Finite-Element Modeling for Crustal Deformation

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Computational Infrastructure for Geodynamics

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Crustal Deformation Modeling

Elasticity problems where geometry does not change significantly

Quasi-static modeling associated with earthquakes

- Strain accumulation associated with interseismic deformation
  - What is the stressing rate on faults X, Y, and Z?
  - Where is strain accumulating in the crust?
- Coseismic stress changes and fault slip
  - What was the slip distribution in earthquake A?
  - How did earthquake A change the stresses on faults X, Y, and Z?
- Post-seismic relaxation of the crust
  - What rheology is consistent with observed post-seismic deformation?
  - Can aseismic creep or afterslip explain the deformation?

Crustal Deformation Modeling

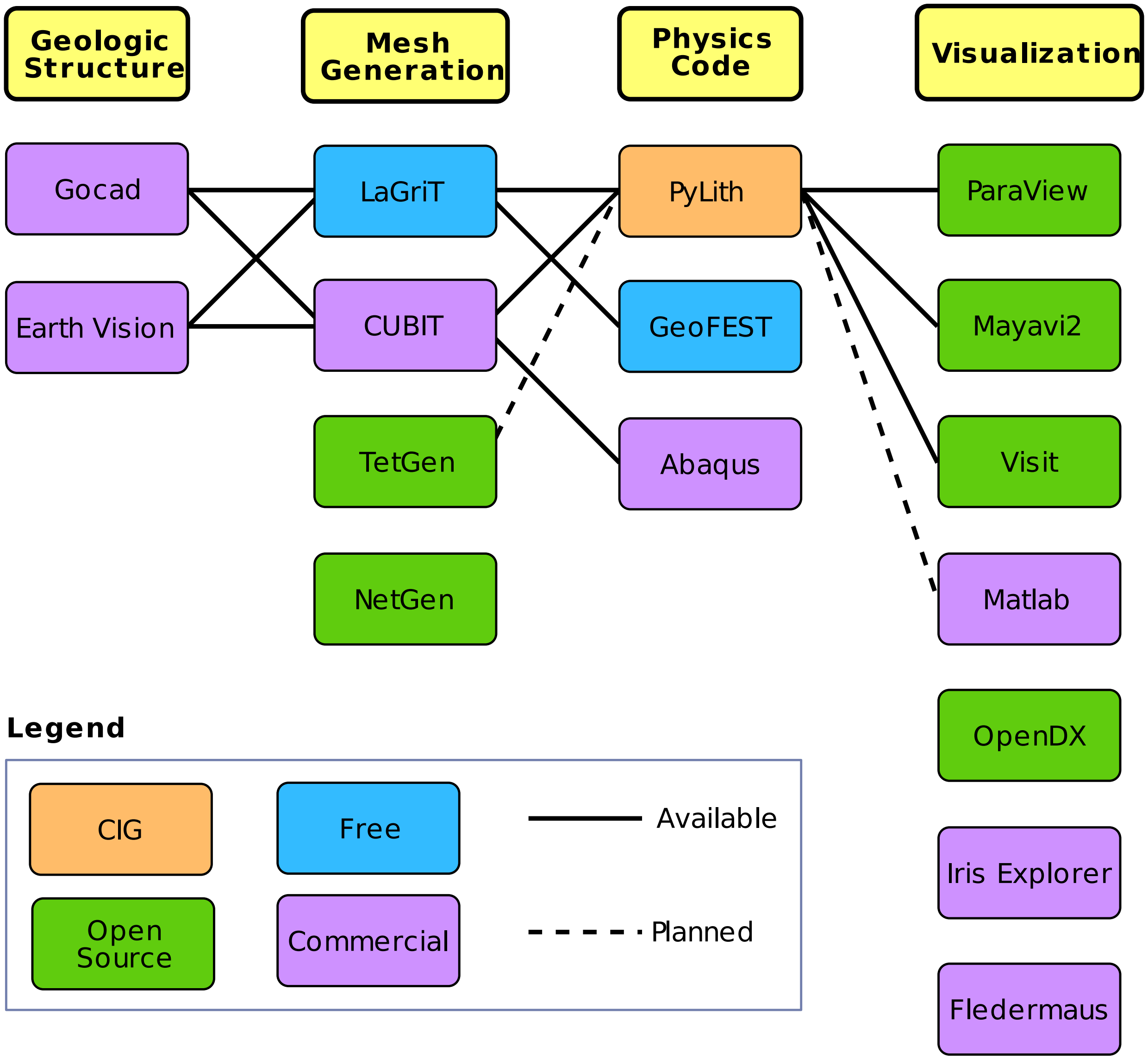
Elasticity problems where geometry does not change significantly

Volcanic deformation associated with magma chambers and/or dikes

- Inflation
  - What is the geometry of the magma chamber?
  - What is the potential for an eruption?
- Eruption
  - Where is the deformation occurring?
  - What is the ongoing potential for an eruption?
- Dike intrusions
  - What the geometry of the intrusion?

Crustal Deformation Modeling

Overview of workflow for typical research problem



Governing Equations

Elasticity equation

$$\sigma_{ij,j} + f_i = \rho \ddot{u} \text{ in } V, \tag{1}$$

$$\sigma_{ij} n_j = T_i \text{ on } S_T, \tag{2}$$

$$u_i = u_i^0 \text{ on } S_u, \text{ and} \tag{3}$$

$$R_{ki}(u_i^+ - u_i^-) = d_k \text{ on } S_f. \tag{4}$$

Multiply by weighting function and integrate over the volume,

$$-\int_V (\sigma_{ij,j} + f_i - \rho \ddot{u}_i)\phi_i \, dV = 0 \tag{5}$$

After some algebra,

$$-\int_V \sigma_{ij}\phi_{i,j} \, dV + \int_{S_T} T_i \phi_i \, dS + \int_V f_i \phi_i \, dV - \int_V \rho \ddot{u}_i \phi_i \, dV = 0 \tag{6}$$

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Governing Equations

Writing the trial and weighting functions in terms of basis (shape) functions,

$$u_i(x_i,t) = \sum_m a_i^m(t) N^m(x_i), \tag{7}$$

$$\phi_i(x_i,t) = \sum_n c_i^n(t) N^n(x_i). \tag{8}$$

After some algebra, the equation for vertex degree of freedom i of vertex n is

$$-\int_V \sigma_{ij} N^n_{,j}\, dV + \int_{S_T} T_i N^n\, dS + \int_V f_i N^n\, dV - \int_V \rho \sum_m \ddot{a}_i^m N^m N^n\, dV = 0 \tag{9}$$

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Governing Equations

Using numerical quadrature we convert the integrals to sums over the cells and quadrature points

$$
\begin{aligned}
&- \sum_{\text{vol cells}} \sum_{\text{quad pts}} \sigma_{ij} N^n_{,j} w_q |J_{\text{cell}}| + \sum_{\text{surf cells}} \sum_{\text{quad pts}} T_i N^n w_q |J_{\text{cell}}| \\
&\quad + \sum_{\text{vol cells}} \sum_{\text{quad pts}} f_i N^n w_q |J_{\text{cell}}| \\
&\quad - \sum_{\text{vol cells}} \sum_{\text{quad pts}} \rho \sum_{m} \ddot{a}_i^m N^m N^n w_q |J_{\text{cell}}| = \vec{0} \qquad (10)
\end{aligned}
$$

Quasi-static Solution

Neglect inertial terms

Form system of algebric equations

$$\underline{A}(t)\vec{u}(t) = \vec{b}(t) \tag{11}$$

where

$$A_{ij}^{nm}(t) = \sum_{\text{vol cells quad pts}} \sum \frac{1}{4} C_{ijkl}(t)(N_{,l}^{m} + N_{,k}^{m})(N_{,j}^{n} + N_{,i}^{n}) w_q |J_{\text{cell}}| \tag{12}$$

$$b_i(t) = \sum_{\text{surf cells quad pts}} \sum T_i(t) N^n w_q |J_{\text{cell}}| + \sum_{\text{vol cells quad pts}} \sum f_i(t) N^n w_q |J_{\text{cell}}| \tag{13}$$

and solve for $\vec{u}(t)$.

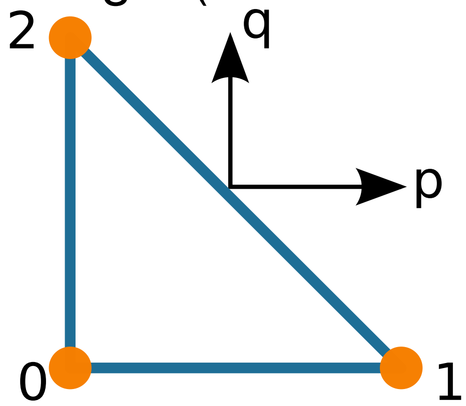
Problem Setup

- Which suite of tools should I use?
  - Is there an analytic or semi-analytic solution?
  - Can I solve a 2-D problem or do I need to solve a 3-D problem?
- Which cell type is appropriate?
  - Can I mesh the geometry of the domain with hexahedral cells?
  - How will the discretization size vary in space?
- What factors will control the resolution that I need?
  - What length scales are important in my problem?
  - What time scales are important?

Basis Functions

$$\vec{u}(\vec{x},t) = \sum_n N^n(\vec{x})\vec{u}^n(t)$$

Triangle (3 basis fns)

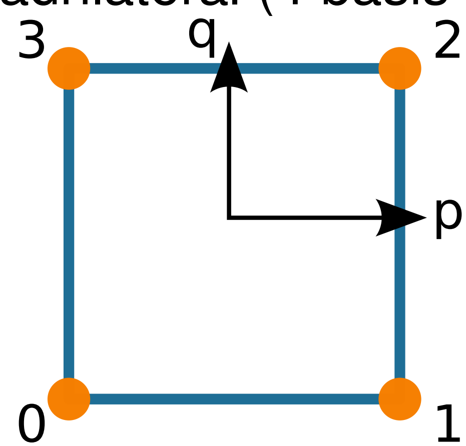


$$N_0 = \frac{1}{2}(-p-q)$$

$$N_1 = \frac{1}{2}(1+p)$$

$$N_2 = \frac{1}{2}(1+q)$$

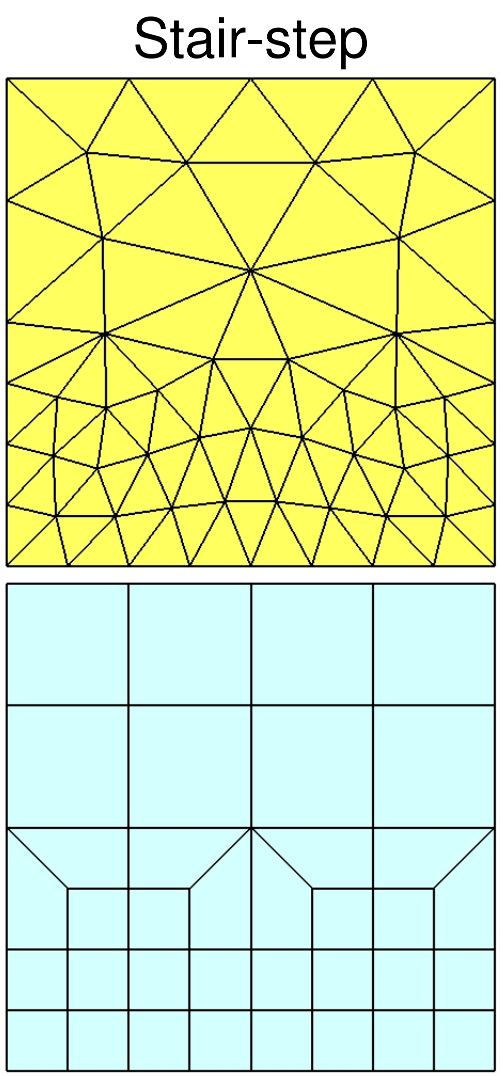
Quadrilateral (4 basis fns)

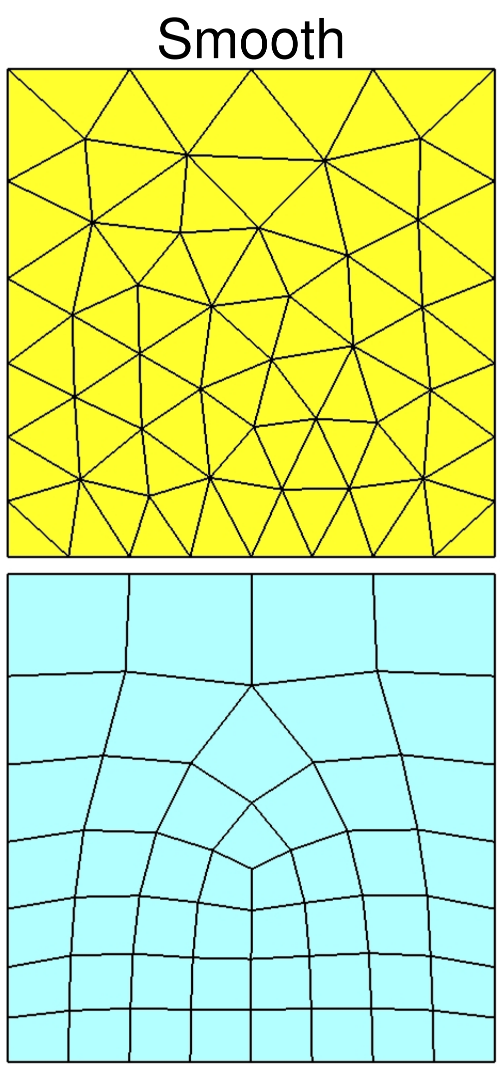


$$N_3 = \frac{1}{4}(1-p)(1+q) \qquad N_2 = \frac{1}{4}(1+p)(1+q)$$

$$N_0 = \frac{1}{4}(1-p)(1-q) \qquad N_1 = \frac{1}{4}(1+p)(1-q)$$

Varying Cell Size





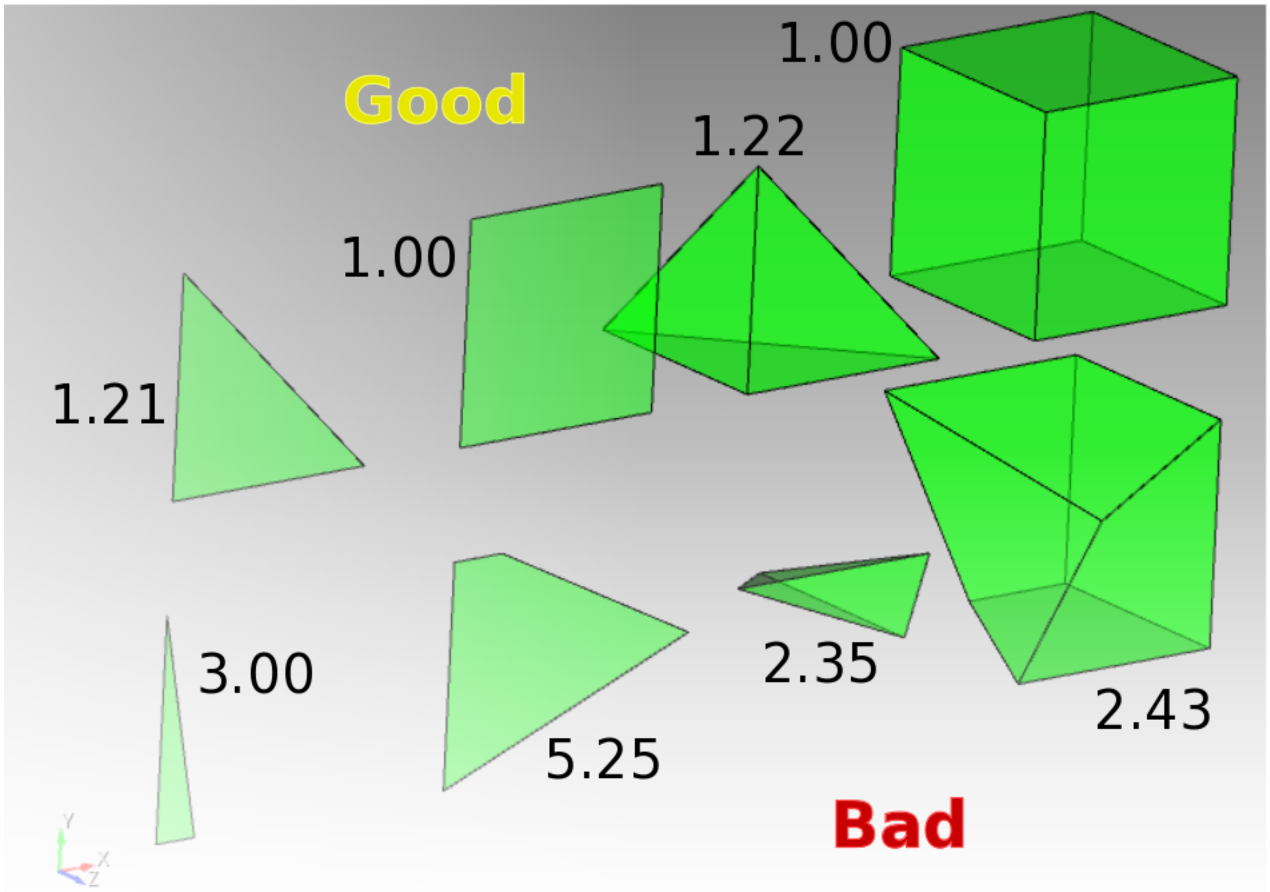
Potential Pitfalls

- Over/under resolving the deformation
  - Poor mesh quality
  - Using resolution much finer than constraints
  - Failing to resolve stress concentrations
- Failing to check the simulation results against intuition
  - Do the results make sense?
  - How close are the results to an anlytical solution?
- Choosing the wrong suite of tools or parameters
  - Nonlinear problems require nonlinear solvers
  - Propagating seismic waves require inertial terms

Poor Mesh Quality

Distorted cells

The most distorted cell controls the rate convergence (quasi-static problems) and the time step (dynamic problems).



Hints, Tips, and Tricks

- Start at the coarsest resolution possible
- Work through the entire problem at a coarse resolution
  - Eliminate obstacles using simple test problems that run quickly
  - Verify workflow is feasible and meets desired objective
- Increase resolution as needed
  - Only run large problems when the kinks are worked out
  - Verify solution is converging
- Double-check inputs and outputs at every stage
  - Did the software do what I think I told it to do?