

①

Probleme de transport neechilibrate

1) Metoda diagonalei

	C_1	C_2	C_3	
D_1	<div>3</div>	<div>1</div>	<div>2</div>	10
D_2	<div>2</div>	<div>4</div>	<div>1</div>	25
	20	10	10	

Pasul 0

oferta: $\sum_{i=1}^2 a_i = 10 + 25 = 35$

cereea: $\sum_{j=1}^3 b_j = 20 + 10 + 10 = 40$

$\Rightarrow \text{oferta} < \text{cereea} \Rightarrow$
 $\Rightarrow \text{P.T.N.}$

• Echilibrăm P.T.N.

\rightarrow transformăm P.T.N. în P.T.E. prin introducerea unui nou depozit de desfacere, D_3^f , ce va avea costurile de transport egale cu zero.

$$40 - 35 = 5 \quad (D_3^f)$$

	C_1	C_2	C_3	
D_1	10	*	*	10 0
D_2	10	10	5	25 15 5 0
D_3	*	*	5	50
	20 10 0	10 0	10 50	

! verificare!

$$v_p = 5$$

$$v.s. = 4$$

Pasul 1

$$\bar{x}_0 = (10, 0, 0, 10, 10, 5, 0, 0, 5) \in \mathbb{R}^9 - \text{s.b. Ned.}$$

$$f(\bar{x}_0) = 10 \cdot 3 + 10 \cdot 2 + 10 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 95 (\text{u.m.})$$

Pasul 2

$$\Delta_{12} = -1 + 4 - 2 + 3 = 4 > 0$$

$$\Delta_{13} = -2 + 1 - 2 + 3 = 0$$

$$\Delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$$

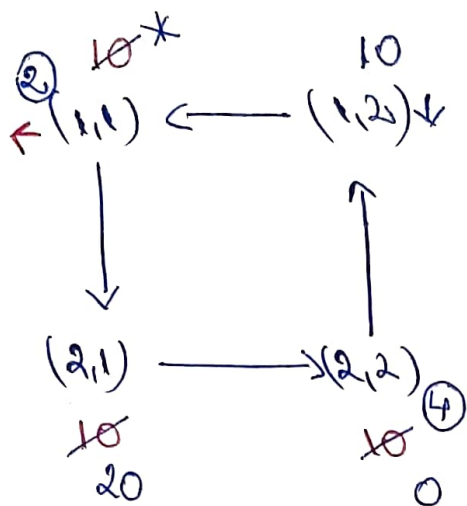
$$\Delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$$

$\nexists \Delta_{ij} > 0 \Rightarrow \bar{x}_0$ Nu este optimă

Pasul 3

$$S_{ke} = \max\{f_{12}; f_{31}; f_{32}\} = f_{12} \Rightarrow x_{12} \downarrow$$

Pasul 4



$$\Theta = \min\{(1,1); (2,2)\} = (1,1) \Rightarrow x_{11} \rightarrow$$

$$\Theta = 10$$

Pasul 5

	3	1	2	
*	10	*	10	
2	4	1	25	
20	0	5		
0	0	0	5	
*	*	5		
20	10	10		

! verificare!

Pasul 6

$$\bar{x}_1 = (0, 10, 0, 20, 0, 5, 0, 0, 5) \in R^9 - \text{s. B. Deg}$$

$$f(\bar{x}_1) = 10 \cdot 1 + 20 \cdot 2 + 0 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 55 \text{ (u.m.)} \leq f(\bar{x}_0)? \text{ Da}$$

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Pasul 7

$$\delta_{11} = -3 + 1 - 4 + 2 = -4$$

$$\delta_{13} = -2 + 1 - 4 + 1 = -4$$

$$\delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$$

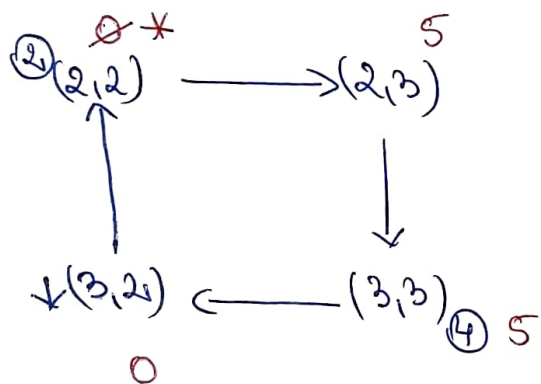
$$\delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$$

$\Rightarrow \exists \delta_{ij} > 0 \Rightarrow \bar{x}_1$ Nu este optimă.

Pasul 8

$$Ske = \max \{ \delta_{31}, \delta_{32} \} = \delta_{32} \Rightarrow x_{32} \downarrow$$

Pasul 9



$$\Theta = \min \{ (2,2), (3,3) \} = (2,2) \Rightarrow (2,2) = x_{22} \rightarrow$$

$$\Theta = 0$$

Pasul 10

	3		1		2	
*		10		*		10
	2		4		1	
20		*		5		25
	0		0		0	
*		0		5		5
	20		10		10	

! verificare!

Pasul 11

$$\bar{x}_2 = (0, 10, 0, 20, 0, 5, 0, 0, 5) \in \mathbb{R}^9 \text{ s.t. Deg.}$$

$$f(\bar{x}_2) = 10 \cdot 1 + 20 \cdot 2 + 5 \cdot 1 + 0 \cdot 5 \cdot 0 = 55 (\text{u.m.}) \leq f(\bar{x}_1)? \underline{\text{Da}}$$

Pasul 12

$$\Delta_{11} = -3 + 1 - 0 + 0 - 1 + 2 = -1$$

$$\Delta_{13} = -2 + 1 - 0 + 0 = -1$$

$$\Delta_{22} = -4 + 0 - 0 + 1 = -3$$

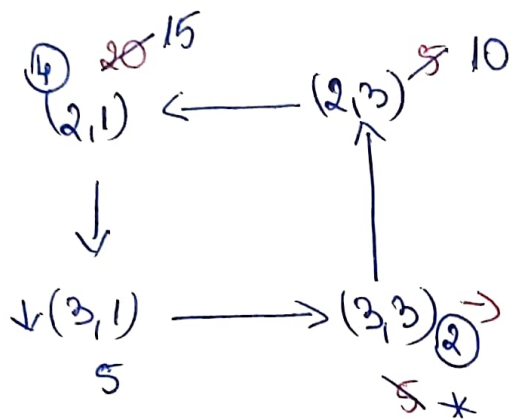
$$\Delta_{31} = -0 + 0 - 1 + 2 = 1 > 0$$

$\Rightarrow \exists \Delta_{ij} > 0 \Rightarrow \bar{x}_2$ este optimă.

Pasul 13

$$S_{ke} = \max \{ f_{31} \} = f_{31} \Rightarrow x_{31} \downarrow$$

Pasul 14



$$\Theta = \min \{ (3,3); (2,1) \} = (3,3) \Rightarrow x_{33} \rightarrow$$
$$\Theta = 5$$

Pasul 15

	3	1	2	
*		10	*	10
15	2	4	1	25
5	0	0	0	5
20	10	10		

! verificare!

$$\bar{x}_3 = (0, 10, 0, 15, 0, 10, 5, 0, 0) \in \mathbb{R}^9 - \text{s.b. Deg},$$

$$f(\bar{x}_3) = 10 + 30 + 10 = 50 \text{ (u.m.)} \leq f(\bar{x}_2)? \underline{\Delta a}$$

$$\Delta_{11} = -3 + 1 - 0 + 0 = -2$$

$$\Delta_{13} = -2 + 1 - 0 + 0 = -1$$

$$\Delta_{22} = -4 + 0 - 0 + 2 = -2$$

$$\Delta_{33} = -0 + 0 - 2 + 1 = -1$$

$\Rightarrow \text{toti } \Delta_{ij} \leq 0 \Rightarrow \bar{x}_3 \text{ este}$
 $\Rightarrow \text{s.o. unică.}$

Concluzia pt. P.T.E.: $\left\{ \begin{array}{l} \text{echilibrat} \\ x_{\text{optim}} = (0, 10, 0, 15, 0, 10, 5, 0, 0) \\ \min f = 50 \text{ (u.m.)} \end{array} \right.$

Concluzia pt. P.T.N.: $\left\{ \begin{array}{l} \text{neechilibrat} \\ x_{\text{optim}} = (0, 10, 0, 15, 0, 10) \\ \min f = 50 \text{ (u.m.)} \end{array} \right.$

②

Probleme de transport meechilibrate

2) Metoda costului minim

	C_1	C_2	
D_1	3	2	10
D_2	1	4	20
D_3	2	1	20
	10	25	

Pasul 0

oferta: $\sum_{i=1}^3 a_i = 10 + 20 + 20 = 50$

cerea: $\sum_{j=1}^2 b_j = 10 + 25 = 35$

\Rightarrow oferta > cerea \Rightarrow P.T.N.

• Echilibrăm P.T.N.

\rightarrow transformăm P.T.N. în P.T.E. prin introducerea unui nou centru de desfacere fictiv, C_3^f , ce va avea costurile de transport egale cu zero.

$$50 - 35 = 15 (C_3^f)$$

	C_1	C_2	C_3^f	
D_1	<div>$*$<div>3</div></div>	<div>$*$<div>2</div></div>	<div>10<div>0</div></div>	10 0
D_2	<div>10<div>1</div></div>	<div>5<div>4</div></div>	<div>5<div>0</div></div>	20 15 50
D_3	<div>$*$<div>2</div></div>	<div>20<div>1</div></div>	<div>$*$<div>0</div></div>	20 0
	10 0	25 50	15 90	

! verificare !

Pasul 1

$$\bar{x}_0 = (0, 0, 10, 10, 5, 5, 0, 20, 0) \in \mathbb{R}^9 - \text{s.b. Ned.}$$

$$f(\bar{x}_0) = 10 \cdot 0 + 10 \cdot 1 + 5 \cdot 4 + 5 \cdot 0 + 20 \cdot 1 = 50 \text{ (u.m.)}$$

Pasul 2

$$\delta_{11} = -3 + 1 - 0 + 0 = -2$$

$$\delta_{12} = -2 + 4 - 0 + 0 = 2 > 0$$

$$\delta_{31} = -2 + 1 - 4 + 1 = -4$$

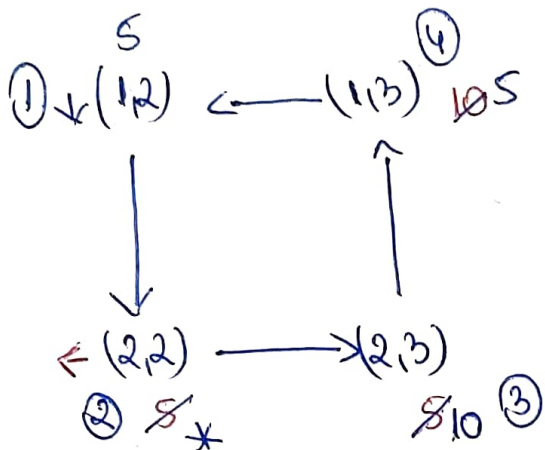
$$\delta_{33} = -0 + 1 - 4 + 0 = -3$$

$\Rightarrow \exists \delta_{ij} > 0 \Rightarrow \bar{x}_0$ nu este optimă

Pasul 3

$$SKE = \max \{f_{12}\} = f_{12} \Rightarrow x_{12} \downarrow$$

Pasul 4



$$\Theta = \min \{(2,2); (1,3)\} = (2,2) \Rightarrow x_{22} \rightarrow$$

$$\Theta = 5$$

Pasul 5

	C_1	C_2	C_3^f	
D_1	<div><div></div><div>3</div></div> *	<div><div></div><div>2</div></div> 5	<div><div></div><div>0</div></div> 5	10
D_2	<div><div></div><div>1</div></div> 10	<div><div></div><div>4</div></div> *	<div><div></div><div>0</div></div> 10	20
D_3	<div><div></div><div>2</div></div> *	<div><div></div><div>1</div></div> 20	<div><div></div><div>0</div></div> *	20
	10	25	15	

! verificare!

Pasul 6

$$\bar{x}_1 = (0, 5, 5, 10, 0, 10, 0, 20, 0) \in \mathbb{R}^9 - \text{s.b. Ned.}$$

$$f(\bar{x}_1) = 10 + 10 + 20 = 40 (\mu.m) \leq f(\bar{x}_0)? \underline{\text{Da}}$$

Pasul 7

$$J_{11} = -3 + 1 - 0 + 0 = -2$$

$$J_{22} = -4 + 2 - 0 + 0 = -2$$

$$J_{31} = -2 + 1 - 0 + 0 - 2 + 1 = -2$$

$$J_{33} = -0 + 1 - 2 + 0 = -1$$

$\Rightarrow \text{totu } J_{ij} \geq 0 \Rightarrow \bar{x}_1 \text{ este s.o. unică.}$

Concluzia pt. P.T.E. $\left\{ \begin{array}{l} \text{echilibrat} \\ x_{\text{optim}} = (0, 5, 5, 10, 0, 10, 0, 20, 0) \\ \min f = 40 (\mu.m.) \end{array} \right.$

Concluzia pt. P.T.N. $\left\{ \begin{array}{l} \text{neechilibrat} \\ x_{\text{optim}} = (\cancel{0, 5, 5, 10, 0, 10}) \\ \min f = 40 (\mu.m.) \end{array} \right.$
 $(0, 5, 10, 0, 0, 20)$