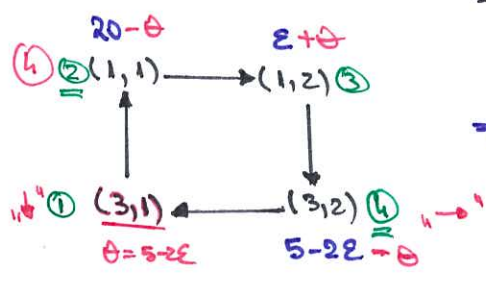
non-degenerate iBAS(!!!)



② We apply the optimality criterion:

$$\left. \begin{aligned} \delta_{13} &= -1 + 2 - 3 + 2 = 0 \\ \delta_{21} &= -3 + 2 - 2 + 4 = 1 > 0 \\ \delta_{23} &= -3 + 2 - 3 + 4 = 0 \\ \delta_{31} &= -1 + 2 - 2 + 3 = 2 > 0 \end{aligned} \right\} \Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_0 \text{ it isn't the optimal solution}$$

③  $\delta_{ij} = \max \{ \delta_{ke} > 0 \} = \max \{ \underbrace{\delta_{21}}_{=1}, \underbrace{\delta_{31}}_{=2} \} = \delta_{31} \Rightarrow "x_{31} \downarrow"$  (becomes a basic component)



$\Rightarrow \theta = \min \{ \underbrace{x_{11}}_{=20}, \underbrace{x_{32}}_{=5-2\varepsilon} \} = 5-2\varepsilon \Rightarrow "x_{32} \rightarrow"$  (becomes a non-basic variable)

⑤

	$s_1$	$s_2$	$s_3$	
$w_1$	15+2ε	5-ε	*	20+ε
$w_2$	*	15+ε	*	15+ε
$w_3$	5-2ε	*	5+3ε	10+ε
	20	20	5+3ε	

$\Rightarrow \bar{x}_1 \begin{cases} \bar{x}_{11} = 15+2\varepsilon; \bar{x}_{12} = 5-\varepsilon; \bar{x}_{22} = 15+\varepsilon; \bar{x}_{31} = 5-2\varepsilon; \bar{x}_{33} = 5+3\varepsilon \\ \bar{x}_{13} = \bar{x}_{21} = \bar{x}_{23} = \bar{x}_{32} = 0 (= " * ") \end{cases}$

$f(\bar{x}_1) = 115 + 10\varepsilon (< f(\bar{x}_0))$

We repeat the steps of the algorithm:

②  $\delta_{13} = -1 + 2 - 1 + 2 = 2 > 0 \Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_1$  - it isn't the optimal solution  $\Rightarrow$  the algorithm continued...