Metoda celor două faze

① $(PPL)\begin{cases} (1)\ \max f(x_1,x_2,x_3) = x_1 - 2x_2 + 3x_3 \\ (2)\ \begin{cases} 2x_1 + x_2 - x_3 \geq 4 \\ -x_1 + 2x_2 + 2x_3 \geq \cancel{8}\ 9 \end{cases} \\ (3)\ x_1, x_2, x_3 \geq 0 \end{cases}$

$\max f = -\min(-f)$

$" \leq " \Rightarrow " + "$

$" \geq " \Rightarrow " - "$

$$(PPL)_s : \begin{cases} (1_s)\ \min(-f)(x_1,x_2,x_3,x_4^c,x_5^c) = -x_1 + 2x_2 - 3x_3 + \\ \qquad + 0\cdot x_4^c + 0\cdot x_5^c \\ (2_s)\begin{cases} 2x_1 + x_2 - x_3 - x_4^c = 4 \\ -x_1 + 2x_2 + 2x_3 - x_5^c = \cancel{8}\ 9 \end{cases} \Rightarrow \overline{A}_s = \begin{array}{c} \begin{array}{cccccc} P_1 & P_2 & P_3 & P_4^c & P_5^c & P_0 \end{array} \\ \left(\begin{array}{ccccc|c} 2 & 1 & -1 & -1 & 0 & 4 \\ -1 & 2 & 2 & 0 & -1 & \cancel{8}\ 9 \end{array}\right) \end{array} \\ (3_s)\ x_1, x_2, x_3, x_4^c, x_5^c \geq 0 \end{cases}$$

$$(PPL)_a : \begin{cases} (1a)\ \min(f_a)(x_6^a, x_7^a) = 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 0\cdot x_4^c + 0\cdot x_5^c + x_6^a + x_7^a \\ (2a)\begin{cases} 2x_1 + x_2 - x_3 - x_4^c + x_6^a = 4 \\ -x_1 + 2x_2 + 2x_3 - x_5^c + x_7^a = \cancel{8}\ 9 \end{cases} \Rightarrow \overline{A}_a = \begin{array}{c} \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4^c & P_5^c & P_6^a & P_7^a & P_0 \end{array} \\ \left(\begin{array}{ccccccc|c} 2 & 1 & -1 & -1 & 0 & 1 & 0 & 4 \\ -1 & 2 & 2 & 0 & -1 & 0 & 1 & \cancel{8}\ 9 \end{array}\right) \end{array} \\ (3a)\ x_1, x_2, x_3, x_4^c, x_5^c, x_6^a, x_7^a \geq 0 \end{cases}$$

v.s. = 0   v.p.

$\Rightarrow \overline{X}_0^a = (0,0,0,0,0,4,\cancel{8}\ 9)^T$ - SBAi$_a$

$$\overline{A}_a = \begin{array}{c} \begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4^c & P_5^c & P_6^a & P_7^a & P_0 \end{array} \\ \left(\begin{array}{ccccccc|c} 2 & 1 & -1 & -1 & 0 & 0 & 1 & 4 \\ -1 & 2 & 2 & 0 & -1 & 1 & 0 & 8 \end{array}\right) \end{array} \Rightarrow \overline{X}_0^a = (0,0,0,0,0,8,4)^T$$

v.s. = 0

Faza I: Tabelul Simplex pt. $(PPL)_a$:

		$(f_a) \rightarrow$	0	0	0	0	0	1	1		col. lui P_0
B	C_B	P_0	P_1	$P_2 \downarrow$	$P_3 \downarrow$	P_4^c	P_5^c	P_6^a	P_7^a	$\theta_i = \frac{P_0}{P_j \downarrow} > 0$	
$\leftarrow P_6^a$	1	4	2	$\boxed{1}$	-1	-1	0	1	0	$\frac{P_0}{P_2 \downarrow}$ $\frac{4}{1}$ (-2)	$\Rightarrow \overline{X}_0^a = (0,0,0,0,0,4,9)^T$
P_7^a	1	$\cancel{8}$ 9	-1	2	2	0	-1	0	1	[illegible]	$f_a(\overline{X}_0^a) = 12$
		$f_a(\overline{X}_0^a) = \cancel{12}$ 13	1	3	1	-1	-1	0	0	$z_j - c_j$	
			$z_1 - c_1$	$z_2 - c_2$							
P_2	0	4	2	1	-1	-1	0	1	0	$\frac{P_0}{P_3 \downarrow}$ * (-4)	$\Rightarrow \overline{X}_1^a = (0,4,0,0,0,0,1)^T$
$\leftarrow P_7^a$	1	$\cancel{0}$ 1	-5	0	$\boxed{4}$	2	-1	-2	1	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	
		$f_a(\overline{X}_1^a) = 1$	-5	0	4	2	-1	-3	0	$z_j - c_j$	
P_2	0	$\frac{17}{4}$	$+\frac{3}{4}$	1	0	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$		$\Rightarrow \overline{X}_2^a = (0,\frac{17}{4},\frac{1}{4},0,0,0,0)^T$
P_3	0	$\frac{1}{4}$	$-\frac{5}{4}$	0	1	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$		$f_a(\overline{X}_2^a) = 0$
		$f_a(\overline{X}_2^a) = 0$	0	0	0	0	0	-1	-1	$z_j - c_j$	

Cl. pt $(PPL)_a$: $\begin{cases} X_{optim}^{artif.} = \overline{X}_2^a = (0, \frac{17}{4}, \frac{1}{4}, 0,0,0,0)^T \\ \min f_a = 0 \end{cases}$

soluție optimă neunică

Faza II: Tabelul Simplex pt $(PPL)_s$:

			-1	2	-3	0	0		ok pe col lui P_0
B	C_B	P_0	P_1	P_2	P_3	P_4^c	P_5^c	$\theta_i = \frac{P_0}{P_j \downarrow} > 0$	
P_2	2	$\frac{17}{4}$	$\frac{3}{4}$	1	0	$-\frac{1}{2}$	$-\frac{1}{4}$		$\Rightarrow \overline{X}_0^s = (0,\frac{17}{4},\frac{1}{4},0,0)^T$
P_3	-3	$\frac{1}{4}$	$-\frac{5}{4}$	0	1	$\frac{1}{2}$	$-\frac{1}{4}$		$-f(\overline{X}_0^s) = \frac{31}{4}$
		$-f(\overline{X}_0^s) = \frac{31}{4}$	$\frac{25}{4}$	0	0	$-\frac{5}{2}$	$\frac{1}{4}$	$z_j - c_j$	

Cl. pt $(PPL)_s$: $\begin{cases} S_O = \emptyset \\ \boxed{\min(-f) = -\infty} \end{cases} \Rightarrow$ Cl. pt $(PPL)_i$: $\begin{cases} S_O = \emptyset \\ \max f = +\infty \end{cases}$

(1) $\max f(x_1, x_2, x_3) = 35x_1 + 55x_2 + 90x_3$ (mii lei)

$35x_1 = 35 \dfrac{\text{mii lei}}{\text{buc}} \cdot x_1 \text{ (buc)}$

sau

$= 35000x_1 + 55000x_2 + 90000x_3$ (lei)

sau $= 350x_1 + 550x_2 + 900x_3$ (sute lei)

$85000x_1 + 125000x_2 + 180000x_3 \leq 200000000$ (lei)

$85x_1 + 125x_2 + 180x_3 \leq 200000$ (mii lei)

P.T.E.

Met. diag.

! verif !

	C_1	C_2	C_3	C_4	
D_1	10 $\boxed{2}$	15 $\boxed{1}$	5 $\boxed{3}$	* $\boxed{4}$	$\cancel{30}$ $\cancel{20}$ $\cancel{5}$ 0
D_2	* $\boxed{1}$	* $\boxed{2}$	10 $\boxed{1}$	* $\boxed{1}$	$\cancel{10}$ 0
D_3	* $\boxed{3}$	* $\boxed{2}$	10 $\boxed{2}$	10 $\boxed{1}$	$\cancel{20}$ $\cancel{10}$ 0
	$\cancel{10}$ 0	$\cancel{15}$ 0	$\cancel{25}$ $\cancel{20}$ $\cancel{10}$ 0	$\cancel{10}$ 0	

$$\sum_{i=1}^{3} a_i = 30+10+20 = 60 \text{ (oferta)}$$

$$\sum_{j=1}^{4} b_j = 10+15+25+10 = 60 \text{ (cererea)}$$

$\Rightarrow$ P.T.E.

$$v.p = m+n-1 = 3+4-1 = \underline{6}$$

$$v.s = m \cdot n - (m+n-1) = 12-6 = \underline{6}$$

$$\begin{cases} \overline{X_0} = (10, 15, 5, 0,\ 0, 0, 10, 0,\ 0, 0, 10, 10)^T - \text{SBAINd} \\ f(\overline{X_0}) = \text{- - -} \ (\text{um}) \end{cases}$$

Met. celor 2 faze:

$$(PPL)\begin{cases}(1)\ \max f(x_1,x_2,x_3)=2x_1-x_2+x_3\\ (2)\begin{cases}x_1+x_2-x_3\le 2\\ 2x_1-x_2+2x_3\ge 5\end{cases}\\ (3)\ x_1,x_2,x_3\ge 0\end{cases}$$

"$\le$" $\Rightarrow$ "+"

"$\ge$" $\Rightarrow$ "−"

$$\boxed{\max f = -\min(-f)}$$

$$\Downarrow$$

$$(PPL)_s\begin{cases}(1s)\ \min(-f)(x_1,x_2,x_3,x_4^c,x_5^c)=-2x_1+x_2-x_3+0\cdot x_4^c+0\cdot x_5^c\\ (2s)\begin{cases}x_1+x_2-x_3+x_4^c=2\\ 2x_1-x_2+2x_3-x_5^c=5\end{cases}\\ (3s)\ x_1,x_2,x_3,x_4^c,x_5^c\ge 0\end{cases}$$

$$\Rightarrow \overline{A}_s=\begin{array}{c}\begin{array}{cccccc}P_1 & P_2 & P_3 & P_4^c & P_5^c & P_0\end{array}\\ \left(\begin{array}{ccccc|c}1 & 1 & -1 & 1 & 0 & 2\\ 2 & -1 & 2 & 0 & -1 & 5\end{array}\right)\end{array}$$

$$\Downarrow$$

$$(PPL)_a\begin{cases}(1a)\ \min(f_a)(x_6^a)=0\cdot x_1+0\cdot x_2+0\cdot x_3+0\cdot x_4^c+0\cdot x_5^c+1\cdot x_6^a\\ (2a)\begin{cases}x_1+x_2-x_3+x_4^c=2\\ 2x_1-x_2+2x_3-x_5^c+x_6^a=5\end{cases}\\ (3a)\ x_1,x_2,x_3,x_4^c,x_5^c,x_6^a\ge 0\end{cases}$$

$$\Rightarrow \overline{A}_a=\begin{array}{c}\begin{array}{ccccccc}P_1 & P_2 & P_3 & P_4^c & P_5^c & P_6^a & P_0\end{array}\\ \left(\begin{array}{cccccc|c}1 & 1 & -1 & 1 & 0 & 0 & 2\\ 2 & -1 & 2 & 0 & -1 & 1 & 5\end{array}\right)\end{array}$$

V.S $=0$ V.P.

$$\Rightarrow \overline{x_0^a}=(0,0,0,2,0,5)^T - \text{SBAi}\,a$$

$\in \mathbb{R}^6$

Faza I: Tabelul Simplex pt $(PPL)_a$:

		$(1a)\to$	0	0	0	0	0	1	
B	C_B	P_0	$P_1\downarrow$	P_2	$P_3\downarrow$	P_4^c	P_5^c	P_6^a	$\theta_i = \frac{P_0}{P_j\downarrow} > 0$
$\leftarrow P_4^c$	0	2	$\boxed{1}$	1	-1	1	0	0	$\frac{P_0}{P_1\downarrow}$: 2 $\cdot(-2)$
P_6^a	1	5	2	-1	2	0	-1	1	$\frac{5}{2}$ +
	$f_a(\overline{X_0}^a) = 5$		2 (z_1-c_1)	-1 (z_2-c_2)	2	0	-1	0	$z_j - c_j$
P_1	0	2	1	1	-1	1	0	0	* +
$\leftarrow P_6^a$	1	1	0	-3	$\boxed{4}$	-2	-1	1	$\frac{P_0}{P_3\downarrow}$: $\frac{1}{4}$ $\cdot\frac{1}{4}$ $\cdot\frac{1}{4}$
	$f_a(\overline{X_1}^a) = 1$		0	-3	4	-2	-1	0	$z_j - c_j$
P_1	0	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	
P_3	0	$\frac{1}{4}$	0	$-\frac{3}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	
	$f_a(\overline{X_2}^a) = 0$		0	0	0	0	0	-1	$z_j - c_j$

de pe col. lui P_0

$\Rightarrow \overline{X_0}^a = (0,0,0,2,0,5)^T$

$f_a(\overline{X_0}^a) = 5$

$\Rightarrow \overline{X_1}^a = (2,0,0,0,0,1)^T$

$f_a(\overline{X_1}^a) = 1$

$\Rightarrow \overline{X_2}^a = (\frac{9}{4}, 0, \frac{1}{4}, 0, 0, 0)^T$

$f_a(\overline{X_2}^a) =$

Cl. pt. $(PPL)_a$: $\begin{cases} X_{optim}^{artif} = \overline{X_2}^a = (\frac{9}{4}, 0, \frac{1}{4}, 0, 0, 0)^T \\ \min f_a = 0 \end{cases}$ S.O. unică

Faza II: Tabelul Simplex pt $(PPL)_s$:

		$(1s)\to$	-2	1	-1	0	0	
B	C_B	P_0	P_1	P_2	P_3	P_4^c	P_5^c	$\theta_i = \frac{P_0}{P_j\downarrow} > 0$
P_1	-2	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	
P_3	-1	$\frac{1}{4}$	0	$-\frac{3}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$	
	$-f(\overline{X_0}^s) = -\frac{19}{4}$		0	$-\frac{3}{4}$	0	$-\frac{1}{2}$	$\frac{3}{4}$	$z_j - c_j$

$\Rightarrow \overline{X_0}^s = (\frac{9}{4}, 0, \frac{1}{4}, 0, 0)^T$

$-f(\overline{X_0}^s) = -\frac{19}{4}$

Cl. pt $(PPL)_s$: $\begin{cases} S_O = \emptyset \\ \min(-f) = -\infty \end{cases}$ $\Rightarrow$ Cl. pt $(PPL)_i$: $\begin{cases} S_O = \emptyset \\ \max f = +\infty \end{cases}$

optim infinit.

$$450 \frac{lei}{\cancel{hl}} \cdot x_1 \cancel{hl} + \dots$$

$$x_1 = 150\,000\,000 \text{ (lei)}$$

$\downarrow$ lei

~~150 (mil lei)~~

$$0{,}02\% x_1 = \frac{0{,}02}{100} x_1 = \frac{2}{10000} x_1 = 0{,}0002\, x_1$$

$$0{,}0002 x_1 + 0{,}0006 x_2 + 0{,}0004 x_3 \le 380\,000 \text{ (lei)} \quad \cdot 10^4$$

$$\boxed{2x_1 + 6x_2 + 4x_3 \le 3800000000 \text{ (lei)}}$$

$$380000 \text{ lei} = 3800000000 \text{ (\% lei)}$$