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In search of the scenario of the star formation process, fragmentation of molecular clouds has been modelled under two different conditions. In the former case, thermal instability along with an equation of state with negative polytropic indices (Viala & Horedt 1974; Leighton 1997; Horedt 2004; Franco et al. 1994; Nath & Vishwakarma 2016) has been considered which give rise to minimum Jeans masses in the range $0.001 M_\odot$ to $8 M_\odot$ for $-6 < n < 0$, $n \neq -1$. In the latter case, an opacity limited fragmentation along with positive polytropic indices is considered which lead to a wide range of Jeans masses starting from field stars and massive stars to star bursts. This might be due to the effect of opacity which is responsible for slow dissipation of heat in a compressed medium.

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Highlights:

- Star formation in molecular clouds has been considered.
- Thermal instability along with opacity and molecular cooling.
- Negative polytropes result in field stars.
- Positive polytropes result in star bursts.

Fragmentation of molecular cloud in a polytropic medium

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Abstract

In search of the scenario of the star formation process, fragmentation of molecular clouds has been modelled under two different conditions. In the former case, thermal instability along with an equation of state with negative polytropic indices (Viala & Horedt 1974; Leighton 1997; Horedt 2004; Franco et al. 1994; Nath & Vishwakarma 2016) has been considered which give rise to minimum Jeans masses in the range $0.001M_{\odot}$ to $8 M_{\odot}$ for $-6 < n < 0$, $n \neq -1$. In the latter case, an opacity limited fragmentation along with positive polytropic indices is considered which lead to a wide range of Jeans masses starting from field stars and massive stars to star bursts. This might be due to the effect of opacity which is responsible for slow dissipation of heat in a compressed medium.

Keywords: interstellar medium, fragmentation, Jeans mass

1. Introduction

The interstellar space, filled with a dilute mixture of charged particles, atoms, molecules and dust grains, is called the interstellar medium (ISM). The physics of the ISM plays a crucial role in many areas of astronomy. The fragmentation process of the interstellar molecular cloud and star formation as well as its structure and evolution is still considered to be a poorly understood problem in astrophysics. The thermal instability is the most famous physical process to understand the

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ISM(Field 1965).

Thermal instability could occur in denser region like molecular cloud and has actually been studied by various authors(de Jong et al. 1980; Gilden 1984; Nejad-Asghar 2011). Molecular clouds are characterized by supersonic velocity dispersions, which are most probably due to turbulence (Larson 1981; Solomon et al. 1987) and the turbulent velocity is highly supersonic (Sabano & Tosa 1985). The opacity limited fragmentation occurs when the rate of energy released due to the collapse exceeds the rate of energy radiation through outside. Then the Jeans mass increases and as a result the Jeans-unstable collapsing clump becomes a Jeans-stable fragment. The density at which this occurs depends on the opacity and the initial temperature of the gas, hence the term 'opacity limited fragmentation' emerges. The most dominant sources of opacity is high density and the minimum fragment mass for gravitational collapse for a dark molecular gas cloud is about $0.006 M_{\odot}$ (Silk 1977) and the opacity limit mass is about $0.007 M_{\odot}$ (Low & Lynden Bell 1976).

In implementing the condition that the rate at which the energy is radiated equals the rate of acquisition of compressional energy and the optical depth across a fragment of scale be unity of a volume element of the collapsing cloud in the temperature -density plane, Silk (1977) computed the adopted parameters of grain materials. Similar work regarding the thermal balance of heating and cooling of different substances along with opacity have been considered by Kanjilal & Basu (1992). In the present work we have considered equation of state of a polytropic gas as $p = K\rho^{1+\frac{1}{n}}$ where, p and ρ are respectively the thermal pressure and gas density and n is the polytropic index. Using this equation with thermal balance equation in Section 2, we found in Section 3, the value of temperature T and the corresponding minimum Jeans masses in the presence of different grains for different values of n . Like the classical polytropes with positive indices the study of polytropes with negative indices is also practically important from the point of view that they can approximate either some models of interstellar gas clouds in the case of $n < -1$ (Shu et al. 1972; Viala 1972; Toci & Galli 2015) or in the case $-1 < n < 0$ (Chavanis 2014) where thermally unstable gas phases of the interstellar

medium have $\frac{\partial p}{\partial \rho} < 0$ (Viala & Horedt 1974) and also in a shock compressed layer (Lou & Gao 2006) as a result of explosive shocks at the central region of Galaxy (Kanjilal & Basu 1991).

In Section 4 we have developed the model of fragmentation governed by opacity. Here we have calculated the Jeans masses for gravitational collapse under positive values of polytropes. Since negative values corresponds to rapid cooling hence rapid increase of density so presence of opacity resists such process. Section 5 outlines discussion.

2. Model and basic equations

In the present model the gravitational collapse of molecular clouds have been considered in a low temperature range ($T < 500$ K). During the collapse it is thus necessary for the thermal energy generated during the collapse, to be dissipated in order to continue the collapse. Now at such low temperature range the particles, excited, must have low excitation potentials (< 0.1 eV). In molecular clouds H atoms and heavy elements do not satisfy the condition. On the contrary rotational levels of molecular hydrogen (Hirasawa 1969; Matsuda et al. 1969; Puy et al. 1999), carbon monoxide (Goldsmith 2001) and similar molecules (Peebles & Dicke 1968; Bally et al. 1987, 1988) and grains (Silk 1977) are excited which therefore act as the effective cooling elements in such cold molecular clouds. Thus we have considered here the thermal balance for most of the heating and cooling processes known to be important in the interstellar medium along with the polytropic equation of state with a negative index in the molecular gas clouds. The modified form of the equation of state for the ideal gas to express the thermal pressure is as follows:

$$p = \frac{R}{\mu_0} \rho T, \quad (1)$$

where R is the Universal gas constant, $\mu = \mu_0 m_H$, μ is the mean molecular weight and m_H is the mass of hydrogen and

T denotes the gas temperature.

A polytropic equation of state is a special case of barotropic equation of state where p and ρ are related as

$$p = K \rho^{1+\frac{1}{n}}, \quad (2)$$

where K is a constant and n is the so-called polytropic index. The values of K are determined from the several initial conditions of temperatures and number densities e.g. $T = 10$ K, 50 K, 90 K and number densities as 10^4 and 10^5 (cm^{-3}) respectively (Herbst & Klemperer 1973; Goldsmith & Langer 1978;

Bally et al. 1987, 1988; Caselli et al. 1999).

Then equations (1) and (2) imply

$$\rho = \left(\frac{RT}{\mu_0 K} \right)^n. \quad (3)$$

We assume that Γ_c is the rate of heating due to cloud compression (Silk 1977) and

$$\Gamma_c = \frac{\rho k T}{\mu} (16\pi G \rho)^{\frac{1}{2}}. \quad (4)$$

We also assume that Λ_g , Λ_{H_2} and Λ_{CO} are the rates of cooling due to grains (Silk 1977), molecular hydrogen (Peebles & Dicke 1968) and cooling due to excitation of rotational levels of CO (Oppenheimer & Dalgarno 1975) respectively and

$$\Lambda_g = 3\sigma T_g^{4+\delta} q \frac{Z\rho}{\rho_g}, \quad (5)$$

where q , δ are grain parameters and T_g , ρ_g , Z are the grain temperature, grain density and fraction of grain by mass of the total density respectively.

$$\Lambda_{H_2} = 2 \times 8.05 \times 10^{-7} T^3 \rho y, \quad (6)$$

for $\rho \geq 10^{-20}$ g cm^{-3} where y is the relative abundance of molecular hydrogen and the adopted value of $y = 0.2$ (Kanjilal & Basu 1992).

$$\Lambda_{CO} = 7 \times 10^{-26} T^{\frac{1}{2}} [1 - \frac{5.3}{T} \exp(-\frac{T}{20})] \exp(-\frac{5.3}{T}) n_{CO} n_{H_2}, \quad (7)$$

where n_{CO} and n_{H_2} are the number densities of CO and H_2 molecules respectively. We assume that the abundance of CO relative to hydrogen as $n_{CO}/2n_{H_2} = 7.3 \times 10^{-5}$ (Morton 1974), which corresponds to the depleted value of carbon abundance. We know that $n_H m_H = \rho$ and $n_H = 2n_{H_2}$. Then $n_{H_2} = \rho/(2 \times 1.67 \times 10^{-24})$. Also $n_{CO} = 7.3\rho/(1.67 \times 10^{-19})$ and $n_{CO}n_{H_2} = 7.3\rho^2/\{2 \times (1.67)^2 \times 10^{-43}\}$ (Morton 1974).

Then the condition of thermal balance reduces to

$$\Lambda_g + \Lambda_{H_2} + \Lambda_{CO} = \Gamma_c. \quad (8)$$

Jeans mass for gravitational collapse at density ρ and temperature T , is given by

$$m_J = \rho^{-\frac{1}{2}} \left(\frac{\pi k T}{\mu G} \right)^{\frac{3}{2}}, \quad (9)$$

where G is the gravitational constant. Here we used three types of grain material and their adopted parameter values (Silk 1977) are given in Table 1. Now the ratio of dust to gas mass in the diffuse ISM of Milky way is around 1 : 100 (Bohlin et al. 1978; Klapdor-Kleingrothaus 2000) but within molecular clouds that ratio depends on the grain size. For 10 μm grain size the ratio varies as 40% around the mean value and more larger grains allow for larger fluctuations (Tricco et al. 2017). Therefore in our entire calculation we take this ratio (viz. Z) as 10^{-3} .

In the presence of three types of grains in the molecular clouds (Silk 1977), we have computed in the following sections, the minimum temperature (T_{min}), maximum density (ρ_{max}), minimum Jeans length (R_J) and minimum Jeans mass ($m_J = m_{min}$) for different values of n . For a typical n we considered the fragment with lowest mass for gravitational collapse leading to star formation. The lowest critical mass for a star to support nuclear fusion is $0.08M_{\odot}$.

(Nakamura et al. 1997). But the fragmentation of interstellar cloud can give rise to gaseous fragments with mass much lower than $0.08M_{\odot}$ and it may be as low as $0.001M_{\odot}$ (Boss 1987, 2001; Kumar 2003; Whitworth & Stamatellos 2006).

3. Minimum mass of a protostellar fragment for negative polytropes

Using equations (4) - (7) in the equation (8) we get,

$$\begin{aligned} \frac{3\sigma q Z \rho}{\rho_g} T_g^{4+\delta} + \frac{7 \times 7.3 \times 10^{17} \times \rho^2}{2 \times (1.67)^2} T^{\frac{1}{2}} \left[1 - \frac{5.3}{T} \exp\left(-\frac{T}{20}\right) \right] \times \exp\left(-\frac{5.3}{T}\right) \\ + 2 \times 8.05 \times 10^{-7} T^3 \rho y = \frac{4kT}{\mu} (\pi G)^{\frac{1}{2}} \rho^{\frac{3}{2}}. \end{aligned} \quad (10)$$

Since $T = T_g$ at low temperature and high density we take $T = T_g = T_{min}(\text{say})$ and $m_J = m_{min}(\text{say})$.

Using equation(3), equation(10) reduces to

$$\begin{aligned} \frac{3\sigma q Z}{\rho_g} T^{4+\delta} + \frac{7 \times 7.3 \times 10^{17}}{2 \times (1.67)^2} T^{\frac{1}{2}} \left(\frac{RT}{K\mu_0} \right)^n \left[1 - \frac{5.3}{T} \exp\left(-\frac{T}{20}\right) \right] \times \exp\left(-\frac{5.3}{T}\right) \\ + 2 \times 8.05 \times 10^{-7} T^3 y = \frac{4kT}{\mu} (\pi G)^{\frac{1}{2}} \left(\frac{RT}{K\mu_0} \right)^{\frac{n}{2}}. \end{aligned} \quad (11)$$

Now solving equation (11) by Newton-Raphson method for different values of the parameters we find the value of T_{min} and substituting T_{min} in equation (3), we have the corresponding value of $\rho = \rho_{max}(\text{say})$. Then substituting T_{min} and ρ_{max} in equation (9), we find the minimum Jeans mass of a fragment $m_J = m_{min}$. The results are shown in the Tables 2 - 3.

This is to be noted that for Olivine and Graphite core and ice mantle the resulting values of Jeans masses and the related parameters are not significantly different from the case of Graphite.

4. Opacity limited fragmentation

Here we assume that the optical depth across a fragment of Jeans length is unity and it is due to the effect of τ_g and τ_{H_2} , where τ_g and τ_{H_2} are respectively the optical depths due to grains and photons emitted in a $J \rightarrow J - 2$ transition of molecular hydrogen. i.e,

$$\tau = \tau_g + \tau_{H_2} = 1 \quad (12)$$

We have not used the opacity caused by CO as it is negligibly small compared to that made by hydrogen (Gaustad 1963). Now,

$$\tau_g = \frac{3}{8} Z q \frac{\rho^{\frac{1}{2}}}{\rho_g} T_g^\delta \left(\frac{\pi k T}{\mu G} \right)^{\frac{1}{2}} \quad (13)$$

(Silk 1977) and

$$\tau_{H_2} = 2 \times 3.1 \times 10^5 y \rho^{\frac{1}{2}} T^{\frac{1}{2}} \quad (14)$$

(Yoshii & Saio 1986). Therefore equation (12) becomes

$$\frac{3}{8} \frac{qZ}{\rho_g} T_g^\delta \left(\frac{\pi k T}{\mu G} \right)^{\frac{1}{2}} + 2 \times 3.1 \times 10^5 y T^{\frac{1}{2}} = \rho^{-\frac{1}{2}} \quad (15)$$

At low temperature and high density the equation(15) with the help of equation(3) reduces to

$$\frac{3}{8} \frac{qZ}{\rho_g} T_g^\delta \left(\frac{\pi k T}{\mu G} \right)^{\frac{1}{2}} + 2 \times 3.1 \times 10^5 y T^{\frac{1}{2}} = \left(\frac{RT}{\mu_0 K} \right)^{-\frac{1}{2}} \quad (16)$$

Solving numerically equation (16) and using equations (3) and (9) we calculate the minimum Jeans mass of a fragment for hierarchical fragmentation (Hoyle 1953) and the corresponding results are given in Tables 4 - 9 respectively for various grain types.

5. Discussion

In the present work we have considered fragmentation of molecular clouds in a polytropic medium (viz. polytropic index n) under two physical environments. In the former case thermal instability associated with equation of state can give rise to Jeans masses in the range $0.001 M_{\odot} - 8 M_{\odot}$ under various initial conditions. Tables 2 - 3 indicate that field stars can form in a medium with negative polytropic indices (viz. n varies in the range, $-6 \leq n < 0$). For a lower value of the initial temperature (e.g. $T = 10 K$) Jeans masses, found, are slightly bigger compared to higher value of initial temperature (e.g. $T = 50 K$). This type of situation can arise in a shock compressed medium (Kanjilal & Basu 1991; Kaplan 1966) where due to radiation from shocked layer at high temperature it is cooled and compressed to a very high density (Kaplan 1966; Zeldovich & Raizer 1966; Reighard & Drake 2007) which then undergoes gravitational instability leading to fragmentation and subsequent collapse, called protostars. These protostars finally become stars of various masses passing through a complicated process associated with accretion as well as further compression.

For $n < -6$ Jeans masses have very small values (viz. $\sim 0.0001 M_{\odot}$) which cannot give birth to stars even after accretion etc. (Silk 1977; Kumar 2003; Whitworth & Stamatellos 2006). Negative polytropes can also approximate some models of interstellar medium for $n < -1$ (Shu et al. 1972; Viala 1972) and for $-1 < n < 0$ (Field 1965) when the medium is thermally unstable having $dp/d\rho < 0$ and convective in stars (Cox & Giuli 1968). Polytropic cylinders are unstable under an external pressure only if $-\infty < n < -1$ (Horedt 1970) and polytropic spheres are unstable under external pressure only if $-\infty \leq n < -1$ (Bonnor 1956; McCrea 1957; Horedt 1970; Viala 1972; Shu et al. 1972) which in part correspond to the present situation. Under explosive spherical shocks molecular rings are produced behind the shock front (Saito 1990; Kanjilal & Basu 1991) which can be considered as a spherical shell or cylinders in a small area. Such molecular

rings have been observed in the central region of our Galaxy (Scoville & Solomon 1975; Gordon & Burton 1976; Cohen & Thaddeus 1977; Solomon et al. 1979).

Regarding the cooling sources we have included grains, molecular hydrogen and CO . CO is an important coolant for most of the molecular ISM (Goldsmith 2001; Goldsmith & Langer 1978; Neufeld & Kaufman 1993; Neufeld et al. 1995; Omukai et al. 2010). These are primary constituents for cooling in molecular clouds (Bally et al. 1987, 1988; Morton 1974; Hocuk et al. 2014), because at low temperature (viz. $T < 500K$) the rotational levels of molecules are only excited (Spitzer et al. 1973; Spitzer & Cochran 1973; Spitzer et al. 1974) and they are used as probing tool to study the properties (e.g. temperature, velocity dispersions etc.) of the molecular clouds .

In the latter case we have considered opacity limited fragmentation in a polytropic medium. For opacity we have considered different types of grains and molecular hydrogen as the opacity sources. Due to the absence of CO absorption lines in some clouds (Barvainis & Antonucci 1994; Gaustad 1963) we have not considered CO as opacity source in the present model. Also CO are depleted by freezing on dust grains and we have already included dust opacity (Caselli et al. 1999; Bergin et al. 2002; Hocuk et al. 2014). The minimum Jeans masses are shown in Tables 4 - 9. The following features have been observed.

- (i) Jeans masses for gravitational collapse varies from field stars and massive stars to star clusters and that is possible only for positive values of the polytropic indices ($1 \leq n \leq 4.5$) unlike the previous case.
- (ii) As the initial temperature decreases Jeans mass decreases with respect to the the same initial number density, grain composition, and polytropic indices.
- (iii) As the initial number density decreases Jeans mass increases

remaining other parameters fixed.

(iv) The minimum temperature is always higher compared to former case.

(v) For higher initial temperature a burst of star formation is more likely and the effect is enhanced in the presence of Olivine and Graphite core and ice mantle.

(vi) The formation of field stars are more likely for higher values of n (viz. $n \sim 4 - 4.5$).

For values $n > 5$ the configuration extends to infinite dimension and we have confined our model to finite configuration (e.g. stars or star clusters) which is possible for positive indices of n . For negative indices Jeans masses are further increased with a very high T_{min} (e.g. $\sim 10^3 K - 10^4 K$) which have not been observed in a star forming molecular clouds, hence become unrealistic situation for study.

The above features can be explained in terms of opacity. In case of opacity limited fragmentation the heat generated during collapse remains trapped in the system in absence of cooling by grains and molecules which increases the temperature leading to higher values of Jeans masses. Hierarchical fragmentation (Hoyle 1953) can lead to formation of star clusters.

Thus in a word we can say that formation of field stars can result out of thermal instability in a negative polytropic medium while on the contrary formation of open clusters is more likely as a result of opacity limited fragmentation in the absence of molecular or grain cooling in a polytropic medium with positive indices.

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Table 1: Different values of grain parameters used in the study (Silk 1977)

Type of grain	δ	$\log q$	$\rho_g (gcm^{-3})$
Graphite	2.03	-1.87	2.25
Olivine	1.96	-3.17	3.5
Graphite core and icemantle	1.05	-0.30	1.0

Table 2: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite for initial temperature $T = 10$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_\odot)
-0.1	10^5	4.28	1.817×10^{-19}	3.014×10^{17}	2.502
-0.1	10^4	4.29	1.817×10^{-20}	9.546×10^{17}	7.945
-0.5	10^5	4.28	2.550×10^{-19}	2.544×10^{17}	2.111
-0.5	10^4	4.29	2.547×10^{-20}	8.061×10^{17}	6.706
-1.5	10^5	4.28	5.955×10^{-19}	1.664×10^{17}	1.380
-1.5	10^4	4.29	5.937×10^{-20}	5.277×10^{17}	4.386
-2.0	10^5	4.28	9.102×10^{-19}	1.346×10^{17}	1.116
-2.0	10^4	4.29	9.071×10^{-20}	4.268×10^{17}	3.546
-2.5	10^5	4.28	1.391×10^{-18}	1.088×10^{17}	0.902
-2.5	10^4	4.28	1.386×10^{-19}	3.452×10^{17}	2.866
-3.0	10^5	4.28	3.252×10^{-18}	7.121×10^{16}	0.590
-3.0	10^4	4.28	3.240×10^{-19}	2.257×10^{17}	1.873
-4.0	10^5	4.28	4.972×10^{-18}	5.759×10^{16}	0.477
-4.0	10^4	4.28	4.955×10^{-19}	1.825×10^{17}	1.514
-5.0	10^5	4.28	1.162×10^{-17}	3.766×10^{16}	0.312
-5.0	10^4	4.28	1.159×10^{-18}	1.193×10^{17}	0.989
-6.0	10^5	4.27	2.717×10^{-17}	2.463×10^{16}	0.204
-6.0	10^4	4.28	2.710×10^{-18}	7.800×10^{16}	0.646

Table 3: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite for initial temperature $T = 50$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_\odot)
-0.1	10^5	4.30	2.134×10^{-19}	2.788×10^{17}	2.324
-0.1	10^4	4.47	2.125×10^{-19}	9.006×10^{17}	7.805
-0.5	10^5	4.29	5.700×10^{-19}	1.703×10^{17}	1.414
-0.5	10^4	4.36	5.654×10^{-20}	5.451×10^{17}	4.603
-1.0	10^5	4.28	1.949×10^{-18}	9.202×10^{16}	0.763
-1.0	10^4	4.30	1.938×10^{-19}	2.926×10^{17}	2.440
-1.5	10^5	4.28	6.665×10^{-18}	4.974×10^{16}	0.412
-1.5	10^4	4.28	6.645×10^{-19}	1.577×10^{17}	1.309
-2.0	10^5	4.28	2.278×10^{-17}	2.690×10^{16}	0.222
-2.0	10^4	4.28	2.275×10^{-18}	8.516×10^{16}	0.706
-2.5	10^5	4.27	7.791×10^{-16}	1.454×10^{16}	0.120
-2.5	10^4	4.28	7.785×10^{-18}	4.602×10^{16}	0.381
-3.0	10^5	4.27	2.663×10^{-16}	7.867×10^{15}	0.065
-3.0	10^4	4.28	2.662×10^{-17}	2.488×10^{16}	0.206
-4.0	10^5	4.27	3.112×10^{-15}	2.301×10^{15}	0.019
-4.0	10^4	4.27	3.112×10^{-16}	7.278×10^{15}	0.060
-5.0	10^5	4.27	3.637×10^{-14}	6.732×10^{14}	0.005
-5.0	10^4	4.27	3.637×10^{-15}	2.129×10^{15}	0.017
-6.0	10^5	4.27	4.250×10^{-13}	1.969×10^{14}	0.001
-6.0	10^4	4.27	4.250×10^{-14}	6.228×10^{14}	0.005

Table 4: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite for initial temperature $T = 10$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_\odot)
1.0	10^5	358	5.992×10^{-18}	4.803×10^{17}	3.337×10^2
1.0	10^4	525	8.771×10^{-19}	1.518×10^{18}	1.544×10^3
1.5	10^5	272	2.379×10^{-17}	2.101×10^{17}	1.110×10^2
1.5	10^4	388	4.037×10^{-18}	6.085×10^{17}	4.573×10^2
2.0	10^5	215	7.753×10^{-17}	1.034×10^{17}	43.170
2.0	10^4	299	1.495×10^{-17}	2.776×10^{17}	1.608×10^2
2.5	10^5	175	2.154×10^{-16}	5.602×10^{16}	19.035
2.5	10^4	238	4.648×10^{-17}	1.406×10^{16}	65.005
3.0	10^5	146	5.254×10^{-16}	3.278×10^{16}	9.300
3.0	10^4	195	1.252×10^{-16}	7.760×10^{16}	29.409
3.5	10^5	124	1.149×10^{-15}	2.046×10^{16}	4.947
3.5	10^4	164	2.995×10^{-16}	4.595×10^{16}	14.610
4.0	10^5	108	2.294×10^{-15}	1.348×10^{16}	2.826
4.0	10^4	140	6.685×10^{-16}	2.887×10^{16}	7.849

Table 5: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite for initial temperature $T = 50$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_\odot)
2.0	10^5	340	7.762×10^{-18}	4.113×10^{17}	2.714×10^2
2.0	10^4	472	1.492×10^{-18}	1.104×10^{18}	1.011×10^3
2.5	10^5	300	2.582×10^{-17}	2.001×10^{17}	1.039×10^2
2.5	10^4	407	6.105×10^{-18}	4.749×10^{17}	3.288×10^2
3.0	10^5	268	2.582×10^{-17}	2.001×10^{17}	1.039×10^2
3.0	10^4	357	6.105×10^{-18}	4.749×10^{17}	3.288×10^2
3.5	10^5	243	4.235×10^{-17}	1.487×10^{17}	70.019
3.5	10^4	318	1.090×10^{-17}	3.354×10^{17}	2.069×10^2
4.0	10^5	222	6.573×10^{-17}	1.142×10^{17}	49.267
4.0	10^4	287	1.826×10^{-17}	2.462×10^{17}	1.371×10^2
4.5	10^5	205	9.737×10^{-17}	9.025×10^{16}	35.974
4.5	10^4	262	2.896×10^{-17}	1.867×10^{17}	94.858

Table 6: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Olivine for initial temperature $T = 10$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_\odot)
2.5	10^5	472	2.562×10^{-15}	2.665×10^{16}	24.383
2.5	10^4	656	5.824×10^{-16}	6.588×10^{16}	83.704
3.0	10^5	362	7.933×10^{-15}	1.326×10^{16}	9.300
3.0	10^4	497	2.050×10^{-15}	3.056×10^{16}	29.409
3.5	10^5	284	2.049×10^{-14}	7.312×10^{15}	4.027
3.5	10^4	387	6.025×10^{-15}	1.573×10^{16}	11.790
4.0	10^5	272	4.507×10^{-14}	4.414×10^{15}	1.948
4.0	10^4	308	1.508×10^{-14}	8.874×10^{15}	5.296

Table 7: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Olivine for initial temperature $T= 50$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_{\odot})
2.0	10^5	1026	7.045×10^{-17}	2.369×10^{17}	4.712×10^2
2.0	10^4	1439	1.383×10^{-17}	6.330×10^{17}	1.764×10^3
2.5	10^5	833	1.897×10^{-16}	1.301×10^{17}	2.101×10^2
2.5	10^4	1144	4.186×10^{-17}	3.245×10^{17}	7.192×10^2
3.0	10^5	694	4.476×10^{-16}	7.732×10^{16}	1.039×10^2
3.0	10^4	936	1.096×10^{-16}	1.813×10^{17}	3.288×10^2
3.5	10^5	590	9.452×10^{-16}	4.906×10^{16}	56.090
3.5	10^4	783	2.545×10^{-16}	1.089×10^{17}	1.652×10^2
4.0	10^5	510	1.817×10^{-15}	3.290×10^{16}	32.534
4.0	10^4	668	5.339×10^{-16}	6.945×10^{16}	89.917
4.5	10^5	448	3.227×10^{-15}	2.313×10^{16}	20.072
4.5	10^4	579	1.027×10^{-15}	4.662×10^{16}	52.331

Table 8: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite core and ice mantle for initial temperature $T= 10$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_{\odot})
1.5	10^5	432	4.758×10^{-18}	1.872×10^{17}	1.569×10^2
1.5	10^4	716	1.012×10^{-17}	5.220×10^{17}	7.243×10^2
2.0	10^5	298	1.485×10^{-16}	8.795×10^{16}	50.791
2.0	10^4	470	3.693×10^{-17}	1.096×10^{17}	2.016×10^2
2.5	10^5	219	3.769×10^{-16}	4.736×10^{16}	20.130
2.5	10^4	332	1.065×10^{-16}	2.214×10^{17}	70.628
3.0	10^5	169	8.173×10^{-16}	2.829×10^{16}	9.300
3.0	10^4	248	2.574×10^{-16}	6.102×10^{16}	29.409
3.5	10^5	136	1.569×10^{-15}	1.830×10^{15}	4.838
3.5	10^4	194	5.424×10^{-16}	3.717×10^{16}	14.003
4.0	10^5	113	2.738×10^{-15}	1.261×10^{16}	2.764
4.0	10^4	157	1.025×10^{-15}	2.432×10^{16}	7.412

Table 9: Values of n , T_{min} , ρ_{max} , R_J and m_{min} for different values of grain parameter for the grain type Graphite core and ice mantle for initial temperature $T= 50$ K, initial number densities 10^4 and 10^5 respectively.

n	Initial no.density (cm^{-3})	T_{min} (K)	ρ_{max} (gcm^{-3})	R_J (cm)	m_{min} (M_{\odot})
2.0	10^5	563	2.121×10^{-17}	3.199×10^{17}	3.490×10^2
2.0	10^4	886	5.253×10^{-18}	8.064×10^{17}	1.384×10^3
2.5	10^5	453	4.133×10^{-17}	2.055×10^{17}	1.804×10^2
2.5	10^4	685	1.161×10^{-17}	4.767×10^{17}	6.326×10^2
3.0	10^5	377	7.212×10^{-17}	1.420×10^{17}	1.0391×10^2
3.0	10^4	552	2.253×10^{-17}	3.073×10^{17}	3.288×10^2
3.5	10^5	323	1.155×10^{-16}	1.039×10^{17}	65.176
3.5	10^4	460	3.949×10^{-17}	2.118×10^{17}	1.887×10^2
4.0	10^5	283	1.730×10^{-16}	7.947×10^{16}	43.653
4.0	10^4	393	6.389×10^{-17}	1.539×10^{17}	1.172×10^2
4.5	10^5	252	2.455×10^{-16}	6.298×10^{16}	30.835
4.5	10^4	343	9.695×10^{-17}	1.167×10^{17}	77.560