CSci 4511W: Intro To AI
Homework 3

1. **Specify if each of the following expressions is a correct representation of the corresponding English statement. If not write in English what the expression says and correct it.**

1. There is a cat in each house → $\forall x \forall y [House(x) \wedge Cat(y) \Rightarrow In(y, x)]$
   ↳ Incorrect: This says "For all x & y, if x is a house & y is a cat, then y is in x"
   Correct: $\forall x\ House(x) \Rightarrow \exists y\ (Cat(y) \wedge In(y, x)$
   ↳ This says "For every house, there exists a cat that is in it"

2. Every dog owner loves all animals → $\exists x \exists y [Person(x) \wedge Dog(y) \wedge Owns(x, y)] \wedge \forall z [Animal(z) \Rightarrow Loves(x, z)]$
   ↳ Incorrect: This says "There exists a person who owns a dog & that person loves all animals"
   Correct: $\forall x [Person(x) \wedge \exists y (Dog(y) \wedge Owns(x, y)) \Rightarrow \forall z\ (Animal(z) \Rightarrow Loves(x, z))]$
   ↳ This says "For every person who owns a dog that person loves all animals"

3. Cats catch Birds → $\forall x \exists y\ Cat(x) \wedge Bird(y) \wedge Catch(x, y)$
   ↳ Incorrect: This says "For all x, there exists a y such that x is a cat, y is a bird, & x catches y"
   Correct: $\forall x [(Cat(x) \Rightarrow \exists y\ (Bird(y) \wedge Catch(x, y))]$
   ↳ This says "For every cat, there exists a bird that it catches"

4. Birds live in all cities → $\forall x\ City(x) \wedge \exists y\ Bird(y) \wedge Lives\ In(y, x)$
   ↳ Incorrect: This says "For all x, x is a city, & there exists a y that is a bird & lives in x"
   Correct: $\forall x [City(x) \Rightarrow \exists y\ (Bird(y) \wedge Lives\ In(y, x))]$
   ↳ This says "For every city, there exists a bird that lives in it"

5. Only birds fly
   ↳ Incorrect: This says "Everything is a bird & flies"
   Correct: $\forall x [Fly(x) \Rightarrow Bird(x)]$
   ↳ This says "If something flies, then it is a bird"

2. Convert the following sentences to CNF:

1. $\forall x [(\exists y\ Loves(x, y) \vee Loves(y, x)) \Rightarrow Happy(x)]$
   ↳ $\forall x [\neg(\exists y\ Loves(x, y) \vee Loves(y, x)) \vee Happy(x)]$
   $\forall x [\forall y (\neg Loves(x, y) \wedge \neg Loves(y, x)) \vee Happy(x)]$
   $\forall x \forall y [(\neg Loves(x, y) \vee Happy(x)) \wedge (\neg Loves(y, x) \vee Happy(x))]$
   $(\neg Loves(x, y) \vee Happy(x)) \wedge (\neg Loves(y, x) \vee Happy(x))$
   $\neg Loves(x, y) \vee Happy(x)$
   $\neg Loves(y, x) \vee Happy(x)$ } Final

2. $\forall x [Happy(x) \Rightarrow (\exists y\ Loves(x, y))]$
   ↳ $\forall x [\neg Happy(x) \vee \exists y\ Loves(x, y)]$ → $y = f(x)$ skolem EI
   $\forall x [\neg Happy(x) \vee Loves(x, f(x))]$
   $\neg Happy(x) \vee Loves(x, f(x))$ ⇝ Final

3. $\exists x\ Person(x) \wedge \forall y [(Dog(y) \wedge Own(x, y)) \Rightarrow Like(y, x)]$
   ↳ $\exists x\ Person(x) \wedge \forall y [\neg(Dog(y) \wedge Own(x, y)) \vee Like(y, x)]$
   $\exists x\ Person(x) \wedge \forall y [\neg Dog(y) \vee \neg Own(x, y) \vee Like(y, x)]$ → $x = a$ skolem EI
   $Person(a) \wedge \forall y [\neg Dog(y) \vee \neg Own(a, y) \vee Like(y, a)]$
   $Person(a)$
   $\neg Dog(y) \vee \neg Own(a, y) \vee Like(y, a)$ } Final

4. $\forall x \exists y \forall z [(City(x) \wedge Cat(z)) \vee (Person(y) \wedge Feeds(y, z))]$
   ↳ $\forall x \exists y \forall z [\neg(City(x) \wedge Cat(z)) \vee (Person(y) \wedge Feeds(y, z))]$
   $\forall x \exists y \forall z [\neg City(x) \wedge \neg Cat(z) \vee (Person(y) \wedge Feeds(y, z))]$
   $\forall x \exists y \forall z [(\neg City(x) \vee \neg Cat(z) \vee Person(y)) \wedge (\neg City(x) \vee \neg Cat(z) \vee Feeds(y, z))]$ → $y = f(x)$ skolem EI
   $\forall x \forall z [(\neg City(x) \vee \neg Cat(z) \vee Person(f(x))) \wedge (\neg City(x) \vee \neg Cat(z) \vee Feeds(f(x), z))]$
   $\neg City(x) \vee \neg Cat(z) \vee Person(f(x))$
   $\neg City(x) \vee \neg Cat(z) \vee Feeds(f(x), z)$ } Final

3. **You are given the following knowledge base in predicate calculus. Use resolution with refutation to derive that "$\exists x\, S(x)$". Specify the value of x that was found by resolution. Show each step in the solution, showing the pair of clauses resolved, the results of unification, and the resolvent.**

Knowledge Base:

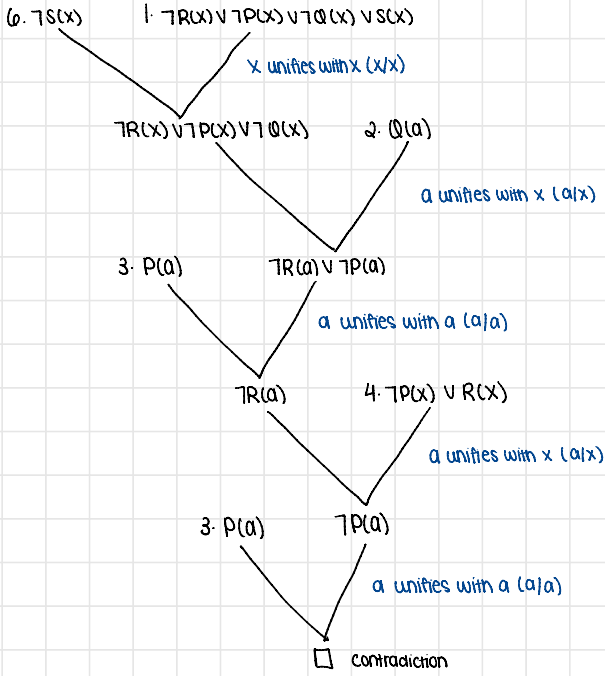
1. $\forall x\, R(x) \wedge P(x) \Rightarrow \neg Q(x) \vee S(x)$ $\longrightarrow$ $\neg R(x) \vee \neg P(x) \vee \neg Q(x) \vee S(x)$
2. $\exists x\, Q(x) \wedge P(x) \rightarrow Q(a) \wedge P(a)$ $\rightsquigarrow$ x=a skolemize
3. $\forall x\, P(x) \Rightarrow R(x) \rightarrow \neg P(x) \vee R(x)$
4. $\forall x\, Q(x) \Rightarrow P(x) \rightarrow \neg Q(x) \vee P(x)$

} CNF

Negated goal: $\neg \exists x\, S(x) \rightarrow \forall x \neg S(x)$ $\rightarrow \neg S(x)$

Clauses:

1. $\neg R(x) \vee \neg P(x) \vee \neg Q(x) \vee S(x)$
2. $Q(a)$
3. $P(a)$
4. $\neg P(x) \vee R(x)$
5. $\neg Q(x) \vee P(x)$
6. $\neg S(x)$



4. Represent the expressions shown down here in predicate calculus using only the predicate Older(x,y) = x is older than y. Use resolution with refutation to show that "John is older than Tom and Harry is not older than Wilt". Show each step in the solution, showing the pair of clauses resolved, the results of unification, and the resolvent.

1. Tom is not older than Karen: $\neg Older(Tom, Karen) \rightarrow \neg Older(Tom, Karen)$
2. Harry is older than Tom: $Older(Harry, Tom)$
3. John is older than Karen: $Older(John, Karen)$

} do not change when converting to CNF

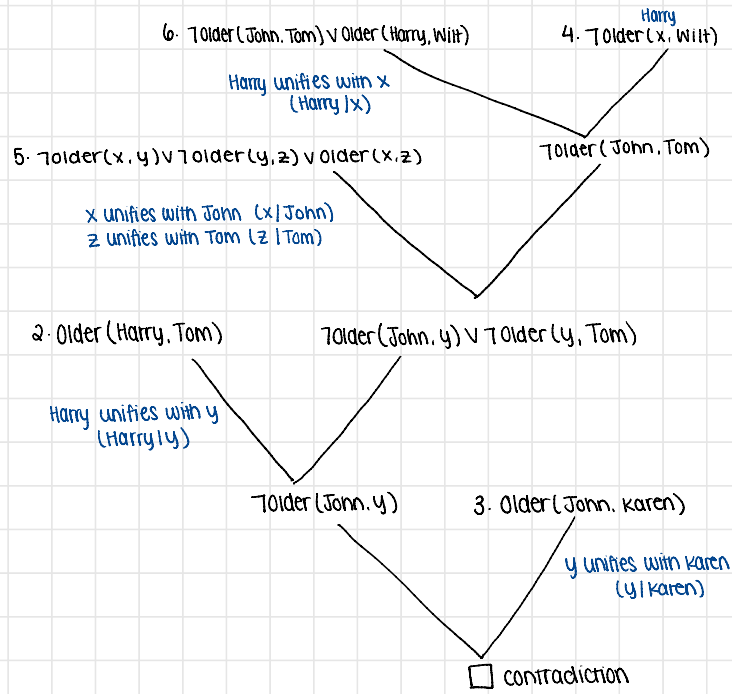
4. No one is older than Wilt: $\forall x\ \neg Older(x, Wilt) \rightarrow \neg Older(x, Wilt)$
5. older is transitive, i.e. $\forall x, y, z\ Older(x,y) \wedge Older(y,z) \Rightarrow Older(x,z) \rightarrow \neg Older(x,y) \vee \neg Older(y,z) \vee Older(x,z)$

} CNF

Goal: $Older(John, Tom) \wedge \neg Older(Harry, Wilt)$
Negated Goal: $\neg Older(John, Tom) \vee Older(Harry, Wilt)$

Clauses

1. $\neg Older(Tom, Karen)$
2. $Older(Harry, Tom)$
3. $Older(John, Karen)$
4. $\neg Older(x, Wilt)$
5. $\neg Older(x,y) \vee \neg Older(y,z) \vee Older(x,z)$
6. $\neg Older(John, Tom) \vee Older(Harry, Wilt)$



5. [30 points] Write the following statements in predicate calculus. Convert the sentences to CNF and show by resolution with refutation that "Silver is faster than Bunny." Show the resolution steps.

1. Horses are faster than dogs: $\forall x \forall y (Horse(x) \wedge Dog(y)) \Rightarrow Faster(x,y) \rightarrow \neg Horse(x) \vee \neg Dog(y) \vee Faster(x,y)$
2. There is a dog that is faster than every rabbit: $\exists x (Dog(x) \wedge \forall y (Rabbit(y) \Rightarrow Faster(x,y))) \rightarrow Dog(d) \wedge (\neg Rabbit(y) \vee Faster(d,y))$ ↪ x = a skolemize
3. Silver is a horse & Bunny is a rabbit: $Horse(Silver) \wedge Rabbit(Bunny)$ } does not change when converting to CNF
4. Faster is transitive: $\forall x \forall y \forall z (Faster(x,y) \wedge Faster(y,z) \Rightarrow Faster(x,z)) \rightarrow \neg Faster(x,y) \vee \neg Faster(y,z) \vee Faster(x,z)$

} CNF

Goal: Faster(Silver, Bunny)

Negated Goal: ¬Faster(Silver, Bunny)

Clauses:

1. $\neg Horse(x) \vee \neg Dog(y) \vee Faster(x,y)$
2. $Dog(d)$
3. $\neg Rabbit(y) \vee Faster(d,y)$
4. $Horse(Silver)$
5. $Rabbit(Bunny)$
6. $\neg Faster(x,y) \vee \neg Faster(y,z) \vee Faster(x,z)$
7. $\neg Faster(Silver, Bunny)$