Example:

a) non-perturbate ETP $\Longrightarrow$ a') perturbate ETP

	S_1	S_2	$\cdots$	S_n	
W_1	c_{11}	c_{12}	$\cdots$	c_{1n}	a_1
W_2	c_{21}	c_{22}	$\cdots$	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
W_m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	a_m
	b_1	b_2	$\cdots$	b_n	

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

	S_1	S_2	$\cdots$	S_n	
W_1	c_{11}	c_{12}	$\cdots$	c_{1n}	$a_1+\varepsilon$
W_2	c_{21}	c_{22}	$\cdots$	c_{2n}	$a_2+\varepsilon$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
W_m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	$a_m+\varepsilon$
	b_1	b_2	$\cdots$	$b_n+m\cdot\varepsilon$	

$$\sum_{i=1}^{m}(a_i+\varepsilon) = \sum_{i=1}^{m} a_i + m\varepsilon = \sum_{j=1}^{n} b_j + m\cdot\varepsilon$$

Examples:

Solve the next TP. If the second BA solution which we will obtain it isn't the optimal solution don't continue the algorithm:

a)

	S_1	S_2	S_3	
W_1	3	2	1	30
W_2	1	4	2	20
W_3	2	1	3	20
	20	15	20	

Because, $\sum_{i=1}^{3} a_i = 70 > \sum_{j=1}^{3} b_j = 55$ we have a NeTP, so we must to equilibrate.

$\Longrightarrow$

①

	S_1	S_2	S_3	S_4^f	
W_1	20 (3)	10 (2)	* (1)	* (0)	30, 10, 0
W_2	* (1)	5 (4)	15 (2)	* (0)	20, 15, 0
W_3	* (2)	* (1)	5 (3)	15 (0)	20, 15, 0
	20, 0	15, 5, 0	20, 5, 0	15, 0	

For ETP we applied the diagonal method to find the iBAS: $\bar{x}_0 \begin{cases} \bar{x}_{11}=20;\ \bar{x}_{12}=10;\ \bar{x}_{22}=5; \\ \bar{x}_{23}=15;\ \bar{x}_{33}=5;\ \bar{x}_{34}=15 \\ \text{the other } \bar{x}_{ij} = {}^{*} = 0 \end{cases}$

$f(\bar{x}_0) = 145$

② We check if $\bar{x}_0$ it's optimal solution:

$$\left.\begin{aligned}
\delta_{13} &= -1+2-4+2 = -1 \\
\delta_{14} &= -0+0-3+2-4+2 = -3 \\
\delta_{21} &= -1+3-2+4 = 4>0 \\
\delta_{24} &= -0+0-3+2 = -1 \\
\delta_{31} &= -2+3-2+4-2+3 = 4>0 \\
\delta_{32} &= -1+4-2+3 = 4>0
\end{aligned}\right\} \Rightarrow (\exists)\,\delta_{ij}>0 \Rightarrow \bar{x}_0 \text{ it isn't optimal solution!!}$$

③ We apply the "input criteria": $\delta_{ij} = \max\{\delta_{kl}>0\} = \max\{\underbrace{\delta_{21}}_{=4}, \underbrace{\delta_{31}}_{=4}, \underbrace{\delta_{32}}_{=4}\} \overset{(!)}{=} \delta_{21} \Rightarrow x_{21}\downarrow$

I choose "δ_{21}"

④ We apply the „output criteria":

$$\begin{tikzcd} ② \overset{20}{(1,1)} \arrow[r] & \overset{10}{(1,2)} ③ \arrow[d] \\ ① \underset{\theta=5}{(2,1)} \arrow[u] & \underset{5}{(2,2)} ④ \arrow[l] \end{tikzcd}$$

$\theta = \min\{x_{ij} / x_{ij}$ been into even no. alloc$\} = \min\{\underbrace{x_{11}}_{=20}, \underbrace{x_{22}}_{=5}\} = 5 \Rightarrow x_{22} \to *$

⑤ We determine the new solution: $\bar{X}_1$

	S_1	S_2	S_3	S_4^f	
W_1	15 [3]	15 [2]	* [1]	* [0]	30
W_2	5 [1]	* [4]	5 [2]	* [0]	20
W_3	* [2]	* [1]	5 [3]	15 [0]	20
	20	15	20	15	

$$\begin{tikzcd} \overset{=15}{\overbrace{20-\theta}} \ (1,1) \arrow[r] & \overset{=15}{\overbrace{10+\theta}} \ (1,2) \arrow[d] \\ „+" \ (2,1) \ \theta=5 \arrow[u] & (2,2) \ \underbrace{5-\theta}_{=0 = \text{out } „*"} \ „\to" \arrow[l] \end{tikzcd}$$

$\bar{X}_1: \begin{cases} \bar{x}_{11}=15, \bar{x}_{12}=15, \bar{x}_{21}=5, \bar{x}_{23}=5, \bar{x}_{33}=5, \bar{x}_{34}=15 & \to \text{basic components} \\ \bar{x}_{13}=\bar{x}_{14}=\bar{x}_{22}=\bar{x}_{24}=\bar{x}_{31}=\bar{x}_{32}=0\ (= „*") & - \text{non-basic (free) components} \end{cases}$

$\underline{f(\bar{X}_1) = 115}\ (< f(\bar{X}_0))$

② We check if $\bar{X}_1$ it is a optimal solution:

$\delta_{13} = -1+2-1+3 = \underline{3 > 0} \Rightarrow (\exists)\delta_{ij} > 0 \Leftrightarrow \bar{X}_1$ - it isn't optimal solution $\Rightarrow$ the algorithm must to continue ...!!

$\vdots$

b)

	S_1	S_2	S_3	
W_1	20 [2]	0 [2]	* [1]	~~20~~, 0
W_2	* [3]	15 [4]	* [3]	~~15~~, 0
W_3	* [1]	5 [3]	5 [2]	~~10~~, ~~5~~, 0
	~~20~~, 0	~~20~~, ~~5~~, 0	~~5~~, 0	

$\left(\sum_{i=1}^{3} a_i = 45 = \sum_{j=1}^{3} b_j\right)$

$\Leftrightarrow \bar{X}_0: \begin{cases} \bar{x}_{11}=20;\ \bar{x}_{12}=0;\ \bar{x}_{22}=15;\ \bar{x}_{32}=5;\ \bar{x}_{33}=5 & - \text{basic components } (m+n-1 = 3+3-1 \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0\ (= „*") \to \text{non-basic components} \end{cases}$

$f(\bar{X}_0) = 125$

$\Downarrow$

it's degenerate BAS(!!!)

(no, it's possible to appear the cycle phenom...

$\Downarrow$

we apply the perturbation method

$\Downarrow$

①

	S_1	S_2	S_3	
W_1	20 [2]	ε [2]	* [1]	$20+\varepsilon$, ε, 0,
W_2	* [3]	$15+\varepsilon$ [4]	* [3]	$15+\varepsilon$, 0
W_3	* [1]	$5-2\varepsilon$ [3]	$5+3\varepsilon$ [2]	$10+\varepsilon$, $5+3\varepsilon$, 0
	~~20~~, 0	~~20~~, $20+\varepsilon$, $5-2\varepsilon$, 0	$5+3\varepsilon$	

$\Rightarrow \bar{X}_0 = \begin{cases} \bar{x}_{11}=20;\ \bar{x}_{12}=\varepsilon;\ \bar{x}_{22}=15+\varepsilon;\ \bar{x}_{32}=5-2\varepsilon;\ \bar{x}_{33}=5+3\varepsilon & \text{basic com...} \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0\ (= „*") & - \text{non-basic components} \end{cases}$

$\underline{f(\bar{X}_0) = 125 + 6\varepsilon}$

$\Downarrow$

non-degenerate iBAS(!!!)

② We apply the „optimal criteria":

$$\left.\begin{array}{l} \delta_{13} = -1+2-3+2 = 0 \\ \underline{\delta_{21}} = -3+2-2+4 = \underline{1>0} \\ \delta_{23} = -3+2-3+4 = 0 \\ \underline{\delta_{31}} = -1+2-2+3 = \underline{2>0} \end{array}\right\} \Rightarrow (\exists)\delta_{ij}>0 \Rightarrow \overline{X}_0 \text{ it isn't the optimal solution}$$

③ $\delta_{ij} = \max\{\delta_{ke}>0\} = \max\{\underbrace{\delta_{21}}_{=1}, \underbrace{\delta_{31}}_{=2}\} = \delta_{31} \Rightarrow$ „$x_{31}\uparrow$" (becomes a basic component)

④
$$\begin{tikzcd} \overset{20-\theta}{② (1,1)} \arrow[r] & \overset{\varepsilon+\theta}{(1,2) ③} \arrow[d] \\ „+" ① \underset{\theta=5-2\varepsilon}{\underline{(3,1)}} \arrow[u] & \underset{5-2\varepsilon-\theta}{(3,2) ④} \arrow[l] \end{tikzcd}$$
„−"

$\Rightarrow \underline{\theta} = \min\{\underbrace{x_{11}}_{=20}, \underbrace{x_{32}}_{=5-2\varepsilon}\} = \underline{5-2\varepsilon} \Rightarrow$ „$x_{32}\rightarrow$" (becomes a non-basic variable)

⑤

	S_1	S_2	S_3	
W_1	$\boxed{2}$ $15+2\varepsilon$	$\boxed{2}$ $5-\varepsilon$	$\boxed{1}$ *	$20+\varepsilon$
W_2	$\boxed{3}$ *	$\boxed{4}$ $15+\varepsilon$	$\boxed{3}$ *	$15+\varepsilon$
W_3	$\boxed{1}$ $5-2\varepsilon$	$\boxed{3}$ *	$\boxed{2}$ $5+3\varepsilon$	$10+\varepsilon$
	20	20	$5+3\varepsilon$	

$$\Rightarrow \overline{X}_1 \begin{cases} \overline{x}_{11} = 15+2\varepsilon;\ \overline{x}_{12} = 5-\varepsilon;\ \overline{x}_{22} = 15+\varepsilon;\ \overline{x}_{31} = 5-2\varepsilon;\ \overline{x}_{33} = 5+3\varepsilon \\ \overline{x}_{13} = \overline{x}_{21} = \overline{x}_{23} = \overline{x}_{32} = 0 \ (= „*") \end{cases}$$

$$\underline{f(\overline{X}_1) = 115+10\varepsilon} \quad (< f(\overline{X}_0))$$

We repeat the steps of the algorithm:

② $\begin{cases} \delta_{13} = -1+2-1+2 = 2>0 \\ \vdots \end{cases} \Rightarrow (\exists)\delta_{ij}>0 \Rightarrow \overline{X}_1$ - it isn't the optimal solution $\Rightarrow$ the algorithm continued...