

$$\textcircled{1} \left\{ \begin{array}{l} (1) \min f(x_1, x_2) = 2x_1 - x_2 \\ (2) \begin{cases} x_1 + 4x_2 \geq 4 \\ 2x_1 - 3x_2 \leq 6 \\ x_1 - x_2 \geq -3 \\ x_1 + x_2 \leq 6 \end{cases} \\ (3) x_1, x_2 \geq 0 \end{array} \right.$$

pos1 :  $S = ?$  (mit. solutionen generale a PPL)

$$\bullet (R_1): x_1 + 4x_2 \geq 4 \Rightarrow (\Delta_1): x_1 + 4x_2 = 4 \Rightarrow \begin{cases} x_1 = 0 \xrightarrow{\Delta_1} 0 + 4x_2 = 4 \Rightarrow x_2 = 1 \Rightarrow P_1(0, 1) \\ x_2 = 0 \xrightarrow{\Delta_1} x_1 + 4 \cdot 0 = 4 \Rightarrow x_1 = 4 \Rightarrow P_2(4, 0) \end{cases}$$

$$O(0,0) \xrightarrow{R_1} 0 + 4 \cdot 0 \geq 4 \Rightarrow 0 \geq 4 \text{ (F)}$$

$$\bullet (R_2): 2x_1 - 3x_2 \leq 6 \Rightarrow (\Delta_2): 2x_1 - 3x_2 = 6 \Rightarrow \begin{cases} x_1 = 0 \xrightarrow{\Delta_2} 2 \cdot 0 - 3x_2 = 6 \Rightarrow x_2 = -2 \Rightarrow P_3(0, -2) \\ x_2 = 0 \xrightarrow{\Delta_2} 2x_1 - 3 \cdot 0 = 6 \Rightarrow x_1 = 3 \Rightarrow P_4(3, 0) \end{cases}$$

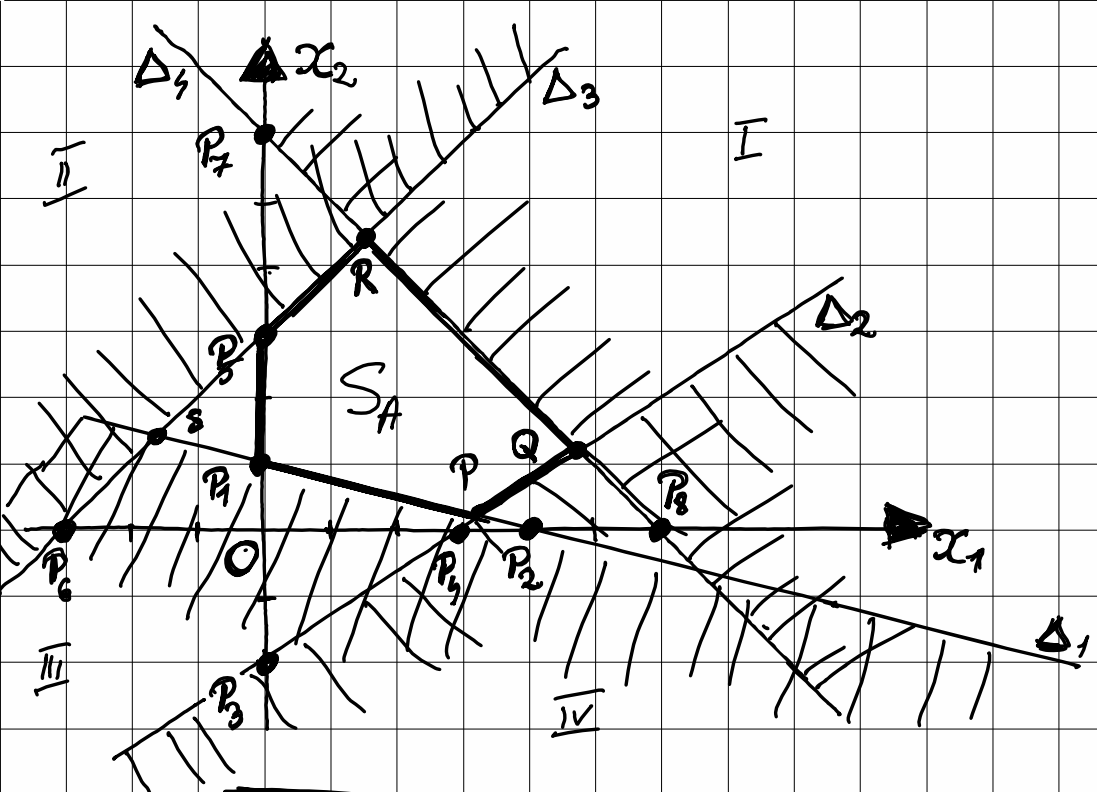
$$O(0,0) \xrightarrow{R_2} 2 \cdot 0 - 3 \cdot 0 \leq 6 \Rightarrow 0 \leq 6 \text{ (A)}$$

$$\bullet (R_3): x_1 - x_2 \geq -3 \Rightarrow (\Delta_3): x_1 - x_2 = -3 \Rightarrow \begin{cases} x_1 = 0 \xrightarrow{\Delta_3} 0 - x_2 = -3 \Rightarrow x_2 = 3 \Rightarrow P_5(0, 3) \\ x_2 = 0 \xrightarrow{\Delta_3} x_1 - 0 = -3 \Rightarrow x_1 = -3 \Rightarrow P_6(-3, 0) \end{cases}$$

$$O(0,0) \xrightarrow{R_3} 0 - 0 \geq -3 \Rightarrow 0 \geq -3 \text{ (A)}$$

$$\bullet (R_4): x_1 + x_2 \leq 6 \Rightarrow (\Delta_4): x_1 + x_2 = 6 \Rightarrow \begin{cases} x_1 = 0 \xrightarrow{\Delta_4} 0 + x_2 = 6 \Rightarrow x_2 = 6 \Rightarrow P_7(0, 6) \\ x_2 = 0 \xrightarrow{\Delta_4} x_1 + 0 = 6 \Rightarrow x_1 = 6 \Rightarrow P_8(6, 0) \end{cases}$$

$$O(0,0) \xrightarrow{R_4} 0 + 0 \leq 6 \Rightarrow 0 \leq 6 \text{ (A)}$$



$$S = [PQRS]$$

pas2 :  $S_A = ?$  (mt. soluțiilor admisibile a PP2)

$$S_A = S \cap \{\text{cadrantul I}\} = [PQR, P_5, P_1]$$

pas3 :  $S_{AB} = ?$  (mt. soluțiilor optime a PP2)

$S_{AB}$  = mt. soluțiilor de lucru admisibile a PP2

$$S_{AB} = \{ \text{vf. mt. } S_A \} = \{P, Q, R, P_5, P_1\}$$

$$\bullet P_1(0,1) \Rightarrow f(P_1) = f(0,1) = 2 \cdot 0 - 1 = \textcircled{-1}$$

$$\bullet P_5(0,3) \Rightarrow f(P_5) = f(0,3) = 2 \cdot 0 - 3 = \textcircled{-3} !$$

$$f(x_1, x_2) = 2x_1 - x_2$$

$$\bullet R = (\Delta_3) \cap (\Delta_4): \begin{cases} x_1 - x_2 = -3 \\ x_1 + x_2 = 6 \end{cases} \Rightarrow \begin{array}{l} \textcircled{+} \\ \hline 2x_1 = 3 \Rightarrow x_1 = \frac{3}{2} \\ \textcircled{-} \\ \hline -2x_2 = -9 \Rightarrow x_2 = \frac{9}{2} \end{array} \Rightarrow R\left(\frac{3}{2}, \frac{9}{2}\right) \Rightarrow$$

$$\Rightarrow f(R) = f\left(\frac{3}{2}, \frac{9}{2}\right) = 2 \cdot \frac{3}{2} - \frac{9}{2} = \left(-\frac{3}{2}\right)$$

$$\bullet Q = (\Delta_2) \cap (\Delta_4): \begin{cases} 2x_1 - 3x_2 = 6 \\ x_1 + x_2 = 6 \end{cases} \Rightarrow \begin{cases} 2x_1 - 3x_2 = 6 \\ -2x_1 - 2x_2 = -12 \end{cases} \Rightarrow \begin{array}{l} \textcircled{+} \\ \hline -5x_2 = -6 \Rightarrow x_2 = \frac{6}{5} \\ \textcircled{-} \\ \hline x_1 + \frac{6}{5} = \frac{5}{5} \Rightarrow x_1 = \frac{24}{5} \end{array}$$

$$\Rightarrow Q\left(\frac{24}{5}, \frac{6}{5}\right) \Rightarrow f(Q) = f\left(\frac{24}{5}, \frac{6}{5}\right) = \frac{24}{5} \cdot 2 - \frac{6}{5} = \left(\frac{52}{5}\right)$$

$$\bullet P = (\Delta_1) \cap (\Delta_2): \begin{cases} x_1 + 4x_2 = 4 \\ 2x_1 - 3x_2 = 6 \end{cases} \Rightarrow \begin{cases} -2x_1 - 8x_2 = -8 \\ 2x_1 - 3x_2 = 6 \end{cases} \Rightarrow \begin{array}{l} \textcircled{+} \\ \hline -11x_2 = -2 \Rightarrow x_2 = \frac{2}{11} \\ \textcircled{-} \\ \hline x_1 + 4 \cdot \frac{2}{11} = \frac{4}{11} \Rightarrow x_1 = \frac{36}{11} \end{array}$$

$$\Rightarrow P\left(\frac{36}{11}, \frac{2}{11}\right) \Rightarrow f(P) = f\left(\frac{36}{11}, \frac{2}{11}\right) = \frac{36}{11} \cdot 2 - \frac{2}{11} = \left(\frac{70}{11}\right)$$

$$S_0 = \{P_5\} \Rightarrow \begin{cases} x_1^{\text{optimal}} = 0 \\ x_2^{\text{optimal}} = 3 \end{cases}, \text{ i.e. min } f(x_1, x_2) = -3$$

$$\textcircled{2} \quad \begin{cases} (1g) \min f(x_1, x_2, x_3) = 2x_1 - x_2 - 3x_3 \\ (2g) \begin{cases} x_1 + x_2 + 2x_3 \leq 10 \\ 3x_1 - x_2 + 2x_3 \leq 8 \end{cases} \\ (3g) x_1, x_2, x_3 \geq 0 \end{cases}$$

pas1: Aducem  $(PPL)_g$  la forma standard:

$$(PPL)_s \quad \begin{cases} (1s) \min f(x_1, x_2, x_3, x_4^c, x_5^c) = 2x_1 - x_2 - 3x_3 + 0 \cdot x_4^c + 0 \cdot x_5^c \\ (2s) \begin{cases} x_1 + x_2 + 2x_3 + x_4^c = 10 \\ 3x_1 - x_2 + 2x_3 + x_5^c = 8 \end{cases} \\ (3s) x_1, x_2, x_3, x_4^c, x_5^c \geq 0 \end{cases}$$

$$\begin{array}{|l} \textcircled{!} \\ 0 \end{array} \left[ \begin{array}{l} " \leq " \Rightarrow " + " \\ " \geq " \Rightarrow " - " \end{array} \right]$$

pas2: Det. o soluție de bază admisibilă inițială  $\bar{X}_0$  (= S.B.A.i.) a  $(PPL)_s$  (ie. a sistemului  $(2s)$ ) cu metoda lui Gauss.

$$(2s) \rightarrow \bar{A} = \begin{pmatrix} P_1 & P_2 & P_3 & P_4^c & P_5^c & P_0 \\ 1 & 1 & 2 & 1 & 0 & 10 \\ 3 & -1 & 2 & 0 & 1 & 8 \\ x_1 & x_2 & x_3 & \textcircled{x_4^c} & \textcircled{x_5^c} & \end{pmatrix} \equiv \bar{A}_{G-J} \xrightarrow[\text{v.s.}=0]{\text{v.p.}=x_4^c, x_5^c} \bar{R}$$

$$\Rightarrow \bar{X}_0 = (0, 0, 0, 10, 8)^T - \text{S.B.} \begin{pmatrix} \textcircled{1} \\ \textcircled{1} \end{pmatrix} H_d = \text{S.B.A.i.} \checkmark$$

$\begin{matrix} \geq 0 & \geq 0 \\ \neq 0 & \neq 0 \end{matrix}$

$$\textcircled{3} \quad \text{(PPL)}_1 \quad \begin{cases} (1_i) \max f(x_1, x_2, x_3) = x_1 + x_2 + 3x_3 \\ (2_i) \begin{cases} 2x_1 + x_2 + 2x_3 = 10 \\ 3x_1 + x_3 \geq 6 \end{cases} \\ (3_i) x_1, x_2, x_3 \geq 0 \end{cases}$$

$$\boxed{\max(f) = -\min(-f)}$$

pas 1:  $(\text{PPL})_s$  !

$$(\text{PPL})_s \quad \begin{cases} (1_s) \min(-f)(x_1, x_2, x_3, x_4^c) = -x_1 - x_2 - 3x_3 + 0 \cdot x_4^c \\ (2_s) \begin{cases} 2x_1 + x_2 + 2x_3 = 10 \\ 3x_1 + x_3 - x_4^c = 6 \end{cases} \\ (3_s) x_1, x_2, x_3, x_4^c \geq 0 \end{cases}$$

pas 2:  $\overline{X}_0 = \text{S.B.A.i} = ?$

$$(2_s) \rightarrow \overline{A} = \begin{pmatrix} p_1 & p_2 & p_3 & p_4^c & p_0 \\ 2 & 1 & 2 & 0 & 10 \\ 3 & 0 & 1 & -1 & 6 \end{pmatrix} \begin{matrix} x_1 & x_2 & x_3 & x_4^c & \\ \hline \end{matrix} \quad \sim \begin{pmatrix} 2 & 1 & 2 & 0 & 10 \\ -3 & 0 & -1 & 1 & -6 \end{pmatrix} \begin{matrix} x_1 & x_2 & x_3 & x_4^c & \\ \hline \end{matrix}$$

$$\Rightarrow \overline{X}_0 = (0, 10, 0, -6)^T - \text{S.B.N.No!} \neq \text{S.B.A.i.}$$

$$\begin{matrix} \geq 0 & \leq 0 \\ \neq 0 & \neq 0 \end{matrix} \rightarrow$$

Hu este utilă !

$\Rightarrow$  couterm altoi S.B , luămol altoi combinație de v.p. :

$$(2s) \rightarrow \bar{A} = \left( \begin{array}{cccc|c} 2 & 1 & 2 & 0 & 10 \\ 3 & 0 & 1 & -1 & 6 \end{array} \right) \begin{array}{l} \leftarrow + \\ \leftarrow - \end{array} \sim$$

$x_1 \quad x_2 \quad x_3 \quad x_4^c$

$$\sim \left( \begin{array}{cccc|c} -1 & 1 & 0 & 2 & -2 \\ 3 & 0 & 1 & -1 & 6 \end{array} \right) \begin{array}{l} \leftarrow -2 \\ \leftarrow 6 \end{array} \equiv \bar{A}_{GJ}^R \Rightarrow \bar{X}_0 = (0, -2, 6, 0)^T$$

$x_1 \quad (x_2) \quad (x_3) \quad x_4^c$

S.B.H. Hd.  $\neq$  SBAi  
! tot nu este bună!

... mai couterăm :

$$(2s) \rightarrow \bar{A} = \left( \begin{array}{cccc|c} 2 & 1 & 2 & 0 & 10 \\ 3 & 0 & 1 & -1 & 6 \end{array} \right) \begin{array}{l} \leftarrow + \\ \leftarrow - \end{array} \sim$$

$x_1 \quad x_2 \quad x_3 \quad x_4^c$

$$\sim \left( \begin{array}{cccc|c} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 6 \\ 1 & 0 & \frac{1}{3} & -\frac{1}{3} & 2 \end{array} \right) \begin{array}{l} \leftarrow 6 \\ \leftarrow 2 \end{array} \sim \left( \begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & -\frac{1}{3} & 2 \\ 0 & 1 & \frac{2}{3} & \frac{2}{3} & 6 \end{array} \right) \equiv \bar{A}_{GJ}^R$$

$(x_1) \quad (x_2) \quad x_3 \quad x_4^c$        $(x_1) \quad (x_2) \quad x_3 \quad x_4^c$

v.p.      v.s = 0

$$\Rightarrow \bar{X}_0 = (2, 6, 0, 0)^T - \text{S.B.A. Hd} = \text{S.B.A. i.} \checkmark$$

$\geq 0 \geq 0$   
 $\neq 0 \neq 0$