Probleme de transport neechilibrate

1) Metoda diagonalei

	C_1	C_2	C_3	
D_1	3	1	2	10
D_2	2	4	1	25
	20	10	10	

Pasul 0

oferta: $\sum_{i=1}^{2} a_i = 10+25 = 35$

cererea: $\sum_{j=1}^{3} b_j = 20+10+10 = 40$

$\Rightarrow$ oferta < cererea $\Rightarrow$ $\Rightarrow$ P.T.N.

- Echilibrăm P.T.N.

  $\rightarrow$ transformăm P.T.N. în P.T.E. prin introducerea unui nou depozit de desfacere, D_3^f, ce va avea costurile de transport egale cu zero.

$40-35=\underline{5}$ (D_3^f)

	C_1	C_2	C_3	
D_1	$\boxed{3}$ 10	$\boxed{1}$ *	$\boxed{2}$ *	$\cancel{10}$ 0
D_2	$\boxed{2}$ 10	$\boxed{4}$ 10	$\boxed{1}$ 5	$\cancel{25}$ $\cancel{15}$ $\cancel{5}$ 0
D_3^f	$\boxed{0}$ *	$\boxed{0}$ *	$\boxed{0}$ 5	$\cancel{5}$ 0
	$\cancel{20}$ $\cancel{10}$ 0	$\cancel{10}$ 0	$\cancel{10}$ $\cancel{5}$ 0	

!verificarea!

v.p. = 5

v.s. = 4

Pasul 1

$\overline{x_0} = (10, 0, 0, 10, 10, 5, 0, 0, 5) \in R^9$ – S.B. Ned.

$f(\overline{x_0}) = 10 \cdot 3 + 10 \cdot 2 + 10 \cdot 4 + 5 \cdot 1 + 5 \cdot 0 = 95$ (u.m.)

Pasul 2

$\delta_{12} = -1 + 4 - 2 + 3 = 4 > 0$

$\delta_{13} = -2 + 1 - 2 + 3 = 0$

$\delta_{31} = -0 + 2 - 1 + 0 = 1 > 0$

$\delta_{32} = -0 + 4 - 1 + 0 = 3 > 0$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{x_0}$ Nu este optimă

Pasul 3

$$\delta_{ke} = \max\{\delta_{12};\delta_{31};\delta_{32}\} = \delta_{12} \Rightarrow x_{12} \downarrow$$

Pasul 4

$$\begin{tikzcd}
\overset{\cancel{10}\,*}{(1,1)}\ 2 \arrow[d] & \overset{10}{(1,2)} \downarrow \arrow[l] \\
\underset{\substack{\cancel{10}\\ 20}}{(2,1)} \arrow[r] & \underset{\substack{\cancel{10}\\ 0}}{(2,2)}\ 4 \arrow[u]
\end{tikzcd}$$

$$\theta = \min\{(1,1);(2,2)\} = (1,1) \Rightarrow x_{11} \rightarrow$$

$$\theta = 10$$

Pasul 5

$\boxed{3}$ *	$\boxed{1}$ 10	$\boxed{2}$ *	10
$\boxed{2}$ 20	$\boxed{4}$ 0	$\boxed{1}$ 5	25
$\boxed{0}$ *	$\boxed{0}$ *	$\boxed{0}$ 5	5
20	10	10	

! verificare !

Pasul 6

$$\bar{x}_1 = (0,10,0,20,\underline{0},5,0,0,5) \in \mathbb{R}^9 - \text{S.B.Deg}$$

$$f(\bar{x}_1) = 10\cdot 1 + 20\cdot 2 + 0\cdot 4 + 5\cdot 1 + 5\cdot 0 = 55(u.m) \leq f(\bar{x}_0)?\ \underline{Da}$$

Pasul 7

$\delta_{11} = -3+1-4+2 = -4$

$\delta_{13} = -2+1-4+1 = -4$

$\delta_{31} = -0+2-1+0 = 1>0$

$\delta_{32} = -0+4-1+0 = 3>0$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{X_1}$ Nu este optimă.

Pasul 8

$SKe = \max\{\delta_{31} ; \delta_{32}\} = \delta_{32} \Rightarrow x_{32} \checkmark$

Pasul 9

$$\begin{tikzcd}
\overset{\cancel{0}\,*}{②(2,2)} \arrow[r] & \overset{5}{(2,3)} \arrow[d] \\
\underset{0}{\checkmark(3,2)} \arrow[u] & (3,3)\,④\ 5 \arrow[l]
\end{tikzcd}$$

$\Theta = \min\{(2,2) ; (3,3)\} = (2,2) \Rightarrow (2,2) = x_{22} \rightarrow$

$\Theta = 0$

Pasul 10

[3] *	[1] 10	[2] *	10
[2] 20	[4] *	[1] 5	25
[0] *	[0] 0	[0] 5	5
20	10	10	

! verificare !

Pasul 11

$\overline{x_2} = (0,10,0,20,0,5,0,\underline{0},5) \in R^9$ – S.B. Deg.

$f(\overline{x_2}) = 10\cdot 1 + 20\cdot 2 + 5\cdot 1 + 0\cdot 5\cdot 0 = 55$ (u.m.) $\leq f(\overline{x_1})$? Da

Pasul 12

$\delta_{11} = -3+1-0+0-1+2 = -1$

$\delta_{13} = -2+1-0+0 = -1$

$\delta_{22} = -4+0-0+1 = -3$

$\delta_{31} = -0+0-1+2 = 1 > 0$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{x_2}$ nu este optimă.

Pasul 13

$SKe = \max\{\delta_{31}\} = \delta_{31} \Rightarrow x_{31} \downarrow$

Pasul 14

$$
\begin{tikzcd}
\overset{4\ \cancel{20}\ 15}{(2,1)} \arrow[d] & \overset{\cancel{5}\ 10}{(2,3)} \arrow[l] \\
\underset{5}{\downarrow (3,1)} \arrow[r] & \underset{\cancel{5}\ *}{(3,3)\ 2 \rightarrow} \arrow[u]
\end{tikzcd}
$$

$\Theta = \min\{(3,3); (2,1)\} = (3,3) \Rightarrow x_{33} \rightarrow$

$\Theta = 5$

Pasul 15

$\boxed{3}$ *	$\boxed{1}$ 10	$\boxed{2}$ *	10
$\boxed{2}$ 15	$\boxed{4}$ *	$\boxed{1}$ 10	25
$\boxed{0}$ 5	$\boxed{0}$ 0	$\boxed{0}$ *	5
20	10	10	

! verificare !

$\overline{x}_3 = (0, 10, 0, 15, 0, 10, 5, \underline{0}, 0) \in R^9$ – S.B.Deg.

$f(\overline{x}_3) = 10 + 30 + 10 = 50$ (u.m) $\leq f(\overline{x}_2)$? Da

$\delta_{11} = -3 + 1 - 0 + 0 = -2$

$\delta_{13} = -2 + 1 - 0 + 0 = -1$

$\delta_{22} = -4 + 0 - 0 + 2 = -2$

$\delta_{33} = -0 + 0 - 2 + 1 = -1$

$\Rightarrow$ toți $\delta_{ij} \leq 0 \Rightarrow \overline{x}_3$ este S.O. unică.

Concluzia pt. P.T.E.: $\begin{cases} x_{\text{optim}}^{\text{echilibrat}} = (0, 10, 0, 15, 0, 10, 5, 0, 0) \\ \min f = 50 \text{ (u.m)} \end{cases}$

Concluzia pt. P.T.N.: $\begin{cases} x_{\text{optim}}^{\text{neechilibrat}} = (0, 10, 0, 15, 0, 10) \\ \min f = 50 \text{ (u.m)} \end{cases}$

(2)

Probleme de transport neechilibrate

2) Metoda costului minim

	C_1	C_2	
D_1	3	2	10
D_2	1	4	20
D_3	2	1	20
	10	25	

Pasul 0

oferta: $\sum_{i=1}^{3} a_i = 10+20+20 = 50$

cererea: $\sum_{j=1}^{2} b_j = 10+25 = 35$

$\Rightarrow$ oferta > cererea $\Rightarrow$ P.T.N.

• Echilibrăm P.T.N.

→ transformăm P.T.N. în P.T.E. prin introducerea unui nou centru de desfacere fictiv, C_3^f, ce va avea costurile de transport egale cu zero.

$$50-35 = 15 \,(C_3^f)$$

	C_1	C_2	C_3f	
D_1	$\boxed{3}$ *	$\boxed{2}$ *	$\boxed{0}$ 10	~~10~~ 0
D_2	$\boxed{1}$ 10	$\boxed{4}$ 5	$\boxed{0}$ 5	~~20~~ ~~15~~ ~~5~~ 0
D_3	$\boxed{2}$ *	$\boxed{1}$ 20	$\boxed{0}$ *	~~20~~ 0
	~~10~~ 0	~~25~~ ~~5~~ 0	~~15~~ ~~5~~ 0	

! verificare !

Pasul 1

$\overline{x_0} = (0,0,10,10,5,5,0,20,0) \in \mathbb{R}^9$ – S.B.Ned.

$f(\overline{x_0}) = 10\cdot 0 + 10\cdot 1 + 5\cdot 4 + 5\cdot 0 + 20\cdot 1 = 50$ (u.m.)

Pasul 2

$\delta_{11} = -3+1-0+0 = -2$

$\delta_{12} = -2+4-0+0 = 2 > 0$

$\delta_{31} = -2+1-4+1 = -4$

$\delta_{33} = -0+1-4+0 = -3$

$\Rightarrow \exists\, \delta_{ij} > 0 \Rightarrow \overline{x_0}$ nu este optimă

Pasul 3

$\delta_{ke} = \max\{\delta_{12}\} = \delta_{12} \Rightarrow x_{12}\downarrow$

Pasul 4

$$\begin{tikzcd}
\textcircled{1}\downarrow \overset{5}{(1,2)} \arrow[d] & (1,3)^{\textcircled{4}}\ \cancel{10}\,5 \arrow[l] \\
(2,2)\ \textcircled{2}\ \cancel{5}\ * \arrow[r] & (2,3)\ \cancel{5}\,10\ \textcircled{3} \arrow[u]
\end{tikzcd}$$

$\theta = \min\{(2,2);(1,3)\} = (2,2) \Rightarrow x_{22}\rightarrow$

$\theta = 5$

Pasul 5

	C_1	C_2	C_3f	
D_1	* (3)	5 (2)	5 (0)	10
D_2	10 (1)	* (4)	10 (0)	20
D_3	* (2)	20 (1)	* (0)	20
	10	25	15	

! verificare !

Pasul 6

$\overline{x}_1 = (0, 5, 5, 10, 0, 10, 0, 20, 0) \in R^9$ - S.B.Ned.

$f(\overline{x}_1) = 10 + 10 + 20 = 40 (u.m) \leq f(\overline{x}_0)$? Da

Pasul 7

$\delta_{11} = -3 + 1 - 0 + 0 = -2$

$\delta_{22} = -4 + 2 - 0 + 0 = -2$

$\delta_{31} = -2 + 1 - 0 + 0 - 2 + 1 = -2$

$\delta_{33} = -0 + 1 - 2 + 0 = -1$

$\Rightarrow$ toți $\delta_{ij} \not\geq 0 \Rightarrow \overline{x}_1$ este S.O. unică.

Conluzia pt. P.T.E. $\begin{cases} x_{optim}^{\text{echilibrat}} = (0, 5, 5, 10, 0, 10, 0, 20, 0) \\ \min f = 40 (u.m.) \end{cases}$

Conluzia pt. P.T.N. $\begin{cases} x_{optim}^{\text{neechilibrat}} = (\cancel{0, 5, 5, 10, 0, 10}) \\ \quad (0, 5, 10, 0, 0, 20) \\ \min f = 40 (u.m). \end{cases}$