Rezolvarea problemelor de programare liniară (PPL) cu 2 necunoscute, cu metoda grafică

$$(PPL)\begin{cases} (1)\ \min f(x_1,x_2) = x_1 + \frac{1}{2}x_2 \\ (2)\begin{cases} x_1 + 4x_2 \geq 4 \ (R_1) \\ 2x_1 - 3x_2 \leq 6 \ (R_2) \\ x_1 - x_2 \geq -3 \ (R_3) \\ x_1 + x_2 \leq 6 \ (R_4) \end{cases} \\ (3)\ x_1, x_2 \geq 0 \end{cases}$$

Pas 1 : $S = ?$ (mț. soluțiilor generale a PPL => mț. soluțiilor sist. (2) dime.

(R_1): $x_1 + 4x_2 \geq 4 \Rightarrow (\Delta_1): x_1 + 4x_2 = 4 \Rightarrow \begin{cases} x_1 = 0 \overset{(\Delta_1)}{\Longrightarrow} 0 + 4\cdot x_2 = 4 \Rightarrow x_2 = 1 \Rightarrow P_1(0,1) \\ x_2 = 0 \overset{(\Delta_1)}{\Longrightarrow} x_1 + 4\cdot 0 = 4 \Rightarrow x_1 = 4 \Rightarrow P_2(4,0) \end{cases}$

$O(0,0) \overset{(R_1)}{\Longrightarrow} 0 + 4\cdot 0 \geq 4 \Rightarrow 0 \geq 4$ (F) => Semiplanul ce conține originea este semiplanul fără soluții pentru (R_1).

(R_2): $2x_1 - 3x_2 \leq 6 \Rightarrow (\Delta_2): 2x_1 - 3x_2 = 6 \Rightarrow \begin{cases} x_1 = 0 \overset{(\Delta_2)}{\Longrightarrow} 2\cdot 0 - 3x_2 = 6 \Rightarrow x_2 = -2 \Rightarrow P_3(0,-2) \\ x_2 = 0 \overset{(\Delta_2)}{\Longrightarrow} 2x_1 - 3\cdot 0 = 6 \Rightarrow x_1 = 3 \Rightarrow P_4(3,0) \end{cases}$

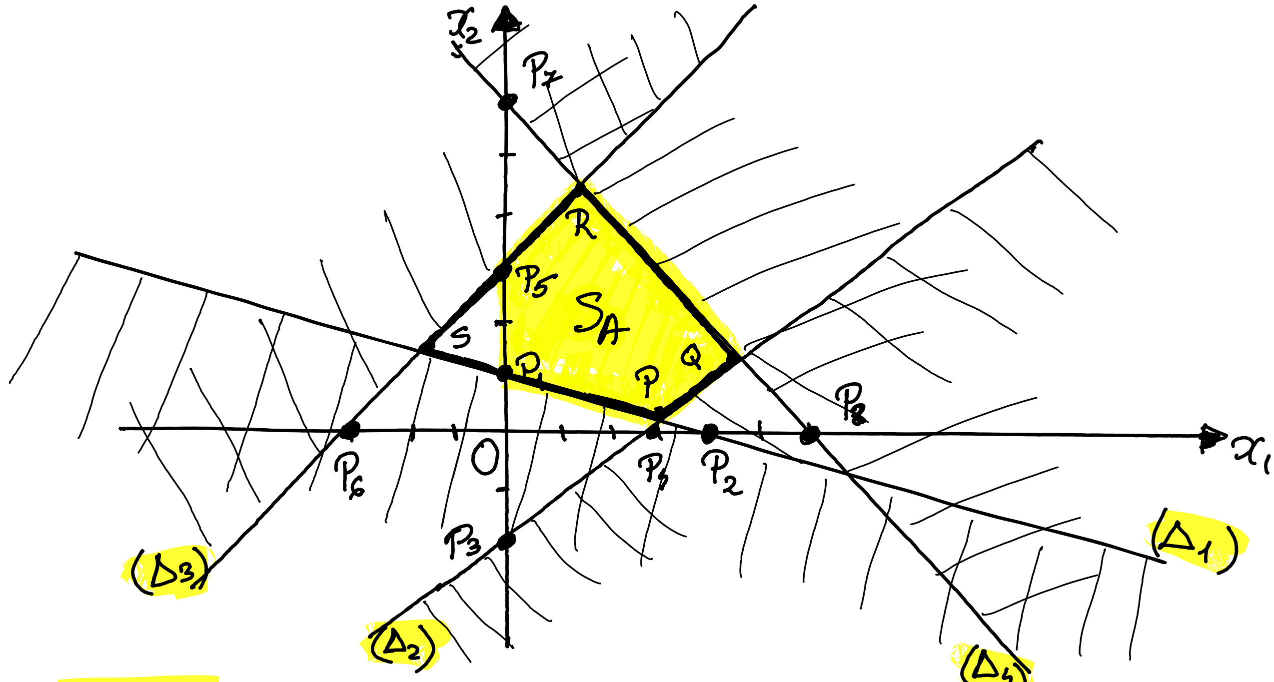
$O(0,0) \overset{(R_2)}{\Longrightarrow} 2\cdot 0 - 3\cdot 0 \leq 6 \Rightarrow 0 \leq 6$ (A) => Semiplanul ce conține originea este semiplanul cu soluțiile pentru (R_2).

(R_3): $x_1 - x_2 \geq -3 \Rightarrow (\Delta_3): x_1 - x_2 = -3 \Rightarrow \begin{cases} x_1 = 0 \Rightarrow 0 - x_2 = -3 \Rightarrow x_2 = 3 \Rightarrow P_5(0,3) \\ x_2 = 0 \Rightarrow x_1 = -3 \Rightarrow P_6(-3,0) \end{cases}$

$O(0,0) \overset{(R_3)}{\Longrightarrow} 0 - 0 \geq -3 \Rightarrow 0 \geq -3$ (A)

(R_4): $x_1 + x_2 \leq 6 \Rightarrow (\Delta_4): x_1 + x_2 = 6 \Rightarrow \begin{cases} x_1 = 0 \Rightarrow x_2 = 6 \Rightarrow P_7(0,6) \\ x_2 = 0 \Rightarrow x_1 = 6 \Rightarrow P_8(6,0) \end{cases}$

$O(0,0) \overset{(R_4)}{\Longrightarrow} 0 + 0 \leq 6 \Rightarrow 0 \leq 6$ (A)



$S = \overline{[PQRS]}$

Pas 2: $S_A = ?$ (mț. soluțiilor admisibile a PPL)

$S_A = S \cap \{\text{cadranul I}\} = \overline{[PQRP_5P_1]}$

Pas 3: $S_O = ?$ (mț. soluțiilor optime a PPL)

S_{AB} = mț. soluțiilor de bază admisibile a PPL

$S_{AB} = \{\text{vf. mț. } S_A\} = \{P, Q, R, P_5, P_1\}$

$$\boxed{(1)\ \min f(x_1,x_2)=x_1+\frac{1}{2}x_2}$$

- $P_1(0,1) \Rightarrow f(P_1)=f(0,1)=0+\frac{1}{2}\cdot 1=\frac{1}{2}=0{,}5$ ✓
- $P_5(0,3) \Rightarrow f(P_5)=f(0,3)=0+\frac{1}{2}\cdot 3=\frac{3}{2}=1{,}5$
- $P=(\Delta_1)\cap(\Delta_2):\begin{cases}x_1+4x_2=4 \quad |\cdot 2\\ 2x_1-3x_2=6\end{cases} \Rightarrow \begin{cases}2x_1+8x_2=8\\ 2x_1-3x_2=6\end{cases}$ $\ominus$ $\overline{/\ \ 11x_2=2} \Rightarrow x_2=\frac{2}{11} \Big| \Rightarrow x_1=\overset{11}{\cancel{4}}-4\cdot\frac{2}{11}=\frac{36}{11}$

$\Rightarrow P\left(\frac{36}{11},\frac{2}{11}\right) \Rightarrow f(P)=f\left(\frac{36}{11},\frac{2}{11}\right)=\frac{36}{11}+\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{11}=\frac{37}{11}\cong 3{,}3$ $\quad x_1+4x_2=4$

- $Q=(\Delta_2)\cap(\Delta_4):\begin{cases}2x_1-3x_2=6\\ x_1+x_2=6 \quad |\cdot 2\end{cases} \Rightarrow \begin{cases}2x_1-3x_2=6\\ 2x_1+2x_2=12\end{cases}$ $\ominus$ $\overline{/\ \ -5x_2=-6} \Rightarrow x_2=\frac{6}{5} \Big| \Rightarrow x_1=\overset{5}{\cancel{6}}-\frac{6}{5}=\frac{24}{5}$

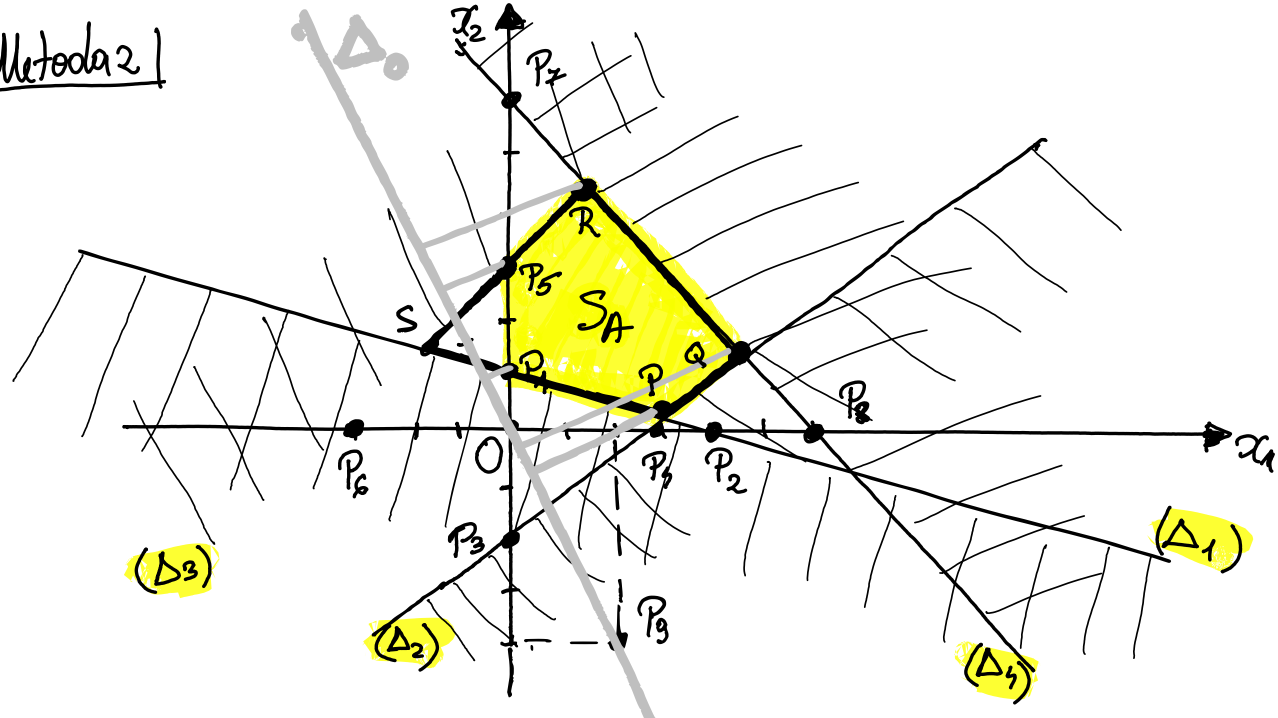
$\Rightarrow Q\left(\frac{24}{5},\frac{6}{5}\right) \Rightarrow f(Q)=f\left(\frac{24}{5},\frac{6}{5}\right)=\frac{24}{5}+\frac{1}{\cancel{2}_1}\cdot\frac{\overset{3}{\cancel{6}}}{5}=\frac{27}{5}=5{,}4$ $\quad x_1+x_2=6$

- $R=(\Delta_3)\cap(\Delta_4):\begin{cases}x_1-x_2=-3\\ x_1+x_2=6\end{cases} \oplus \Rightarrow 2x_1=3 \Rightarrow x_1=\frac{3}{2}$ $\ominus$ $\overline{/\ \ -2x_2=-9} \Rightarrow x_2=\frac{9}{2} \Big| \Rightarrow$

$\Rightarrow R\left(\frac{3}{2},\frac{9}{2}\right) \Rightarrow f(R)=f\left(\frac{3}{2},\frac{9}{2}\right)=\frac{^{2/}3}{2}+\frac{1}{2}\cdot\frac{9}{2}=\frac{15}{4}=3{,}75$

$S_\Theta=\{P_1\}$ cu $\begin{cases}x_1^{optim}=0\\ x_2^{optim}=1\end{cases}$, iar $\min f(x_1,x_2)=0{,}5$

Metoda 2



(1) $\min f(x_1, x_2) = \underbrace{x_1 + \frac{1}{2}x_2}$

$\Downarrow$

$(\Delta_0): x_1 + \frac{1}{2}x_2 = 0 : \begin{cases} x_1 = 0 \overset{\Delta_0}{\Longrightarrow} 0 + \frac{1}{2} \cdot x_2 = 0 \Rightarrow x_2 = 0 \Rightarrow O(0,0) \\ x_1 = 2 \overset{\Delta_0}{\Longrightarrow} 2 + \frac{1}{2} \cdot x_2 = 0 \Rightarrow \frac{1}{2}x_2 = -2 \Rightarrow \end{cases}$

$\Rightarrow x_2 = -2 : \frac{1}{2} = -2 \cdot \frac{2}{1} = -4 \Rightarrow P_9(2,-4)$

$$\boxed{S_O = \{P_1\} \text{ cu } \begin{cases} x_1^{optim} = 0 \\ x_2^{optim} = 1 \end{cases}, \ \min f(x_1, x_2) = 0,5}$$