Probleme de transport neechilibrate

1) Metoda diagonalei

	C_1	C_2	C_3	
D_1	3	1	2	10
D_2	2	4	1	25
	20	10	10	

Pasul 0

oferta: $\sum_{i=1}^{2} a_i = 10+25=35$

cererea: $\sum_{j=1}^{3} b_j = 20+10+10=40$

$\Rightarrow$ oferta $<$ cererea $\Rightarrow$

$\Rightarrow$ P.T.N.

• Echilibrăm P.T.N.

$\rightarrow$ transformăm P.T.N. în P.T.E. prin introducerea unui nou depozit de desfacere, D_3^f, ce va avea costurile de transport egale cu zero.

$40-35=\underline{5}$ (D_3^f)

	C_1	C_2	C_3	
D_1	$\boxed{3}$ 10	$\boxed{1}$ *	$\boxed{2}$ *	~~10~~ 0
D_2	$\boxed{2}$ 10	$\boxed{4}$ 10	$\boxed{1}$ 5	~~25~~ ~~15~~ ~~5~~ 0
D_3^f	$\boxed{0}$ *	$\boxed{0}$ *	$\boxed{0}$ 5	~~5~~ 0
	~~20~~ ~~10~~ 0	~~10~~ 0	~~10~~ ~~5~~ 0	

!verificarea!

V.P. = 5

V.S. = 4

Pasul 1

$\overline{x_0} = (10, 0, 0, 10, 10, 5, 0, 0, 5) \in \mathbb{R}^9$ - S.B. Ned.

$f(\overline{x_0}) = 10 \cdot 3 + 10 \cdot 2 + 10 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 95$ (u.m.)

Pasul 2

$\delta_{12} = -1 + 4 - 2 + 3 = 4 > 0$

$\delta_{13} = -2 + 1 - 2 + 3 = 0$

$\delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$

$\delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{x_0}$ Nu este optimă

Pasul 3

$\delta_{ke} = \max\{\delta_{12}; \delta_{31}; \delta_{32}\} = \delta_{12} \Rightarrow x_{12} \downarrow$

Pasul 4

$$\begin{tikzcd}
\overset{\cancel{10}\ *}{\textcircled{2}\ (1,1)} \arrow[d] & \overset{10}{(1,2)\downarrow} \arrow[l] \\
\underset{\substack{\cancel{10} \\ 20}}{(2,1)} \arrow[r] & \underset{\substack{\cancel{10} \\ 0}}{(2,2)\ \textcircled{4}} \arrow[u]
\end{tikzcd}$$

$\Theta = \min\{(1,1); (2,2)\} = (1,1) \Rightarrow x_{11} \rightarrow$

$\Theta = 10$

Pasul 5

$\boxed{3}$ *	$\boxed{1}$ 10	$\boxed{2}$ *	10
$\boxed{2}$ 20	$\boxed{4}$ 0	$\boxed{1}$ 5	25
$\boxed{0}$ *	$\boxed{0}$ *	$\boxed{0}$ 5	5
20	10	10	

! verificare !

Pasul 6

$\overline{x_1} = (0, 10, 0, 20, \underline{0}, 5, 0, 0, 5) \in R^9$ - S.B.Deg

$f(\overline{x_1}) = 10 \cdot 1 + 20 \cdot 2 + 0 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 55$ (u.m) $\leq f(\overline{x_0})$? Da

Pasul 7

$\delta_{11} = -3+1-4+2 = -4$

$\delta_{13} = -2+1-4+1 = -4$

$\delta_{31} = -0+2-1+0 = 1>0$

$\delta_{32} = -0+4-1+0 = 3>0$

$\Rightarrow \exists\, \delta_{ij}>0 \Rightarrow \overline{X_1}$ nu este optimă.

Pasul 8

$S_{ke} = \max\{\delta_{31};\delta_{32}\} = \delta_{32} \Rightarrow x_{32}\downarrow$

Pasul 9

$$
\begin{tikzcd}
2\,(2,2)^{\cancel{0}\,*} \arrow[r] & (2,3)^{5} \arrow[d] \\
\downarrow(3,2)_{0} \arrow[u] & (3,3)\,4\ \ 5 \arrow[l]
\end{tikzcd}
$$

$\Theta = \min\{(2,2);(3,3)\} = (2,2) \Rightarrow (2,2) = x_{22} \rightarrow$

$\Theta = 0$

Pasul 10

[3] *	[1] 10	[2] *	10
[2] 20	[4] *	[1] 5	25
[0] *	[0] 0	[0] 5	5
20	10	10	

! verificare!

Pasul 11

$\bar{x}_2 = (0,10,0,20,0,5,0,0,5) \in R^9$ - s.B. Deg.

$f(\bar{x}_2) = 10\cdot1+20\cdot2+5\cdot1+0\cdot5\cdot0 = 55$ (u.m.) $\leq f(\bar{x}_1)$? Da

Pasul 12

$\delta_{11} = -3+1-0+0-1+2 = -1$

$\delta_{13} = -2+1-0+0 = -1$

$\delta_{22} = -4+0-0+1 = -3$

$\delta_{31} = -0+0-1+2 = 1>0$

$\Rightarrow \exists\, \delta_{ij}>0 \Rightarrow \bar{x}_2$ nu este optimă.

Pasul 13

$S_{Ke} = \max\{\delta_{31}\} = \delta_{31} \Rightarrow x_{31} \downarrow$

Pasul 14

$$\begin{tikzcd}
\overset{\cancel{20}\ 15}{(2,1)} \ (4) \arrow[d] & \overset{\cancel{5}\ 10}{(2,3)} \arrow[l] \\
\downarrow (3,1) \ \underset{5}{} \arrow[r] & (3,3) \ (2) \ \underset{\cancel{5}\ *}{} \arrow[u]
\end{tikzcd}$$

$\Theta = \min\{(3,3);(2,1)\} = (3,3) \Rightarrow x_{33} \rightarrow$

$\Theta = 5$

Pasul 15

[3] *	[1] 10	[2] *	10
[2] 15	[4] *	[1] 10	25
[0] 5	[0] 0	[0] *	5
20	10	10	

! verificare !

$\overline{x_3} = (0, 10, 0, 15, 0, 10, 5, 0, 0) \in \mathbb{R}^9$ - S.B.Deg.

$f(\overline{x_3}) = 10 + 30 + 10 = 50$ (u.m) $\leq f(\overline{x_2})$? Da

$\delta_{11} = -3 + 1 - 0 + 0 = -2$

$\delta_{13} = -2 + 1 - 0 + 0 = -1$

$\delta_{22} = -4 + 0 - 0 + 2 = -2$

$\delta_{33} = -0 + 0 - 2 + 1 = -1$

$\Rightarrow$ toți $\delta_{ij} \leq 0 \Rightarrow \overline{x_3}$ este S.O. unică.

Concluzia pt. P.T.E.: $\begin{cases} x_{optim}^{echilibrat} = (0, 10, 0, 15, 0, 10, 5, 0, 0) \\ \min f = 50 \text{ (u.m)} \end{cases}$

Concluzia pt. P.T.N.: $\begin{cases} x_{optim}^{neechilibrat} = (0, 10, 0, 15, 0, 10) \\ \min f = 50 \text{ (u.m)} \end{cases}$

2

Probleme de transport neechilibrate

2) Metoda costului minim

	C_1	C_2	
D_1	[3]	[2]	10
D_2	[1]	[4]	20
D_3	[2]	[1]	20
	10	25	

Pasul 0

oferta: $\sum_{i=1}^{3} a_i = 10+20+20=50$

cererea: $\sum_{j=1}^{2} b_j = 10+25=35$

$\Rightarrow$ oferta > cererea $\Rightarrow$ P.T.N.

• Echilibrăm P.T.N.

$\rightarrow$ transformăm P.T.N. în P.T.E. prin introducerea unui nou centru de desfacere fictiv, C_3^f, ce va avea costurile de transport egale cu zero.

$$50-35=15\,(C_3^f)$$

	C_1	C_2	C_3^f	
D_1	$\boxed{3}$ *	$\boxed{2}$ *	$\boxed{0}$ 10	~~10~~ 0
D_2	$\boxed{1}$ 10	$\boxed{4}$ 5	$\boxed{0}$ 5	~~20~~ ~~15~~ ~~5~~ 0
D_3	$\boxed{2}$ *	$\boxed{1}$ 20	$\boxed{0}$ *	~~20~~ 0
	~~10~~ 0	~~25~~ ~~5~~ 0	~~15~~ ~~5~~ 0	

! verificare !

Pasul 1

$\overline{x_0} = (0,0,10,10,5,5,0,20,0) \in \mathbb{R}^9$ - S.B.Ned.

$f(\overline{x_0}) = 10\cdot 0 + 10\cdot 1 + 5\cdot 4 + 5\cdot 0 + 20\cdot 1 = 50$ (u.m.)

Pasul 2

$\delta_{11} = -3+1-0+0 = -2$

$\delta_{12} = -2+4-0+0 = 2 > 0$

$\delta_{31} = -2+1-4+1 = -4$

$\delta_{33} = -0+1-4+0 = -3$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{x_0}$ nu este optimă

Pasul 3

$\delta_{ke} = \max\{\delta_{12}\} = \delta_{12} \Rightarrow x_{12}\downarrow$

Pasul 4

$$\begin{tikzcd}
① \downarrow \overset{5}{(1,2)} \arrow[d] & (1,3)^{④} \; \cancel{10}\,5 \arrow[l] \\
\leftarrow (2,2) \; ② \; \cancel{5}\,* \arrow[r] & (2,3) \; \cancel{5}\,10 \; ③ \arrow[u]
\end{tikzcd}$$

$\theta = \min\{(2,2) ; (1,3)\} = (2,2) \Rightarrow x_{22} \rightarrow$

$\theta = 5$

Pasul 5

	C_1	C_2	C_{3f}	
D_1	[3] *	[2] 5	[0] 5	10
D_2	[1] 10	[4] *	[0] 10	20
D_3	[2] *	[1] 20	[0] *	20
	10	25	15	

! verificare !

Pasul 6

$\bar{x}_1 = (0,5,5,10,0,10,0,20,0) \in R^9$ - s.b.ned.

$f(\bar{x}_1) = 10+10+20 = 40 (u.m) \leq f(\bar{x}_0)$? Da

Pasul 7

$\delta_{11} = -3+1-0+0 = -2$

$\delta_{22} = -4+2-0+0 = -2$

$\delta_{31} = -2+1-0+0-2+1 = -2$

$\delta_{33} = -0+1-2+0 = -1$

$\Rightarrow$ toți $\delta_{ij} \leq 0 \Rightarrow \bar{x}_1$ este s.o. unică.

Concluzia pt. P.T.E. $\begin{cases} x_{optim}^{echilibrat} = (0,5,5,10,0,10,0,20,0) \\ \min f = 40 (u.m.) \end{cases}$

Concluzia pt. P.T.N. $\begin{cases} x_{optim}^{neechilibrat} = (0,5,10,0,0,20) \\ \min f = 40 (u.m.) \end{cases}$