# P.T.

| | $C_1$ | $C_2$ | |
|---|---|---|---|
| $D_1$ | $x_{11}$ $\boxed{1}$ | $x_{12}$ $\boxed{3}$ | 20 |
| $D_2$ | $x_{21}$ $\boxed{2}$ | $x_{22}$ $\boxed{2}$ | 10 |
| $D_3$ | $x_{31}$ $\boxed{3}$ | $x_{32}$ $\boxed{1}$ | 30 |
| | 20 | 20 | |

$$\sum_{i=1}^{3} a_i = 20+10+30 = 60 \text{ (oferta)}$$
$$\sum_{j=1}^{2} b_j = 20+20 = 40 \text{ (cererea)}$$
$\Rightarrow$ P.T.N.

Vom echilibra P.T. prin introducerea unui nou centru de desfacere fictiv ($C_3^f$)

of > cer $\Rightarrow + C^f$

of < cer $\Rightarrow + D^f$

| | $C_1$ | $C_2$ | $C_3^f$ | |
|---|---|---|---|---|
| $D_1$ | * $\boxed{1}$ | 0 $\boxed{3}$ | 20 $\boxed{0}$ | ~~20~~ 0 |
| $D_2$ | 10 $\boxed{2}$ | * $\boxed{2}$ | * $\boxed{0}$ | ~~10~~ 0 |
| $D_3$ | 10 $\boxed{3}$ | 20 $\boxed{1}$ | * $\boxed{0}$ | ~~30~~ ~~10~~ 0 |
| | ~~20~~ ~~10~~ 0 | ~~20~~ 0 | ~~20~~ 0 | |

60-40

of = 60
cer = 60 $\Rightarrow$ PTE

! verificare

Det $\overline{X_0}$ - SBAI cu met. costurilor minime.

$$v.p = \underset{\text{nr. depozite}}{m} + \underset{\text{nr. centrelor}}{n} - 1 = 3+3-1 = \underline{5}$$

$$\overline{X_0} = (0, \underline{0}, \underline{20}, \underline{10}, 0, 0, \underline{10}, \underline{20}, 0) - \text{SBAI } \Delta \text{ de ord I.}$$

Pentru a evita apariția fenomenului de ciclaj vom folosi metoda perturbării

| | $C_1$ | $C_2$ | $C_{3f}$ | |
|---|---|---|---|---|
| $D_1$ | $10-\varepsilon$ [1] | $*$ [3] | $10+2\varepsilon$ [0] | ~~$20+\varepsilon$~~ ~~$10-\varepsilon$~~ $0$ |
| $D_2$ | $*$ [2] | $*$ [2] | $10+\varepsilon$ [0] | ~~$10+\varepsilon$~~ $0$ |
| $D_3$ | $10+\varepsilon$ [3] | $20$ [1] | $*$ [0] | ~~$30+\varepsilon$~~ ~~$10+\varepsilon$~~ $0$ |
| | ~~$20$~~ ~~$10+\varepsilon$~~ $0$ | ~~$30$~~ $0$ | ~~$20+3\varepsilon$~~ ~~$10+2\varepsilon$~~ $0$ | |

$$\left.\begin{aligned} of &= 60+3\varepsilon \\ cer &= 60+3\varepsilon \end{aligned}\right| \Rightarrow PTE$$

! verif. ! v.p $= \underline{5}$

$$20+3\varepsilon-(10+\varepsilon) = 20+3\varepsilon-10-\varepsilon = 10+2\varepsilon$$

1) Det. $\overline{X_0}$ - SBAiNd cu met. costurilor minime.

$$\overline{X_0} = (10-\varepsilon, 0, 10+2\varepsilon, 0, 0, 10+\varepsilon, 10+\varepsilon, 20, 0) \text{ - SBAiND}$$

$$f(\overline{X_0}) = (10-\varepsilon)\cdot 1 + (10+\varepsilon)\cdot 3 + 20\cdot 1$$
$$= 10-\varepsilon + 30+3\varepsilon + 20 = 60+2\varepsilon \text{ (u.m)}.$$

2) $\overline{X_0}$ e s.o?

$$\left.\begin{aligned} \delta_{12} &= -3+1-3+1 = -4 \\ \delta_{21} &= -2+0-0+1 = -1 \\ \delta_{22} &= -2+0-0+1-3+1 = -3 \\ \delta_{33} &= -0+0-1+3 = \underline{2 > 0} \end{aligned}\right| \xrightarrow{\exists\, \delta_{ij} > 0} \overline{X_0} \text{ nu e S.O.}$$

3) Crt. de intrare: $\delta_{kl} = \max\{\delta_{ij} > 0\} = \max\{\delta_{33}\}$

$\Rightarrow$ „$x_{33}$ ↓"

4) Crt. de ieșire:

Desenăm traseul var. care intră în bază: „$x_{33}$ ↓"

(3) $(1,1)$: $10-\varepsilon+\theta = \underline{20}$ ← (2) $(1,3)$: $10+2\varepsilon-\theta = 10+2\varepsilon-(10+\varepsilon) = \underline{\varepsilon}$ !

(4) $(3,1)$: $10+\varepsilon-\theta = \underline{*}$ → (1) $(3,3)$: $\underline{\theta = 10+\varepsilon}$

$$\theta = \min\{\underbrace{x_{13}}_{10+2\varepsilon}, \underbrace{x_{31}}_{10+\varepsilon}\}$$

$$\theta = 10+\varepsilon$$

$\Rightarrow$ „$x_{31}$ →"

5) Noul tabel al P.T:

| | $C_1$ | $C_2$ | $C_3^f$ | |
|---|---|---|---|---|
| $D_1$ | $20\ \boxed{1}$ | $\times\ \boxed{3}$ | $\varepsilon\ \boxed{0}$ | $20+\varepsilon$ |
| $D_2$ | $\times\ \boxed{2}$ | $\times\ \boxed{2}$ | $10+\varepsilon\ \boxed{0}$ | $10+\varepsilon$ |
| $D_3$ | $\times\ \boxed{3}$ | $20\ \boxed{1}$ | $10+\varepsilon\ \boxed{0}$ | $30+\varepsilon$ |
| | $20$ | $20$ | $20+3\varepsilon$ | |

!! verif !!

$v.p = 5$

1') Noua S.B.A.Nd:

$\overline{X_1} = (20, 0, \varepsilon, 0, 0, 10+\varepsilon, 0, 20, 10+\varepsilon)$ SBANd

$f(\overline{X_1}) = 20 \cdot 1 + 20 \cdot 1 = 40\ (u.m) < f(\overline{X_0})$

$< f(\overline{X_0})$

!

2') $\overline{X_1}$ - e s.o ??

$\delta_{12} = -3+1-0+0 = -2$

$\delta_{21} = -2+0-0+1 = -1$

$\delta_{22} = -2+1-0+0 = -1$

$\delta_{31} = -3+0-0+1 = -2$

toți $\delta_{ij} < 0$

$\overline{X_1}$ e s.o. unică.

Cl. P.T. inițiale (P.T.N. $\xrightarrow{\text{elimin } C_3^f}$ neperturbată $\xrightarrow{\varepsilon = 0}$)

$\left\{ \begin{array}{l} \overline{X}^{\text{inițial}}_{\text{optim}} = (20, 0, 0, 0, 0, 20) \text{ - s.o. unică} \\ \min f = 40 \end{array} \right.$

Cu met. diag:

(colțul din N-V)

| | $C_1$ | $C_2$ | $C_3^f$ | |
|---|---|---|---|---|
| $D_1$ | $20\ \boxed{1}$ | $\varepsilon\ \boxed{3}$ | $\times\ \boxed{0}$ | ~~$20+\varepsilon$~~ ~~$\varepsilon$~~ $0$ |
| $D_2$ | $\times\ \boxed{2}$ | $10+\varepsilon\ \boxed{2}$ | $\times\ \boxed{0}$ | ~~$10+\varepsilon$~~ $0$ |
| $D_3$ | $\times\ \boxed{3}$ | $10-2\varepsilon\ \boxed{1}$ | $20+3\varepsilon\ \boxed{0}$ | ~~$30+\varepsilon$~~ ~~$20+3\varepsilon$~~ $0$ |
| | ~~$20$~~ SBAIK,) $0$ | ~~$20$~~ ~~$20-\varepsilon$~~ ~~$10-2\varepsilon$~~ $0$ | ~~$20+3\varepsilon$~~ $0$ | |

$\overline{X_0} = (20, \varepsilon, 0, 0, 10+\varepsilon, 0, 0, 10-2\varepsilon, 20+3\varepsilon)$

$f(\overline{X_0}) = 50 + 3\varepsilon$

(2) $\begin{cases} f: \mathbb{R}^3 \to \mathbb{R} \\ f(x,y,z) = x^3yz^2 + xy^2 + xz^3 + 2x \end{cases}$ $P(\underset{x}{-1}, \underset{y}{1}, \underset{z}{1})$

a) Calculați derivatele parțiale de ordinul I:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^3yz^2 + xy^2 + xz^3 + 2x\right) =$$

$$= yz^2 \cdot 3x^2 + y^2 \cdot 1 + z^3 \cdot 1 + 2 \cdot 1 = \boxed{3x^2yz^2 + y^2 + z^3 + 2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^3yz^2 + xy^2 + xz^3 + 2x\right) =$$

$$= x^3z^2 \cdot 1 + x \cdot 2y + 0 + 0 = \boxed{x^3z^2 + 2xy}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left(x^3yz^2 + xy^2 + xz^3 + 2x\right) =$$

$$= x^3y \cdot 2z + 0 + x \cdot 3z^2 + 0 = \boxed{2x^3yz + 3xz^2}$$

$$\frac{\partial f}{\partial x}(P) = 3 \cdot (-1)^2 \cdot 1 \cdot 1^2 + 1^2 + 1^3 + 2 = 7$$

$$\frac{\partial f}{\partial y}(P) = (-1)^3 \cdot 1^2 + 2 \cdot (-1) \cdot 1 = -3$$

$$\frac{\partial f}{\partial z}(P) = -5$$

b) Scrieți diferențiala de ordinul I:

$$df(x,y,z) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$\overset{a)}{=} (3x^2yz^2 + y^2 + z^3 + 2)\,dx + (x^3z^2 + 2xy)\,dy + (2x^3yz + 3xz^2)\,dz$$

$$df(P) = 7\,dx + (-3)\,dy + (-5)\,dz = 7\,dx - 3\,dy - 5\,dz$$

$x, y, z$

c) Calculați derivatele parțiale de ordinul II:

$$\frac{\partial^2 f}{\partial x^2} \overset{def}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \overset{a)}{=} \frac{\partial}{\partial x}\left(3x^2yz^2+y^2+z^3+2\right) = 3yz^2\cdot 2x+0+0+0 = \boxed{6xyz^2}$$

$$\frac{\partial^2 f}{\partial y^2} \overset{def}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \overset{a)}{=} \frac{\partial}{\partial y}\left(x^3z^2+2xy\right) = 0+2x\cdot 1 = \boxed{2x}$$

$$\frac{\partial^2 f}{\partial z^2} \overset{def}{=} \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) \overset{a)}{=} \frac{\partial}{\partial z}\left(2x^3yz+3xz^2\right) = 2x^3y\cdot 1+3x\cdot 2z = \boxed{2x^3y+6xz}$$

$$\frac{\partial^2 f}{\partial x\partial y} \overset{def}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \overset{a)}{=} \frac{\partial}{\partial x}\left(x^3z^2+2xy\right) = z^2\cdot 3x^2+2y\cdot 1 = \boxed{3x^2z^2+2y} = \frac{\partial^2 f}{\partial y\partial x}$$

$$\frac{\partial^2 f}{\partial y\partial x} \overset{def}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \dots$$

$$\frac{\partial^2 f}{\partial x\partial z} \overset{def}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial x}\left(2x^3yz+3xz^2\right) = 2yz\cdot 3x^2+3z^2\cdot 1 = \boxed{6x^2yz+3z^2} = \frac{\partial^2 f}{\partial z\partial x}$$

$$\frac{\partial^2 f}{\partial z\partial x} \overset{def}{=} \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right) = \dots$$

$$\frac{\partial^2 f}{\partial y\partial z} \overset{def}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right) \overset{a)}{=} \frac{\partial}{\partial y}\left(2x^3yz+3xz^2\right) = 2x^3z\cdot 1+0 = \boxed{2x^3z} = \frac{\partial^2 f}{\partial z\partial y}$$

$$\frac{\partial^2 f}{\partial z\partial y} \overset{def}{=} \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right) = \dots$$

$P(-1,1,1)$

$\frac{\partial^2 f}{\partial x^2}(P) = 6\cdot(-1)\cdot 1\cdot 1^2 = -6$

$\frac{\partial^2 f}{\partial y^2}(P) = -2$

$\frac{\partial^2 f}{\partial z^2}(P) = -8$

$\frac{\partial^2 f}{\partial x \partial y}(P) = \frac{\partial^2 f}{\partial y \partial x}(P) = 5$

$\frac{\partial^2 f}{\partial x \partial z}(P) = \frac{\partial^2 f}{\partial z \partial x}(P) = 9$

$\frac{\partial^2 f}{\partial y \partial z}(P) = \frac{\partial^2 f}{\partial z \partial y}(P) = -2$

d) Scrieți diferențiala de ordinul II

$$d^2 f(x,y,z) \overset{def}{=} \frac{\partial^2 f}{\partial x^2} d^2x + \frac{\partial^2 f}{\partial y^2} dy^2 + \frac{\partial^2 f}{\partial z^2} d^2z + 2\cdot \frac{\partial^2 f}{\partial x \partial y} dx dy$$
$$+ 2\cdot \frac{\partial^2 f}{\partial x \partial z} dx dz + 2\cdot \frac{\partial^2 f}{\partial y \partial z} dy dz \overset{c)}{=}$$
$$= (6xyz^2)d^2x + (2x)d^2y + (2x^3y + 6xz)d^2z + 2\cdot(3x^2z^2 + 2y)dxdy$$
$$+ 2\cdot(6x^2yz + 3z^2)dxdz + 2\cdot(2x^3z)dydz$$

$$d^2 f(P) = -6d^2x - 2d^2y - 8d^2z + 2\cdot 5 dx dy + 2\cdot 9 dx dz +$$
$$+ 2\cdot(-2) dy dz = -6d^2x - 2d^2y - 8d^2z + 10 dx dy +$$
$$+ 18 dx dz - 4 dy dz$$

e) Aduceți forma pătratică găsită la d) $\left(d^2 f(P)\right)$ la forma canonică.

$$d^2f(P) = -6d^2x - 2d^2y - 8d^2z + 10\,dx\,dy + 18\,dx\,dz - 4\,dy\,dz$$
$$= a_{11}d^2x + a_{22}d^2y + a_{33}d^2z + 2\cdot a_{12}\,dx\,dy + 2\cdot a_{13}\,dx\,dz +$$
$$+ 2\cdot a_{23}\,dy\,dz$$

$a_{12} = a_{21} = \frac{10}{2}$

$$A = \begin{array}{c|ccc} & dx & dy & dz \\ \hline dx & a_{11} & a_{12} & a_{13} \\ dy & a_{21} & a_{22} & a_{23} \\ dz & a_{31} & a_{32} & a_{33} \end{array} = \begin{pmatrix} -6 & \frac{10}{2} & \frac{18}{2} \\ \frac{10}{2} & -2 & -\frac{4}{2} \\ \frac{18}{2} & -\frac{4}{2} & -8 \end{pmatrix} = \begin{pmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix}$$

$a_{12} = a_{21}$

$a_{13} = a_{31}$

$a_{23} = a_{32}$

Met lui Jacobi: $\Delta_0 = 1$

$\Delta_1 = -6$

$$\Delta_2 = \begin{vmatrix} -6 & 5 \\ 5 & -2 \end{vmatrix} = (-6)\cdot(-2) - 5\cdot 5 = 12 - 25 = -13$$

$$\Delta_3 = \begin{vmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{vmatrix} \begin{matrix} \\ \\ \\ -6 & 5 & 9 \\ 5 & -2 & -2 \end{matrix} = (-6)\cdot(-2)\cdot(-8) + 5\cdot(-2)\cdot 9 + 9\cdot 5\cdot(-2) -$$
$$- 9\cdot(-2)\cdot 9 - (-2)\cdot(-2)\cdot(-6) - (-8)\cdot 5\cdot 5$$
$$= -96 - 90 - 90 + 162 + 24 + 200$$
$$= 110$$

$\Rightarrow$ toți $\Delta_i \neq 0, i = \overline{1,3}$

$$d^2f(P) = \frac{\Delta_0}{\Delta_1}dy_1^2 + \frac{\Delta_1}{\Delta_2}dy_2^2 + \frac{\Delta_2}{\Delta_3}dy_3^2 =$$

$\Delta_1 \neq 0 \qquad \Delta_2 \neq 0 \qquad \Delta_3 \neq 0$

$$= \frac{1}{-6}dy_1^2 + \frac{-6}{-13}dy_2^2 + \frac{-13}{110}dy_3^2 = \boxed{-\frac{1}{6}dy_1^2 + \frac{6}{13}dy_2^2 - \frac{13}{110}dy_3^2}$$

$\lambda_1 < 0 \quad \lambda_2 > 0 \quad \lambda_3 < 0$

$d^2f(P)$ este nedefinită ca semn. f. can.

$$\frac{13}{6}\cdot ? + \frac{11}{2} = 0$$

Met lui Gauss:

$$
\begin{array}{cc}
 & \begin{array}{ccc} dx & dy & dz \end{array} \\
\begin{array}{c} dx \\ dy \\ dz \end{array} & A = \begin{pmatrix} \boxed{-6} & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix}
\end{array}
\Big/ \cdot \frac{5}{6} \Big/ \cdot \frac{3}{2}
\sim
\begin{array}{cc}
 & \begin{array}{ccc} dx & dy & dz \end{array} \\
\begin{array}{c} dx \\ dy \\ dz \end{array} & \begin{pmatrix} -6 & 5 & 9 \\ 0 & \boxed{\frac{13}{6}} & \frac{11}{2} \\ 0 & \frac{11}{2} & \frac{11}{2} \end{pmatrix}
\end{array}
\Big/ \cdot \left(-\frac{33}{13}\right) \sim
$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} dx & dy & dz \end{array} \\
\begin{array}{c} dy_1 \\ dy_2 \\ dy_3 \end{array} & \begin{pmatrix} -6 & 5 & 9 \\ 0 & \frac{13}{6} & \frac{11}{2} \\ 0 & 0 & -\frac{110}{13} \end{pmatrix} = A^1
\end{array}
$$

$$\frac{11}{2}\cdot\left(-\frac{33}{13}\right) + \frac{11}{2} = \frac{11}{2}\left(-\frac{33}{13} + \frac{13}{1}\right) = \frac{11}{2}\cdot\left(-\frac{20}{13}\right) = -\frac{110}{13}$$

$$\frac{1}{\frac{13}{6}} = 1 : \frac{13}{6} = 1\cdot\frac{6}{13} = \frac{6}{13}$$

$a_{11}^1 = -6$

$a_{22}^1 = \frac{13}{6}$

$a_{33}^1 = -\frac{110}{13}$

$$d^2f(P) = \frac{1}{a_{11}^1}dy_1^2 + \frac{1}{a_{22}^1}dy_2^2 + \frac{1}{a_{33}^1}dy_3^2$$

$$= \frac{1}{-6}dy_1^2 + \frac{1}{\frac{13}{6}}dy_2^2 + \frac{1}{-\frac{110}{13}}dy_3^2$$

$$= \boxed{\underbrace{-\frac{1}{6}}_{\alpha_1<0}dy_1^2 + \underbrace{\frac{6}{13}}_{\alpha_2>0}dy_2^2 \underbrace{-\frac{13}{110}}_{\alpha_3<0}dy_3^2} \quad \text{f.can.} \Rightarrow$$

$\Rightarrow d^2f(P)$ este nedefinită ca semn, nu păstrează semn constant.

cu
$$\begin{cases} dy_1 = -6dx + 5dy + 9dz \\ dy_2 = \frac{13}{6}dy + \frac{11}{2}dz \\ dy_3 = -\frac{110}{13}dz \end{cases}$$

Dacă:

$$A = \begin{array}{c|ccc} & dx & dy & dz \\ dx & \boxed{0} & 2 & -2 \\ dy & 2 & 1 & -1 \\ dz & -2 & -1 & 2 \end{array} \quad \begin{array}{c|ccc} & dy & dx & dz \\ dy & \boxed{1} & 2 & -1 \\ dx & 2 & 0 & -2 \\ dz & -1 & -2 & 2 \end{array}$$

$L$

$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & -2 \\ -2 & -1 & 2 \end{pmatrix}$$

$C$

~~dx dy dz~~

dy dx dz

$$\begin{array}{c|ccc} dy_1 & 1 & 2 & -1 \\ dy_2 & 0 & -4 & 0 \\ dy_3 & 0 & 0 & 1 \end{array} = A'$$

$$\begin{cases} dy_1 = 2\,dx + 1\cdot dy - 1\,dz \\ dy_2 = -4\,dx \\ dy_3 = 1\,dz \end{cases}$$

$$d^2f(?) = \frac{1}{1}dy_1^2 + \frac{1}{-4}dy_2^2 + \frac{1}{1}dy_3^2$$

$$= \underbrace{1}_{\alpha_1>0}\cdot dy_1^2 \underbrace{-\frac{1}{4}}_{\alpha_2<0} dy_2^2 + \underbrace{1}_{\alpha_3>0}\,dy_3^2$$

nedef. ca semn.

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

nedef. ca semn.

$$\to d^2f(??) = \underbrace{\frac{1}{1}}_{\alpha_1>0}dy_1^2 \underbrace{-\frac{1}{4}}_{\alpha_2<0}dy_2^2 + \underbrace{0}_{\alpha_3=0}\cdot dy_3^2$$

$$\frac{1}{0}dy_3^2 \text{ trece în } 0\cdot dy_3^2$$

4

curs 12 pag 3

f) Scrieți matricea Hessianăi:

$$H(x,y,z) \overset{def.}{=} \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix} \overset{c)}{=}$$

$$= \begin{pmatrix} 6xyz^2 & 3x^2z^2+2y & 6x^2yz+3z^2 \\ 3x^2z^2+2y & 2x & 2x^3z \\ 6x^2yz+3z^2 & 2x^3z & 2x^3y+6xz \end{pmatrix}.$$

$$H(P) = H(\underset{x}{-1}, \underset{y}{1}, \underset{z}{1}) = \begin{pmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix}$$

//

## (3) Determinați punctele de extrem local.

$$\begin{cases} f:\mathbb{R}^2 \to \mathbb{R} \\ f(x,y) = 3xy + x^2 - xy^3 - 12 \end{cases}$$

pas 1) Calculăm derivatele parțiale de ordinul I:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(3xy + x^2 - xy^3 - 12\right) = 3y\cdot 1 + 2x - y^3\cdot 1 + 0 = \boxed{3y + 2x - y^3}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(3xy + x^2 - xy^3 - 12\right) = 3x\cdot 1 + 0 - x\cdot 3y^2 - 0 = \boxed{3x - 3xy^2}$$

pas 2) Determinăm punctele critice (staționare) rezolvând următorul sistem:

$$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases} \xrightarrow{\text{pas 1)}} \begin{cases} 3y + 2x - y^3 = 0 \\ 3x - 3xy^2 = 0 \;\; /:3 \end{cases} \Leftrightarrow \begin{cases} 3y + 2x - y^3 = 0 \\ x - xy^2 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 3y + 2x - y^3 = 0 \\ x\cdot(1 - y^2) = 0 \end{cases}$$

$$\begin{cases} 3y + 2x - y^3 = 0 \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} 3y + 2\cdot 0 - y^3 = 0 \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} y\cdot(3 - y^2) = 0 \\ x = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow P_1(0,0)$$

$$\begin{cases} 3 - y^2 = 0 \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} y^2 = 3 \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} y = \pm\sqrt{3} \\ x = 0 \end{cases} \Rightarrow P_2(0,\sqrt{3}),\; P_3(0,-\sqrt{3})$$

$$\begin{cases} 3y + 2x - y^3 = 0 \\ 1 - y^2 = 0 \end{cases} \Leftrightarrow \begin{cases} 3y + 2x - y^3 = 0 \\ y^2 = 1 \end{cases} \Leftrightarrow \begin{cases} 3y + 2x - y^3 = 0 \\ y = \pm 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 3y + 2x - y^3 = 0 \\ y = 1 \end{cases} \Leftrightarrow \begin{cases} 3\cdot 1 + 2x - 1^3 = 0 \\ y = 1 \end{cases} \Leftrightarrow \begin{cases} 2x = -2 \\ y = 1 \end{cases} \Leftrightarrow \begin{cases} x = -1 \\ y = 1 \end{cases} \Rightarrow P_4(-1,1)$$

$$\begin{cases} 3y + 2x - y^3 = 0 \\ y = -1 \end{cases} \Leftrightarrow \begin{cases} 3\cdot(-1) + 2x - (-1)^3 = 0 \\ y = -1 \end{cases} \Leftrightarrow \begin{cases} 2x = 2 \\ y = -1 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = -1 \end{cases} \Rightarrow P_5(1,-1)$$

Am obținut 5 puncte critice (staționare):

$P_1(0,0)$; $P_2(0,\sqrt{3})$; $P_3(0,-\sqrt{3})$; $P_4(-1,1)$; $P_5(1,-1)$

pas 3) Calculăm derivatele parțiale de ordinul II:

$$\frac{\partial^2 f}{\partial x^2} \overset{def}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \overset{pas 1)}{=} \frac{\partial}{\partial x}\left(3y + 2x - y^3\right) = 0 + 2\cdot 1 - 0 = \boxed{2}$$

$$\frac{\partial^2 f}{\partial y^2} \overset{def}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \overset{pas 1)}{=} \frac{\partial}{\partial y}\left(3x - 3xy^2\right) = 0 - 3x\cdot 2y = \boxed{-6xy}$$

$$\frac{\partial^2 f}{\partial x \partial y} \overset{def}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \overset{pas 1)}{=} \frac{\partial}{\partial x}\left(3x - 3xy^2\right) = 3\cdot 1 - 3y^2\cdot 1 = \boxed{3-3y^2}$$

$$\overset{!}{=} \quad \frac{\partial^2 f}{\partial y \partial x} \overset{def}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \overset{pas 1)}{=} \frac{\partial}{\partial y}\left(3y + 2x - y^3\right) = 3\cdot 1 + 0 - 3y^2 = \boxed{3-3y^2}$$

pas 4) Scriem matricea Hessiană:

$$H(x,y) \overset{def}{=} \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} \overset{pas 3)}{=} \begin{pmatrix} 2 & 3-3y^2 \\ 3-3y^2 & -6xy \end{pmatrix}$$

pas 5) Calculăm matricea Hessiană în toate punctele critice găsite la pasul 2) și apoi stabilim natura lor. (met. lui Jacobi).

• $P_1(0,0) \Rightarrow H(P_1) = H(\underset{x}{0},\underset{y}{0}) \overset{pas 4)}{=} \begin{pmatrix} 2 & 3-3\cdot 0^2 \\ 3-3\cdot 0^2 & -6\cdot 0\cdot 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 0 \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{cases} \Delta_1 = 2 \neq 0 \\ \Delta_2 = \begin{vmatrix} 2 & 3 \\ 3 & 0 \end{vmatrix} = 2\cdot 0 - 3\cdot 3 = -9 \neq 0 \end{cases} \xRightarrow[+,\,-]{\text{Th. 2 din curs 13}} P_1(0,0) \text{ este punct de inflexiune (pct. SA) (nu este pct. de extrem local)}$$

$$\left|\begin{matrix} a & b \\ c & d \end{matrix}\right| = a\cdot d - b\cdot c$$

$$\left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right| = a\cdot e\cdot i + d\cdot h\cdot c + g\cdot b\cdot f - c\cdot e\cdot g - f\cdot h\cdot a - i\cdot b\cdot d$$

$$\begin{matrix} a & b & c \\ d & e & f \end{matrix}$$

- $P_2(0,\sqrt{3}) \Rightarrow H(P_2) = H(0,\sqrt{3}) \xrightarrow{pas\,1)} \begin{pmatrix} 2 & -6 \\ -6 & 0 \end{pmatrix} \Rightarrow$

$\xrightarrow[\text{Jacobi}]{\text{met lui}} \begin{cases} \Delta_1 = 2 \neq 0 \\ \Delta_2 = -36 \neq 0 \end{cases} \xrightarrow{+,-} P_2$ e pct. şa

cu met lui Gauss: $A = H(P_2) = \begin{pmatrix} \boxed{2} & -6 \\ -6 & 0 \end{pmatrix} \cdot 3 \sim$

$\sim \begin{pmatrix} 2 & -6 \\ 0 & -18 \end{pmatrix} = A' \Rightarrow d^2 f(P_2) = \frac{1}{a'_{11}} dy_1^2 + \frac{1}{a'_{22}} dy_2^2$

$= \frac{1}{2} dy_1^2 + \frac{1}{-18} dy_2^2$

$= \boxed{\underbrace{\frac{1}{2}}_{\alpha_1 > 0} dy_1^2 - \underbrace{\frac{1}{18}}_{\alpha_2 < 0} dy_2^2}$ f. can.

$$\begin{aligned} 3 - 3y^2 &= 3 - 3\cdot\sqrt{3}^2 \\ &= 3 - 3\cdot 3 \\ &= 3 - 9 \\ &= -6 \end{aligned}$$

$d^2 f$ este nedefinită ca semn

// curs 13, Th 1

$P_2(0,\sqrt{3})$ este pct. şa.

- $P_3$ ...
- $P_4$ ...
- $P_5$ ...

④

| | $C_1$ | $C_2$ | $C_3$ | |
|---|---|---|---|---|
| $D_1$ | 1 | 3 | 2 | 20 |
| $D_2$ | 2 | 1 | 3 | 20 |
| | 20 | 10 | 30 | |

$\sum_{i=1}^{2} a_i = 20+20=40$ (oferta)

$\sum_{j=1}^{3} b_j = 20+10+30=60$ (cererea)

$\Rightarrow$ PTN

Echilibrăm P.T. prin introducerea unui depozit fictiv, $D_3^f$ (ce are costurile de transport egale cu 0).

oferta < cererea $\Rightarrow + D^f$
oferta > cererea $\Rightarrow + C^f$

| | $C_1$ | $C_2$ | $C_3$ | |
|---|---|---|---|---|
| $D_1$ | 20 [1] | $\varepsilon$ [3] | x [2] | ~~20+ε~~ ~~ε~~ 0 |
| $D_2$ | x [2] | $10-\varepsilon$ [1] | $10+2\varepsilon$ [3] | ~~20+ε~~ ~~10+2ε~~ 0 |
| $D_3^f$ | x [0] | x [0] | $20+\varepsilon$ [0] | ~~20+ε~~ 0 |
| | ~~20~~ 0 | ~~10~~ ~~10-ε~~ 0 | ~~30+3ε~~ ~~20+ε~~ 0 | |

60−40

Pt. a evita apariția fen. de ciclaj folosim met. perturbării:

oferta $=60+3\varepsilon$
cererea $=60+3\varepsilon$ $\Rightarrow$ PTE

$(20+\varepsilon)-(10-\varepsilon)=20+\varepsilon-10+\varepsilon$
$=10+2\varepsilon$

pas 1) $\overline{X_0}=?$ cu met. diagonalei (colțul de N-V).

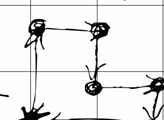
$\overline{X_0}=(20,\varepsilon,0,0,10-\varepsilon,10+2\varepsilon,0,0,20+\varepsilon)$ − SBAINOD

$f(\overline{X_0})=20\cdot1+\varepsilon\cdot3+(10-\varepsilon)\cdot1+(10+2\varepsilon)\cdot3$
$=20+3\varepsilon+10-\varepsilon+30+6\varepsilon=60+8\varepsilon$ (u.m)

$Vp=m+n-1=3+3-1=\underline{5}$

pas 2) $\overline{X_0}=$ e S.O. ??

$\delta_{13}=-2+3-1+3=3>0$
$\delta_{21}=-2+1-3+1=-3$
$\delta_{31}=-0+0-3+1-3+1=-4$
$\delta_{32}=-0+0-3+1=-2$



$\exists\,\delta_{ij}>0 \Rightarrow \overline{X_0}$ nu e S.O.

pas3) $\delta_{kl} = \max\{\delta_{ij} > 0\} = \max\{\delta_{13}\} \Rightarrow$ „$x_{13}$ ⊕"

pas4)

$\varepsilon - \theta = \cancel{\varepsilon}$ $\qquad$ $\theta = \varepsilon$

$$\begin{tikzcd}
4 \; (1,2) & & (1,3) \; 1 \arrow[d] \\
3 \; (2,2) \arrow[u] & & (2,3) \; 2 \arrow[ll]
\end{tikzcd}$$

$10-\varepsilon+\theta = 10$ $\qquad$ $10+2\varepsilon-\theta = 10+\varepsilon$

$\theta = \varepsilon$

„$x_{12}$ ⊖"

pas5)

| | $C_1$ | $C_2$ | $C_3$ | |
|---|---|---|---|---|
| $D_1$ | $20^{\,1}$ | $*^{\,3}$ | $\varepsilon^{\,2}$ | $20+\varepsilon$ |
| $D_2$ | $*^{\,2}$ | $10^{\,1}$ | $10+\varepsilon^{\,3}$ | $20+\varepsilon$ |
| $D_3^f$ | $*^{\,0}$ | $*^{\,0}$ | $20+\varepsilon^{\,0}$ | $20+\varepsilon$ |
| | 20 | 10 | $30+3\varepsilon$ | |

! verif.

pas1')
$$\begin{cases} \overline{X_1} = (20, 0, \varepsilon, 0, 10, 10+\varepsilon, 0, 0, 20+\varepsilon) \text{ SBA-Nd} \\ f(\overline{X_1}) = 60+5\varepsilon \text{ (u.m)} < f(\overline{X_0}) \end{cases}$$

pas2') $\overline{X_1} = ?$ s.o ?

$\delta_{12} = -3$
$\delta_{21} = 0$
$\delta_{31} = -1$
$\delta_{32} = -2$

toți $\delta_{ij} \leq 0 \Longrightarrow$ $\overline{X_1}$ e soluție optimă, dar nu este unică.

$$\text{Cl}: \begin{cases} \overline{X}^{\,\text{initial}}_{\text{optim}} = X^{\text{PT Nnepert}}_{\text{optim}} \xrightarrow{\text{elimin } D_3^f,\ \varepsilon=0} = (20, 0, 0, 0, 10, 10) \\ \min f = 60 \text{ (u.m.)} \end{cases}$$

s.o. nemunică