Assignment 5

1. This problem addresses the difficulty inherent in linear prediction analysis for high-pitchedspeakers,

   (a) Suppose $h[n]$ is the impulse response of an all-pole system where,

   $$H(z) = \frac{1}{1 - \sum_{k=1}^{p} \alpha_k z^{-k}}$$

   So that

   $$h[n] = \sum_{k=1}^{p} \alpha_k h[n-k] + \delta[n]$$

   Hint: Multiply both sides of the difference equation by $h[n-i]$ and sum over n. Note that $h[n]$ is causal.

   (b) Assume s[n] is a periodic waveform, given by

   $$s[n] = \sum_{k=-\infty}^{\infty} h[n-kP]$$

   where P is the pitch period. Show that the autocorrelation of $s[n]$, windowed over multiple pitch periods, consists of periodically repeated replicas of $r_s[\tau]$, i.e.,

   $$r_s[\tau] = \sum_{k=-\infty}^{\infty} r_h[\tau - kP]$$

   but with decreasing amplitude due to the window.

   (c) Using your result in parts (a) and (b), explain the difference between your result in part(a) and the normal equations for the autocorrelation method using the windowed speechsignal s[n].

   (d) Using your results from parts (b) and (c), explain why linear prediction analysis is moreaccurate for low-pitched speakers than high-pitched speakers.

2. (MATLAB) In this exercise, use the voiced speech signal *speech1_10k* (at 10000 samples/s)in the workspace ex5M1.mat. This problem illustrates the autocorrelation method of linearprediction.

   (a) Window *speech1_10k* with a 25-ms Hamming window. Compute the autocorrelation ofthe resulting windowed signal and plot.

   (b) Assume that two resonances represent the signal and model the vocal tract with 4 poles.Set up the autocorrelation matrix R_n, using your result from part (a). The autocorrelationmatrix is of dimension 4×4.

   (c) Solve for the linear predictor coefficients by matrix inversion.

   (d) Plot the log-magnitude of the resulting frequency response:

   $$H(\omega) = \frac{A}{1 - \sum_{k=1}^{p} \alpha_k e^{-j\omega k}}$$

   where the gain is given by $A^2 = E_n = r_n[0] - \sum_{k=1}^{p} \alpha_k r_n[k]$ Compare your result with the log-magnitude of the Fourier transform of the windowed signal. What similarities and differences do you observe?

(e) Using your estimates of the predictor coefficients from part (c), compute the prediction error sequence associated with *speech1_10k* and plot. From the prediction error sequence, what conclusions might one draw about the model (i.e., all-pole/impulse-train-driven) and estimation accuracy?

3. (MATLAB) In this problem you will use your results from previous problem to perform speech synthesis of the speech waveform *speech1_10k* in the workspace *ex5Ml.mat*.

   (a) Using your estimates of the predictor coefficients from previous problem, compute an estimate of the vocal tract impulse response.

   (b) Using the prediction error sequence you computed in previous problem, estimate an average pitch period of the voiced sequence *speech1_10k*.

   (c) Using your results from parts (a) and (b), synthesize an estimate of *speech1_10k*. How does your waveform estimate differ from the original? Consider the minimum-phase nature of the impulse response estimate.

   (d) Using the MATLAB number generator *randm.m*, synthesize the ”whispered” counterpart to your voiced synthesis of part (c). Using the MATLAB *sound.m* function, listen to your two estimates and compare to the original.