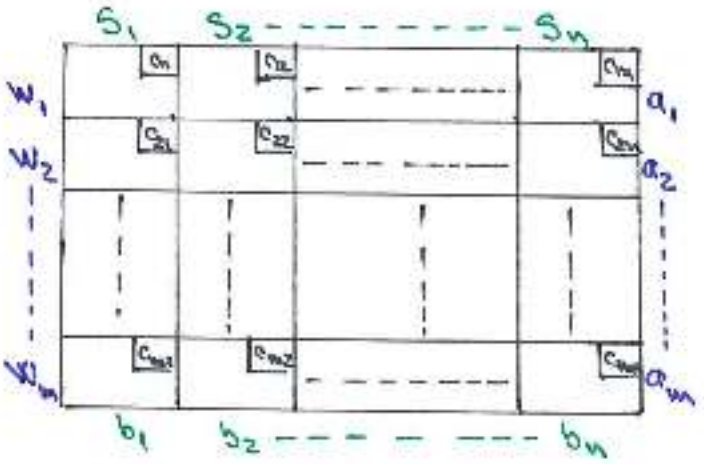
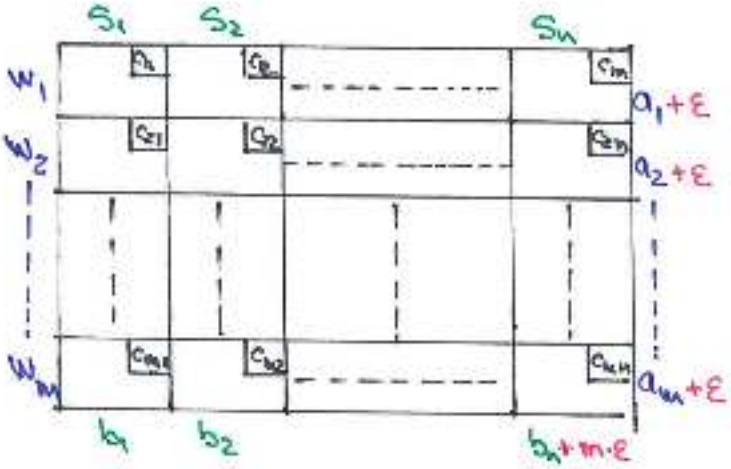
Example:

a) non-perturbate ETP ⟹ a') perturbate ETP



$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

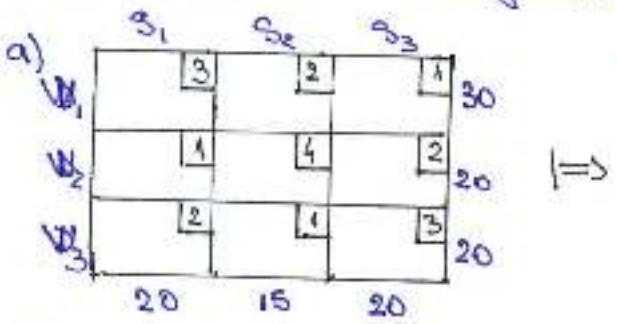


$\sum_{i=1}^{m}(a_i+\varepsilon) \equiv \sum_{i=1}^{m} a_i + m\varepsilon = \sum_{j=1}^{n} b_j + m\varepsilon$ #

Examples:

Solve the next TP. if the second BA. solution which we will obtain it isn't the optimal solution don't continue the algorithm:

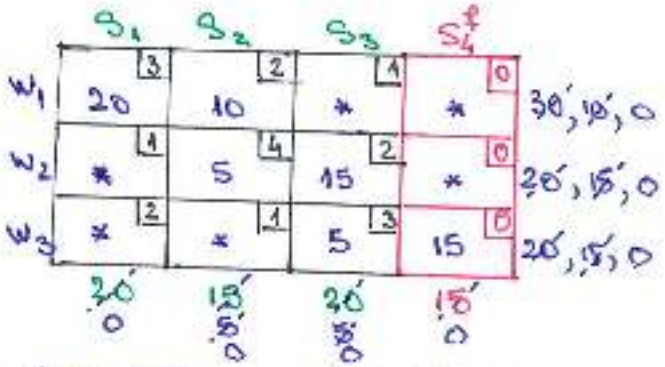
a)



Because, $\sum_{i=1}^{3} a_i = 70 > \sum_{j=1}^{3} b_j = 55$

we have a NeTP, so we must to equilibrate.

①



For ETP we applied the diagonal method to find the iBAS: $\bar{x}_0$ { $\bar{x}_{11}=20$; $\bar{x}_{12}=10$; $\bar{x}_{22}=5$; $\bar{x}_{23}=15$; $\bar{x}_{33}=5$; $\bar{x}_{34}=15$; the other $\bar{x}_{ij}=*, *_{ij}=0$

$f(\bar{x}_0) = 145$

② We check if $\bar{x}_0$ it's optimal solution:

$\delta_{13} = -1+2-4+2 = -1$

$\delta_{14} = -0+0-3+2-4+2 = -3$

$\delta_{21} = -1+3-2+4 = 4 > 0$

$\delta_{24} = -0+0-3+2 = -1$

$\delta_{31} = -2+3-2+4-2+3 = 4 > 0$

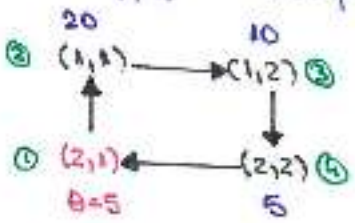
$\delta_{32} = -1+4-2+3 = 4 > 0$

$\Rightarrow (\exists)\delta_{ij} > 0 \Rightarrow \bar{x}_0$ it isn't optimal solution !!

∃ choose $\delta_{21,11}$

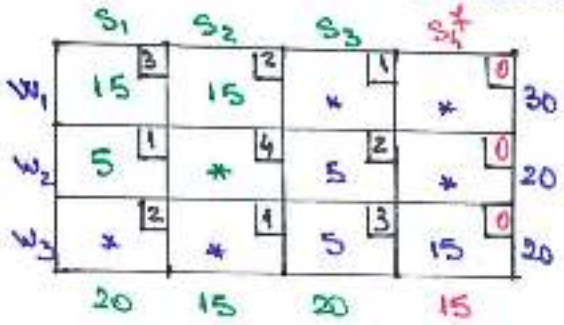
③ We apply the simplex criteria: $\delta_{ij} = \max\{\delta_{ke} > 0\} = \max\{\underbrace{\delta_{21}}_{=4}, \underbrace{\delta_{31}}_{=4}, \underbrace{\delta_{32}}_{=4}\} \overset{(!)}{=} \delta_{21} \Rightarrow x_{21}$ ↓

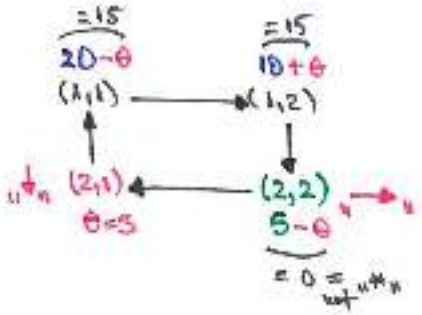
④ We apply the „output criteria":



$\theta = \min\{x_{ij} / x_{ij} \text{ been into even no. cells}\} = \min\{\underbrace{x_{11}}_{=20}, \underbrace{x_{22}}_{=5}\} = 5 \Rightarrow x_{22} \to$

⑤ We determine the new solution: $\bar{X}_1$





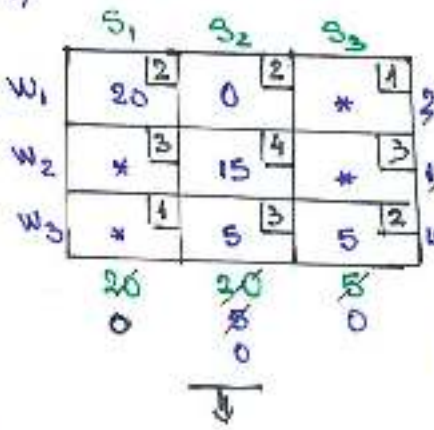
$\bar{X}_1: \begin{cases} \bar{x}_{11}=15,\ \bar{x}_{12}=15,\ \bar{x}_{21}=5,\ \bar{x}_{23}=5,\ \bar{x}_{33}=5,\ \bar{x}_{34}=15 \to \text{basic components} \\ \bar{x}_{13}=\bar{x}_{14}=\bar{x}_{22}=\bar{x}_{24}=\bar{x}_{31}=\bar{x}_{32}=0\ (=\text{"*"}) - \text{non-basic (free) components} \end{cases}$

$f(\bar{X}_1) = 115\ (< f(\bar{X}_0))$

② We check if $\bar{X}_1$ it is a optimal solution:

$\bar{\delta}_{13} = -1+2-1+3 = 3 \geq 0 \Rightarrow (\forall)\bar{\delta}_{ij} \geq 0 \Leftrightarrow \bar{X}_1$ - it isn't optimal solution $\Rightarrow$ the algorithm must to continue ... !!

b)



$\left(\sum a_i = 45 = \sum b_j\right)$

$\Rightarrow \bar{X}_0: \begin{cases} \bar{x}_{11}=20;\ \bar{x}_{12}=0;\ \bar{x}_{22}=15,\ \bar{x}_{32}=5;\ \bar{x}_{34}=5 - \text{basic components} \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0\ (=\text{"*"}) \Rightarrow \text{non-basic components} \end{cases}$ (number = 3+3-1 = 5)

$f(\bar{X}_0) = 125$

↓

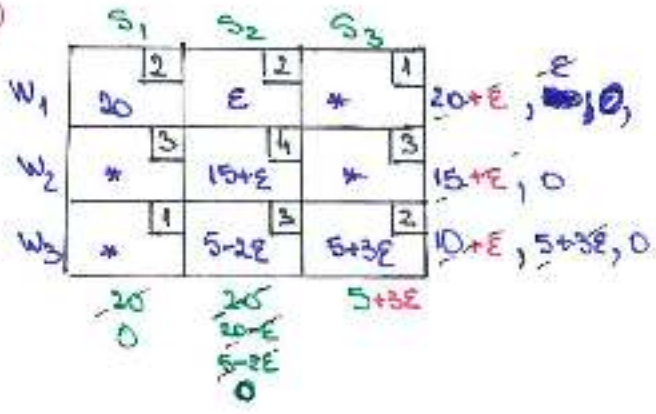
it's degenerate BAS(!!!)

(no, it's possible to appear the cycle phenom.)

!!

we apply the perturbation method

①



$\Rightarrow \bar{X}_0 = \begin{cases} \bar{x}_{11}=20;\ \bar{x}_{12}=\varepsilon;\ \bar{x}_{22}=15+\varepsilon;\ \bar{x}_{32}=5-2\varepsilon;\ \bar{x}_{33}=5+3\varepsilon \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{31}=0\ (=\text{"*"}) - \text{non-basic components} \end{cases}$ ← basic components

$f(\bar{X}_0) = 125 + 6\varepsilon$

↓

non-degenerate BAS(!!!)

② We apply the „optimal criteria":

$\delta_{13} = -1+2-3+2 = 0$

$\underline{\delta_{21}} = -3+2-2+4 = \underline{1>0}$

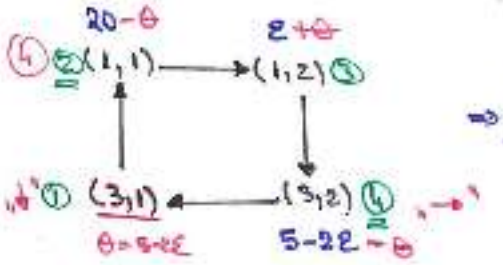
$\delta_{23} = -3+2-3+4 = 0$

$\underline{\delta_{31}} = -1+2-2+3 = \underline{2>0}$

$\Rightarrow (\exists)\delta_{ij}>0 \Rightarrow \bar{X}_0$ it isn't the optimal solution

③ $\delta_{ij}\max\{\delta_{ke}>0\} = \max\{\underbrace{\delta_{21}}_{=1}, \underbrace{\delta_{31}}_{=2}\} = \delta_{31} \Rightarrow$ „x_{31}↑" (becomes a basic component)

④



$\Rightarrow \underline{\theta} = \min\{\underbrace{x_{11}}_{=20}, \underbrace{x_{32}}_{=5-2\varepsilon}\} = \underline{5-2\varepsilon} \Rightarrow$ „x_{32}→" (becomes a non-basic variable)

⑤

	S_1	S_2	S_3	
W_1	2: $15+2\varepsilon$	2: $5-\varepsilon$	1: *	$20+\varepsilon$
W_2	3: *	4: $15+\varepsilon$	3: *	$15+\varepsilon$
W_3	1: $5-2\varepsilon$	3: *	2: $5+3\varepsilon$	$10+\varepsilon$
	20	20	$5+3\varepsilon$	

$\Rightarrow \bar{X}_1 \begin{cases} \bar{x}_{11}=15+2\varepsilon;\ \bar{x}_{12}=5-\varepsilon;\ \bar{x}_{22}=15+\varepsilon;\ \bar{x}_{31}=5-2\varepsilon;\ \bar{x}_{33}=5+3\varepsilon \\ \bar{x}_{13}=\bar{x}_{21}=\bar{x}_{23}=\bar{x}_{32}=0\ (=\text{„}*\text{"}) \end{cases}$

$\underline{f(\bar{X}_1) = 115+10\varepsilon}\ (< f(\bar{X}_0))$

We repeat the steps of the algorithm:

② $\begin{cases} \delta_{13} = -1+2-1+2 = 2>0 \Rightarrow (\exists)\delta_{ij}>0 \Rightarrow \bar{X}_1 \text{ - it isn't the optimal solution} \Rightarrow \text{the algorithm continues...} \\ ! \end{cases}$