

Example:

a) non-perturbate ETP

	s_1	s_2	...	s_n	
w_1	c_{11}	c_{12}	...	c_{1n}	a_1
w_2	c_{21}	c_{22}	...	c_{2n}	a_2
...
w_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
	b_1	b_2	...	b_n	

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

a') perturbate ETP

	s_1	s_2	...	s_n	
w_1	c_{11}	c_{12}	...	c_{1n}	$a_1 + \epsilon$
w_2	c_{21}	c_{22}	...	c_{2n}	$a_2 + \epsilon$
...
w_m	c_{m1}	c_{m2}	...	c_{mn}	$a_m + \epsilon$
	b_1	b_2	...	$b_n + m \cdot \epsilon$	

$$\sum_{i=1}^m (a_i + \epsilon) = \sum_{i=1}^m a_i + m \cdot \epsilon = \sum_{j=1}^n b_j + m \cdot \epsilon$$

Examples:

Solve the next TP. if the second solution which we obtain it isn't the optimal solution don't continue the algorithm:

a)

	s_1	s_2	s_3	
w_1	3	2	1	30
w_2	1	4	2	20
w_3	2	1	3	20
	20	15	20	

Because, $\sum_{i=1}^3 a_i = 70 > \sum_{j=1}^3 b_j = 55$
we have a No TP, so we must to equilibrate.

①

	s_1	s_2	s_3	s_4^f	
w_1	20	10	*	*	30, 10, 0
w_2	*	5	15	*	20, 15, 0
w_3	*	*	5	15	20, 15, 0
	20	15	20	15	

For ETP we applied the diagonal method to find the iBAS: \bar{x}_0 $\begin{cases} \bar{x}_{11}=20; \bar{x}_{12}=10; \bar{x}_{22}=5; \\ \bar{x}_{23}=15; \bar{x}_{33}=5; \bar{x}_{34}=15 \end{cases}$
the other $\bar{x}_{ij} = * = 0$
 $f(\bar{x}_0) = 145$

② We check if \bar{x}_0 it's optimal solution:

$$\delta_{13} = -1 + 2 - 4 + 2 = -1$$

$$\delta_{14} = -0 + 0 - 3 + 2 - 4 + 2 = -3$$

$$\delta_{21} = -1 + 3 - 2 + 4 = 4 > 0$$

$$\delta_{24} = -0 + 0 - 3 + 2 = -1$$

$$\delta_{31} = -2 + 3 - 2 + 4 - 2 + 3 = 4 > 0$$

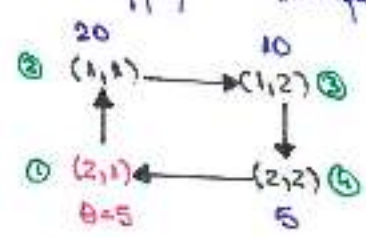
$$\delta_{32} = -1 + 4 - 2 + 3 = 4 > 0$$

$\Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_0$ it isn't optimal solution!!

I choose δ_{21}

③ We apply the input criteria: $\delta_{ij} = \max \{ \delta_{ij} > 0 \} = \max \{ \delta_{21}, \delta_{31}, \delta_{32} \} \stackrel{(!)}{=} \delta_{21} \Rightarrow x_{21} \downarrow$

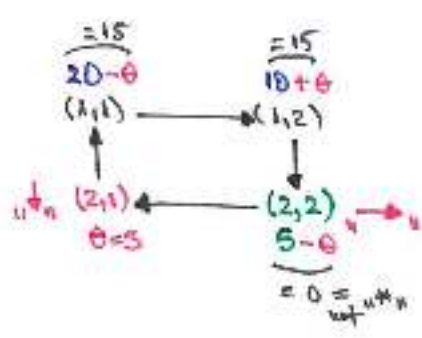
④ We apply the "output criteria":



$$\theta = \min \{ x_{ij} \mid x_{ij} \text{ been into even no. cells} \} = \min \{ \underbrace{x_{11}}, \underbrace{x_{22}} \} = 5 \Rightarrow x_{22} \rightarrow 0$$

⑤ We determine the new solution: \bar{x}_1

	s_1	s_2	s_3	s_4	
w_1	15	15	*	*	30
w_2	5	*	5	*	20
w_3	*	*	5	15	20
	20	15	20	15	



$$\bar{x}_1: \begin{cases} \bar{x}_{11}=15, \bar{x}_{12}=15, \bar{x}_{21}=5, \bar{x}_{23}=5, \bar{x}_{33}=5, \bar{x}_{34}=15 \rightarrow \text{basic components} \\ \bar{x}_{13}=\bar{x}_{14}=\bar{x}_{22}=\bar{x}_{24}=\bar{x}_{31}=\bar{x}_{32}=0 (= "x") \rightarrow \text{non-basic (free) components} \end{cases}$$

$$f(\bar{x}_1) = 115 (< f(\bar{x}_0))$$

⑥ We check if \bar{x}_1 it is a optimal solution:

$$\delta_{13} = -1 + 2 - 1 + 3 = 3 > 0 \Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_1 \text{ it isn't optimal solution} \Rightarrow \text{the algorithm must to continue ... !!}$$

b)

	s_1	s_2	s_3	
w_1	20	0	*	30, 0
w_2	*	15	*	15, 0
w_3	*	5	5	10, 5, 0
	20	20	5	

($\sum w_i x_i = 45 = \sum s_j b_j$)

$$\bar{x}_0: \begin{cases} \bar{x}_{11}=20; \bar{x}_{12}=0; \bar{x}_{22}=15; \bar{x}_{32}=5; \bar{x}_{34}=5 \rightarrow \text{basic component} \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0 (= "x") \rightarrow \text{non-basic components} \end{cases}$$

$$f(\bar{x}_0) = 125$$

it's degenerate BAS(!!!)

(no, it's possible to appear the cycle phenomenon!)

we apply the perturbation method

①

	s_1	s_2	s_3	
w_1	20	ϵ	*	$20+\epsilon, \epsilon, 0$
w_2	*	$15+\epsilon$	*	$15+\epsilon, 0$
w_3	*	$5-2\epsilon$	$5+3\epsilon$	$10+\epsilon, 5+3\epsilon, 0$
	20	$20-\epsilon$	$5+3\epsilon$	

$$\Rightarrow \bar{x}_0: \begin{cases} \bar{x}_{11}=20; \bar{x}_{12}=\epsilon; \bar{x}_{22}=15+\epsilon; \bar{x}_{32}=5-2\epsilon; \bar{x}_{33}=5+3\epsilon \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0 (= "x") \rightarrow \text{non-basic components} \end{cases}$$

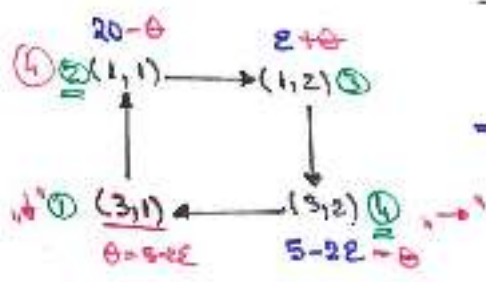
$$f(\bar{x}_0) = 125 + 6\epsilon$$

non-degenerate BAS(!!!)

② we apply the optimality criteria:

$$\left. \begin{aligned} \delta_{13} &= -1 + 2 - 3 + 2 = 0 \\ \delta_{21} &= -3 + 2 - 2 + 4 = 1 > 0 \\ \delta_{23} &= -3 + 2 - 3 + 4 = 0 \\ \delta_{31} &= -1 + 2 - 2 + 3 = 2 > 0 \end{aligned} \right\} \Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_0 \text{ it isn't the optimal solution}$$

③ $\delta_j = \max \{ \delta_{ij} > 0 \} = \max \{ \underbrace{\delta_{21}}_{=1}, \underbrace{\delta_{31}}_{=2} \} = \delta_{31} \Rightarrow x_{31} \downarrow$ (becomes a basic component)



$\Rightarrow \theta = \min \{ \underbrace{x_{11}}_{=20}, \underbrace{x_{32}}_{=5-2\varepsilon} \} = 5-2\varepsilon \Rightarrow x_{32} \rightarrow 0$ (becomes a non-basic variable)

⑤

	s_1	s_2	s_3	
w_1	15+2ε	5-ε	*	20+ε
w_2	*	15+ε	*	15+ε
w_3	5-2ε	*	5+3ε	10+ε
	20	20	5+3ε	

$\Rightarrow \bar{x}_1 \begin{cases} \bar{x}_{11} = 15+2\varepsilon; \bar{x}_{12} = 5-\varepsilon; \bar{x}_{22} = 15+\varepsilon; \bar{x}_{31} = 5-2\varepsilon; \bar{x}_{32} = 5+3\varepsilon \\ \bar{x}_{13} = \bar{x}_{21} = \bar{x}_{23} = \bar{x}_{33} = 0 (= " * ") \end{cases}$

$f(\bar{x}_1) = 115 + 10\varepsilon (< f(\bar{x}_0))$

we repeat the steps of the algorithm:

② $\delta_{13} = -1 + 2 - 1 + 2 = 2 > 0 \Rightarrow (\exists) \delta_{ij} > 0 \Rightarrow \bar{x}_1$ - it isn't the optimal solution, the algorithm continues...