Rezolvarea problemelor de programare liniară (PPL) cu 2 necunoscute, cu metoda grafică

$$(PPL)\begin{cases} (1)\ \min f(x_1,x_2) = x_1 + \frac{1}{2}x_2 \\ (2)\begin{cases} x_1+4x_2 \geq 4 & (R_1) \\ 2x_1-3x_2 \leq 6 & (R_2) \\ x_1-x_2 \geq -3 & (R_3) \\ x_1+x_2 \leq 6 & (R_4) \end{cases} \\ (3)\ x_1,x_2 \geq 0 \end{cases}$$

Pas 1 : $S = ?$ (mulț. soluțiilor generale a PPL => mulț. soluțiilor sist. (2) dime.

(R_1): $x_1+4x_2 \geq 4 \Rightarrow (\Delta_1): x_1+4x_2=4 \Rightarrow \begin{cases} x_1=0 \xRightarrow{(\Delta_1)} 0+4\cdot x_2=4 \Rightarrow x_2=1 \Rightarrow P_1(0,1) \\ x_2=0 \xRightarrow{(\Delta_1)} x_1+4\cdot 0=4 \Rightarrow x_1=4 \Rightarrow P_2(4,0) \end{cases}$

$O(0,0) \xRightarrow{(R_1)} 0+4\cdot 0 \geq 4 \Rightarrow 0 \geq 4$ (F) => Semiplanul ce conține originea este semiplanul fără soluții pentru (R_1).

(R_2): $2x_1-3x_2 \leq 6 \Rightarrow (\Delta_2): 2x_1-3x_2=6 \Rightarrow \begin{cases} x_1=0 \xRightarrow{(\Delta_2)} 2\cdot 0-3x_2=6 \Rightarrow x_2=-2 \Rightarrow P_3(0,-2) \\ x_2=0 \xRightarrow{(\Delta_2)} 2x_1-3\cdot 0=6 \Rightarrow x_1=3 \Rightarrow P_4(3,0) \end{cases}$

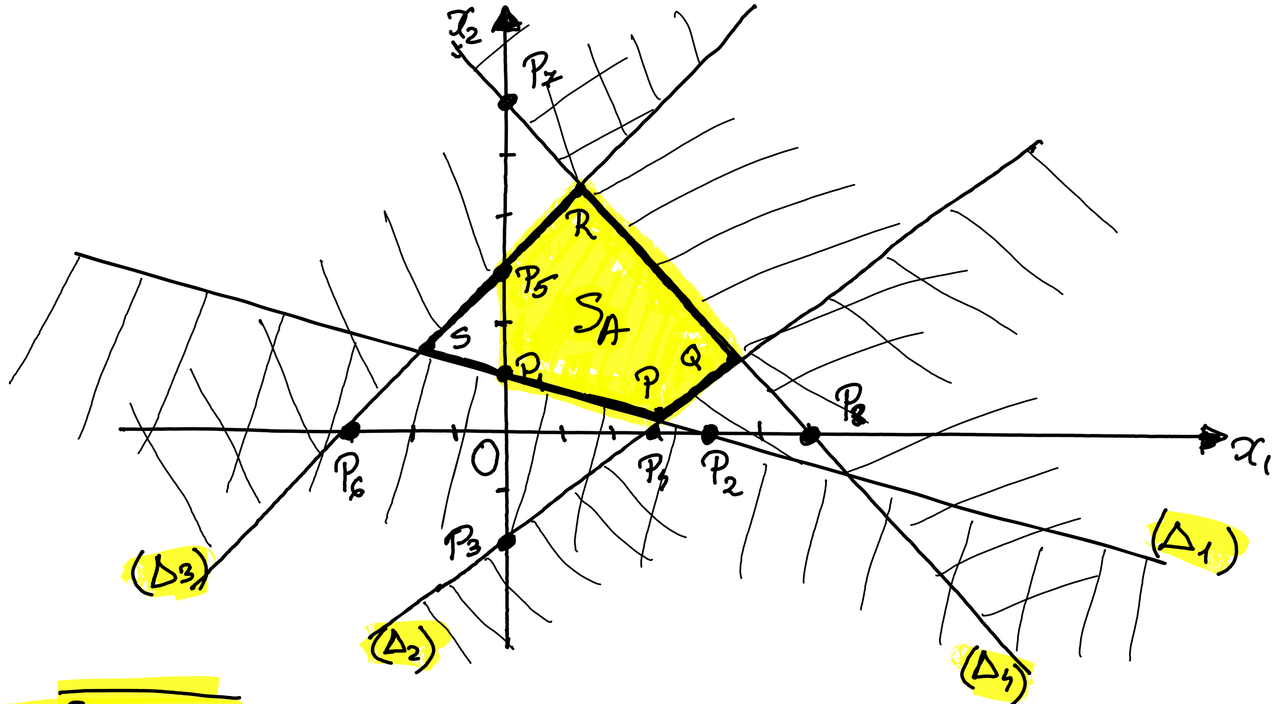
$O(0,0) \xRightarrow{(R_2)} 2\cdot 0-3\cdot 0 \leq 6 \Rightarrow 0 \leq 6$ (A) => Semiplanul ce conține originea este semiplanul cu soluțiile pentru (R_2).

(R_3): $x_1-x_2 \geq -3 \Rightarrow (\Delta_3): x_1-x_2=-3 \Rightarrow \begin{cases} x_1=0 \Rightarrow 0-x_2=-3 \Rightarrow x_2=3 \Rightarrow P_5(0,3) \\ x_2=0 \Rightarrow x_1=-3 \Rightarrow P_6(-3,0) \end{cases}$

$O(0,0) \xRightarrow{(R_3)} 0-0 \geq -3 \Rightarrow 0 \geq -3$ (A)

(R_4): $x_1+x_2 \leq 6 \Rightarrow (\Delta_4): x_1+x_2=6 \Rightarrow \begin{cases} x_1=0 \Rightarrow x_2=6 \Rightarrow P_7(0,6) \\ x_2=0 \Rightarrow x_1=6 \Rightarrow P_8(6,0) \end{cases}$

$O(0,0) \xRightarrow{(R_4)} 0+0 \leq 6 \Rightarrow 0 \leq 6$ (A)



$S = \overline{[PQRS]}$

Pas 2 : $S_A = ?$ (mț. soluțiilor admisibile a PPL)

$S_A = S \cap \{\text{cadranul I}\} = \overline{[PQRP_5P_1]}$

Pas 3 : $S_O = ?$ (mț. soluțiilor optime a PPL)

S_{AB} = mț. soluțiilor de bază admisibile a PPL

$S_{AB} = \{\text{vf. mț. } S_A\} = \{P, Q, R, P_5, P_1\}$

$$\boxed{(1)\ \min f(x_1, x_2) = x_1 + \frac{1}{2} x_2}$$

- $P_1(0,1) \Rightarrow f(P_1) = f(0,1) = 0 + \frac{1}{2}\cdot 1 = \frac{1}{2} = 0{,}5$ ✓
- $P_5(0,3) \Rightarrow f(P_5) = f(0,3) = 0 + \frac{1}{2}\cdot 3 = \frac{3}{2} = 1{,}5$
- $P = (\Delta_1) \cap (\Delta_2)$: $\begin{cases} x_1 + 4x_2 = 4 \quad /\cdot 2 \\ 2x_1 - 3x_2 = 6 \end{cases} \Rightarrow \begin{cases} 2x_1 + 8x_2 = 8 \\ 2x_1 - 3x_2 = 6 \end{cases}$

$(-)\ \ /\ 11x_2 = 2 \Rightarrow x_2 = \frac{2}{11}$ $\Big| \Rightarrow x_1 = 4 - 4\cdot\frac{2}{11} = \frac{36}{11}$ ($\overset{11}{4}$)

$x_1 + 4x_2 = 4$

$\Rightarrow P\left(\frac{36}{11}, \frac{2}{11}\right) \Rightarrow f(P) = f\left(\frac{36}{11}, \frac{2}{11}\right) = \frac{36}{11} + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{11} = \frac{37}{11} \simeq 3{,}3$

- $Q = (\Delta_2) \cap (\Delta_4)$: $\begin{cases} 2x_1 - 3x_2 = 6 \\ x_1 + x_2 = 6 \quad /\cdot 2 \end{cases} \Rightarrow \begin{cases} 2x_1 - 3x_2 = 6 \\ 2x_1 + 2x_2 = 12 \end{cases}$

$(-)\ \ /\ -5x_2 = -6 \Rightarrow x_2 = \frac{6}{5}$ $\Big| \Rightarrow x_1 = 6 - \frac{6}{5} = \frac{24}{5}$ ($\overset{5/}{6}$)

$x_1 + x_2 = 6$

$\Rightarrow Q\left(\frac{24}{5}, \frac{6}{5}\right) \Rightarrow f(Q) = f\left(\frac{24}{5}, \frac{6}{5}\right) = \frac{24}{5} + \frac{1}{\cancel{2}_1}\cdot\frac{\overset{3}{\cancel{6}}}{5} = \frac{27}{5} = 5{,}4$

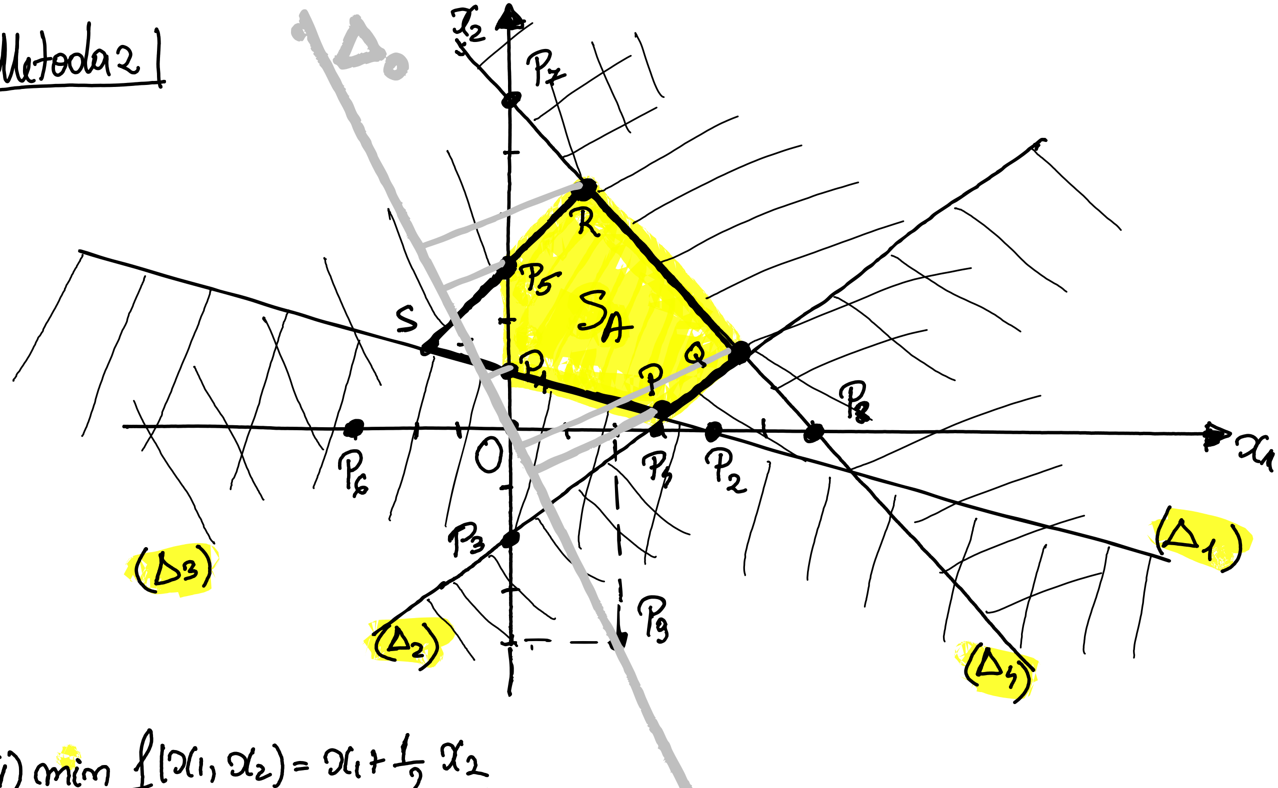
- $R = (\Delta_3) \cap (\Delta_4)$: $\begin{cases} x_1 - x_2 = -3 \\ x_1 + x_2 = 6 \end{cases}$ $\Big> (+) \Rightarrow 2x_1 = 3 \Rightarrow x_1 = \frac{3}{2}$ $\Big| \Rightarrow$

$(-)\ \ /\ -2x_2 = -9 \Rightarrow x_2 = \frac{9}{2}$

$\Rightarrow R\left(\frac{3}{2}, \frac{9}{2}\right) \Rightarrow f(R) = f\left(\frac{3}{2}, \frac{9}{2}\right) = \frac{^{2/}3}{2} + \frac{1}{2}\cdot\frac{9}{2} = \frac{15}{4} = 3{,}75$

$S_O = \{P_1\}$ cu $\begin{cases} x_1^{optim} = 0 \\ x_2^{optim} = 1 \end{cases}$, iar $\min f(x_1, x_2) = 0{,}5$

Metoda 2



(1) $\min f(x_1, x_2) = x_1 + \frac{1}{2} x_2$

$(\Delta_0): x_1 + \frac{1}{2} x_2 = 0 : \begin{cases} x_1 = 0 \overset{\Delta_0}{\Longrightarrow} 0 + \frac{1}{2} \cdot x_2 = 0 \Rightarrow x_2 = 0 \Rightarrow O(0,0) \\ x_1 = 2 \overset{\Delta_0}{\Longrightarrow} 2 + \frac{1}{2} \cdot x_2 = 0 \Rightarrow \frac{1}{2} x_2 = -2 \Rightarrow \end{cases}$

$\Rightarrow x_2 = -2 : \frac{1}{2} = -2 \cdot \frac{2}{1} = -4 \Rightarrow P_9(2,-4)$

$$\boxed{S_O = \{P_1\} \text{ cu } \begin{cases} x_1^{optim} = 0 \\ x_2^{optim} = 1 \end{cases}, \min f(x_1, x_2) = 0{,}5}$$