Problem 1

Prove that for a LPC model described by

$$Q(z) = 1 - a_1 z^{-1} - a_2 z^{-2} \cdots - a_n z^{-n}$$

The poles migrate towards the unit circle as $a_n \to \infty$. Sketch the same on the unit circle.

Solution.
We know that any rational expression can either be expressed in PoS or SoP form, so coverting to PoS form,

$$\begin{aligned} Q(z) &= 1 - a_1 z^{-1} - a_2 z^{-2} \cdots - a_n z^{-n} \\ &= (1 - p_1 z^{-1})(1 - p_2 z^{-1}) \cdots (1 - p_n z^{-1}) \end{aligned}$$

where $\forall i \in \{1, \cdots n\}, p_i \in \mathcal{C}$

$$\prod_{k=1}^{n} p_k = a_n$$

$$\prod_{k=1}^{n} |p_k| = |a_n| \to \infty$$

Therefore poles aren't moving towards the unit circle, but some of the poles are diverging. ∎

Problem 2

Show that the phase derivative $\dot{\angle} X(\omega)$ of the Fourier transform $X(\omega)$ of a sequence $x[n]$ can be obtained through real and imaginary parts of $X(\omega), X_r(\omega)$ and $X_i(\omega)$, respectively, as

$$\dot{\angle} X(\omega) = \frac{X_r(\omega)\dot{X}_i(\omega) - X_i(\omega)\dot{X}_r(\omega)}{|X(\omega)|^2}$$

Where $|X(\omega)|$, the Fourier transform magnitude of $x[n]$, is assumed non-zero.

Solution.
We define $\angle X(\omega) = \arctan\{\frac{X_i(\omega)}{X_r(\omega)}\}$. Taking simple derivative w.r.t. ω and by chain rule

$$\begin{aligned}\frac{d\angle X(\omega)}{d\omega} &= \frac{1}{1+\frac{X_i^2}{X_r^2}} \frac{X_r\dot{X}_i - X_i\dot{X}_r}{X_r^2} \\ &= \frac{X_r\dot{X}_i - X_i\dot{X}_r}{|X|^2}(\omega)\end{aligned}$$

■

Problem 3

Programming Assignment: (MATLAB) In this problem, you use the speech waveform *speech1_10k* (at 10000 samples/s) in the workspace ex6Ml.mat. The exercise works through a problem in homomorphic de-convolution, leading to the method of homomorphic prediction.

1. Window *speech1_10k* with a 25-ms Hamming window. Using a 1024-point FFT, compute the real cepstrum of the windowed signal and plot. For a clear view of the real cepstrum, set the first cepstral value to zero (which is the DC component of the log-spectral magnitude) and plot only the first 256 cepstral values.

2. Estimate the pitch period (in samples and in milliseconds) from the real cepstrum by locating a distinct peak in the quefrency region.

3. Extract the first 50 low-quefrency real cepstral values using a lifter of the form

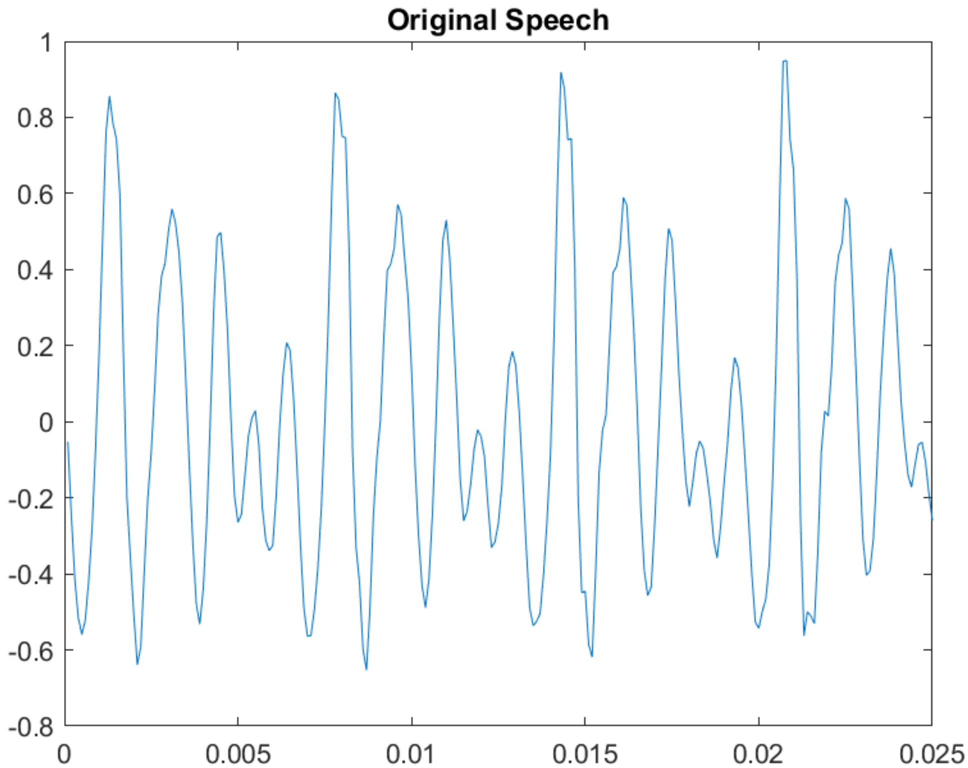
$$l[n] = \begin{cases} 1, & n = 0 \\ 2, & 1 \leq n \leq 49 \\ 0, & \text{otherwise} \end{cases}$$

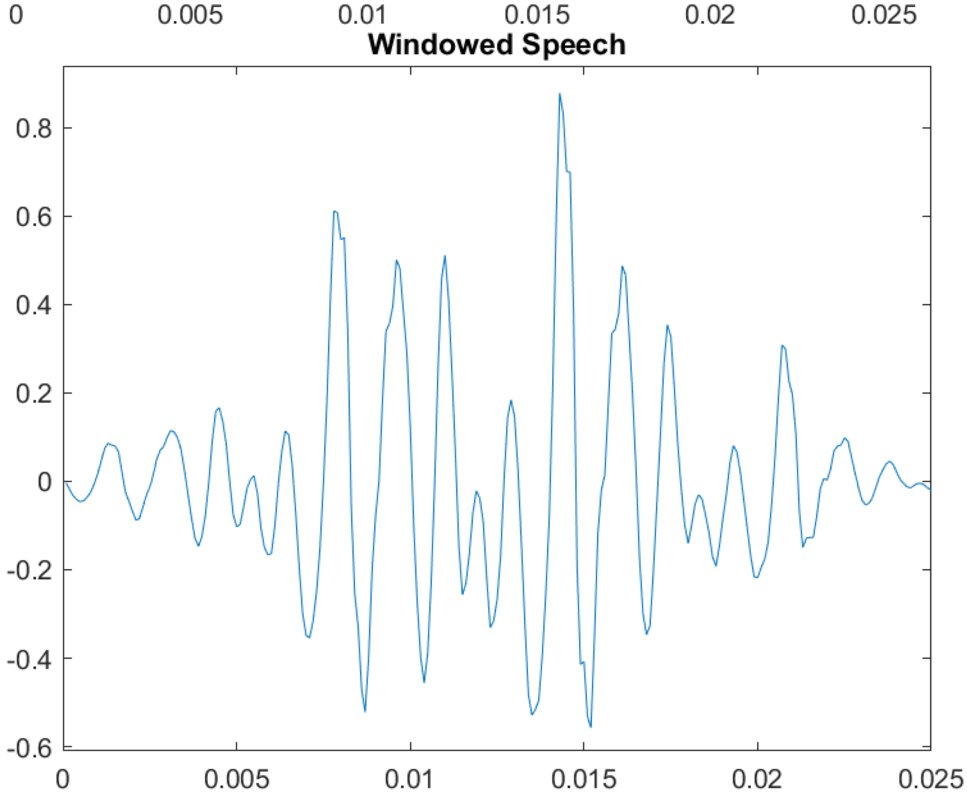
   Then Fourier transform (using 1024-point FFT) and plot the first 512 samples of the resulting log-magnitude and phase.

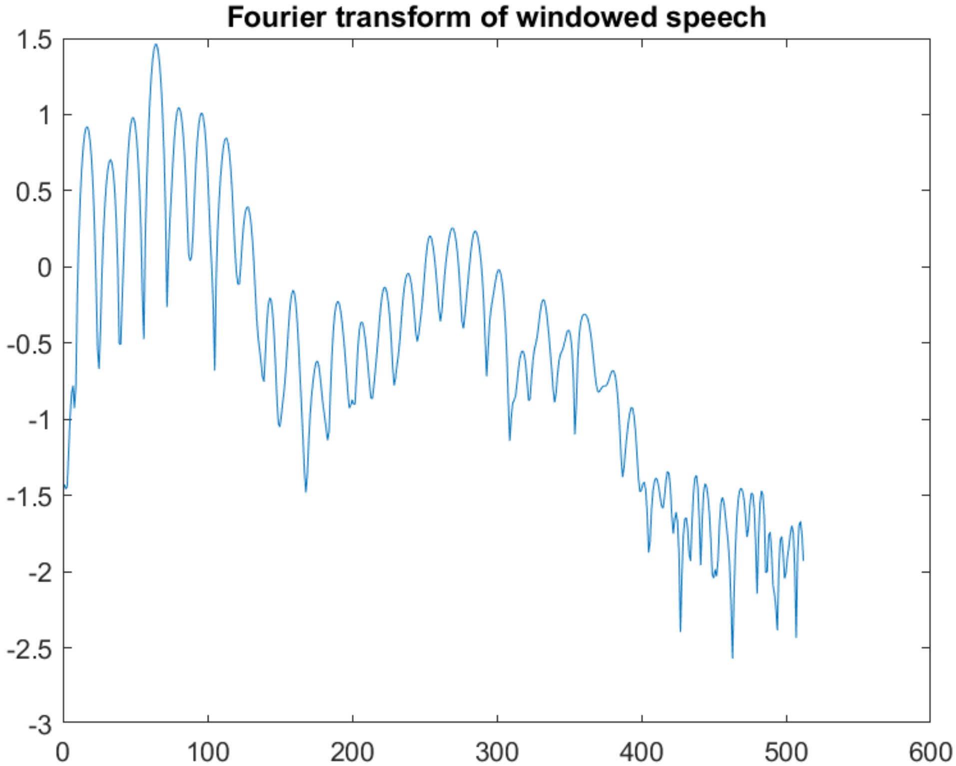
4. Compute and plot the minimum-phase impulse response using your result from part (c). Plot just the first 200 samples to obtain a clear view. Does the impulse response resemble one period of the original waveform? If not, then why not?

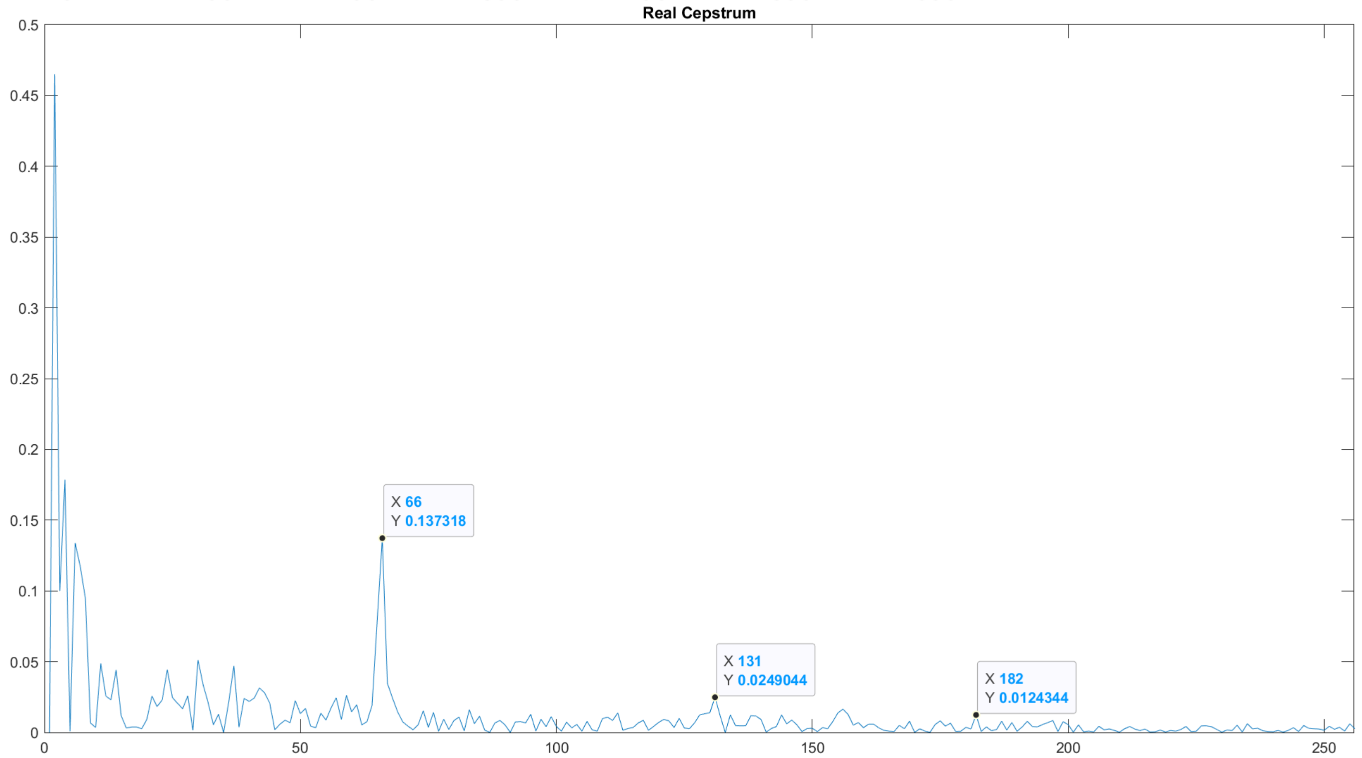
5. Use your estimate of the pitch period in samples from part (b) to form a periodic unit sample train, thus simulating an ideal glottal pulse train. Make the length of the pulse train 4 pitch periods. Convolve this pulse train with your 200-sample impulse response estimate from part (d) and plot. You have now synthesized a minimum-phase counter part to the possibly mixed-phase vowel *speech1_10k*. What are the differences in your construction and the original waveform?

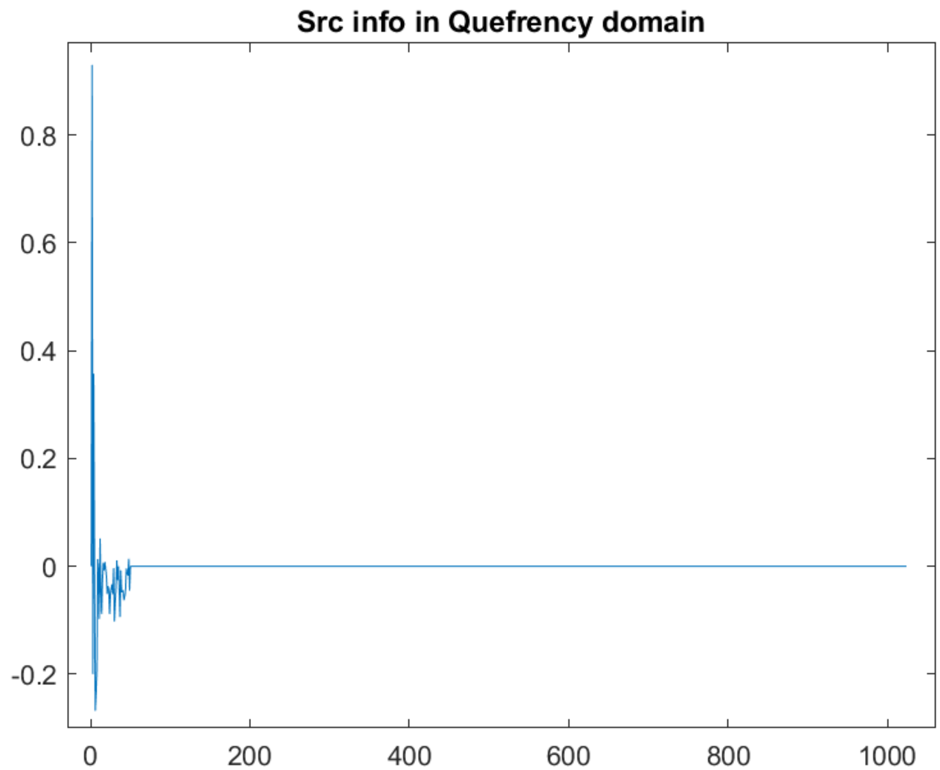
Solution.
The pitch period found by Cepstral analysis is $\sim 66ms$. The minimum-phase impulse response doesn't look similar to one period of the original waveform, because we lost the phase part of the original waveform while taking the real cepstrum. ■

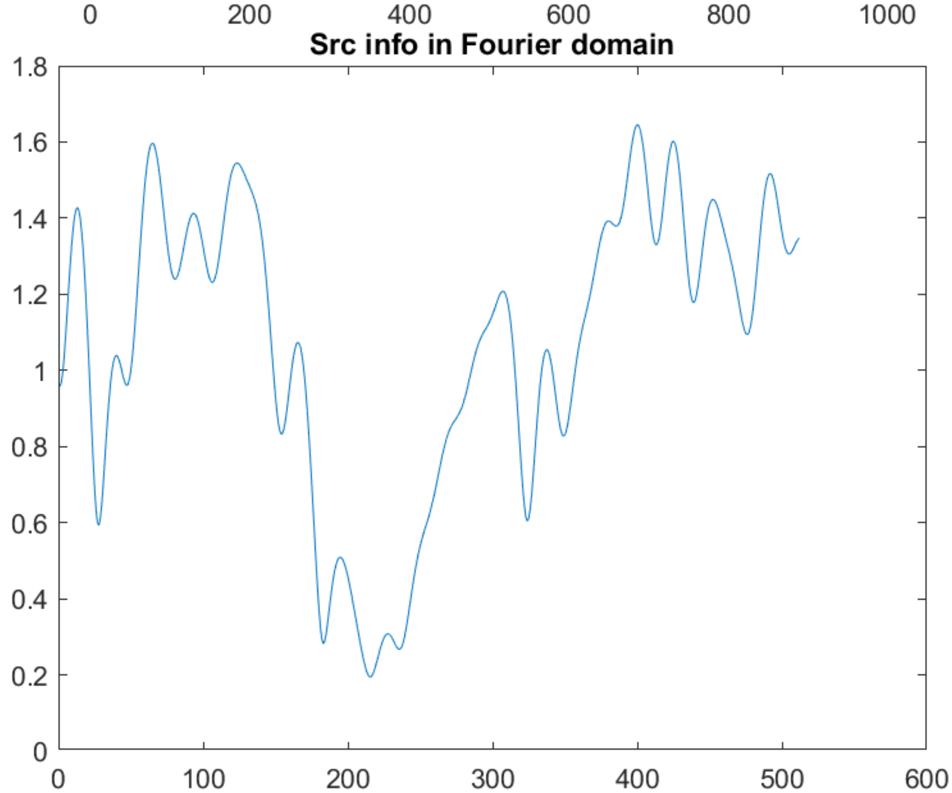

Original Speech
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.005
0.01
0.015
0.02
0.025

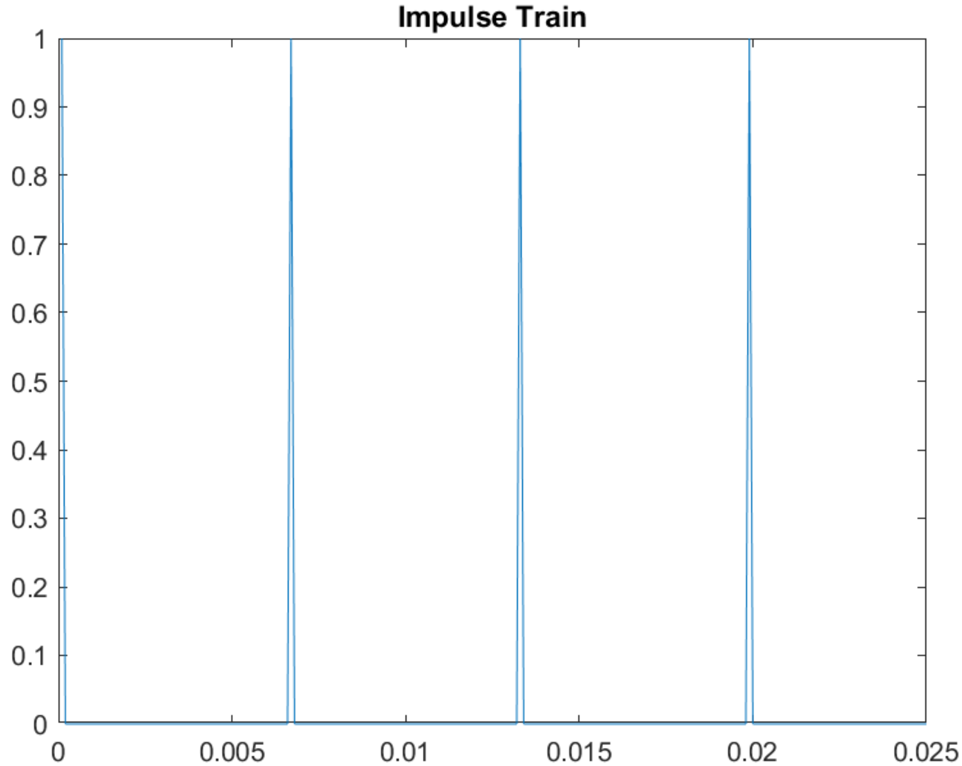

Windowed Speech
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.005
0.01
0.015
0.02
0.025

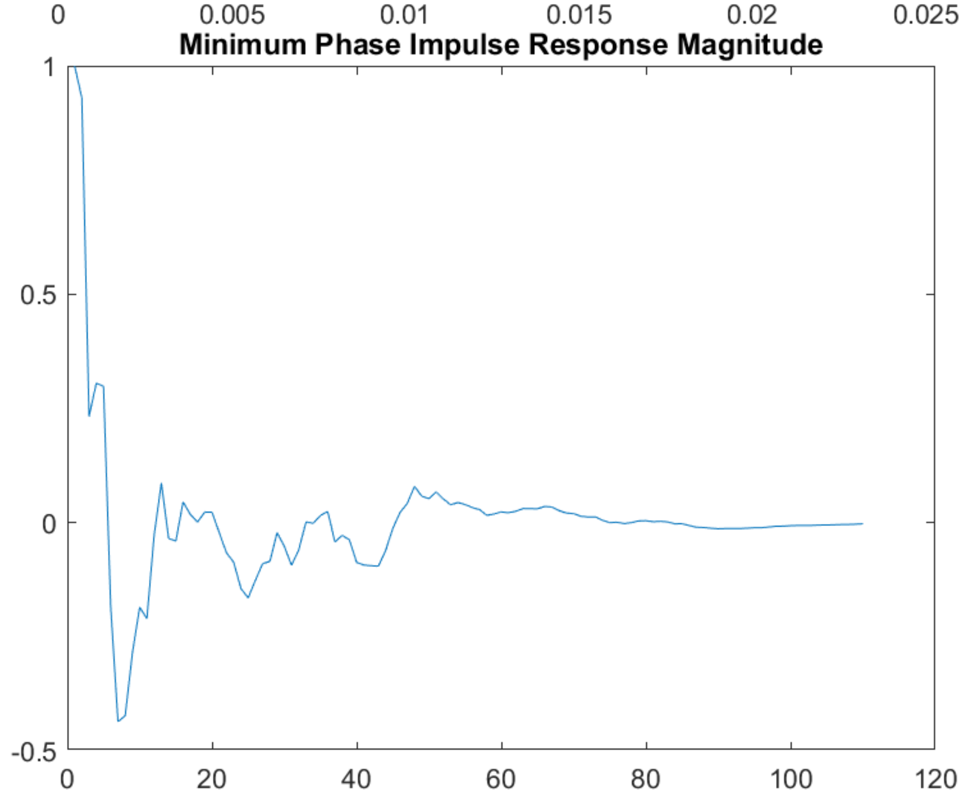

Fourier transform of windowed speech
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
100
200
300
400
500
600

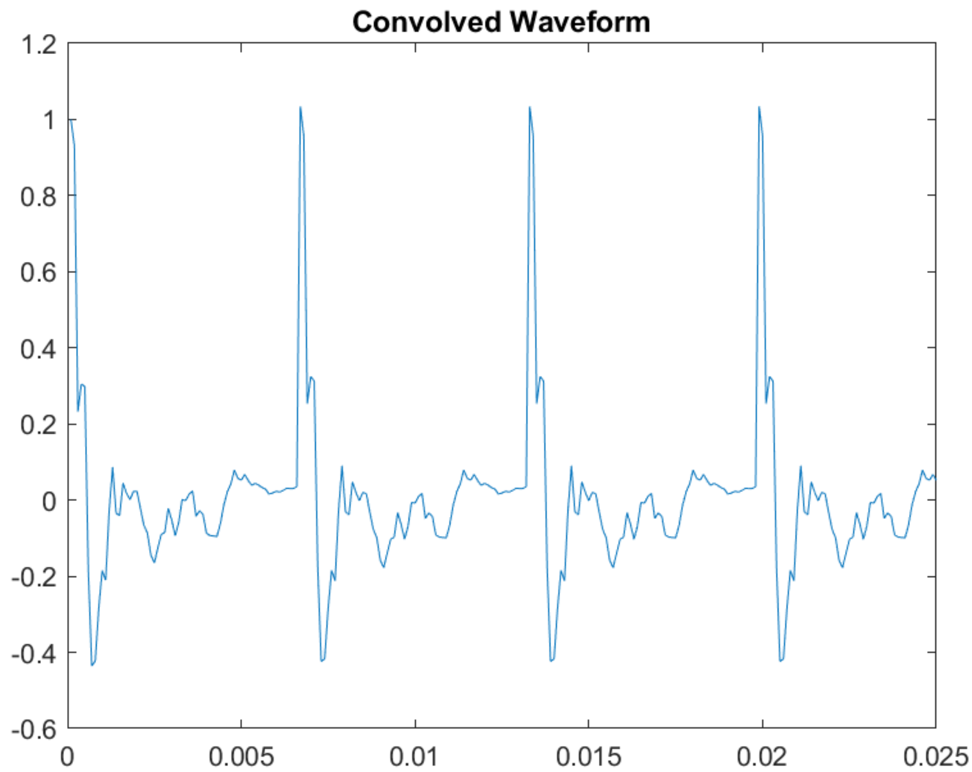

Real Cepstrum
X 66
Y 0.137318
X 131
Y 0.0249044
X 182
Y 0.0124344
0
50
100
150
200
250
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05


Src info in Quefrency domain
0.8
0.6
0.4
0.2
0
-0.2
0
200
400
600
800
1000


Src info in Fourier domain
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600


Impulse Train
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.005
0.01
0.015
0.02
0.025


Minimum Phase Impulse Response Magnitude
1
0.5
0
-0.5
0
20
40
60
80
100
120



MATLAB Code

```
[speech, Fs] = audioread('./speech1_10k.wav');
range = (1:length(speech))/Fs;
plot(range, speech);
title('Original Speech');
window_duration = 0.025;
shift_duration = 0.015;
window = hamming(window_duration*Fs);
shift = shift_duration*Fs;
w_speech = zeros(1, length(window) * ...
    ceil(length(speech)/length(window)));
w_speech(1:length(speech)) = speech;
% speech = w_speech;
num_shifts = ceil((length(speech) - length(window))/shift);
num_windows = num_shifts+1;
windowed_speech = zeros(num_windows, length(window));
for i=0:num_windows-1
    windowed_speech(i+1, :) = ...
    speech(i*shift+1:(i*shift)+length(window)) .* window';
end
range_win = (1:length(window))/Fs;
for i=1:num_windows
```

```
    figure;
    plot(range_win+(i-1)*window_duration, speech(i, :));
end
title('Windowed Speech')
% xlim([0.0000 0.0250])
% ylim([-0.61 0.94])
freq = fft(speech,1024);
plot(log10(abs(freq(1:512))));
title('Fourier transform of windowed speech')
rcep = ifft(log10(abs(freq)));
rcep(1) = 0;
plot((1:256), abs(rcep(1:256)));
title('Real Cepstrum')
xlim([0,256])
lifter = zeros(1, length(rcep));
lifter(1) = 1;
lifter(2:49) = 2;
srcInfo = rcep.*lifter;
plot(srcInfo);
title('Src info in Quefrency domain')
srcFft = fft(srcInfo);
plot(abs(srcFft(1:512)));
title('Src info in Fourier domain')

% minResponse = ifft(exp(srcFft));
minResponse = ifft(exp(fft(srcInfo)));
% minResponse(1)=0;
plot(minResponse(1:110));
title('Minimum Phase Impulse Response Magnitude')

% Pitch period is 66
impTrain = zeros(1, length(speech));
for i=1:66:length(speech)
    impTrain(i)=1;
end
plot(range_win, impTrain)
title('Impulse Train');

convolved_speech = conv(minResponse(1:110), impTrain, "full");
plot(range_win, convolved_speech(1:length(speech)));
title('Convolved Waveform');
```