CSci 4511W: Intro To AI
Homework 3

1. **Show if the following sentences are valid, satisfiable, or invalid using truth tables and equivalency rules. Show the truth tables in your submission to make clear how you produced the answers.**

1. $(A \land B) \lor \neg B$

A	B	$\neg B$	$(A \land B) \lor \neg B$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Valid: No because it is not true in all interpretations (false when A=F, B=T)
Satisfiable: Yes because it is true in some interpretations (ex. A=T, B=T)
Invalid:
This expression is satisfiable, but not valid (invalid)

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2. $(A \land B) \Rightarrow (A \Rightarrow B)$

A	B	$(A \land B)$	$(A \Rightarrow B)$ $(\neg A \lor B)$	$(A \land B) \Rightarrow (A \Rightarrow B)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

valid: yes because the formula is true in all interpretations
satisfiable: yes since it's valid, it's also satisfiable
Invalid: No, because it's always true
This expression is valid & satisfiable

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3. $(A \Rightarrow B) \Rightarrow ((A \land C) \Rightarrow B)$

A	B	C	$A \Rightarrow B$ $(\neg A \lor B)$	$A \land C$	$(A \land C) \Rightarrow B$ $\neg(A \land C) \lor B$ $\neg A \lor \neg C \lor B$	$(A \Rightarrow B) \Rightarrow ((A \land C) \Rightarrow B)$ $(\neg A \lor B) \Rightarrow (\neg A \lor \neg C \lor B)$ $\neg(\neg A \lor B) \lor (\neg A \lor \neg C \lor B)$ $(A \land \neg B) \lor \neg A \lor \neg C \lor B$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Valid: Yes because the formula is true in all interpretations
Satisfiable: Yes because it is true in some cases
Invalid: No, because it's always true
This expression is valid & satisfiable

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4. $(A \Rightarrow B) \Rightarrow (\neg A \Rightarrow \neg B)$

A	B	$(A \Rightarrow B)$ $(\neg A \lor B)$	$\neg A \Rightarrow \neg B$ $A \lor \neg B$	$(A \Rightarrow B) \Rightarrow (\neg A \Rightarrow \neg B)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Valid: No because it is false when A=F, B=T
Satisfiable: Yes because it is true in at least one interpretation
Invalid: Yes because it is false in at least one case
This expression is satisfiable, but not valid (invalid)

2. **For each of the following expressions, state briefly if it is a correct representation in propositional calculus of the sentence "If the dog is hungry and it does not have food, then the dog is upset." or not and explain why. The propositions used in the sentences should have an obvious interpretation.**

1. $DogHungry \land \neg HasFood \land DogUpset$
↳ Dog is Hungry, does not have food, & is upset

Here all conditions must be true so its a conjunction rather than an implication
Doesn't follow an if-then Structure

2. $(DogHungry \lor \neg HasFood) \Rightarrow DogUpset$
↳ If dog is hungry or it does not have food, then dog is upset"

This states if EITHER the dog is hungry or it does not have food, then it is upset which is not specific enough
We are also looking for and, not or

3. $(DogHungry \land \neg HasFood) \Rightarrow DogUpset$
↳ If dog is hungry and it does not have food, then dog is upset

This matches the given sentence
Correct propositional calculus & captures the if-then structure with the proper condition

4. $DogUpset \Rightarrow (DogHungry \land \neg HasFood)$
↳ If dog is upset then dog is hungry and does not have food

This is the converse of the actual correct sentence

5. $\neg HasFood \Rightarrow DogHungry \land DogUpset$
↳ If dog does not have food, then dog is hungry & upset

Incorrect order

3. Convert these expressions to CNF

1. $(Z \land (W \land M)) \Rightarrow P$
↳ $\neg(Z \land (W \lor M)) \lor P$
$(\neg Z \lor \neg(W \lor M)) \lor P$
$(\neg Z \lor (\neg W \land \neg M)) \lor P$
$(\neg Z \lor \neg W \lor P) \land (\neg Z \lor \neg M \lor P)$ ⤳ Final

2. $(B \lor (A \land C)) \Rightarrow \neg A$
↳ $\neg(B \lor (A \land C)) \lor \neg A$
$(\neg B \land \neg(A \land C)) \lor \neg A$
$(\neg B \land (\neg A \lor \neg C)) \lor \neg A$
$(\neg B \lor \neg A) \land (\neg B \lor \neg C) \land \neg A$ ⤳ Final

3. $\neg(\neg B \Rightarrow D)$
↳ $\neg(\neg(\neg B) \lor D)$
$\neg(B \lor D)$
$\neg B \land \neg D$ ⤳ Final

4. **You are given the following sentence in propositional calculus: $(P \Rightarrow Q) \lor (R \Rightarrow S)$. Prove using resolution with refutation that $(P \Rightarrow S) \lor (R \Rightarrow Q)$**

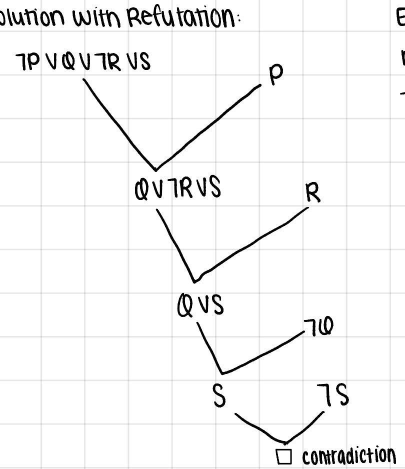
Given: $(P \Rightarrow Q) \lor (R \Rightarrow S)$
$(\neg P \lor Q) \lor (\neg R \lor S)$
$(\neg P \lor Q \lor \neg R \lor S)$

Goal: $(P \Rightarrow S) \lor (R \Rightarrow Q)$
$(\neg P \lor S) \lor (\neg R \lor Q)$
$(\neg P \lor S \lor \neg R \lor Q)$

Negation: $\neg(\neg P \lor S \lor \neg R \lor Q)$
$P \land \neg S \land R \land \neg Q$

Conversion into CNF:
1. P
2. $\neg S$
3. R
4. $\neg Q$
5. $\neg P \lor Q \lor \neg R \lor S$

Resolution with Refutation:



Ended up at a contradiction which means our assumption (negation of the goal) must be false, meaning the original goal is true.
Therefore, we've proved $(P \Rightarrow S) \lor (P \Rightarrow Q)$ using resolution w/ refutation

5. **Convert the following set of propositional clauses to CNF and prove, by resolution with refutation, that "Pleasant" is entailed by the knowledge base. Show the steps in the resolution proof.**

Convert to CNF:

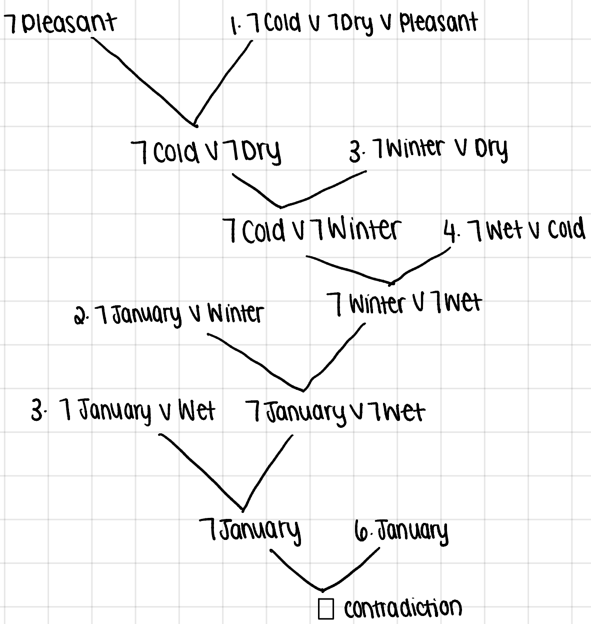
1. (Cold ∧ Dry) ⇒ Pleasant
   ↳ ¬Cold ∨ ¬Dry ∨ Pleasant
2. January ⇒ (Winter ∧ Wet)
   ↳ (¬January ∨ Winter) ∧ (¬January ∨ Wet)
3. Winter ⇒ Dry
   ↳ ¬Winter ∨ Dry
4. Wet ⇒ Cold
   ↳ ¬Wet ∨ Cold
5. January

Prove: Pleasant

Negate: ¬Pleasant

Knowledge Base:

1. ¬Cold ∨ ¬Dry ∨ Pleasant
2. ¬January ∨ Winter
3. ¬January ∨ Wet
4. ¬Winter ∨ Dry
5. ¬Wet ∨ Cold
6. January
7. ¬Pleasant



Ended up at a contradiction which means our assumption (negation of the goal) must be false, meaning the original goal is true.
↳ Therefore, Pleasant has been proved using resolution w/ refutation

5. **Answer the following questions precisely, explaining your reasoning:**
    a. **Is it true that it is always possible to prove that a sentence in propositional logic is entailed or not entailed by the knowledge base? Explain.**
    Yes, it is true. This is because propositional logic is decidable, which means that for any given knowledge base (KB) and query (α), we can determine whether KB entails α or KB does not entail α for all possible interpretations using truth tables or resolution. Since propositional logic has a finite number of potential variables, we can check all models of assignments to determine entailment.

    b. **Is modus ponens complete or not? What does completeness mean? Why is it important?**
    In general, modus ponens is not complete. Completeness means that an inference algorithm can derive any sentence entailed by the knowledge base (KB). If KB entails query (α), then the inference algorithm can derive α from KB. For modus ponens to be complete, it should be able to prove that everything is logically true, which is not the case. Modus ponens can only derive true conclusions from true premises, but it cannot derive all possible true conclusions. Completeness is important because it ensures that the inference algorithm is powerful enough to discover all the logical conclusions of our knowledge base. If an algorithm is incomplete, we might miss important conclusions.

    c. **Suppose you use resolution with refutation to prove that KB |= α. Does this mean that KB is valid? or is α valid? Explain clearly why or why not.**
    Resolution with refutation proves entailment, KB |= α, not the validity of KB or α. Resolution with refutation proves entailment showing that the sentence is unsatisfiable, leading to a contradiction. For a sentence to be considered valid, it would mean it is true in all possible models. KB could be true in some models and false in others, which means it is not valid. Resolution shows that in every model where KB is true, α is also true, not that KB itself is always true. α might only be true when KB is true, so when KB is false, α is also false, hence not valid. α is valid only if it is true in all models, not just when KB is true. Therefore, resolution with refutation proves entailment, not the validity of KB or α.

Note: Used Grammarly for sentence structure and grammar purposes.
Sources:
- Class Textbook
- https://quillbot.com/blog/reasoning/modus-ponens/