

$$\text{oferta} = 10 + 25 = 35$$

$$\text{curea} = 20 + 20 + 10 = 50$$

$\text{oferta} < \text{curea}$
 PTH

	C ₁	C ₂	C ₃	
D ₁	3	1	2	10
	X ₁₁	X ₁₂	X ₁₃	
D ₂	2	4	1	25
	X ₂₁	X ₂₂	X ₂₃	
	20	20	10	

$$(1g) \min f(x_{11}, x_{12}, \dots, x_{23}) = 3 \cdot x_{11} + 1 \cdot x_{12} + 2 \cdot x_{13} + 2 \cdot x_{21} + 4 \cdot x_{22} + 1 \cdot x_{23}$$

$$\begin{cases} (1g) & x_{11} + x_{12} + x_{13} = 10 \\ (2g) & x_{21} + x_{22} + x_{23} = 25 \\ (3g) & x_{11} + x_{21} \leq 20 \\ & x_{12} + x_{22} \leq 20 \\ & x_{13} + x_{23} \leq 10 \\ & x_{11}, x_{12}, \dots, x_{23} \geq 0 \end{cases}$$

$$\begin{pmatrix} + x_{11}^c \\ + x_{12}^c \\ + x_{13}^c \\ + x_{21}^c \\ + x_{22}^c \\ + x_{23}^c \end{pmatrix}$$

$$(1g) \min f(x_{11}, \dots, x_{23}) = 3 \cdot x_{11} + 1 \cdot x_{12} + \dots + 0 \cdot x_{23}$$

PTE

	C ₁	C ₂	C ₃	
D ₁	3	1	2	10
	X ₁₁	X ₁₂	X ₁₃	
D ₂	2	4	1	25
	X ₂₁	X ₂₂	X ₂₃	
	20	20	10	

$$\begin{cases} (1g) & \min f(x_{11}, \dots, x_{23}) = 3 \cdot x_{11} + x_{12} + x_{13} = 10 \\ (2g) & x_{21} + x_{22} + x_{23} = 25 \\ & x_{31} + x_{32} + x_{33} = 15 \\ & x_{11} + x_{21} + x_{31} = 20 \\ & x_{12} + x_{22} + x_{32} = 20 \\ & x_{13} + x_{23} + x_{33} = 10 \end{cases}$$

$\text{oferta} = 50$
 $\text{curea} = 50$

$$(3g) \quad x_{11}, x_{12}, \dots, x_{23} \geq 0$$

$$(2) \begin{cases} x_{11} + x_{12} + x_{13} = 10 \\ x_{21} + x_{22} + x_{23} = 25 \\ x_{31} + x_{32} + x_{33} = 15 \\ x_{11} + x_{21} + x_{31} = 20 \\ x_{12} + x_{22} + x_{32} = 20 \\ x_{13} + x_{23} + x_{33} = 10 \end{cases}$$

$$\Rightarrow \overline{A}_5 =$$

$$\overline{A}_5 = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

+

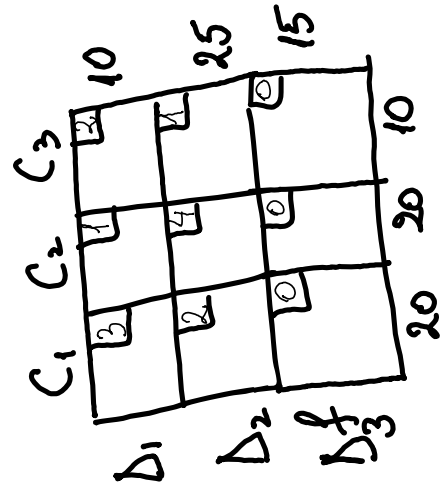
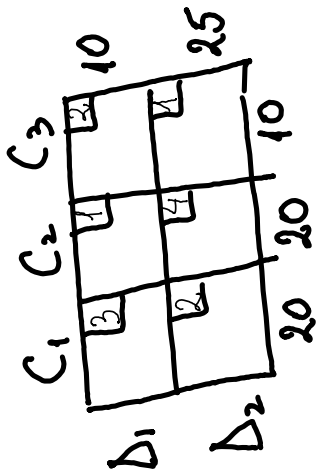
1.(-1)

2. i2

1. -1

SPFA

2



	C_1	C_2	C_3	
D_1	10	3	1	10
D_2	10	2	15	25
D_3	*	5	10	15

o) clăt. diag. , $\overline{X_0} = ?$ smi

$C_i \rightarrow$ linia
Linia coloana

! verificari ! \sum linie

\sum col
"O-uri" la D
"O-uri" la C

$\overline{X_0} = (10, 0, 0, 10, 15, 10, 0, 5, 10)$ smi

$f(\overline{X_0}) = 30 + 20 + 60 = 110$ u.m.

nr. var. prime =
 $m + n - 1 = 3 + 3 - 1$
 $= 5$ v.p.

nr. var. dec = $m \cdot n - \text{var.}$
 $= 9 - 5 = 4$ v. dec

$$\overline{X_0} = ? \quad \text{SMA?}$$

staf i Hol

$$\bar{X}_0 = (0, 10, 0, 15, 10, 0, 5, 10)$$

074-1611

$$c_{-1} = -2 + 0 - 0 + 2$$

04-01-25

$$\Theta = \min \{ \widehat{x_{22}}, \frac{10}{5} \} = 5 \Rightarrow x_{31} \rightarrow$$

2

$$= 2 \times 0$$

3) Get. i. ex. 12.

$x_{32} \rightarrow$

$$\frac{15 + \theta = 20}{10 - \theta = 5}$$

$$(2,1) \leftarrow (2,2) \textcircled{2}$$

A hand-drawn diagram of a cell. It features a large, irregular outer boundary representing the cell membrane. Inside, there is a smaller, roughly circular nucleus with a darker, textured interior. To the right of the nucleus is a large, clear, oval-shaped vacuole. A small, dark, irregular shape is located near the bottom left of the cell, possibly representing a chloroplast or another organelle. The drawing is simple and appears to be a student sketch.

$\rightarrow (3, 2, 1)$

(15)

$$Y = a - b$$

4

5) How Total

! verify!

	C ₁	C ₂	C ₃	
D ₁	*	3	10	*
D ₂	20	2	4	*
D ₃	*	0	5	10
	20	20	10	

$5 - \theta = *$ $\theta = 5$
 $(2,2) \rightarrow (2,3) \textcircled{1}$
 $(3,2) \leftarrow (3,3) \textcircled{2}$
 $5 + \theta = 10$ $10 - \theta = 5$

5') How Total.

	C ₁	C ₂	C ₃	
D ₁	*	3	10	*
D ₂	20	2	4	*
D ₃	*	0	5	10
	20	20	10	

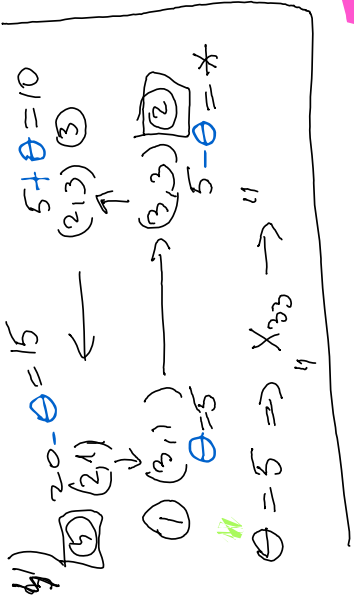
1) $\bar{X}_1 = (0, 10, 0, 20, 5, 0, 0, 5, 10)$ sum A
 $f(\bar{X}_1) = 10 + 10 + 20 = 70 \leq f(\bar{X}_0)$

2) $\delta_{11} = -3 + 1 - 4 + 2 = -4 < 0$ 3') Get int
 $\delta_{13} = -2 + 0 - 0 + 1 = -1 < 0$ 11
 $\delta_{23} = -1 + 0 - 0 + 4 = 3 > 0$ 23
 $\delta_{31} = -0 + 2 - 4 + 0 = -2 < 0$ 11

$\theta = \min \left\{ \underbrace{x_{33}}_{10}, \underbrace{x_{23}}_{5} \right\} = 5 \Rightarrow$

11) $\bar{X}_2 = (0, 10, 0, 20, 0, 5, 0, 10, 5)$ sum A
 $f(\bar{X}_2) = 55 \leq f(\bar{X}_1)$

21) $\delta_{11} = -3 + 1 - 0 + 0 - 1 + 2 = -1 < 0$
 $\delta_{13} = -2 + 0 - 0 + 1 = -1 < 0$
 $\delta_{22} = -4 + 1 - 0 + 0 = -3 < 0$
 $\delta_{31} = -0 + 0 - 1 + 2 = 1 > 0$
 31) $x_{31} \uparrow$



1st) $\overline{X}_3 = (0, 10, 0, 15, 0, 19, 5, 1, 0, 0)$ s.n.a. ①
 $f(\overline{X}_3) = 50 \leq f(\overline{X}_2)$

2nd) $\delta_{11} = -2 < 0$
 $\delta_{13} = -241 - 040 - 241 = -2 < 0$
 $\delta_{22} = -2 < 0$
 $\delta_{33} = -1 < 0$
 $\Rightarrow \overline{X}_3 - 0 \text{ s.o. } u$

$\overline{PTE} \equiv \overline{X}_3 = (1) \text{ s.o. } u.$
 $\text{opt PTE: } \begin{cases} \overline{X}_\theta \\ \min f = 50 \text{ u.m.} \end{cases}$

$\text{opt PTH: } \begin{cases} \overline{X}_\theta^i = (0, 10, 0, 15, 0, 10) \text{ s.o. } u. \\ \min f = 50 \text{ u.m.} \end{cases}$

(2)

	C_1	C_2	
D_1	X_{11}	X_{12}	10
D_2	X_{21}	X_{22}	20
D_3	X_{31}	X_{32}	30
	20	20	

X_{ij} limit
varian

$$\text{efunda} = 10 + 2 \times 30 = 60 \neq 10 \Rightarrow \text{PTH}$$
$$\text{variana} = 20 + 20 = 40$$

$$3 \cdot X_{11} + 2 \cdot X_{12} + 1 \cdot X_{21} + 4 \cdot X_{22} + 2 \cdot X_{31} + 1 \cdot X_{32}$$

(1g) $\min f(X_{11}, X_{12}, \dots, X_{32}) = 3 \cdot X_{11} + 2 \cdot X_{12} + 1 \cdot X_{21} + 4 \cdot X_{22} + 2 \cdot X_{31} + 1 \cdot X_{32}$

(2g) $\begin{cases} X_{11} + X_{12} \leq 10 \Rightarrow + X_{11}^C \\ X_{21} + X_{22} \leq 20 \Rightarrow + X_{21}^C \\ X_{31} + X_{32} \leq 30 \Rightarrow + X_{31}^C \\ X_{11} + X_{21} + X_{31} = 20 \\ X_{12} + X_{22} + X_{32} = 20 \end{cases}$

(3g) $X_{11}, X_{12}, \dots, X_{32} \geq 0$

Echilibriu PT4 prin introducerea unui nou activ. (Cst)

(PTE)

	C ₁	C ₂	C ₃	
D ₁	X ₁₁	X ₁₂	X ₁₃	10
D ₂	X ₂₁	X ₂₂	X ₂₃	20
D ₃	X ₃₁	X ₃₂	X ₃₃	30
	20	20	20	

(1s) $\min f(X_{11}, X_{12}, \dots, X_{33}) = 3X_{11} + 2X_{12} + 0 \cdot X_{13} + 1 \cdot X_{21} + 4X_{22} + 0 \cdot X_{23} + 2 \cdot X_{31} + 1 \cdot X_{32} + 0 \cdot X_{33}$

(2s)
$$\begin{cases} X_{11} + X_{12} + X_{13} = 10 \\ X_{21} + X_{22} + X_{23} = 20 \\ X_{31} + X_{32} + X_{33} = 30 \\ X_{11} + X_{21} + X_{31} = 20 \\ X_{12} + X_{22} + X_{32} = 20 \\ X_{13} + X_{23} + X_{33} = 20 \end{cases} \Rightarrow$$

(3s) $X_{11}, X_{12}, \dots, X_{33} \geq 0$

$$\bar{A}_5 = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{21} & X_{22} & X_{23} & X_{31} & X_{32} & X_{33} \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 20 & 30 & 20 & 20 & 20 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

1
10
20
30
20
20
20
20
20
1
smf

oferta = 60 = cererea \Rightarrow PTE

	C_1	C_2	C_3	
D_1	3	2	0	10
D_2	1	4	0	20
D_3	2	1	0	30
	20	20	20	

	C_1	C_2	C_3	C_f
D_1	10	3	2	0
D_2	10	1	4	0
D_3	*	2	1	0

~~10~~ ~~20~~ ~~30~~
~~10~~ ~~10~~ ~~0~~
~~10~~ ~~0~~

1) $\overline{X_0} - SPAT_i$, $\overline{X_0} = ?$
 ce met. diagonală
 (celt, ul de $N-v$).

$$\overline{X_0} = (10, 0, 0, 10, 10, 0, 0, 10, 20) - SPAT_i$$

$$f(\overline{X_0}) = 30 + 10 + 10 + 10 = 80 (u.m.)$$

! verific !

\sum linii

\sum coloane

" 0-uri " la Δ

" 0-uri " la C

$$nr. \text{ var. prime} = m + n - 1 = 3 + 3 - 1 = 5$$

$$nr. \text{ var. sec} = m \cdot n - \text{var. prime} = 9 - 5 = 4$$

	C_1	C_2	C_3	C_j
D_1	*	3	2	10
D_2	20	1	4	*
D_3	0	2	1	10
	20	20	20	30

0 val!

1) $\overline{X_0} = ?$, $\overline{X_0} = SMA_i$
 cu met. costului minim.

$$\overline{X_0} = (0, 0, \underline{10}, \underline{20}, 0, 0, 0, \underline{20}, \underline{10})$$

SMA

$$f(\overline{X_0}) = 20 + 20 = 40 \text{ (u.m.)}$$

	C_1	C_2	C_3	C_f
D_1	10	3	2	10
D_2	10	4	1	20
D_3	20	2	1	30

1) $\overline{X_0} - \text{sm} A_i, \overline{X_0} = ?$
 cu met. diagonali
 (calcul de $H-v$).

$$\overline{X_0} = (10, 0, 0, 10, 10, 0, 0, 10, 20) - \text{sm} A_i$$

$$f(\overline{X_0}) = 30 + 10 + 40 + 10 = 80 \text{ (u.m.)}$$

3) Get. introduce \Rightarrow Get. exitu.

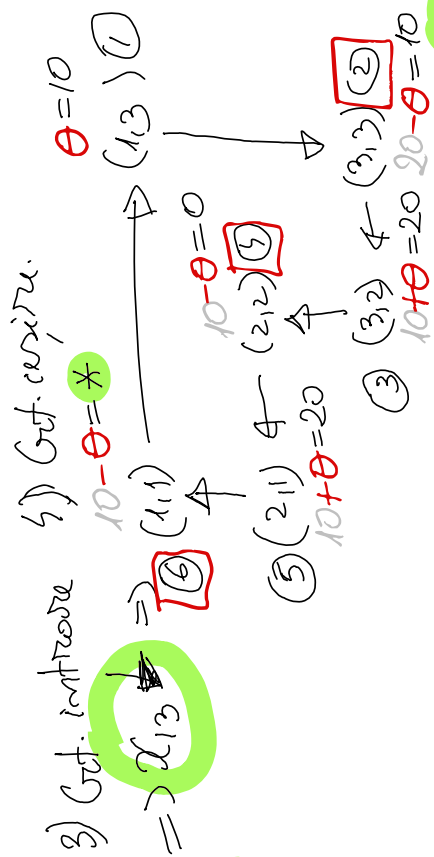
2) $\overline{X_0} = \text{S.O.} ??$

$$\delta_{12} = -2 + 4 - 1 + 3 = 4 > 0$$

$$\delta_{13} = -0 + 0 - 1 + 4 - 1 + 3 = 5 > 0$$

$$\delta_{23} = -0 + 4 - 1 + 0 = 3 > 0$$

$$\delta_{31} = -2 + 1 - 4 + 1 = -4 \leq 0$$



$\theta = \min \{ \underbrace{X_{33}}, \underbrace{X_{22}}, \underbrace{X_{11}} \} = 10 \Rightarrow \underline{\underline{X_{11}}}$

! verif.

5)

	C_1	C_2	C_3	f
D_1	*	3	2	10
D_2	20	1	4	*
D_3	*	2	1	10
	20	20	20	30

3') $X_{23} = 0$

4') $0 - \theta = * \Rightarrow \theta = 0$

$(2,12) \rightarrow (2,13) \text{ (1)}$

$(3,12) \leftarrow (3,13) \text{ (2)}$

$20 + \theta = 20 \quad 10 - \theta = 10$
 $\theta = \min \{ \widehat{X_{23}}, \widehat{X_{22}} \} = 0 \Rightarrow X_{22} \rightarrow 0$

1') $\overline{X_1} = (0, 10, 20, 0, 0, 0, 20, 10)$
 SBA Degenerata (1 pivot de ciclay)
 $f(\overline{X_1}) = 20 + 20 = 40 \leq f(\overline{X_0})$

2') $\overline{X_1} \in \text{so} \quad ??$

$f_{11} = -3 + 0 - 0 + 1 - 4 + 1 = -5 \leq 0$

$f_{12} = -2 + 1 - 0 + 0 = -1 \leq 0$

$f_{23} = -0 + 4 - 1 + 0 = 3 > 0 \Rightarrow \overline{X_1} \text{ nu e s.o.}$

$f_{31} = -2 + 1 - 4 + 1 = -4 \leq 0$

5)

	C_1	C_2	C_3	f
D_1	*	3	2	10
D_2	20	1	4	*
D_3	*	2	1	10
	20	20	20	30

! verif.

	C_1	C_2	C_3	C_f
D_1	*	3	2	10
D_2	20	1	4	0
D_3	*	2	1	10
	20	20	20	30

D. verif.

$$1'') \overline{X_2} = (0, 0, 1, 0, 2, 0, 0, 1, 2, 0, 1, 0) \text{ S.O. A. D. } \textcircled{1}$$

$$f(\overline{X_2}) = 40 \leq f(\overline{X_1})$$

$$2'') \overline{X_2} \in \text{S.O.}?$$

$$g_{11} = -2 \leq 0$$

$$g_{12} = -1 \leq 0$$

$$g_{22} = -3 \leq 0$$

$$g_{31} = -1 \leq 0$$

$$\therefore \Rightarrow \overline{X_2} \in \text{S.O. } \underline{\underline{U.}}$$

$$\text{Q. PTE} \left\{ \overline{X_\Theta}^{\text{PTE}} = \overline{X_2} = (\dots) \textcircled{1} \right. \\ \left. \min f = 40 \text{ u.m.} \right.$$

$$\text{Q. PTH} \left\{ \overline{X_\Theta}^{\text{PTH}} = \overline{X_\Theta}^i = (0, 0, 2, 0, 0, 1, 2, 0) \text{ S.O. u.} \right. \\ \left. \min f = 40 \text{ u.m.} \right.$$