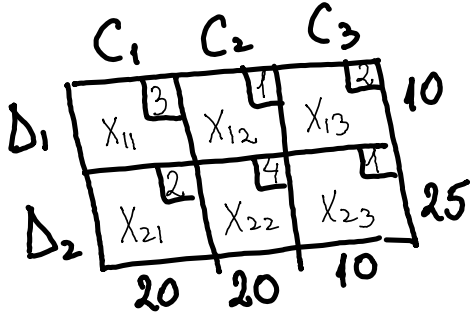


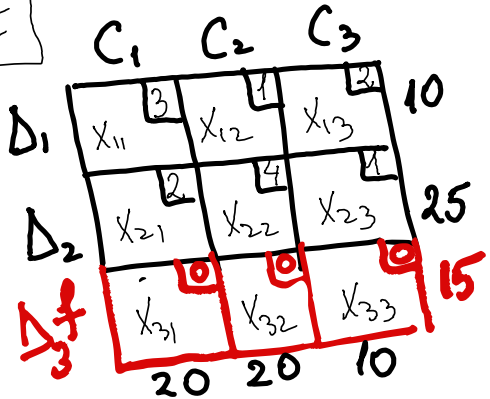
oferta $= 10+25 = 35 \neq$
cerere $= 20+20+10 = 50$
PTH oferta < cerere

$$f(x_{11}, x_{12}, \dots, x_{23}) = 3\cdot x_{11} + 1\cdot x_{12} + 2\cdot x_{13} + 2\cdot x_{21} + 4\cdot x_{22} + 1\cdot x_{23}$$

$$(PPL_g)\begin{cases} (1g)\ \min f(x_{11}, x_{12}, \dots, x_{23}) \\ (2g)\begin{cases} x_{11}+x_{12}+x_{13} = 10 \\ x_{21}+x_{22}+x_{23} = 25 \\ x_{11}+x_{21} \leq 20 \\ x_{12}+x_{22} \leq 20 \\ x_{13}+x_{23} \leq 10 \end{cases} \\ (3g)\ x_{11}, x_{12}, \dots, x_{23} \geq 0 \end{cases}$$

$$\begin{pmatrix} + x_i^c \\ + x_j^c \\ + x_k^c \end{pmatrix}$$

PTE



$\Rightarrow (PPL)_S$

oferta = 50
cerere = 50

$$(1s)\ \min f(x_{11}, \dots, x_{33}) = 3\cdot x_{11} + 1\cdot x_{12} + \dots + 0\cdot x_{33}$$

$$(2s)\begin{cases} x_{11}+x_{12}+x_{13} = 10 \\ x_{21}+x_{22}+x_{23} = 25 \\ x_{31}+x_{32}+x_{33} = 15 \\ x_{11}+x_{21}+x_{31} = 20 \\ x_{12}+x_{22}+x_{32} = 20 \\ x_{13}+x_{23}+x_{33} = 10 \end{cases}$$

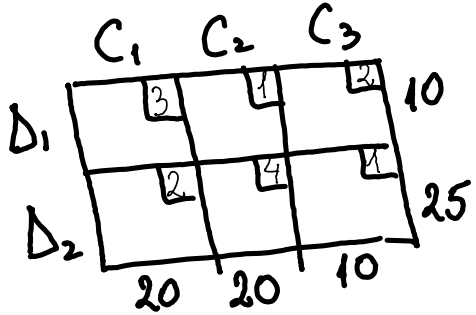
$$(3s)\ x_{11}, x_{12}, \dots, x_{33} \geq 0$$

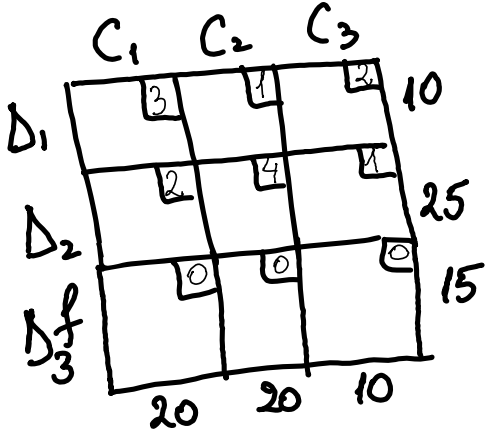
(2)
$$\begin{cases} x_{11}+x_{12}+x_{13}=10 \\ x_{21}+x_{22}+x_{23}=25 \\ x_{31}+x_{32}+x_{33}=15 \\ x_{11}+x_{21}+x_{31}=20 \\ x_{12}+x_{22}+x_{32}=20 \\ x_{13}+x_{23}+x_{33}=10 \end{cases} \Rightarrow \overline{A_5} =$$

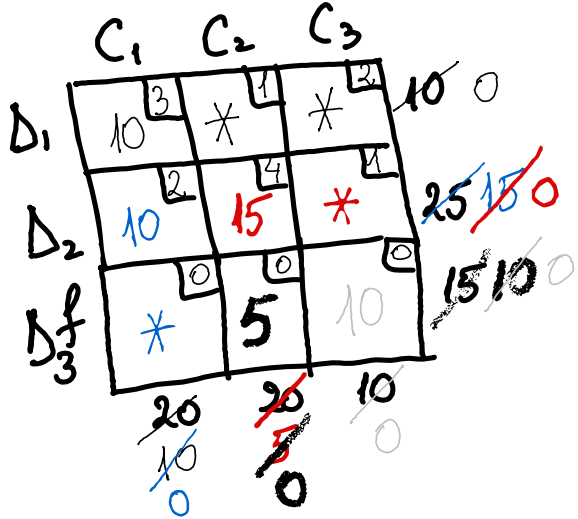
$$\overline{A_5} = \left(\begin{array}{ccccccccc|c} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} & \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 15 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 10 \end{array}\right) \quad \begin{matrix} + \\ \cdot(-1) \end{matrix}$$

$$\sim \left(\begin{array}{c} \\ \\ \\ \\ \\ \\ -10 \end{array}\right) \quad ?!\ \ddot{\cdot}$$

SR(A)i


C1 C2 C3
D1
D2
3 1 2
2 4 1
10
25
20 20 10


C1 C2 C3
D1
D2
D3
3 1 2
2 4 1
0 0 0
10
25
15
20 20 10



o) met. diag. , $\overline{X_0}$ = ? SBAI

C_{ij}

↙ Linia ↘ coloana

! verificări ! Σ linie

Σ col

"0-uri" la D

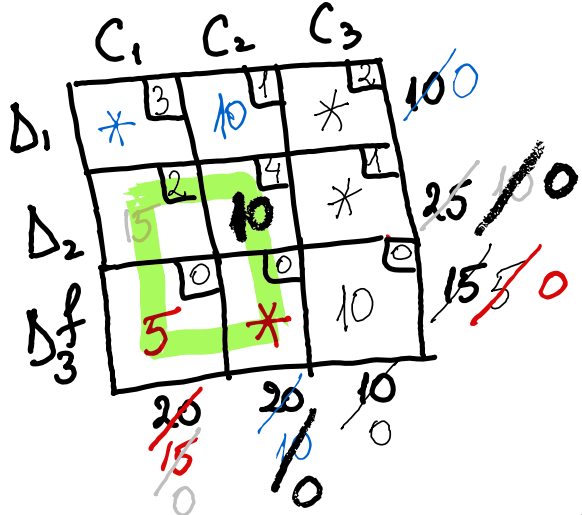
"0-uri" la C

$$\begin{cases} \overline{X_0} = (\underline{10}, 0, 0, \underline{10}, \underline{15}, 0, 0, \underline{5}, \underline{10}) \text{ SBAI Hol} \\ f(\overline{X_0}) = 30 + 20 + 60 = 110 \text{ u.m.} \end{cases}$$

nr. var. princ = $m + n - 1 = 3 + 3 - 1 = 5$ v.p.

nr. var sec = $m \cdot n -$ var. princ

$= 9 - 5 = 4$ v. sec



o) Cu met. cost. min.
$\overline{X_0}$ = ? SBA i

var prim = 3+3-1 = 5

$\overline{X_0} = (0, \underline{10}, 0, \underline{15}, \underline{10}, 0, \underline{5}, 0, \underline{10})$ SBA i Ho

$f(\overline{X_0}) = 10 + 30 + 40 = 80$ u.m

1) $\overline{X_0}$ - es. o?

$\delta_{11} = -3+1-4+2 = -4 < 0$

$\delta_{13} = -2+0-0+2-4+1 = -3 < 0$

$\delta_{23} = -1+2-0+0 = 1 > 0$;

$\delta_{32} = -0+4-2+0 = 2 > 0$;

2) $\delta_{32} = 2 > 0$ cost. intrare.

x_{32} ↓

3) Cost. ieșire.

$15+\theta = 20$ $10-\theta = 5$

(3,2) ← (2,1) ← (3,2)

(3,1) → (3,2)

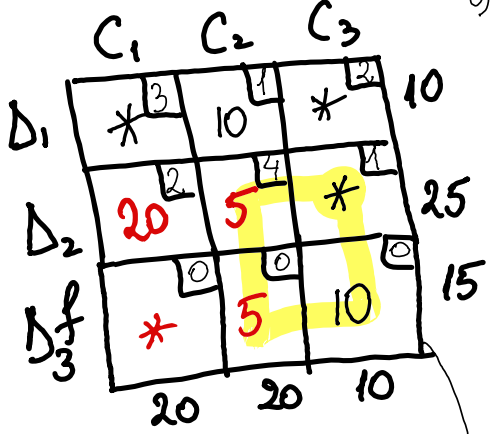
$\theta = 5$

$5-\theta = *$

$\theta = \min\{\underbrace{x_{22}}_{10}, \underbrace{x_{31}}_{5}\} = 5 \Rightarrow x_{31}$ →

5) Nord Tabul

! verif !



1) $\overline{X_1} = (0, 10, 0, 20, 5, 0, 0, 5, 10)$ SNA

$f(\overline{X_1}) = 10 + 40 + 20 = 70 \leq f(\overline{X_0})$

2) $\delta_{11} = -3+1-4+2 = -4 < 0$

$\delta_{13} = -2+0-0+1 = -1 < 0$

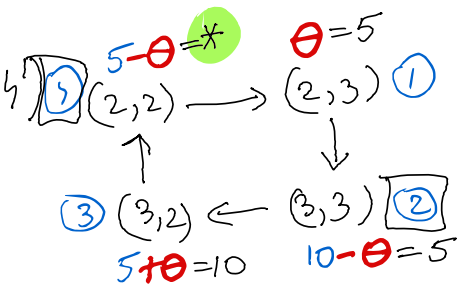
$\delta_{23} = -1+0-0+4 = 3 > 0$

$\delta_{31} = -0+2-4+0 = -2 < 0$

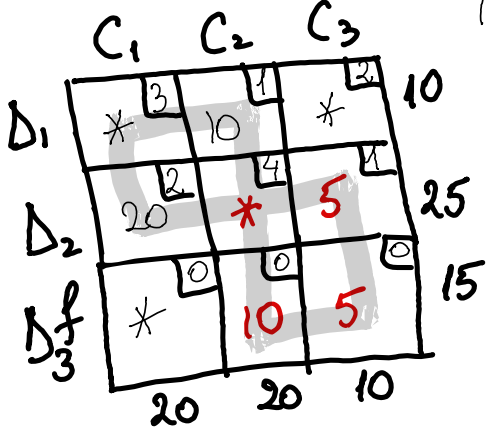
3) Crit. int $\Rightarrow x_{23}$ ↓ "

$\theta = \min\{\underbrace{x_{33}}_{10}, \underbrace{x_{22}}_{5}\} = 5 \Rightarrow x_{22}$ →

4)



5') Nord Tabul.



1") $\overline{X_2} = (0, 10, 0, 20, 0, 5, 0, 10, 5)$ SNA

$f(\overline{X_2}) = 55 \leq f(\overline{X_1})$

2") $\delta_{11} = -3+1-0+0-1+2 = -1 < 0$

$\delta_{13} = -2+0-0+1 = -1 < 0$

$\delta_{22} = -4+1-0+0 = -3 < 0$

$\delta_{31} = -0+0-1+2 = 1 > 0$

3") x_{31} ↓

4')

(5) (2,1) $20 - \theta = 15$ ← (2,3) (3) $5 + \theta = 10$

↓ ↑

(1) (3,1) $\theta = 5$ → (3,3) (2) $5 - \theta = *$

$\theta = 5 \Rightarrow x_{33} \rightarrow$ "

5'')

	C_1	C_2	C_3	
D_1	[3] *	[1] 10	[2] *	10
D_2	[2] 15	[4] *	[1] 10	25
D_f / D_3	[0] 5	[0] 10	[0] *	15
	20	20	10	

!verif.

1''') $\begin{cases} \overline{X_3} = (0,10,0,15,0,10,5,10,0) \text{ SBA } (1) \\ f(\overline{X_3}) = 50 \leq f(\overline{X_2}) \end{cases}$

2''') $\delta_{11} = -2 < 0$

$\delta_{13} = -2+1-0+0-2+1 = -2 < 0$

$\delta_{22} = -2 < 0$

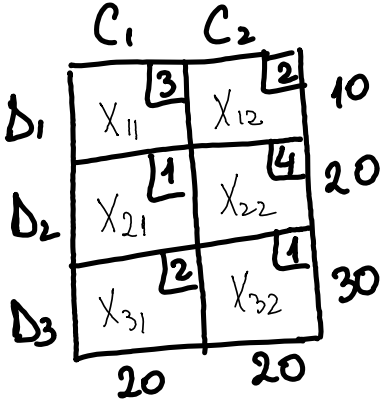
$\delta_{33} = -1 < 0$

$\Rightarrow \overline{X_3}$ - S.O. U

Cl. pt PTE : $\begin{cases} \overline{X_0}^{PTE} = \overline{X_3} = (1) \text{ S.O.U.} \\ \min f = 50 \text{ u.m.} \end{cases}$

Cl. pt PTN (P inițială) : $\begin{cases} \overline{X_0}^{PTH} = \overline{X_0}^{i} = (0,10,0,15,0,10) \text{ S.O.U} \\ \min f = 50 \text{ u.m.} \end{cases}$

(2)



oferta = 10+20+30 = 60 $\neq$ | $\Rightarrow$ PTN

cerere = 20+20 = 40

$$f(X_{11}, X_{12}, \dots, X_{32}) = 3 \cdot X_{11} + 2 \cdot X_{12} + 1 \cdot X_{21} + 4 \cdot X_{22} + 2 \cdot X_{31} + 1 \cdot X_{32}$$

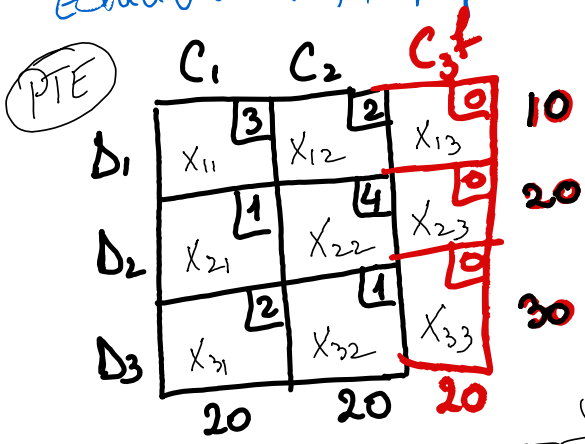
(PPL)

(1g) $\min f(X_{11}, X_{12}, \dots, X_{32})$

(2g)
$$\begin{cases} X_{11} + X_{12} \leq 10 \Rightarrow (+X_i^C) \\ X_{21} + X_{22} \leq 20 \Rightarrow (+X_j^C) \\ X_{31} + X_{32} \leq 30 \Rightarrow (+X_k^C) \\ X_{11} + X_{21} + X_{31} = 20 \\ X_{12} + X_{22} + X_{32} = 20 \end{cases}$$

(3g) $X_{11}, X_{12}, \dots, X_{32} \geq 0$

Echilibrăm PTH prin introducerea unui nou centru (C_3') fictiv.



oferta = 60 = cerere $\Rightarrow$ PTE

$$(1_S)\ \min f(x_{11}, x_{11}, \dots, x_{33}) = 3x_{11} + 2x_{12} + 0\cdot x_{13} + 1\cdot x_{21} + 4x_{22} + 0\cdot x_{23} + 2\cdot x_{31} + 1\cdot x_{32} + 0\cdot x_{33}$$

$$(2_S)\ \begin{cases} x_{11}+x_{12}+x_{13} = 10 \\ x_{21}+x_{22}+x_{23} = 20 \\ x_{31}+x_{32}+x_{33} = 30 \\ x_{11}+x_{21}+x_{31} = 20 \\ x_{12}+x_{22}+x_{32} = 20 \\ x_{13}+x_{23}+x_{33} = 20 \end{cases} \Longrightarrow$$

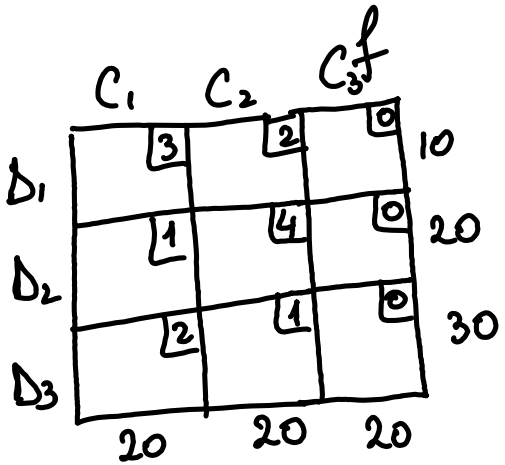
$$(3_S)\ x_{11}, x_{12}, \dots, x_{33} \geq 0$$

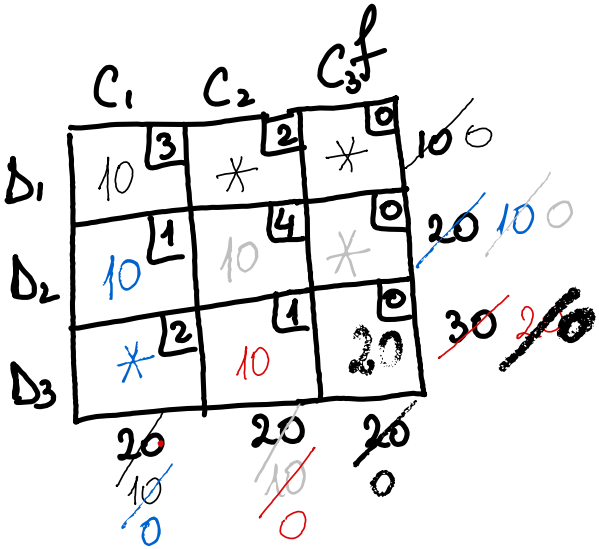
$$\Rightarrow \overline{A}_S = \begin{array}{c|ccccccccc|c} & x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} & \\ \hline & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \\ & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 20 \\ & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 30 \\ & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 20 \\ & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 20 \\ & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 20 \end{array} \quad \cdot(-1) \sim \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{c} -10 \\ 20 \\ 30 \\ 20 \\ 20 \\ 20 \end{array}$$

!?!

SMA!


C1
C2
C3f
D1
D2
D3
3
2
0
10
1
4
0
20
2
1
0
30
20
20
20



1) $\overline{X_0}$ - SMA i , $\overline{X_0}$ = ?
cu met. diagonalei
(colțul de N-V).

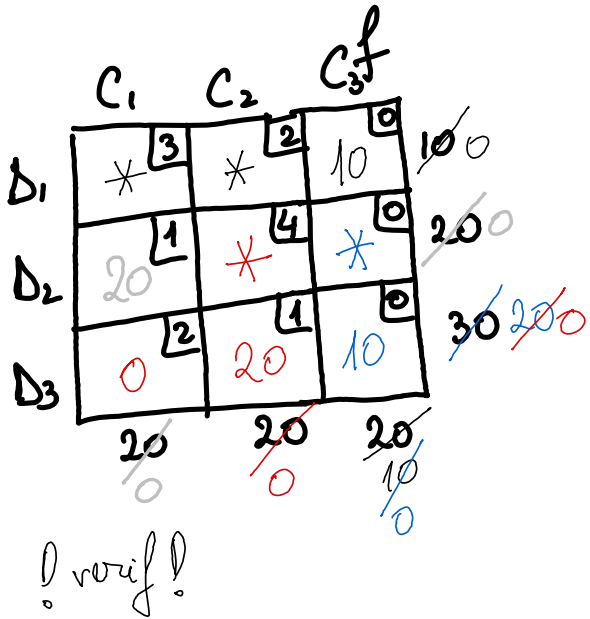
$\overline{X_0} = (10, 0, 0, 10, 10, 0, 0, 10, 20)$ - SMA i

$f(\overline{X_0}) = 30 + 10 + 40 + 10 = 80$ (u.m)

! verif ! $\sum$ linii
$\sum$ coloane
„0-uri" la D
„0-uri" la C

nr. var. princ = $m + n - 1 = 3 + 3 - 1 = 5$

nr. var. sec = $m \cdot n$ - var. princ = $9 - 5 = 4$
„*"

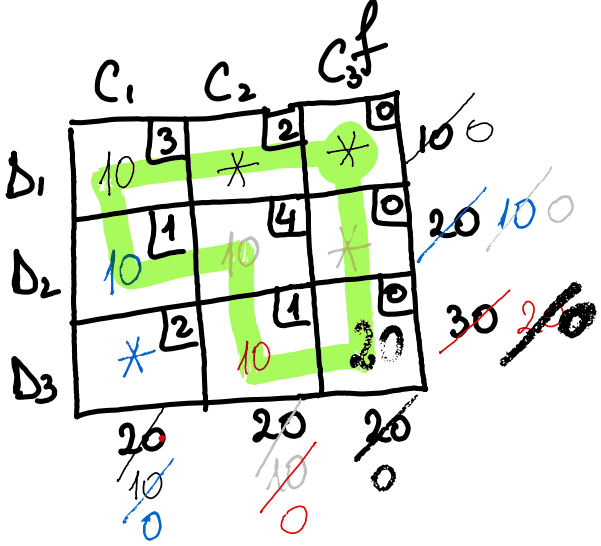


! verif !

1) $\overline{X_0} = ?$, $\overline{X_0}$ = SBA-i
cu met. costului minim.

$\overline{X_0} = (0, 0, \underline{10}, \underline{20}, 0, 0, \underline{0}, \underline{20}, \underline{10})$
SBA-i

$f(\overline{X_0}) = 20 + 20 = 40$ (u.m)



1) $\overline{X_0}$ – SNAI, $\overline{X_0}$ = ?
cu met. diagonalei
(colțul de N-V).

$\overline{X_0} = (10, 0, 0, 10, 10, 0, 0, 10, 20)$ – SNAI

$f(\overline{X_0}) = 30 + 10 + 40 + 10 = 80$ (u.m)

2) $\overline{X_0}$ = S.O. ??

$\Delta_{12} = -2 + 4 - 1 + 3 = 4 > 0$

$\Delta_{13} = -0 + 0 - 1 + 4 - 1 + 3 = 5 > 0$

$\Delta_{23} = -0 + 4 - 1 + 0 = 3 > 0$

$\Delta_{31} = -2 + 1 - 4 + 1 = -4 \leq 0$

3) Crit. intrare $\Rightarrow x_{13}$

4) Crit. ieșire.

$10 - \theta = *$

$\theta = 10$

(1,3) (1)

(1,1) (6)

$10 - \theta = 0$

(2,1) (5) $10 + \theta = 20$

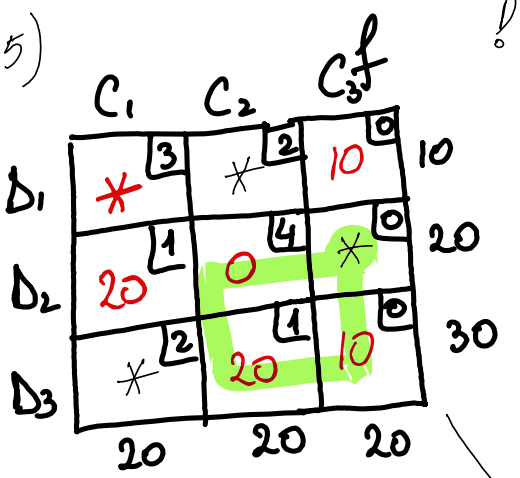
(2,2) (4)

(3,2) (3) $10 + \theta = 20$

(3,3) (2) $20 - \theta = 10$

$\theta = \min\{\underbrace{X_{33}}_{20}, \underbrace{X_{22}}_{10}, \underbrace{X_{11}}_{10}\} = 10 \Rightarrow X_{11}$

5)



! verif.

1') $\overline{X_1} = (0,0,\underline{10},\underline{20},\underline{0},0,0,\underline{20},\underline{10})$

SBA Degenerata (! pericol de ciclaj)

$f(\overline{X_1}) = 20+20 = 40 \leq f(\overline{X_0})$

2') $\overline{X_1}$ e SO ??

$\Delta_{11} = -3+0-0+1-4+1 = -5 \leq 0$

$\Delta_{12} = -2+1-0+0 = -1 \leq 0$

$\Delta_{23} = -0+4-1+0 = 3 > 0$ $\therefore \Rightarrow \overline{X_1}$ nu e S.O.

$\Delta_{31} = -2+1-4+1 = -4 \leq 0$

3') X_{23}

4') $0-\theta = -\theta$ $\quad \theta = 0$

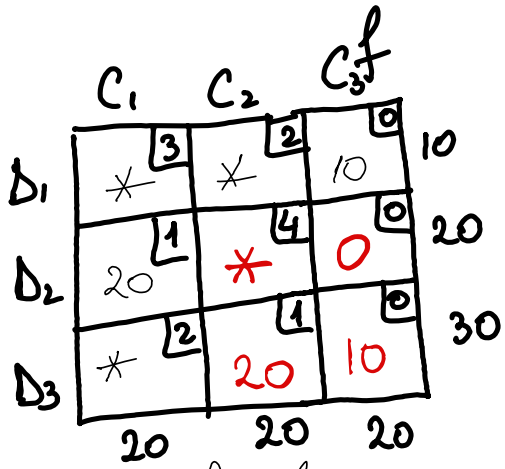
(3) (2,2) $\longrightarrow$ (2,3) (1)

(3) (3,2) $\longleftarrow$ (3,3) (2)

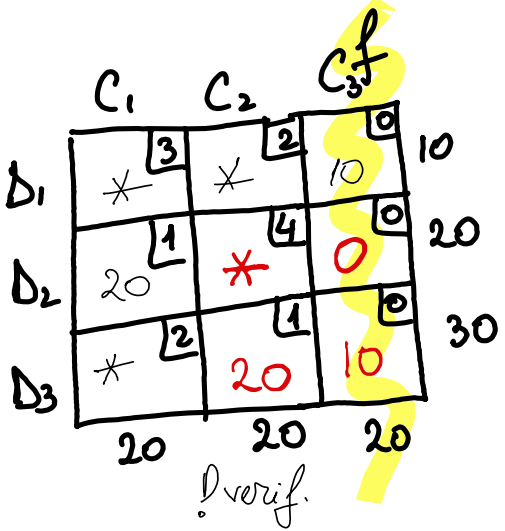
$20+\theta = 20$ $\quad 10-\theta = 10$

$\theta = \min\{\underbrace{X_{33}}_{10}, \underbrace{X_{22}}_{0}\} = 0 \Rightarrow X_{22} \rightarrow$ "

5')



! verif.



1'') $\overline{X_2} = (0,0,\underline{10},\underline{20},0,\underline{0},0,\underline{20},10)$ ①
SBA D.
$f(\overline{X_2}) = 40 \leq f(\overline{X_1})$

2'') $\overline{X_2}$ e s.o. ?
$f_{11} = -2 \leq 0$
$f_{12} = -1 \leq 0$ ☺ $\Rightarrow \overline{X_2}$ e s.o. U.
$f_{22} = -3 \leq 0$
$f_{31} = -1 \leq 0$

Cl. PTE $\begin{cases} \overline{X_0}^{PTE} \equiv \overline{X_2} = (\dots\dots) \text{①} \\ \min f = 40 \text{ u.m.} \end{cases}$

Cl. PTH $\begin{cases} \overline{X_0}^{PTH} \equiv \overline{X_0}^{i} = (0,0,\underline{20},0,0,\underline{20}) \text{ S.O.U.} \\ \min f = 40 \text{ u.m.} \end{cases}$