

1)

# Probleme de transport neechilibrate

## 1) Metoda diagonalei

	$C_1$	$C_2$	$C_3$	
$D_1$	<div>3</div>	<div>1</div>	<div>2</div>	10
$D_2$	<div>2</div>	<div>4</div>	<div>1</div>	25
	20	10	10	

### Pasul 0

oferta:  $\sum_{i=1}^2 a_i = 10 + 25 = 35$

cereea:  $\sum_{j=1}^3 b_j = 20 + 10 + 10 = 40$

$\Rightarrow \text{oferta} < \text{cereea} \Rightarrow$   
 $\Rightarrow \text{P.T.N.}$

### • Echilibrăm P.T.N.

$\rightarrow$  transformăm P.T.N. în P.T.E. prin introducerea unui nou depozit de desfacere,  $D_3^f$ , ce va avea costurile de transport egale cu zero.

$$40 - 35 = 5 \quad (D_3^f)$$

	$C_1$	$C_2$	$C_3$	
$D_1$	10 <span style="float: right;">3</span>	* <span style="float: right;">1</span>	* <span style="float: right;">2</span>	<del>10</del> 0
$D_2$	10 <span style="float: right;">2</span>	10 <span style="float: right;">4</span>	5 <span style="float: right;">1</span>	<del>25</del> <del>15</del> $\neq 0$
$D_3$	* <span style="float: right;">0</span>	* <span style="float: right;">0</span>	5 <span style="float: right;">0</span>	80
	<del>20</del> $\neq 0$	<del>10</del> 0	<del>10</del> $\neq 0$	

! verificare!

$$v_p = 5$$

$$v_s = 4$$

Pasul 1

$$\bar{x}_0 = (10, 0, 0, 10, 10, 5, 0, 0, 5) \in \mathbb{R}^9 - \text{s.b. Ned.}$$

$$f(\bar{x}_0) = 10 \cdot 3 + 10 \cdot 2 + 10 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 95 (\mu.m.)$$

Pasul 2

$$\delta_{12} = -1 + 4 - 2 + 3 = 4 > 0$$

$$\delta_{13} = -2 + 1 - 2 + 3 = 0$$

$$\delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$$

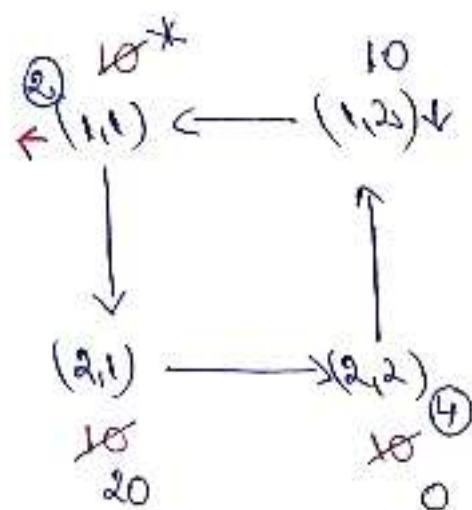
$$\delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$$

$\nexists \delta_{ij} > 0 \Rightarrow \bar{x}_0$  Nu este optimă

Pasul 3

$$S_{ke} = \max \{ f_{12}; f_{31}; f_{32} \} = f_{12} \Rightarrow x_{12} \downarrow$$

Pasul 4



$$\Theta = \min \{ (1,1); (2,2) \} = (1,1) \Rightarrow x_{11} \rightarrow$$

$$\Theta = 10$$

Pasul 5

	3	1	2	
*	10	*	10	
2	4	1		
20	0	5	25	
0	0	0		
*	*	5	5	
20	10	10		

! verificare !

Pasul 6

$$\bar{x}_1 = (0, 10, 0, 20, 0, 5, 0, 0, 5) \in R^9 - \text{s.B. Deg}$$

$$f(\bar{x}_1) = 10 \cdot 1 + 20 \cdot 2 + 0 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 55 \text{ (u.m.)} \leq f(\bar{x}_0)? \text{ Da}$$

Pg 2

### Pasul 7

$$\delta_{11} = -3 + 1 - 4 + 2 = -4$$

$$\delta_{13} = -2 + 1 - 4 + 1 = -4$$

$$\delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$$

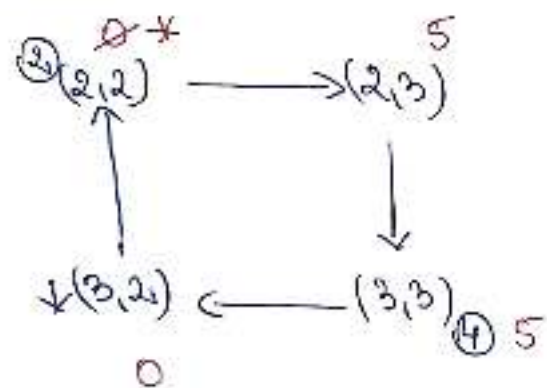
$$\delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$$

$\Rightarrow \exists \delta_{ij} > 0 \Rightarrow \bar{x}_1$  Nu este optimă.

### Pasul 8

$$Ske = \max \{ \delta_{31}, \delta_{32} \} = \delta_{32} \Rightarrow x_{32} \downarrow$$

### Pasul 9



$$\Theta = \min \{ (2,2), (3,3) \} = (2,2) \Rightarrow (2,2) = x_{22} \rightarrow$$

$$\Theta = 0$$

### Pasul 10

	3		1		2	
*		10		*	10	
	2		4		1	
20		*		5	25	
	0		0		0	
*		0		5	5	
20		10		10		

! verificare!

### Pasul 11

$$\bar{x}_2 = (0, 10, 0, 20, 0, 5, 0, 0, 5) \in \mathbb{R}^9 \text{ s.t. } \Delta g.$$

$$f(\bar{x}_2) = 10 \cdot 1 + 20 \cdot 2 + 5 \cdot 1 + 0 \cdot 5 \cdot 0 = 55 (n.m.) \leq f(\bar{x}_1)? \text{ Da}$$

### Pasul 12

$$\Delta_{11} = -3 + 1 - 0 + 0 - 1 + 2 = -1$$

$$\Delta_{13} = -2 + 1 - 0 + 0 = -1$$

$$\Delta_{22} = -4 + 0 - 0 + 1 = -3$$

$$\Delta_{31} = -0 + 0 - 1 + 2 = 1 > 0$$

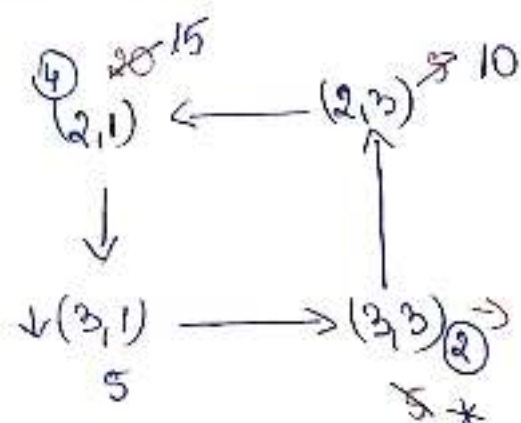
$\Rightarrow \exists \Delta_{ij} > 0 \Rightarrow \bar{x}_2$  este optimă.



### Pasul 13

$$S_{ke} = \max \{ f_{31} \} = f_{31} \Rightarrow x_{31} \downarrow$$

### Pasul 14



$$\Theta = \min \{ (3,3); (2,1) \} = (3,3) \Rightarrow x_{33} \rightarrow$$

$$\Theta = 5$$

### Pasul 15

	3	1	2	
*		10	*	10
15	2	4	1	25
*		*	10	
5	0	0	0	5
			*	
20	10	10		

! verificare!

$$\bar{x}_3 = (0, 10, 0, 15, 0, 10, 5, 0, 0) \in \mathbb{R}^9 - \text{s.b. Neg.},$$

$$f(\bar{x}_3) = 10 + 30 + 10 = 50 \text{ (u.m.)} \leq f(\bar{x}_2)? \underline{\Delta a}$$

$$\Delta_{11} = -3 + 1 - 0 + 0 = -2$$

$$\Delta_{12} = -2 + 1 - 0 + 0 = -1$$

$$\Delta_{22} = -4 + 0 - 0 + 2 = -2$$

$$\Delta_{33} = -0 + 0 - 2 + 1 = -1$$

$\Rightarrow \text{toti } \Delta_{ij} \leq 0 \Rightarrow \bar{x}_3 \text{ este s.o. unică.}$

Concluzia pt. P.T.E.:  $\left\{ \begin{array}{l} \text{echilibrat} \\ x_{\text{optim}} = (0, 10, 0, 15, 0, 10, 5, 0, 0) \\ \min f = 50 \text{ (u.m.)} \end{array} \right.$

Concluzia pt. P.T.N.:  $\left\{ \begin{array}{l} \text{neechilibrat} \\ x_{\text{optim}} = (0, 10, 0, 15, 0, 10) \\ \min f = 50 \text{ (u.m.)} \end{array} \right.$

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# Probleme de transport meechilibrade

## 2) metoda costului minimu

	$C_1$	$C_2$	
$D_1$	3	2	10
$D_2$	1	4	20
$D_3$	2	1	20
	10	25	

### Pasul 0

oferta:  $\sum_{i=1}^3 a_i = 10 + 20 + 20 = 50$

cereea:  $\sum_{j=1}^2 b_j = 10 + 25 = 35$

$\Rightarrow \text{oferta} > \text{cereea} \Rightarrow \text{P.T.N.}$

• Echilibrăm P.T.N.

$\rightarrow$  transformăm P.T.N. în P.T.E. prin introducerea unui nou centru de desfacere fictiv,  $C_3^f$ , ce va avea costurile de transport egale cu zero.

$$50 - 35 = 15 (C_3^f)$$



	$C_1$	$C_2$	$C_3^f$	
$D_1$	<div><div>*</div><div>3</div></div>	<div><div>*</div><div>2</div></div>	<div><div>10</div><div>0</div></div>	<del>10</del> 0
$D_2$	<div><div>10</div><div>1</div></div>	<div><div>5</div><div>4</div></div>	<div><div>5</div><div>0</div></div>	<del>20</del> 15 50
$D_3$	<div><div>*</div><div>2</div></div>	<div><div>20</div><div>1</div></div>	<div><div>*</div><div>0</div></div>	<del>20</del> 0
	<del>10</del> 0	<del>25</del> 50	<del>15</del> 90	! verificare !

Pasul 1

$$\bar{x}_0 = (0, 0, 10, 10, 5, 5, 0, 20, 0) \in \mathbb{R}^9 - \text{s.b. Ned.}$$

$$f(\bar{x}_0) = 10 \cdot 0 + 10 \cdot 1 + 5 \cdot 4 + 5 \cdot 0 + 20 \cdot 1 = 50 \text{ (u.m.)}$$

Pasul 2

$$\delta_{11} = -3 + 1 - 0 + 0 = -2$$

$$\delta_{12} = -2 + 4 - 0 + 0 = 2 > 0$$

$$\delta_{31} = -2 + 1 - 4 + 1 = -4$$

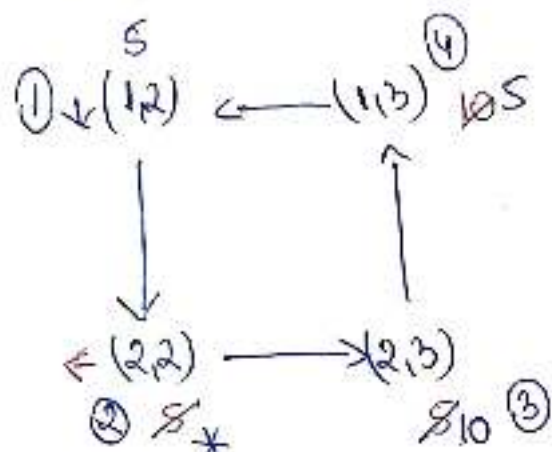
$$\delta_{33} = -0 + 1 - 4 + 0 = -3$$

$\Rightarrow \exists \delta_{ij} > 0 \Rightarrow \bar{x}_0$  nu este optimă

Passo 3

$$SKE = \max\{f_{12}\} = f_{12} \Rightarrow x_{12} \downarrow$$

Passo 4



$$\Theta = \min\{(2,2); (1,3)\} = (2,2) \Rightarrow x_{22} \rightarrow$$

$\Theta = 5$

Passo 5

	$C_1$	$C_2$	$C_3^f$	
$D_1$	<div><div></div><div>3</div><div>*</div></div>	<div><div></div><div>2</div><div>5</div></div>	<div><div></div><div>0</div><div>5</div></div>	10
$D_2$	<div><div></div><div>1</div><div>10</div></div>	<div><div></div><div>4</div><div>*</div></div>	<div><div></div><div>0</div><div>10</div></div>	20
$D_3$	<div><div></div><div>2</div><div>*</div></div>	<div><div></div><div>1</div><div>20</div></div>	<div><div></div><div>0</div><div>*</div></div>	20
	10	25	15	

! verificare!

### Posul 6

$$\bar{x}_1 = (0, 5, 5, 10, 0, 10, 0, 20, 0) \in \mathbb{R}^9 - \text{s. b. Ned.}$$

$$f(\bar{x}_1) = 10 + 10 + 20 = 40 (\mu.m) \leq f(\bar{x}_0)? \underline{\text{Da}}$$

### Posul 7

$$J_{11} = -3 + 1 - 0 + 0 = -2$$

$$J_{22} = -4 + 2 - 0 + 0 = -2$$

$$J_{33} = -2 + 1 - 0 + 0 - 2 + 1 = -2$$

$$J_{38} = -0 + 1 - 2 + 0 = -1$$

$\Rightarrow$  totu  $J_{ij} \leq 0 \Rightarrow \bar{x}_1$  este s.o. unică.

$$\text{Concluzia pt. P.T.E.} \left\{ \begin{array}{l} \text{echilibrat} \\ x_{\text{optim}} = (0, 5, 5, 10, 0, 10, 0, 20, 0) \\ \min f = 40 (\mu.m.) \end{array} \right.$$

$$\text{Concluzia pt. P.T.N.} \left\{ \begin{array}{l} \text{neechilibrat} \\ x_{\text{optim}} = (\underline{0, 5, 5, 10, 0, 10}) \\ \min f = 40 (\mu.m.) \end{array} \right. \quad \begin{array}{l} (0, 5, 10, 0, 0, 20) \end{array}$$