CS F211
Data Structures and Algorithms
Assignment - 11

Recap

Allowed Languages: **C**

April 17, 2024

General Tips

- Try to use functions as much as possible in your code. Functions increase reusability and the pass-by-value feature provides a significant help sometimes. Modularizing your code also helps you to debug efficiently.
- Use `scanf` to read characters/strings from STDIN. Avoid using `getchar`, `getc` or `gets`. Try to read up about character suppression in `scanf` as it will be very helpful in some of the problems.
- Use `printf` instead of `putc`, `putchar` or `puts` to print character/string output on STDOUT.
- Indent your code appropriately and use proper variable names. These increase readability and writability of the code. Also, Use comments wherever necessary.
- Use a proper IDEs like Sublime Text or VSCode as they help to run and test your code on multiple test-cases easily.
- **Note:** Kindly try to do all of these questions by yourself at least once. Spend some time thinking about it, or trying to code it instead of directly asking help of your friends or searching it up online. This helps you understand the question, allowing you to solve further questions which are not in the scope of this Assignment yourself.

A: Matrix Exponentiation

You have done binary exponentiation before, matrix exponentiation works in a similar way. We need to find M^p where M is a matrix rather than a number. We define $M^2 = M * M$ and thus $M^p = M * M * \ldots M * M$ where M is repeated p times.

Given a matrix M of dimensions NxN, find M^p using Matrix Exponentiation technique.

Input

The first line of input contains two space separated integers N and p ($1 \leq N \leq 10^4, 1 \leq p \leq 10^9$). The following N lines of input each contains N integers each describing rows of matrix M. ($1 \leq M_{ij} \leq 10^9$).

Output

The output should contain N lines, each describing rows of the matrix M^p.

```
input
4 2
5 5 5 5
5 5 5 5
5 5 5 5
5 5 5 5

output
100 100 100 100
100 100 100 100
100 100 100 100
100 100 100 100
```

```
input
3 1000000000
1 0 0
0 1 0
0 0 1

output
1 0 0
0 1 0
0 0 1
```

B: Union Find

Initially there are n distinct nodes. You are also given q queries, each of which represents 2 nodes being joined by adding an edge between them, find the total number of connected components in the graph after each query.

Input

The first line of input contains two space separated integers n and q ($1 \leq n, q \leq 10^5$).
The following q lines of input each contains 2 integers u and v, which represent the nodes being joined ($1 \leq u, v \leq n$).

Output

The output should contain q integers representing the number of connected components after the corresponding query.

---

input
```
8 7
1 2
2 3
3 4
4 5
5 6
6 7
7 8
```

output
```
7 6 5 4 3 2 1
```

---

input
```
8 6
1 1
2 2
3 3
1 2
2 3
1 3
```

output
```
8 8 8 7 6 6
```

C: Heaps?

You are given an array of n integers.You are supposed to perform $n-1$ operations on the array. In one operation you can:

- Delete one element from the array, and add the value of the deleted element to all other elements in the array.

Find the maximum possible value of the last element after all the operations.

Input

The first line of input contains two space separated integers n ($1 \leq n \leq 10^5$).
The second line of input contains n integers $a_1, a_2, \ldots, a_n$ ($1 \leq x \leq 10^3$).

Output

The output should contain one integer, the maximum possible value of the last element.

```
input
6
2 2 2 6 6 6

output
92

explanation
2 2 2 3 3 3 -> 5 5 5 6 6 -> 11 11 11 12 -> 23 23 23 -> 46 46 -> 92
```

```
input
4
2 2 2 2

output
16

explanation
2 2 2 2 -> 4 4 4 -> 8 8 -> 16
```

D: 2D prefix sum

You are given an $n \times n$ matrix. Each cell contains an integer.
Your task is to perform q queries of the form: what is sum of the integers in a given rectangle?

Input

The first input line has two integers n and q: the size of the matrix and the number of queries ($1 \leq n \leq 10^4, 1 \leq q \leq 10^5$).
Then, there are n lines describing the matrix. Each line has n integers corresponding to the row of the matrix ($1 \leq M_{ij} \leq 10^9$).
Finally, there are q lines describing the queries. Each line has four integers y_1, x_1, y_2, x_2 corresponding to the corners of a rectangle ($1 \leq y_1, x_1, y_2, x_2 \leq n$).

Output

The q_{th} line of the output should contain the sum of integers inside the corresponding rectangle.

```
input
3 1
1 2 3
4 5 6
7 8 9

output
45
```

```
input
4 3
2 5 1 8
3 9 4 2
7 1 6 3
8 2 5 4
1 1 2 2
2 3 3 4
3 1 4 3

output
19
21
29
```

E: AND Nom is Back

Nom has one last problem set up for you guys before he leaves you forever from this course. One day he was doing DD and realised that when he's using a lot of consecutive numbers, he's getting their bitwise AND as 0. He got confused as to what to do next.

Initially, he was working with all the numbers between l and r, both inclusive. The bitwise AND of all the numbers could be zero or non-zero. Help him find the minimum possible numbers that you can remove from this range to make the bitwise AND non-zero.

Input

The first line of input contains two integers l and r $(1 \leq l \leq r \leq 2 * 10^5)$

Output

The output contains one integer representing the minimum numbers to be removed.

---

input
4 5

output
0

---

input
2 8

output
3

---

input
100000 200000

output
31072

F: Fractional Knapsack

You are a thief planning to rob a store. However, you can only carry a limited weight in your bag. The store contains n items, each with a certain weight w_i and value v_i. Your objective is to maximize the total value of items you can carry without exceeding the weight limit W of your bag.

Given the weight and value of each item, implement an algorithm to determine the maximum total value you can obtain. You are allowed to take fractional units of items.

Input

The input consists of two integers n and W ($1 \leq n \leq 10^3, 1 \leq W \leq 10^6$), representing the number of items in the store and the maximum weight your bag can carry, respectively.
This is followed by n lines, each containing two integers w_i and v_i ($1 \leq w_i \leq 10^6, 1 \leq v_i \leq 10^6$), representing the weight and value of the i^{th} item.

Output

Output a single real number, representing the maximum total value you can obtain by optimally selecting items and possibly taking fractional units of items.

```
input
3 50
20 100
10 60
30 120

output
240.0
```

```
input
4 10
5 20
4 18
2 14
3 12

output
48.0
```

G: Ternary Search

You are given a function $f(x)$ defined over the real numbers. This function has exactly one maximum in the range $[a, b]$. The function increases from a till its maximum and then decreases from there onwards till b. Your task is to find the value of x where the maximum occurs with precision up to 10^{-6}.

Input

The input consists of two real numbers a and b ($-10^9 \leq a < b \leq 10^9$), representing the range. Additionally, you will be given the coefficients of the function $f(x)$, which is a polynomial of degree at most 3.
These coefficients will be given in the form of four real numbers A, B, C, and D ($-10^9 \leq A, B, C, D \leq 10^9$), where $f(x) = Ax^3 + Bx^2 + Cx + D$.
Note that the function will always satisfy the constraint provided in the question.

Output

Output a single real number, the value of x where the maximum of $f(x)$ occurs, rounded to 6 decimal places.

---

input
```
0 2
-1 0 1 1
```

output
```
1.385000
```

---

input
```
0 1
1 -2 1 0
```

output
```
0.148000
```

H: Nodes at a distance in BST

You are given a complete binary tree with N nodes. Each node is labeled with a unique integer from 1 to N. Your task is to determine the number of nodes that are exactly at distance x from a given node k in the tree.

A complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

Input

The first line of input contains three integers N, k, and x ($1 \leq k \leq N \leq 10^5, 0 \leq x \leq 10^5$).
The second line of input contains N space-separated integers, representing the labels of the nodes in the tree.

Output

Output a single integer, representing the number of nodes that are exactly at distance x from the given node k.

---

input
```
7 3 1
1 5 3 6 2 7 4
```

output
```
3
```

---

input
```
10 8 2
7 2 9 1 6 8 4 5 10 3
```

output
```
0
```
