

Metoda celor două faze

① (1) $\max f(x_1, x_2, x_3) = x_1 - 2x_2 + 3x_3$

(PPL) (2) $\begin{cases} 2x_1 + x_2 - x_3 \geq 4 \\ -x_1 + 2x_2 + 2x_3 \geq 8 \end{cases}$

(3) $x_1, x_2, x_3 \geq 0$

$\max f = -\min(-f)$

" \leq " \Rightarrow " $+$ "

" \geq " \Rightarrow " $-$ "

(PPL)_S :

(1_S) $\min(-f)(x_1, x_2, x_3, x_4^c, x_5^c) = -x_1 + 2x_2 - 3x_3 + 0 \cdot x_4^c + 0 \cdot x_5^c$

(2_S) $\begin{cases} 2x_1 + x_2 - x_3 - x_4^c = 4 \\ -x_1 + 2x_2 + 2x_3 - x_5^c = 8 \end{cases}$

(3_S) $x_1, x_2, x_3, x_4^c, x_5^c \geq 0$

$\Rightarrow \bar{A}_S = \begin{pmatrix} 2 & 1 & -1 & -1 & 0 & 4 \\ -1 & 2 & 2 & 0 & -1 & 8 \end{pmatrix}$

(PPL)_a :

(1_a) $\min(f_a)(x_6^a, x_7^a) = 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4^c + 0 \cdot x_5^c + x_6^a + x_7^a$

(2_a) $\begin{cases} 2x_1 + x_2 - x_3 - x_4^c + x_6^a = 4 \\ -x_1 + 2x_2 + 2x_3 - x_5^c + x_7^a = 8 \end{cases}$

(3_a) $x_1, x_2, x_3, x_4^c, x_5^c, x_6^a, x_7^a \geq 0$

$\Rightarrow \bar{A}_a = \begin{pmatrix} 2 & 1 & -1 & -1 & 0 & 1 & 0 & 4 \\ -1 & 2 & 2 & 0 & -1 & 0 & 1 & 8 \end{pmatrix}$

V.S = 0 V.P.

$\Rightarrow \bar{X}_0 = (0, 0, 0, 0, 0, 4, 8)^T$ - SBA_{1a}

$\bar{A}_a = \begin{pmatrix} 2 & 1 & -1 & -1 & 0 & 1 & 0 & 4 \\ -1 & 2 & 2 & 0 & -1 & 0 & 1 & 8 \end{pmatrix} \Rightarrow \bar{X}_0 = (0, 0, 0, 0, 0, 8, 4)^T$

V.S = 0

Faza I: Tabelul Simplex pt. (PPL) a: col. lui P_0

		(P ₀)									
		P ₀	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇		
P ₆	1	4	2	1	-1	-1	0	1	0	$\theta = \frac{P_0}{P_2} > 0$	
P ₇	1	8	-1	2	2	0	-1	0	1	$\Rightarrow \bar{X}_0 = (0, 0, 0, 0, 0, 0, 0, 0)^T$	
		$f_0(X_0) = 12$	1	3	1	-1	-1	0	0	$f_0(X_0) = 12$	
		$z_j - c_j$									
P ₂	0	4	2	1	-1	-1	0	1	0	$\Rightarrow \bar{X}_1 = (0, 0, 0, 0, 0, 0, 0, 0)^T$	
P ₇	1	0	-5	0	2	-1	-2	1	1	$\Rightarrow \bar{X}_1 = (0, 0, 0, 0, 0, 0, 0, 0)^T$	
		$f_0(X_1) = 1$	-5	0	4	2	-1	-3	0	$z_j - c_j$	
P ₂	0	$\frac{17}{4}$	$+\frac{3}{4}$	1	0	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\Rightarrow \bar{X}_2 = (0, \frac{17}{4}, \frac{1}{4}, 0, 0, 0, 0, 0)^T$	
P ₃	0	$\frac{1}{4}$	$-\frac{5}{4}$	0	1	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$f_0(X_2) = 0$	
		$f_0(X_2) = 0$	0	0	0	0	0	-1	-1	$z_j - c_j$	

Cl. pt (PPL) a: $X_{\text{optimal}} \equiv \bar{X}_2 = (0, \frac{17}{4}, \frac{1}{4}, 0, 0, 0, 0, 0)^T$
 Soluție optimă unică
 $\min f_0 = 0$

Faza II: Tabelul Simplex pt (PPL) s:

		-1 2 -3 0 0						
		P ₁	P ₂	P ₃	P ₄	P ₅		
P ₂	2	$\frac{3}{4}$	1	0	$-\frac{1}{2}$	$-\frac{1}{4}$	$\Rightarrow \bar{X}_0 = (0, \frac{17}{4}, \frac{1}{4}, 0, 0, 0, 0, 0)^T$	
P ₃	-3	$-\frac{5}{4}$	0	1	$\frac{1}{2}$	$-\frac{1}{4}$	$f_0(X_0) = \frac{31}{4}$	
		$\frac{25}{4}$	0	0	$-\frac{5}{2}$	$\frac{1}{4}$	$z_j - c_j$	

Cl. pt (PPL) s: $S_\emptyset = \emptyset \Rightarrow \min(f) = -\infty \Rightarrow \text{Cl. pt (PPL) i: } S_\emptyset = \emptyset, \max f = +\infty$

$$(1) \max f(x_1, x_2, x_3) = 35x_1 + 55x_2 + 90x_3 \text{ (mil li)}$$

$$35x_1 = 35 \text{ mil li} \cdot x_1 \text{ (bus)}$$

$$\text{~~35000 } x_1 + 5000 x_2 + 90000 x_3 \text{ (li)}~~$$

$$\text{~~350 } x_1 + 550 x_2 + 900 x_3 \text{ (sat li)}~~$$

$$85000x_1 + 125000x_2 + 180000x_3 \leq 200000000 \text{ (li)}$$

$$85x_1 + 125x_2 + 180x_3 \leq 200000 \text{ (mil li)}$$

P.T.E.

	C ₁	C ₂	C ₃	C ₄	
D ₁	10 <u>2</u>	15 <u>4</u>	5 <u>3</u>	* <u>4</u>	30 20 50
D ₂	* <u>4</u>	* <u>2</u>	10 <u>1</u>	* <u>1</u>	100
D ₃	* <u>3</u>	* <u>2</u>	10 <u>2</u>	10 <u>1</u>	20 100
	10 0	15 0	25 20 100	10 0	

met. diag.

! verif !

$$\sum_{i=1}^3 a_i = 30 + 10 + 20 = 60 \text{ (efort)} \quad \Rightarrow \text{P.T.E.}$$

$$\sum_{j=1}^4 b_j = 10 + 15 + 25 + 10 = 60 \text{ (work)} \quad \Rightarrow \text{P.T.E.}$$

$$v.p = m + m - 1 = 3 + 4 - 1 = \underline{6}$$

$$v.s = m \cdot m - (m + m - 1) = 12 - 6 = \underline{6}$$

$$\left(\begin{array}{l} \textcircled{X_0} = (10, 15, 5, 0, 0, 0, 10, 0, 0, 0, 10, 10)^T - \text{SBAIND} \\ f(X_0) = \dots (\text{um}) \end{array} \right.$$

cllet. color 2 faze:

(PPL)

$$\begin{cases} (1) \max f(x_1, x_2, x_3) = 2x_1 - x_2 + x_3 \\ (2) \begin{cases} x_1 + x_2 - x_3 \leq 2 \\ 2x_1 - x_2 + 2x_3 \geq 5 \end{cases} \\ (3) x_1, x_2, x_3 \geq 0 \end{cases}$$

$$\begin{aligned} \leq & \Rightarrow + \\ \geq & \Rightarrow - \end{aligned}$$

$$\max f = -\min(-f)$$

(PPL)_s

$$\begin{cases} (1_s) \min(-f)(x_1, x_2, x_3, x_4^c, x_5^c) = -2x_1 + x_2 - x_3 + 0 \cdot x_4^c + 0 \cdot x_5^c \\ (2_s) \begin{cases} x_1 + x_2 - x_3 + x_4^c = 2 \\ 2x_1 - x_2 + 2x_3 - x_5^c = 5 \end{cases} \\ (3_s) x_1, x_2, x_3, x_4^c, x_5^c \geq 0 \end{cases}$$

$$\Rightarrow \bar{A}_s = \begin{matrix} & P_1 & P_2 & P_3 & P_4^c & P_5^c & P_6 \\ \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 2 \\ 2 & -1 & 2 & 0 & -1 & 5 \end{pmatrix} \end{matrix}$$

(PPL)_a

$$\begin{cases} (1_a) \min f_a(x_6^a) = 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4^c + 0 \cdot x_5^c + 1 \cdot x_6^a \\ (2_a) \begin{cases} x_1 + x_2 - x_3 + x_4^c = 2 \\ 2x_1 - x_2 + 2x_3 - x_5^c + x_6^a = 5 \end{cases} \\ (3_a) x_1, x_2, x_3, x_4^c, x_5^c, x_6^a \geq 0 \end{cases}$$

$$\Rightarrow \bar{A}_a = \begin{matrix} & P_1 & P_2 & P_3 & P_4^c & P_5^c & P_6^a \\ \begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 0 & 2 \\ 2 & -1 & 2 & 0 & -1 & 1 & 5 \end{pmatrix} \end{matrix}$$

V.S=0

v.p.

$$\Rightarrow \bar{x}_0^a = (0, 0, 0, 2, 0, 5) \begin{matrix} T \\ \in R^6 \end{matrix} = SBA_1 a$$

Forma I: Tabelul Simplex pt (PPL)_a:

		(13)	0	0	0	0	0	1		
B	C _B	P ₀	P ₁	P ₂	P ₃	P ₄ ^c	P ₅ ^c	P ₆ ^a	$\theta = \frac{P_0}{P_{j \downarrow}} > 0$	
P ₄ ^c	0	2	1	-1	1	0	0	0	$\frac{2}{1} = 2$	
P ₆ ^a	1	5	2	-1	2	0	-1	1	$\frac{5}{2} = 2.5$	
		$f_a(\bar{x}_0) = 5$	$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$	$z_4 - c_4$	$z_5 - c_5$	$z_6 - c_6$		
P ₁	0	2	1	1	-1	1	0	0		
P ₆ ^a	1	1	0	-3	4	-2	-1	1	$\frac{1}{1} = 1$	
		$f_a(\bar{x}_1) = 1$	$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$	$z_4 - c_4$	$z_5 - c_5$	$z_6 - c_6$		
P ₁	0	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$		
P ₃	0	$\frac{1}{4}$	0	$-\frac{3}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$		
		$f_a(\bar{x}_2) = 0$	$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$	$z_4 - c_4$	$z_5 - c_5$	$z_6 - c_6$		
		0	0	0	0	0	-1	0		

Cl. pt. (PPL)_a: $\begin{cases} X_{\text{optimal}} = \bar{x}_2 = \left(\frac{9}{4}, 0, \frac{1}{4}, 0, 0, 0\right)^T \\ \min f_a = 0 \end{cases}$ S.O. = nu exista

Forma II: Tabelul Simplex pt (PPL)_s:

		(13)	-2	1	-1	0	0		
B	C _B	P ₀	P ₁	P ₂	P ₃	P ₄ ^c	P ₅ ^c	$\theta = \frac{P_0}{P_{j \downarrow}} > 0$	
P ₁	-2	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$		
P ₃	-1	$\frac{1}{4}$	0	$-\frac{3}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$		
		$f(\bar{x}_0) = -\frac{19}{4}$	$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$	$z_4 - c_4$	$z_5 - c_5$		
		0	$-\frac{3}{4}$	0	$-\frac{1}{2}$	$\frac{3}{4}$	0		

Cl. pt (PPL)_s: $\begin{cases} S_0 = \emptyset \\ \min(-f) = -\infty \end{cases}$ \Rightarrow Cl. pt (PPL)_i: $\begin{cases} S_0 = \emptyset \\ \max f = +\infty \end{cases}$
 optim infinit.

$$450 \frac{\text{lei}}{\text{hl}} \cdot x_1 \text{ hl} \neq \dots$$

$$x_1 \geq 150\,000\,000 \text{ (lei)}$$

lei ~~150 (mill lei)~~

$$\left(0,02\% x_1 \right) = \frac{0,02}{100} x_1 = \frac{2}{10000} x_1 = \underline{\underline{0,0002 x_1}}$$

$$0,0002 x_1 + 0,0006 x_2 + 0,0004 x_3 \leq 380\,000 \text{ (lei)}$$

$$\left\{ 2x_1 + 6x_2 + 4x_3 \leq 380\,000\,000\,0 \text{ (lei)} \right\} \cdot 10^4$$

$$380\,000 \text{ lei} = 38\,000\,00 (\% \text{ lei})$$