

P.T.

|                | C <sub>1</sub>               | C <sub>2</sub>               |    |
|----------------|------------------------------|------------------------------|----|
| D <sub>1</sub> | x <sub>11</sub> <sup>1</sup> | x <sub>12</sub> <sup>3</sup> | 20 |
| D <sub>2</sub> | x <sub>21</sub> <sup>2</sup> | x <sub>22</sub> <sup>2</sup> | 10 |
| D <sub>3</sub> | x <sub>31</sub> <sup>3</sup> | x <sub>32</sub> <sup>1</sup> | 30 |
|                | 20                           | 20                           |    |

$$\sum_{i=1}^3 a_i = 20 + 10 + 30 = 60 \text{ (oferta)}$$

$$\sum_{j=1}^2 b_j = 20 + 20 = 40 \text{ (cerere)}$$

⇒ P.T.N.

Vom echilibra P.T. prin introducerea unui nou centru de distribuție fictiv (C<sub>3</sub><sup>f</sup>)

$$of > cer \Rightarrow + C^f$$

$$of < cer \Rightarrow + D^f$$

|                | C <sub>1</sub>      | C <sub>2</sub>     | C <sub>3</sub> <sup>f</sup> |                  |
|----------------|---------------------|--------------------|-----------------------------|------------------|
| D <sub>1</sub> | * <sup>1</sup>      | 0 <sup>3</sup>     | 20 <sup>4</sup>             | <del>20</del> 0  |
| D <sub>2</sub> | 10 <sup>2</sup>     | * <sup>2</sup>     | * <sup>2</sup>              | <del>10</del> 0  |
| D <sub>3</sub> | 10 <sup>3</sup>     | 20 <sup>1</sup>    | * <sup>4</sup>              | <del>30</del> 10 |
|                | <del>20</del><br>10 | <del>20</del><br>0 | <del>20</del><br>0          |                  |

$$\begin{aligned} of &= 60 \\ cer &= 60 \end{aligned} \Rightarrow P.T.E$$

! verificare

Det  $\bar{X}_0$  - SBAi cu met. costurilor minime.

$$v.p = m + n - 1 = 3 + 3 - 1 = \underline{5}$$

$\uparrow$  nr. depozite       $\nwarrow$  nr. centrelor

$$\bar{X}_0 = (0, \underline{0}, \underline{20}, \underline{10}, \underline{0}, \underline{10}, \underline{20}, 0) - \text{SBAi } \Delta \text{ de ord I.}$$

Pentru a evita aparitia fenomenului de ciclaaj vom folosi metoda portarilor:

|       | $C_1$                           | $C_2$             | $C_3$   |                                       |   |
|-------|---------------------------------|-------------------|---|---------------------------------------|---|
| $D_1$ | $10-\varepsilon$ <sup>1</sup>   | $*$ <sup>3</sup>  | $10+2\varepsilon$ <sup>0</sup>                | $20+\varepsilon$ $10-\varepsilon$ $0$ | $df = 60+3\varepsilon$  |
| $D_2$ | $*$ <sup>2</sup>                | $*$ <sup>2</sup>  | $10+\varepsilon$ <sup>0</sup>                 | $10+\varepsilon$ $0$                  | $\Rightarrow$ PTE<br>$cr = 60+3\varepsilon$   |
| $D_3$ | $10+\varepsilon$ <sup>3</sup>   | $20$ <sup>1</sup> | $*$ <sup>0</sup>                              | $30+\varepsilon$ $10+\varepsilon$ $0$ | $\text{! verif. ! v.p.} = 5$  |
|       | $20$<br>$10+\varepsilon$<br>$0$ | $20$<br>$0$       | $20+3\varepsilon$<br>$10+2\varepsilon$<br>$0$ |                                       | $20+3\varepsilon - (10+\varepsilon) = 20+3\varepsilon - 10 - \varepsilon = 10+2\varepsilon$ |

1) Det.  $\bar{X}_0$  - SBAiNd cu met. costurilor minime.

$$\bar{X}_0 = (10-\varepsilon, 0, 10+2\varepsilon, 0, 0, 10+\varepsilon, 10+\varepsilon, 20, 0) - \text{SBAiNd}$$

$$f(\bar{X}_0) = (10-\varepsilon) \cdot 1 + (10+2\varepsilon) \cdot 3 + 20 \cdot 1$$

$$= 10 - \varepsilon + 30 + 6\varepsilon + 20 = 60 + 5\varepsilon \text{ (u.m.)}$$

2)  $\bar{X}_0$  e S.O?

$$S_{12} = -3 + 1 - 3 + 1 = -4$$

$$S_{21} = -2 + 0 - 0 + 1 = -1$$

$$S_{22} = -2 + 0 - 0 + 1 - 3 + 1 = -3$$

$$S_{33} = -0 + 0 - 1 + 3 = 2 > 0$$

$$\nexists \delta_{ij} > 0$$

$\bar{X}_0$  nu e S.O.

3) Get. de intrare:  $\delta_{kl} = \max \{ \delta_{ij} > 0 \} = \max \{ \delta_{33} \}$   
 $\Rightarrow$  "  $x_{33} \downarrow$  "

4) Get. de usire:

Desenăm traseul var. care intră în lucru: "  $x_{33} \downarrow$  "

$$\textcircled{3} \quad (1,1) \leftarrow \begin{matrix} 10-\varepsilon+0=20 \\ 10+\varepsilon-\theta=10 \end{matrix}$$

$$\textcircled{2} \quad (1,3) \leftarrow \begin{matrix} 10+2\varepsilon-\theta=10+2\varepsilon-(10+\varepsilon)=\varepsilon \\ 10+\varepsilon-\theta=10 \end{matrix}$$

$$\theta = \min \{ x_{13}, x_{31} \}$$

$$\begin{matrix} 10+2\varepsilon & 10+\varepsilon \\ \hline \end{matrix}$$

$$\textcircled{4} \quad (3,1) \leftarrow \begin{matrix} 10+\varepsilon-\theta=10 \\ 10+\varepsilon-\theta=10 \end{matrix}$$

$$\theta = 10+\varepsilon$$

$$\Rightarrow$$
 "  $x_{31} \rightarrow$  "

5) Noua tabelă al P.T:

|       | $C_1$           | $C_2$           | $C_3$                      |               |
|-------|-----------------|-----------------|----------------------------|---------------|
| $D_1$ | 20 <sup>1</sup> | * <sup>3</sup>  | $\epsilon$ <sup>0</sup>    | $20+\epsilon$ |
| $D_2$ | * <sup>2</sup>  | * <sup>2</sup>  | $10+\epsilon$ <sup>0</sup> | $10+\epsilon$ |
| $D_3$ | * <sup>3</sup>  | 20 <sup>1</sup> | $10+\epsilon$ <sup>0</sup> | $30+\epsilon$ |
|       | 20              | 20              | $20+3\epsilon$             |               |

! verific!

$$v.p=5$$

1') Noua S.B.A.Hol:

$$\bar{X}_1 = (20, 0, \epsilon, 0, 0, 10+\epsilon, 0, 20, 10+\epsilon) \text{ S.B.A.Hol/}$$

$$f(\bar{X}_1) = 20 \cdot 1 + 20 \cdot 1 = 40 \text{ (u.m)} < f(\bar{X}_0)$$

2')  $\bar{X}_1$  - e s.o.??

$$d_{12} = -3 + 1 - 0 + 0 = -2$$

$$d_{21} = -2 + 0 - 0 + 1 = -1$$

$$d_{22} = -2 + 1 - 0 + 0 = -1$$

$$d_{31} = -3 + 0 - 0 + 1 = -2$$

$$\text{tot } d_{ij} < 0$$

$\bar{X}_1$  e s.o.

unică.

Cl. P.T. inițială (P.T.N perturbată)  $\Rightarrow$  elimin  $C_3$   $\Rightarrow \epsilon=0$

$$\left\{ \begin{array}{l} \bar{X}_{\text{optimal}}^{\text{inițial}} = (20, 0, 0, 0, 0, 20) - \text{s.o. unică.} \\ \min f = 40 \end{array} \right.$$

Cu met. olap:

(câștigul de N-V)

|       | $C_1$           | $C_2$                       | $C_3$                       |                             |
|-------|-----------------|-----------------------------|-----------------------------|-----------------------------|
| $D_1$ | 20 <sup>1</sup> | $\epsilon$ <sup>3</sup>     | * <sup>0</sup>              | $20+\epsilon$ $\epsilon=0$  |
| $D_2$ | * <sup>2</sup>  | $10+\epsilon$ <sup>2</sup>  | * <sup>0</sup>              | $10+\epsilon$ $\epsilon=0$  |
| $D_3$ | * <sup>3</sup>  | $10-2\epsilon$ <sup>1</sup> | $20+3\epsilon$ <sup>0</sup> | $30+\epsilon$ $20+\epsilon$ |

$$\bar{X}_0 = (20, \epsilon, 0, 0, 10+\epsilon, 0, 0, 10-2\epsilon, 20+3\epsilon) \text{ S.B.A.Hol}$$

$$f(\bar{X}_0) = 50 + 3\epsilon$$

②  $f: \mathbb{R}^3 \rightarrow \mathbb{R}$   
 $f(x, y, z) = x^3 y z^2 + x y^2 + x z^3 + 2x$   $P(-1, 1, 1)$   
 $\begin{matrix} & & & \uparrow & \uparrow & \uparrow \\ & & & x & y & z \end{matrix}$

a) Calculați derivatele parțiale de ordinul I:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (x^3 y z^2 + x y^2 + x z^3 + 2x) =$$

$$= y z^2 \cdot 3x^2 + y^2 \cdot 1 + z^3 \cdot 1 + 2 \cdot 1 = \boxed{3x^2 y z^2 + y^2 z^3 + 2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (x^3 y z^2 + x y^2 + x z^3 + 2x) =$$

$$= x^3 z^2 \cdot 1 + x \cdot 2y + 0 + 0 = \boxed{x^3 z^2 + 2xy}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} (x^3 y z^2 + x y^2 + x z^3 + 2x) =$$

$$= x^3 y \cdot 2z + 0 + x \cdot 3z^2 + 0 = \boxed{2x^3 y z + 3x z^2}$$

$$\frac{\partial f}{\partial x}(P) = 3 \cdot (-1)^2 \cdot 1 \cdot 1^2 + 1^2 + 1^3 + 2 = 7$$

$$\frac{\partial f}{\partial y}(P) = (-1)^3 \cdot 1^2 + 2 \cdot (-1) \cdot 1 = -3$$

$$\frac{\partial f}{\partial z}(P) = -5$$

b) Scrieți diferențiala de ordinul I:

$$df(x, y, z) = \left( \frac{\partial f}{\partial x} \right) dx + \left( \frac{\partial f}{\partial y} \right) dy + \left( \frac{\partial f}{\partial z} \right) dz$$

$$\stackrel{a)}{=} (3x^2 y z^2 + y^2 + z^3 + 2) dx + (x^3 z^2 + 2xy) dy + (2x^3 y z + 3x z^2) dz$$

$$df(P) = 7 dx + (-3) dy + (-5) dz = 7 dx - 3 dy - 5 dz$$

$x, y, z$

c) Calculati derivatele partiiale de ordinul II:

$$\frac{\partial^2 f}{\partial x^2} \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) \stackrel{a)}{=} \frac{\partial}{\partial x} (3x^2yz^2 + y^2z^3 + 2) = 3yz^2 \cdot 2x + 0 + 0 = \boxed{6xyz^2}$$

$$\frac{\partial^2 f}{\partial y^2} \stackrel{\text{def}}{=} \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} (x^3z^2 + 2xy) = 0 + 2x \cdot 1 = \boxed{2x}$$

$$\frac{\partial^2 f}{\partial z^2} \stackrel{\text{def}}{=} \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial z} \right) = \frac{\partial}{\partial z} (2x^3yz + 3xz^3) = 2x^3y \cdot 1 + 3x \cdot 2z^2 = \boxed{2x^3y + 6xz^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \stackrel{a)}{=} \frac{\partial}{\partial x} (x^3z^2 + 2xy) = z^2 \cdot 3x^2 + 2y \cdot 1 = \boxed{3x^2z^2 + 2y} = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y \partial x} \stackrel{\text{def}}{=} \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \dots$$

$$\frac{\partial^2 f}{\partial x \partial z} \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial z} \right) = \frac{\partial}{\partial x} (2x^3yz + 3xz^3) = 2yz \cdot 3x^2 + 3z^3 \cdot 1 = \boxed{6x^2yz + 3z^3} = \frac{\partial^2 f}{\partial z \partial x}$$

$$\frac{\partial^2 f}{\partial z \partial x} \stackrel{\text{def}}{=} \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial x} \right) = \dots$$

$$\frac{\partial^2 f}{\partial y \partial z} \stackrel{\text{def}}{=} \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial z} \right) \stackrel{a)}{=} \frac{\partial}{\partial y} (2x^3yz + 3xz^3) = 2x^3z \cdot 1 + 0 = \boxed{2x^3z} = \frac{\partial^2 f}{\partial z \partial y}$$

$$\frac{\partial^2 f}{\partial z \partial y} \stackrel{\text{def}}{=} \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial y} \right) = \dots$$

$$P(-1, 1, 1)$$

$$\frac{\partial^2 f}{\partial x^2}(P) = 6 \cdot (-1) \cdot 1 \cdot 1^2 = -6$$

$$\frac{\partial^2 f}{\partial y^2}(P) = -2$$

$$\frac{\partial^2 f}{\partial z^2}(P) = -8$$

$$\frac{\partial^2 f}{\partial x \partial y}(P) = \frac{\partial^2 f}{\partial y \partial x}(P) = 5$$

$$\frac{\partial^2 f}{\partial x \partial z}(P) = \frac{\partial^2 f}{\partial z \partial x}(P) = 9$$

$$\frac{\partial^2 f}{\partial y \partial z}(P) = \frac{\partial^2 f}{\partial z \partial y}(P) = -2$$

d) Scrieti diferentiaza de ordinul II

$$d^2 f(x, y, z) \stackrel{\text{def}}{=} \left( \frac{\partial^2 f}{\partial x^2} \right) dx^2 + \left( \frac{\partial^2 f}{\partial y^2} \right) dy^2 + \left( \frac{\partial^2 f}{\partial z^2} \right) dz^2 + 2 \cdot \left( \frac{\partial^2 f}{\partial x \partial y} \right) dx dy + 2 \cdot \left( \frac{\partial^2 f}{\partial x \partial z} \right) dx dz + 2 \cdot \left( \frac{\partial^2 f}{\partial y \partial z} \right) dy dz \stackrel{(*)}{=}$$

$$= (6xy z^2) \underline{dx^2} + (2x) d^2 y + (2x^3 y + 6xz) dz^2 + 2 \cdot (3x^2 z^2 + 2y) dx dy + 2 \cdot (6x^2 y z + 3z^2) dx dz + 2 \cdot (2x^3 z) dy dz$$

$$d^2 f(P) = -6 dx^2 - 2 d^2 y - 8 dz^2 + 2 \cdot 5 dx dy + 2 \cdot 9 dx dz + 2 \cdot (-2) dy dz = -6 dx^2 - 2 d^2 y - 8 dz^2 + 10 dx dy + 18 dx dz - 4 dy dz$$

e) Aduceti forma patratica gasita la d) ( $d^2 f(P)$ ) la forma canonică.

$$d^2 f(P) = -6d^2 x - 2d^2 y - 8d^2 z + 10dx dy + 18dx dz - 6y dz$$

$$= a_{11}d^2 x + a_{22}d^2 y + a_{33}d^2 z + 2 \cdot a_{12}dx dy + 2 \cdot a_{13}dx dz + 2 \cdot a_{23}dy dz$$

$$H = \begin{pmatrix} \frac{dx}{dy} & \frac{dx}{dz} & \frac{dy}{dz} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} -6 & \frac{10}{2} & \frac{18}{2} \\ \frac{10}{2} & -2 & -\frac{1}{2} \\ \frac{18}{2} & -\frac{1}{2} & -8 \end{pmatrix} = \begin{pmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix}$$

$$a_{12} = a_{21}$$

$$a_{13} = a_{31}$$

$$a_{23} = a_{32}$$

Met lui Jacobi:  $\Delta_0 \equiv 1$

$$\Delta_1 = -6$$

$$\Delta_2 = \begin{vmatrix} -6 & 5 \\ 5 & -2 \end{vmatrix} = (-6) \cdot (-2) - 5 \cdot 5 = 12 - 25 = -13$$

$$\Delta_3 = \begin{vmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{vmatrix} = (-6) \cdot (-2) \cdot (-8) + 5 \cdot (-2) \cdot 9 + 9 \cdot 5 \cdot (-2) - 9 \cdot (-2) \cdot 9 - (-2) \cdot (-2) \cdot (-6) - (-8) \cdot 5 \cdot 5$$

$$= -96 - 90 - 90 + 162 + 24 + 200$$

$$= 110$$

toți  $\Delta_i \neq 0, i=1,2,3$

$$d^2 f(P) = \frac{\Delta_0}{\Delta_1} dy_1^2 + \frac{\Delta_1}{\Delta_2} dy_2^2 + \frac{\Delta_2}{\Delta_3} dy_3^2$$

$$\Delta_1 \neq 0$$

$$\Delta_2 \neq 0$$

$$\Delta_3 \neq 0$$

$$= \frac{1}{-6} dy_1^2 + \frac{-6}{-13} dy_2^2 + \frac{-13}{110} dy_3^2 = \underbrace{\left(-\frac{1}{6}\right)}_{\Delta_1 < 0} dy_1^2 + \underbrace{\left(\frac{6}{13}\right)}_{\Delta_2 > 0} dy_2^2 + \underbrace{\left(-\frac{13}{110}\right)}_{\Delta_3 < 0} dy_3^2$$

$d^2 f(P)$  este definită ca c.m.m.

f.c.m.

Metoda lui Gauss:

$$\frac{13}{6} \cdot \frac{1}{2} + \frac{11}{2} = 0$$

$$A \begin{pmatrix} dx & dy & dz \\ -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix} \begin{matrix} \cdot \frac{5}{6} \\ \cdot \frac{3}{2} \\ + \end{matrix}$$

$$\begin{pmatrix} dx & dy & dz \\ -6 & 5 & 9 \\ 0 & \frac{13}{6} & \frac{11}{2} \\ 0 & \frac{11}{2} & \frac{11}{2} \end{pmatrix} \begin{matrix} \\ \\ \cdot \left(-\frac{33}{13}\right) \end{matrix}$$

$$\begin{pmatrix} dx & dy & dz \\ dy_1 & -6 & 5 & 9 \\ dy_2 & 0 & \frac{13}{6} & \frac{11}{2} \\ dy_3 & 0 & 0 & -\frac{110}{13} \end{pmatrix} = A'$$

$$a'_{11} = -6$$

$$a'_{22} = \frac{13}{6}$$

$$a'_{33} = -\frac{110}{13}$$

$$\begin{aligned} & \frac{11}{2} \cdot \left(-\frac{33}{13}\right) + \frac{11}{2} = \\ & = \frac{11}{2} \left(-\frac{33}{13} + \frac{13}{1}\right) = \\ & = \frac{11}{2} \cdot \left(-\frac{20}{13}\right) = -\frac{110}{13} \end{aligned}$$

$$\frac{1}{\frac{13}{6}} = 1 \cdot \frac{6}{13} = 1 \cdot \frac{6}{13} = \frac{6}{13}$$

$$d^2 f(P) = \frac{1}{a'_{11}} dy_1^2 + \frac{1}{a'_{22}} dy_2^2 + \frac{1}{a'_{33}} dy_3^2$$

$$= \frac{1}{-6} dy_1^2 + \frac{1}{\frac{13}{6}} dy_2^2 + \frac{1}{-\frac{110}{13}} dy_3^2$$

$$= \left[ \underbrace{-\frac{1}{6} dy_1^2}_{d_1 < 0} + \underbrace{\frac{6}{13} dy_2^2}_{d_2 > 0} + \underbrace{-\frac{13}{110} dy_3^2}_{d_3 < 0} \right] f. \text{com.} \Rightarrow$$

$\Rightarrow d^2 f(P)$  este nedefinită ca semn, nu poate fi zero sau semnul constant.

$$\text{cu } \begin{cases} dy_1 = -6 dx + 5 dy + 9 dz \\ dy_2 = \frac{13}{6} dy + \frac{11}{2} dz \\ dy_3 = -\frac{110}{13} dz \end{cases}$$



Bevor:

$$A = \begin{pmatrix} \frac{dx}{dy} & \frac{dy}{dz} & \frac{dz}{dx} \\ 0 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{dy}{dx} & \frac{dx}{dz} & \frac{dz}{dy} \\ 1 & 2 & -1 \\ 2 & 0 & -2 \\ -1 & -2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & -2 \\ -2 & -1 & 2 \end{pmatrix}$$

~~dx dy dz~~

dy dx dz

$$\begin{pmatrix} dy_1 & 1 & 2 & -1 \\ dy_2 & 0 & -1 & 0 \\ dy_3 & 0 & 0 & 1 \end{pmatrix} \equiv A'$$

$$\begin{cases} dy_1 = 2 dx + 1 \cdot dy - 1 dz \\ dy_2 = -1 dx \\ dy_3 = 1 dz \end{cases}$$

$$d^2 f(?) = \frac{1}{1} dy_1^2 + \frac{1}{-1} dy_2^2 + \frac{1}{1} dy_3^2$$

$$= 1 \cdot dy_1^2 - \frac{1}{1} dy_2^2 + 1 dy_3^2$$

indef. co  
Semm.

$$\lambda_1 > 0 \quad \lambda_2 < 0 \quad \lambda_3 > 0$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

indef. co  
Semm.

$$\rightarrow d^2 f(??) = \frac{1}{1} dy_1^2 - \frac{1}{1} dy_2^2 + 0 \cdot dy_3^2$$

$$\lambda_1 > 0 \quad \lambda_2 < 0 \quad \lambda_3 = 0$$

$$\frac{1}{0} dy_3^2 \text{ trace in } 0 \cdot dy_3^2$$

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f) Scrieti matricea Hessiană:

$$H(x, y, z) \stackrel{\text{def.}}{=} \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix} = c)$$

$$= \begin{pmatrix} 6xy z^2 & 3x^2 z^2 + 2y & 6x^2 y z + 3z^2 \\ 3x^2 z^2 + 2y & 2x & 2x^3 z \\ 6x^2 y z + 3z^2 & 2x^3 z & 2x^2 y + 6xz \end{pmatrix}$$

$$H(P) = H(-1, 1, 1) = \begin{pmatrix} -6 & 5 & 9 \\ 5 & -2 & -2 \\ 9 & -2 & -8 \end{pmatrix}$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ x & y & z \end{matrix}$

//



Am obținut 5 puncte critice (stationare) :

$$P_1(0,0); P_2(0,\sqrt{3}); P_3(0,-\sqrt{3}); P_4(-1,1); P_5(1,-1)$$

pas3) Calculăm derivatele parțiale de ordinul II :

$$\frac{\partial^2 f}{\partial x^2} \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) \stackrel{\text{pas2}}{=} \frac{\partial}{\partial x} (3y + 2x - y^3) = 0 + 2 \cdot 1 - 0 = \boxed{2}$$

$$\frac{\partial^2 f}{\partial y^2} \stackrel{\text{def}}{=} \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) \stackrel{\text{pas2}}{=} \frac{\partial}{\partial y} (3x - 3xy^2) = 0 - 3x \cdot 2y = \boxed{-6xy}$$

$$\frac{\partial^2 f}{\partial x \partial y} \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \stackrel{\text{pas2}}{=} \frac{\partial}{\partial x} (2x - 3xy^2) = 2 \cdot 1 - 3y^2 \cdot 1 = \boxed{2 - 3y^2}$$

$$\stackrel{!}{=} \frac{\partial^2 f}{\partial y \partial x} \stackrel{\text{def}}{=} \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) \stackrel{\text{pas2}}{=} \frac{\partial}{\partial y} (3y + 2x - y^3) = 3 \cdot 1 + 0 - 3y^2 = \boxed{3 - 3y^2}$$

pas4) Scriem matricea Hessiană :

$$H(x,y) \stackrel{\text{def}}{=} \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} \stackrel{\text{pas3}}{=} \begin{pmatrix} 2 & 2 - 3y^2 \\ 2 - 3y^2 & -6xy \end{pmatrix}$$

pas5) Calculăm matricea Hessiană în toate punctele critice găsite la pasul 2) și apoi stabilim natura lor. (met. lui Jacobi).

$$\bullet P_1(0,0) \Rightarrow H(P_1) = H(0,0) \stackrel{\text{pas4}}{=} \begin{pmatrix} 2 & 2 - 3 \cdot 0^2 \\ 2 - 3 \cdot 0^2 & -6 \cdot 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 0 \end{pmatrix} \Rightarrow$$

$\begin{matrix} \nearrow x \\ \nearrow y \end{matrix}$

$$\Rightarrow \begin{cases} \Delta_1 = 2 \neq 0 \\ \Delta_2 = \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} = 2 \cdot 0 - 2 \cdot 2 = -4 \neq 0 \end{cases}$$

$\xrightarrow{+2-2}$  Th. 2 din curs 13  
 $\Rightarrow P_1(0,0)$  este punct de inflexiune (pt. SA) (nu este pt. de extrem. local)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot e \cdot i + d \cdot h \cdot c + g \cdot b \cdot f - c \cdot e \cdot g - f \cdot h \cdot a - i \cdot b \cdot d$$

•  $P_2(0, \sqrt{3}) \Rightarrow H(P_2) = H(0, \sqrt{3}) \xrightarrow{\text{pasu}} \begin{pmatrix} 2 & -6 \\ -6 & 0 \end{pmatrix} \Rightarrow$

met hui  
Jacobi  $\begin{cases} \Delta_1 = 2 \neq 0 \\ \Delta_2 = -36 \neq 0 \end{cases} \xrightarrow{+2} P_2 \text{ pct. } pa$

cu met hui Gauss:  $A \equiv H(P_2) = \begin{pmatrix} \boxed{2} & -6 \\ -6 & 0 \end{pmatrix} \xrightarrow{/3} \sim$

$\sim \begin{pmatrix} 2 & -6 \\ 0 & -18 \end{pmatrix} \equiv A' \Rightarrow d^2 f(P_2) = \frac{1}{a'_{11}} dy_1^2 + \frac{1}{a'_{22}} dy_2^2$

$\begin{aligned} 3 - 3y^2 &= 3 - 3 \cdot \sqrt{3}^2 \\ &= 3 - 3 \cdot 3 \\ &= 3 - 9 \\ &= \underline{\underline{-6}} \end{aligned}$

$\begin{aligned} &= \frac{1}{2} dy_1^2 + \frac{1}{-18} dy_2^2 \\ &= \left[ \frac{1}{2} dy_1^2 - \frac{1}{18} dy_2^2 \right] \text{ f. can. } \end{aligned}$

$\underbrace{\frac{1}{2} > 0 \quad \frac{1}{-18} < 0}}_{d^2 f \text{ este nedefinită ca semn}}$

(curs 13, Th 1)

$P_2(0, \sqrt{3})$  este pct.  $pa$ .

- $P_3 \dots$
- $P_4 \dots$
- $P_5 \dots$

|            |       |       |       |    |
|------------|-------|-------|-------|----|
|            | $C_1$ | $C_2$ | $C_3$ |    |
| $\Delta_1$ | 20    | 10    | 30    | 20 |
| $\Delta_2$ | 10    | 20    | 10    | 20 |

$$\sum_{i=1}^2 \alpha_i = 20 + 20 = 40 \quad (\text{oferta})$$

$$\sum_{j=1}^3 b_j = 20 + 10 + 30 = 60 \quad (\text{cerere})$$

Echilibriu P.T. prin introducerea unui depozit fictiv,  $\Delta_3^f$  (ce are costurile de transport egale cu 0).

oferta < cerere  $\Rightarrow + \Delta^f$   
 oferta > cerere  $\Rightarrow - \Delta^f$

|              |          |                    |                     |                    |
|--------------|----------|--------------------|---------------------|--------------------|
|              | $C_1$    | $C_2$              | $C_3$               |                    |
| $\Delta_1$   | 20       | $\varepsilon$      | $\times$            | $20 + \varepsilon$ |
| $\Delta_2$   | $\times$ | $10 - \varepsilon$ | $10 + 2\varepsilon$ | $20 + \varepsilon$ |
| $\Delta_3^f$ | $\times$ | $\times$           | $20 + \varepsilon$  | $20 + \varepsilon$ |
|              | 20       | 10                 | 30                  | 60                 |

Pt. a evita aparitia fetei de  
 cicaj folosim met.

particularizarii:

$$\begin{aligned} \text{oferta} &= 60 + 3\varepsilon \\ \text{cerere} &= 60 + 3\varepsilon \end{aligned} \quad \text{PTE}$$

$$(20 + \varepsilon) - (10 - \varepsilon) = 20 + \varepsilon - 10 + \varepsilon = 10 + 2\varepsilon$$

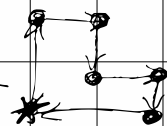
pas 1)  $\bar{X}_0$  cu met. diagonalului (caltul de H-V).

$$\bar{X}_0 = (20, \varepsilon, 0, 0, 10 - \varepsilon, 10 + 2\varepsilon, 0, 0, 20 + \varepsilon) - \text{S.B.A.H.V.}$$

$$f(\bar{X}_0) = 20 \cdot 1 + \varepsilon \cdot 3 + (10 - \varepsilon) \cdot 1 + (10 + 2\varepsilon) \cdot 3 = 20 + 3\varepsilon + 10 - \varepsilon + 30 + 6\varepsilon = 60 + 8\varepsilon \quad (\text{u.m})$$

$$v_p = m + n - 1 = 3 + 3 - 1 = 5$$

pas 2)  $\bar{X}_0 = \text{e.s.o.}??$



$$s_{13} = -2 + 3 - 1 + 3 = 3 > 0$$

$$s_{21} = -2 + 1 - 3 + 1 = -3$$

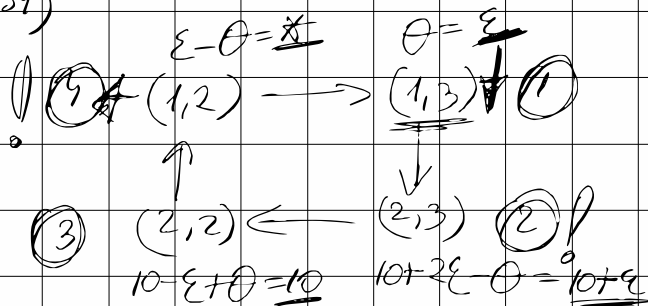
$$s_{31} = -0 + 0 - 3 + 1 - 3 + 1 = -4$$

$$s_{32} = -0 + 0 - 3 + 1 = -2$$

$\exists s_{ij} > 0 \Rightarrow \bar{X}_0$  nu e s.o.

pass 3)  $S_{KE} = \max \{f_{ij} > 0\} = \max \{f_{13}\} \Rightarrow x_{13} \downarrow$

pass 4)



$\theta = \varepsilon$   
 $x_{12} \rightarrow$

pass 5)

|       | $C_1$           | $C_2$           | $C_3$                           |                    |
|-------|-----------------|-----------------|---------------------------------|--------------------|
| $D_1$ | 20 <sup>1</sup> | * <sup>3</sup>  | $\varepsilon$ <sup>2</sup>      | $20 + \varepsilon$ |
| $D_2$ | * <sup>2</sup>  | 10 <sup>1</sup> | $10 + \varepsilon$ <sup>3</sup> | $20 + \varepsilon$ |
| $D_3$ | * <sup>1</sup>  | * <sup>2</sup>  | $20 + \varepsilon$ <sup>1</sup> | $20 + \varepsilon$ |
|       | 20              | 10              | $30 + 3\varepsilon$             |                    |

! verif.

pass 1')

$\bar{x}_1 = (20, 0, \varepsilon, 0, 10, 10 + \varepsilon, 0, 0, 20 + \varepsilon)$  SATISFIED  
 $f(\bar{x}_1) = 60 + 5\varepsilon$  (u.m.)  $< f(\bar{x}_0)$

pass 2')  $\bar{x}_1 = ?$  s.o?

$S_{12} = -3$   
 $S_{21} = 0$   
 $S_{31} = -1$   
 $S_{32} = -2$

$\forall i, j: S_{ij} \leq 0 \Rightarrow \bar{x}_1$  e soluti<sup>o</sup> optimiz<sup>o</sup>,  
 dar  $\varepsilon = 0$  este unic<sup>o</sup>.  
 elimin<sup>o</sup>  $\varepsilon$

Q:  $\bar{x}_{\text{optimal}} = x_{\text{optimal}} = (20, 0, 0, 0, 10, 10)$   
 $\min f = 60$  (u.m.)  
 s.o. maximiz<sup>o</sup>