① $(PPL)\begin{cases}(1)\ \min f(x_1,x_2)=2x_1-x_2\\ (2)\begin{cases}x_1+4x_2\geq 4\\ 2x_1-3x_2\leq 6\\ x_1-x_2\geq -3\\ x_1+x_2\leq 6\end{cases}\\ (3)\ x_1,x_2\geq 0\end{cases}$

<u>pas 1</u> : S = ? (mulțimea soluțiilor generale a PPL)

• $(R_1): x_1+4x_2\geq 4 \Rightarrow (\Delta_1): x_1+4x_2=4 \Rightarrow \begin{cases}x_1=0 \xRightarrow{\Delta_1} 0+4x_2=4 \Rightarrow x_2=1 \Rightarrow\\ \Rightarrow P_1(0,1)\\ x_2=0 \xRightarrow{\Delta_1} x_1+4\cdot 0=4 \Rightarrow x_1=4 \Rightarrow\\ \Rightarrow P_2(4,0)\end{cases}$

$O(0,0) \xRightarrow{R_1} 0+4\cdot 0\geq 4 \Rightarrow 0\geq 4\ \underline{(F)}$

• $(R_2): 2x_1-3x_2\leq 6 \Rightarrow (\Delta_2): 2x_1-3x_2=6 \Rightarrow \begin{cases}x_1=0 \xRightarrow{\Delta_2} 2\cdot 0-3x_2=6 \Rightarrow x_2=-2 \Rightarrow\\ \Rightarrow P_3(0,-2)\\ x_2=0 \xRightarrow[\Delta_2]{} 2x_1-3\cdot 0=6 \Rightarrow x_1=3\\ \Rightarrow P_4(3,0)\end{cases}$

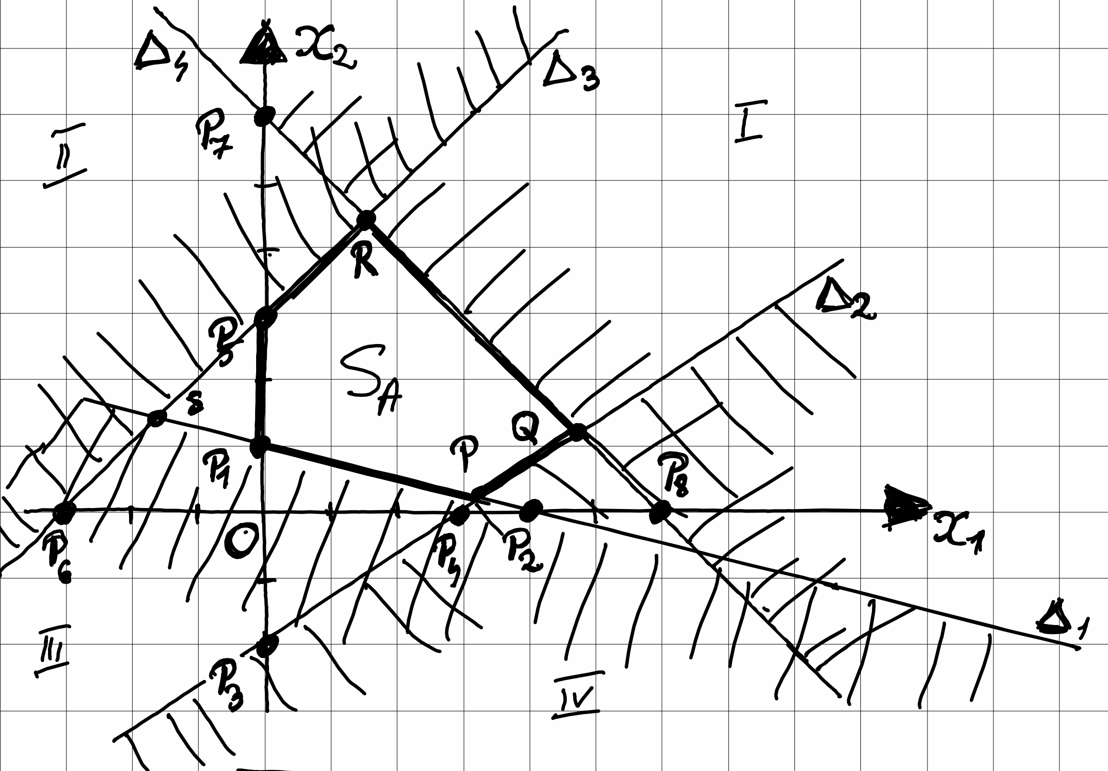
$O(0,0) \xRightarrow{R_2} 2\cdot 0-3\cdot 0\leq 6 \Rightarrow 0\leq 6\ \underline{(A)}$

• $(R_3): x_1-x_2\geq -3 \Rightarrow (\Delta_3): x_1-x_2=-3 \Rightarrow \begin{cases}x_1=0 \xRightarrow{\Delta_3} 0-x_2=-3 \Rightarrow x_2=3 \Rightarrow\\ \Rightarrow P_5(0,3)\\ x_2=0 \xRightarrow{\Delta_3} x_1-0=-3 \Rightarrow x_1=-3\\ \Rightarrow P_6(-3,0)\end{cases}$

$O(0,0) \xRightarrow{R_3} 0-0\geq -3 \Rightarrow 0\geq -3\ \underline{(A)}$

• $(R_4): x_1+x_2\leq 6 \Rightarrow (\Delta_4): x_1+x_2=6 \Rightarrow \begin{cases}x_1=0 \xRightarrow{\Delta_4} 0+x_2=6 \Rightarrow x_2=6 \Rightarrow\\ \Rightarrow P_7(0,6)\\ x_2=0 \xRightarrow[\Delta_4]{} x_1+0=6 \Rightarrow x_1=6 \Rightarrow\\ \Rightarrow P_8(6,0)\end{cases}$

$O(0,0) \xRightarrow{R_4} 0+0\leq 6 \Rightarrow 0\leq 6\ \underline{(A)}$



$S = \overline{[PQRS]}$

pas 2 : $S_A = ?$ (mt. soluțiilor admisibile a PPL)

$S_A = S \cap \{\text{cadranul I}\} = [PQRP_5P_1]$

pas 3 : $S_0 = ?$ (mt. soluțiilor optime a PPL)

S_{AB} = mt. soluțiilor de bază admisibile a PPL

$S_{AB} = \{\text{vf. mt. } S_A\} = \{P, Q, R, P_5, P_1\}$

- $P_1(0,1) \Rightarrow f(P_1) = f(0,1) = 2\cdot 0 - 1 = -1$
- $P_5(0,3) \Rightarrow f(P_5) = f(0,3) = 2\cdot 0 - 3 = -3$!

$\boxed{f(x_1, x_2) = 2x_1 - x_2}$

- $R = (\Delta_3) \cap (\Delta_4)$: $\begin{cases} x_1 - x_2 = -3 \\ x_1 + x_2 = 6 \end{cases}$

$\oplus$ $2x_1 \; / = 3 \Rightarrow x_1 = \frac{3}{2}$

$\ominus$ $/ \; -2x_2 = -9 \Rightarrow x_2 = \frac{9}{2}$

$\Rightarrow R\left(\frac{3}{2}, \frac{9}{2}\right) \Rightarrow$

$\Rightarrow f(R) = f\left(\frac{3}{2}, \frac{9}{2}\right) = 2 \cdot \frac{3}{2} - \frac{9}{2} = -\frac{3}{2}$

- $Q = (\Delta_2) \cap (\Delta_4)$: $\begin{cases} 2x_1 - 3x_2 = 6 \\ x_1 + x_2 = 6 \quad / \cdot (-2) \end{cases} \Rightarrow \begin{cases} 2x_1 - 3x_2 = 6 \\ -2x_1 - 2x_2 = -12 \end{cases}$

$\oplus$ $/ \; -5x_2 = -6 \Rightarrow x_2 = \frac{6}{5}$

$x_1 + \frac{6}{5} = {}^{5/}6 \Rightarrow x_1 = \frac{24}{5}$

$\Rightarrow Q\left(\frac{24}{5}, \frac{6}{5}\right) \Rightarrow f(Q) = f\left(\frac{24}{5}, \frac{6}{5}\right) = \frac{24}{5} \cdot 2 - \frac{6}{5} = \frac{42}{5}$

- $P = (\Delta_1) \cap (\Delta_2)$: $\begin{cases} x_1 + 4x_2 = 4 \; / \cdot (-2) \\ 2x_1 - 3x_2 = 6 \end{cases} \Rightarrow \begin{cases} -2x_1 - 8x_2 = -8 \\ 2x_1 - 3x_2 = 6 \end{cases}$

$\oplus$ $/ \; -11x_2 = -2 \Rightarrow x_2 = \frac{2}{11}$

$x_1 + 4 \cdot \frac{2}{11} = {}^{11/}4 \Rightarrow x_1 = \frac{36}{11}$

$\Rightarrow P\left(\frac{36}{11}, \frac{2}{11}\right) \Rightarrow f(P) = f\left(\frac{36}{11}, \frac{2}{11}\right) = \frac{36}{11} \cdot 2 - \frac{2}{11} = \frac{70}{11}$

$$\boxed{S_0 = \{P_5\} \Rightarrow \begin{cases} x_1^{optim} = 0 \\ x_2^{optim} = 3 \end{cases}, \text{ iar. } \min f(x_1, x_2) = -3}$$

② $(PPL)_g$ i
$$\begin{cases} (1g) & \min f(x_1,x_2,x_3) = 2x_1 - x_2 - 3x_3 \\ (2g) & \begin{cases} x_1+x_2+2x_3 \le 10 \\ 3x_1 - x_2 + 2x_3 \le 8 \end{cases} \\ (3g) & x_1, x_2, x_3 \ge 0 \end{cases}$$

<u>pas 1</u> : Aducem $(PPL)_g$ la forma standard :

$$(PPL)_s \begin{cases} (1_s) & \min f(x_1, x_2, x_3, x_4^c, x_5^c) = 2x_1 - x_2 - 3x_3 + 0\cdot x_4^c + 0\cdot x_5^c \\ (2_s) & \begin{cases} x_1+x_2+2x_3+x_4^c = 10 \\ 3x_1 - x_2 + 2x_3 + x_5^c = 8 \end{cases} \\ (3s) & x_1, x_2, x_3, x_4^c, x_5^c \ge 0 \end{cases}$$

①	„ $\le$ " $\Rightarrow$ „ $+$ "
①	„ $\ge$ " $\Rightarrow$ „ $-$ "

<u>pas 2</u> : Det. o soluție de bază admisibilă inițială $\overline{X_0}$ (= S.B.A.i.) a $\underline{(PPL)_s}$ (ie. a sistemului (2_s)) cu metoda lui Gauss.

v.p $= x_4^c, x_5^c$

$$(2_s) \longrightarrow \overline{A} = \begin{array}{c} \begin{array}{cccccc} P_1 & P_2 & P_3 & P_4^c & P_5^c & P_0 \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 1 & 2 & 1 & 0 & 10 \\ 3 & -1 & 2 & 0 & 1 & 8 \end{array}\right) \\ \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4^c & x_5^c \end{array} \end{array} \equiv \overline{A}_{G-J}^{R} \xrightarrow[V.S=0]{V.S=x_1,x_2,x_3}$$

$$\Rightarrow \overline{X_0} = (0,0,0,\underset{\substack{\ge 0\\ \ne 0}}{10},\underset{\substack{\ge 0\\ \ne 0}}{8})^T - \text{S.B.A.Nd} = \text{S.B.A.i.} \checkmark$$

③

$$(PPL)_i \begin{cases} (1_i)\ \max f(x_1,x_2,x_3) = x_1+x_2+3x_3 \\ (2_i)\ \begin{cases} 2x_1+x_2+2x_3=10 \\ 3x_1+x_3 \geq 6 \end{cases} \\ (3_i)\ x_1,x_2,x_3 \geq 0 \end{cases}$$

$$\boxed{\max(f) = -\min(-f)}$$

pas 1: $(PPL)_s$

$$(PPL)_s \begin{cases} (1_s)\ \min(-f)(x_1,x_2,x_3,x_4^c) = -x_1-x_2-3x_3+0\cdot x_4^c \\ (2_s)\ \begin{cases} 2x_1+x_2+2x_3=10 \\ 3x_1+x_3-x_4^c=6 \end{cases} \\ (3_s)\ x_1,x_2,x_3,x_4^c \geq 0 \end{cases}$$

pas 2: $\overline{X_0} = S.B.A.i = ?$

$$(2_s) \longrightarrow \overline{A} = \begin{array}{cccc|c} P_1 & P_2 & P_3 & P_4^c & P_0 \\ 2 & 1 & 2 & 0 & 10 \\ 3 & 0 & 1 & \boxed{-1} & 6 \\ x_1 & x_2 & x_3 & x_4^c & \end{array} \Big| \cdot(-1) \sim \begin{array}{cccc|c} 2 & 1 & 2 & 0 & 10 \\ -3 & 0 & -1 & 1 & -6 \\ x_1 & x_2 & x_3 & x_4^c & \end{array} \equiv \overline{A}_{s.j}$$

v.p.

$$\Rightarrow \overline{X_0} = (0, 10, 0, -6)^T - S.B.N., N_d \neq S.B.A.i.$$

≥ 0 <0

$\neq 0$ $\neq 0$

Nu este utilă!

$\Rightarrow$ căutăm altă S.B, luând altă combinație de v.p. :

$$(2s) \to \bar{A} = \begin{array}{cccc|c} \multicolumn{5}{l}{} \end{array}\left(\begin{array}{cccc|c} 2 & 1 & 2 & 0 & 10 \\ 3 & 0 & \boxed{1} & -1 & 6 \end{array}\right)\begin{array}{l} + \\ \big| \cdot (-2) \end{array} \sim$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4^c \end{array}$$

$$\sim \left(\begin{array}{cccc|c} -4 & 1 & 0 & 2 & -2 \\ 3 & 0 & 1 & -1 & 6 \end{array}\right) \equiv \bar{A}^R_{G-J} \Rightarrow \bar{X}_0 = (0,-2,6,0)^T$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4^c \end{array}$$

S.B.N. Nd. $\neq$ SBAi

! tot nu este bună !

.... mai căutăm :

$$(2s) \to \bar{A} = \left(\begin{array}{cccc|c} 2 & 1 & 2 & 0 & 10 \\ \boxed{3} & 0 & 1 & -1 & 6 \end{array}\right)\begin{array}{l} + \\ \big| \cdot \frac{1}{3} \big| \cdot \left(-\frac{2}{3}\right) \end{array} \sim$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4^c \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 0 & 1 & \frac{4}{3} & \frac{2}{3} & 6 \\ 1 & 0 & \frac{1}{3} & -\frac{1}{3} & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & -\frac{1}{3} & 2 \\ 0 & 1 & \frac{4}{3} & \frac{2}{3} & 6 \end{array}\right) \equiv \bar{A}^R_{G-J}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4^c \end{array} \qquad \begin{array}{cccc} x_1 & x_2 & x_3 & x_4^c \end{array}$$

v.p.   v.s. = 0

$$\Rightarrow \bar{X}_0 = (2, 6, 0, 0)^T - \text{S.B.A. Nd} = \text{S.B.A.i.} \checkmark$$

$\geq 0 \; \geq 0$

$\neq 0 \; \neq 0$